



Review Review of Traffic Assignment and Future Challenges

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Abstract: The problem of traffic assignment consists of determining the routes taken by the users of transportation infrastructure. This problem has been the subject of numerous studies, particularly in analyzing scenarios for developing road infrastructure and pricing strategies. This paper reviews the major progress in the field. Accordingly, it shows that the evolution of intelligent transportation systems and the emergence of connected and autonomous vehicles present new challenges to classical approaches for solving the traffic assignment problem. It addresses two major perspectives: digital twins coupled with artificial intelligence to help decision-makers, and rule-based policy to offer users fair and efficient itineraries while respecting infrastructure capacity.

Keywords: traffic assignment; user equilibrium; intelligent transportation systems; smart cities

1. Introduction

As megalopolises expand and residents increasingly rely on motor vehicles, the challenges of congestion, energy issues, and pollution become major concerns that must be addressed in the coming years. The data-driven trend shows that congestion progressively worsens yearly in most major cities, as demonstrated by navigation system-based statistics (https://www.tomtom.com/traffic-index/ranking/, accessed on 10 November 2023, Copyright © 2023 TomTom International BV, all rights reserved). Given this mounting pressure, cities require robust traffic prediction and simulation tools to assist them in finding solutions.

The modeling of road traffic encompasses various elements, including driver behavior, road configurations, and the diversity of vehicle flows. The experiences of drivers depend on the type of network they use. Interest in this complex field of research has been growing since the early 20th century. Traffic modeling empowers road network authorities to enhance infrastructure management by investing in new facilities, optimizing vehicle flow, preventing congestion, and rapidly detecting incidents or accidents for prompt resolution. Two complementary approaches are essential for modeling traffic within a road network: flow modeling, which describes the evolution of traffic flows on road segments, and traffic assignment modeling, which explains how users choose their routes within a network.

Traffic assignment is of particular interest in the field of traffic engineering as it constitutes a crucial step in the processes of traffic simulation and transportation forecasting. In general, traffic assignment involves defining the routes of users based on transportation supply and demand. It anticipates the number of vehicles using each segment of the transportation network during a given time period. The transportation demand is described as the number of users wishing to travel from a set of origin nodes to a set of destination nodes during the studied period. The supply is the transportation network, which refers to the road network with its characteristics, including connections between roads, flow-speed curves of roads, number of lanes, intersection regulation, etc.

Traffic assignment has a century of research and over half a century of practical application. Traffic modeling and simulation software development has significantly contributed to the use of traffic assignment results in transportation network investment



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Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). projects. The results of traffic assignment enable the assessment of various planning scenarios, such as the construction of a bridge, the renovation of a railway line, tolling on a highway, or simply a change in traffic direction. The evaluation of the results assesses the impact on transportation network users, traffic conditions, and the environment.

Numerous literature reviews on traffic assignment abound in academic literature. Special emphasis has been placed on traffic assignment from various perspectives, including traffic control [1], the convergence of simulation-based assignment [2], and the used models of traffic flow [3]. More recently, other notable areas include the application of traffic assignment for sustainable road traffic [4,5], evolutionary game theory applied to traffic [6], and bridging user equilibrium and optimal solutions in static traffic assignment [7], among others.

In contrast to the aforementioned papers, this review adopts a more generalist approach. Its primary purpose is to offer a broad introduction to the field of traffic assignment, catering to readers of all expertise levels. The review highlights significant contributions that constitute the domain's foundational pillars of theory and methodology, drawing on well-established and widely recognized fundamental contributions in the field. Importantly, this literature review avoids being application-specific. It refrains from delving into detailed discussions of traffic models, particular frameworks of transportation forecasting, or specific applications in reality. Instead, it focuses on introducing key contributions in the field and outlines future directions based on technological advancements in transportation. More precisely, it revisits the fundamental concepts of traffic assignment and juxtaposes them with the advancements in intelligent transportation systems in order to address promising directions.

The rest of the paper is organized as follows. Section 2 presents the problem statement, gives the scope of Section 3, and ends with an illustrative example. This section aims to introduce the reader to the most important direction of the research in the domain. Section 3 presents the concept of traffic assignment and how the problem could be solved in a decentralized manner. Section 4 focuses on extending the model to handle public transportation and policies for improving traffic conditions. A discussion on emerging techniques and the challenge regarding traffic assignments in the upcoming years is presented in Section 5, which focuses mainly on the impact of the progression of intelligent transportation systems. Finally, the paper is concluded in Section 6.

2. Preview

The problem of traffic assignment has been a subject of study for a century [8]. It has yielded a plethora of intricacies and specialized scenarios. To maintain clarity and coherence and avoid delving into specific cases in the field, this section delineates the scope of Section 3 of the paper. This preliminary section will first present the problem statement, the mode of transportation, and the primary user's criterion under examination in Section 3 of the paper. It ends with an illustrative example that will be referenced throughout the article.

2.1. Problem Statement

Traffic assignment deals with transportation traffic. It involves the following *input* elements:

- Traffic supply: Road network and/or public transportation services as well as their corresponding behaviors.
- Traffic demand: A flow matrix indicating the demand volume between each origindestination (*o*-*d*) pair.

The most extensively researched traffic assignment problem pertains to road traffic, aiming to estimate traffic demand for individual road segments and movements at intersections, including right and left turn movements. In the case of public transportation demand, the goal is to determine the number of passengers for each bus or rail line, along with estimates of boarding, alighting, and connecting passengers at each station. In other

words, the *output* of traffic assignment specifies the routes used for each *o*–*d* pair, with the corresponding volume.

This problem was initially formulated for regular vehicles in 1920 by Pigou [8]. The author distinguishes between two classes of traffic assignment problems. The first is centralized, involving the optimal allocation of a fleet of vehicles on a transportation network. The entity responsible for the fleet's mission optimizes the vehicle assignment according to its specific objectives. The second class, presented in Section 3 of the document, is decentralized. The assignment results from the optimization that each user of the transportation network performs for their benefit. This second class of problems gained particular attention in the literature following Wardrop's contribution [9], who meritoriously formulated the problem. In the following, unless stated otherwise, the term "traffic assignment problem" refers to the second class of problems.

2.2. User Criteria

We commonly refer to a generalized cost that influences user choice. The generalized cost is defined as follows:

Definition 1 (Generalized cost). *The generalized cost is the sum of the monetary and non-monetary costs incurred by a road user in making a journey:*

- The monetary costs, such as the costs of fuel consumption, vehicle's depreciation and maintenance, toll, and transport fare.
- The non-monetary costs, such as the travel time and user's perception of comfort and convenience.

While the generalized cost aims to meticulously capture user behavior, the predominant focus in most studies revolves around evaluating the influence of delays on route selection. Users may also take into account factors such as energy consumption and tolls. However, when examining travel time, it becomes apparent that it possesses unique characteristics compared to other terms of the generalized costs. Firstly, time is the criterion that most accurately represents the interplay between different user's choices. The travel time of a road segment increases with the growth of the number of vehicles. In contrast, the toll for a particular section is generally a constant, unaffected by the number of users of that segment, at least in a short-term horizon. Secondly, extended journey duration is a result of congestion. This congestion, in turn, increases both energy consumption and road user discomfort. Finally, monetary costs are often integrated into the decision-making process to compensate for the travel time. For example, toll considerations are studied to address environmental pollution [4,10–12]. In these studies, the toll price is computed to influence the route choices based on travel time.

2.3. Mode of Transport

Road traffic assignment research has garnered significant attention due to its clear illustration of the relationship between the number of vehicles using the same road and the resulting travel time. This interdependence emerges as congestion and delays manifest with an increasing number of vehicles sharing a particular road segment. Similar considerations apply to public transportation; however, there are some distinct nuances to consider. Firstly, there is often less flexibility in choosing alternative routes. Secondly, the implications of route selection become pronounced when the transport system approaches its capacity, leading passengers to wait for the next available bus, tram, or train. Consequently, as is also generally assumed for individual vehicles, travel times extend as demand on the transportation line intensifies. Notably, this increase occurs progressively in increments. For the sake of clarity, we will focus on road traffic assignment, addressing other modes of transportation in the second part of the paper.

2.4. Illustrative Example

To illustrate the previously communicated scope of the studied traffic assignment problems, a concrete example is presented in the following. Let us consider the road network presented in Figure 1. This network represents the traffic supply. It has one origin and one destination, named *o* and *d*, respectively. There are two itineraries to link *d* to *o*. Each itinerary, k_i , is a road segment, a_i , with a given capacity, Q_{max}^i , and a function relating the travel time, $c_i(r_i)$, to the flow, r_i . We have for each road segment r_i , the following characteristics:

In road segment a_1 , the capacity, $Q_{max}^1 = 1800$ vph, and the travel time behave as follows:

$$c_1(r_1) = 10 \left(1 + 2 \left(\frac{r_1}{1800} \right)^2 \right). \tag{1}$$

• In road segment a_2 , the capacity, $Q_{max}^2 = 3600$ vph, and the travel time behave as follows:

$$c_2(r_2) = 20 \left(1 + 2 \left(\frac{r_2}{3600} \right)^2 \right).$$
⁽²⁾

The time in Equations (1) and (2) is given in minutes. The presented time functions for both roads are based on *BPR* [13] curves, which are widely used to model traffic behavior. One can notice from Equations (1) and (2) that when both roads are empty, the users spend 10 min and 20 min for traveling a_1 and a_2 , respectively. Naturally, a_1 is more attractive without considering the demand.



Figure 1. Example of a network of two alternative roads.

Let us examine a scenario involving 3000 vehicles destined to travel from origin o to destination d within a one-hour time interval. We denote this demand as q^{od} , and consequently, $q^{od} = 3000$ vph. Figure 2 illustrates all possible scenarios of traffic assignment. This figure gives c_1 and c_2 , according to r_1 and r_2 . Note that, from the problem statement, we have $r_1 + r_2 = q^{od}$. From this figure, three assignments can be distinguished:

- Shortest path (All-or-nothing): Without considering Equations (1) and (2) as though both roads are devoid of traffic, all vehicles follow road *a*₁ because it is the shortest path. However, since travel time increases with traffic flow, an all-or-nothing assignment cannot be adopted to predict road use. In this figure, the solution of the all-or-nothing assignment is presented by a green circle. According to Equation (1), the users spend more than an hour to go from *o* to *d*.
- Optimal assignment (Centralized): If we consider that energy consumption is proportional to travel time, the manager of a fleet of the 3000 vehicles has a strong incentive to minimize the average travel time. The red curve in Figure 2 presents the average travel time. The optimal solution is represented by the red circles. One can note that, with this solution, the vehicles taking road *a*₁ spend less travel time than the ones using road *a*₂.
- Fair assignment (Decentralized): In this configuration, each vehicle optimizes its itinerary, so that there is no better solution. The fair assignment is represented by the yellow circle.



Figure 2. Illustration of three approaches of traffic assignment: All-or-nothing (Green point: $r_1 = 3000$ vph, $r_2 = 0$ vph, $c_{average} = c_1 = 66$ min, $c_2 = 20$ min), Optimal (Red points: $r_1 = 1368$ vph, $r_2 = 1632$ vph, $c_{average} = 25$ min, $c_1 = 21$ min, $c_2 = 28$ min), and Fair (Yellow point: $r_1 = 1609$ vph, $r_2 = 1391$ vph, $c_{average} = c_1 = c_2 = 26$ min).

This example clearly illustrates the difference between centralized and decentralized assignments. Assuming a cost proportional to travel time, the discrepancy in the sums of travel times between decentralized and centralized assignments is generally referred to as the cost of anarchy [14].

3. Traffic Assignment: Theoretical Background

Research on traffic assignment can be classified in various ways, such as being based on problem formulation, expected objectives, methodologies, models used, and assumptions about demand, etc. In the following, we consider the classification commonly used in the literature, which distinguishes approaches based on the consideration of traffic dynamics over time. Thus, we first present a static traffic assignment, introducing concepts, definitions, and problem formulations. We then move on to dynamic assignment, where the concepts and algorithms used will be discussed.

3.1. Static Traffic Assignment (STA)

The assignment is categorized as static when the assumption is made that users sharing the same road segment have the same experience during the designated time interval under study. This is exactly the assumption taken in the curves presented in Figure 2 used in the illustrative example (see Section 2.4), where the vehicles using the same road are assumed to experience the same travel time. This type of assignment aligns well with a macroscopic traffic model. In this context, flow-speed curves do not account for variations in speeds between the first and last vehicles traversing the segment during the specified time period. The same can be said for the *BPR* curves. This assumption shapes how the traffic supply is modeled. All that remains is to define the behavior of the decentralized demand, formulate the problem analytically, and solve it numerically. In the following, we present the two major assumptions about the demand behavior and the resolution methods. Within the context of *STA*, both static and stochastic user equilibrium assumptions are the most used in the literature [15].

3.1.1. User Equilibrium (UE): Definition and Formulation of the Problem

In the illustrative example, it is evident that the fair assignment (depicted by the yellow circle in Figure 2) aligns with the preferences of rational road users, each striving to

minimize their travel time. Indeed, we are dealing with independent users, where each user seeks their own interest. The first Wardrop principle describes the behavior of users within this assignment. This principle is given by the following definition:

Definition 2 (First Wardrop Equilibrium Principle). *An assignment is said to be at equilibrium according to the first Wardrop principle if the following two conditions are satisfied:*

- For each origin-destination pair, o-d, the generalized costs of each utilized route are less than or equal to those of alternative (unused) routes.
- If multiple routes are used for an o-d pair, their generalized costs are equal.

According to the definition of the Wardrop principle, it is clear that no user has an interest in changing their route alone. Indeed, if they modify their route, they increases the cost of their journey. This principle is also known as a *UE*.

Several mathematical formulations of Wardrop's first principle can be found in the literature. In the following, we present the one given in [16]. Let there be a network of vertices that includes demand origins and destinations, denoted, respectively, as $o \in O$ and $d \in D$. Let a constant demand, q^{od} , be assigned to a set of routes, K^{od} . Noting the flow assigned to route $k^i n K^{od}$ by r_k^{od} , we have

$$\sum_{k \in K^{od}} r_k^{od} = q^{od}.$$
(3)

Equation (3) reflects the fact that all demand from o to d is affected. The Wardrop's first principle of UE is written as follows:

$$\begin{aligned} c(k') &= \min_{k \in K^{od}} (c(k)) \quad \Rightarrow \quad r_k^{\prime od} \ge 0\\ c(k') &> \min_{k \in K^{od}} (c(k)) \quad \Rightarrow \quad r_k^{\prime od} = 0, \end{aligned}$$
(4)

where c(k) is the generalized cost of route *k*.

Let us note from Equation (4) that the first principle is not written in the classical form of an optimization problem, i.e., an objective function under constraints. Although the formulation given in Equation (4) is common, the contribution of Beckmann et al. [17] has enabled us to reformulate the problem by distinguishing between each segment of the road network.

Let us consider the road network represented as a graph, G = (S, A), where *S* denotes the set of vertices, $s \in S$, and *A* represents the set of directed arcs, $a \in A$. The vertices stand for points of interest and intersections. We have $O \subset S$ and $D \subset S$. Each road segment is modeled by a directed arc, *a*, connecting two vertices, s_i and s_j , where $i \neq j$. Each route, k^{od} , consists of a set of successive arcs, $a \in I_k^{od}$. Therefore, we have

$$c(k^{od}) = \sum_{a \in I_k^{od}} c(a),$$
(5)

where c(a) denotes the generalized cost of arc *a* (route segment). This Equation (5) states that the generalized cost of a route is the sum of the costs of its constituent arcs. This is true in the case where the generalized cost is the travel time; see Section 2.2. For all *o*–*d* and all k^{od} routes, the calculation of r_k^{od} defines the flow through each arc, *a*, of the graph, *G*. Let us denote r_a , the flow assigned to arc *a*. Thus, we have

$$r_a = \sum_{I_k^{od} \ni a} r_k^{od}.$$
 (6)

In other words, the flow assigned to arc *a* is equal to the sum of the flows of all routes that pass through this arc.

The static traffic assignment at equilibrium involves solving the following optimization problem: The problem (7) is a convex optimization problem as it involves minimizing an integral of an increasing function over a compact domain. Note that, from (7), it is possible to abstract away the routes simply by adding flow conservation constraints. In other words, constraints (3), (5), and (6) can be replaced by the following constraints for each vertex *s*:

$$\begin{cases} \sum_{a \in A_{s}^{+}} r_{a} = \sum_{a' \in A_{s}^{-}} r_{a'} \\ \sum_{a \in A_{o}^{+}} r_{a} = \sum_{d \in D} q^{od} \\ \sum_{a \in A_{d}^{-}} r_{a} = \sum_{o \in O} q^{od} \end{cases},$$
(8)

With A_s^+ and A_s^- denoting, respectively, the sets of outgoing and incoming arcs of vertex *s*, and with constraint (8), it is evident that the solution obtained from the problem no longer provides the users' routes. It substitutes them with the number of users per road segment. Thus, for a static traffic assignment problem, the *UE* problem aims to find the optimal flow rates on road segments. The solution allows for multiple possible route configurations [18].

To show the approach, let us consider the illustrative example given in Section 2.4 Figure 2. The *UE* problem given in Equations (3) and (4) consists of finding r_1 and r_2 such that $q_1 + q_2 = q^{od}$ and $t_1 = t_2$. The alternative problem formulation is

$$\min \int_{0}^{r_{1}} \left(10 \left(1 + 2 \left(\frac{z_{1}}{1800} \right)^{2} \right) \right) dz + \int_{0}^{r_{2}} \left(20 \left(1 + 2 \left(\frac{z_{2}}{3600} \right)^{2} \right) \right) dz$$
s.t. $r_{1} + r_{2} = 3000 \text{ vph}$
 $r_{1} \ge 0, r_{2} \ge 0$

$$(9)$$

With only these two roads ($r_2 = 3000 - r_1$), it is possible to obtain the analytical solution to problem (9) by searching where the derivative of the function equals zero:

$$10\left(1+2\left(\frac{r_{1}}{1800}\right)^{2}\right)-20\left(1+2\left(\frac{3000-r_{1}}{3600}\right)^{2}\right) = 0$$

$$r_{1} = 600\left(\sqrt{59}-5\right)^{2}$$

$$r_{1} = 1609 \text{ vph}$$
(10)

We draw the reader's attention to the first line of Equation (10). This expression is obtained by taking the derivative of the function to be minimized. It is worth mentioning that setting the derivative function to zero is equivalent to expressing $c_1 - c_2 = 0$. If there are only two available roads, solving the problem analytically is straightforward. However, numerical approaches, such as gradient descent-based methods, are needed to find a solution when dealing with multiple road segments.

3.1.2. Stochastic User Equilibrium (SUE)

The Stochastic assignment was introduced to address issues arising from the *UE* assumption [19] within the framework of the static assignment. For various reasons, users might have imperfections and variations in their perceived generalized costs of travel. One reason is that users may have different experiences in the same segment during the study period. Another reason for this is that users do not have perfect knowledge of the costs associated with different options. Hence, a route is chosen based on a probability relative

to its travel time. More precisely, the stochastic assignment is based on the following principles [20,21]:

- All reasonable options can be chosen, even if their probability of selection is very low (In [20], the author describes the concept)
- If two options have the same cost, the probability of selection is the same.
- The probability of choosing options depends on their costs: a route with a higher cost has a lower probability of being chosen.
- The user of the SUE model must have some control over the probability of diverting routes.

The principles of *SUE* incorporate a term of random error in the generalized cost of the route. Generally, this term is assumed to follow Gumbel, normal, and Weibull distributions, which correspond, respectively, to choice models based on logit, probit, and weibit models [22–24]. The logit-based model is the most commonly used in the literature on traffic assignment ([24,25]). According to the logit model, the probability of choosing route *k* is as follows:

$$P_{k} = \frac{\exp\left(-\theta \cdot c(k)\right)}{\sum_{i \in K^{od}} \exp\left(-\theta \cdot c(i)\right)},\tag{11}$$

where θ is the diversion factor. This is the controlling variable for variations in users' perceptions of travel costs. When $\theta = 0$, all routes will have an equal probability of being selected. As $\theta \to \infty$, the least costly route will be chosen. With this probability expression, Fisk [23] extended the formulation of the *UE* problem (7) to encompass the multinomial logit model of *SUE*. The problem is formulated as follows:

$$\min_{k_a} \sum_{a \in A} \int_0^{r_a} c_a(z) dz + \frac{1}{\theta} \sum_{o \in O} \sum_{d \in D} \sum_{k \in K^{od}} r_k^{od} \ln\left(r_k^{od}\right)$$
s.t. (8)
$$r_k^{od} \ge 0, r_a \ge 0$$
(12)

Other works have extended the objective function [26–29] of the problem posed in (12). This extension aims to consider the overlap between routes by introducing a conditional probability of route choice.

3.1.3. Approaches to Solving Static Assignment

To solve problems (7) and (12), several studies have relied on the Generalized Nonlinear Optimization Algorithm by Frank and Wolfe (FW) (1956) [30]. The minimization problem of a convex function under linear constraints is approximated from a feasible solution through manageable linear optimization problems using the simplex algorithm. The main drawback of Frank and Wolfe's algorithm is its slow convergence rate [31]. To improve algorithm convergence in the context of traffic assignment, related methods have been proposed in [32–35]. In addition, Refs. [36,37] have proposed methods based on restricted simplicial decomposition (DSR) to handle large networks by decomposing the assignment problem into sub-problems. Other iterative approaches are suggested in the literature. Some are based on routes [38–40]. This approach begins with an assignment solution where all used routes and their associated flows are known. Iteratively, flows may shift from high-cost routes to low-cost routes to reach equilibrium. In [38], for each o-d pair considered sequentially in a cyclic order, flows are shifted from the highest-cost route to the lowest-cost route until both routes have the same cost. When the cost derivatives at the links between the routes are known, they have been used to estimate flow movements toward the minimum-cost route for each o-d pair [37,41]. Other approaches are origin-based [17,40,42] or destination-based [43,44]. These approaches pose a challenge when circuits exist. The authors in [18] propose an origin-based search approach, limiting solutions to acyclic ones. The quasi-Newton method is employed to effectively shift flows at each iteration and eliminate residual flows. Additionally, the authors in [45] proposed

a Barzilai–Borwein-based approach to expedite convergence, moving flows from costlier to less costly routes [46]. Apart from the differences in the techniques for allocating flows to the least-cost routes and the stopping criteria, the resolution of static traffic assignment works according to an iterative approach, as presented in the Algorithm 1.

Algorithm 1 Static Traffic Assignment

- Initialization: Calculate initial routes (e.g., all-or-nothing assignment).
 - Iterations with a stopping criterion:
 - Road network loading: Load demand onto the road network along the routes and obtain generalized costs (travel time: congestion).
 - Generation of choice set: Compute new routes based on the network's updated generalized costs (travel time).
 - Choice: Allocate demand among the routes based on the updated generalized route costs (The allocation strategy varies depending on the optimization approach used, e.g., quasi-Newton method). This involves transferring a portion of the flow to the least costly routes.

3.2. Dynamic Traffic Assignment (DTA)

Dynamic traffic assignment (*DTA*) considers changes in the perception of travel costs based on demand dynamics, choices, traffic conditions, and network characteristics. Unlike static traffic assignment, the vehicle's departure time significantly influences the perceived cost and, consequently, route choice. It is commonly accepted that a dynamic assignment model should include the following elements:

- A traffic model in which congestion (travel time) varies over time.
- A time-varying demand.
- An equilibrium that is based on experienced travel cost, not instantaneous travel cost.

DTA has been studied for over forty years, and numerous research projects have been carried out [47–55]. The first approaches were based on the formulation of the dynamic assignment problem in the form of mathematical programming ([56–59]) or optimal control ([60–62]). More recently, models with greater resolution in terms of time steps, vehicle batches, and traffic interaction [3] are being used.

Macroscopic traffic models can be used for estimating the generalized costs experienced by users [13,63–69]. However, mesoscopic [70] and microscopic [71–74] models are more suitable. The main advantage of using these models lies in their ability to accurately represent more detailed traffic phenomena, such as shock waves, expansion waves, and queue spillback, thereby enhancing the realism of the assignment [4]. However, we draw the reader's attention to the fact that the microscopic simulation-based traffic assignment raises many difficulties, such as convergence issues [2] and deadlock, that impede simulated vehicles from reaching their destination (see Figure 3A). Over the last decade, DTA has garnered significant attention due to the increased accuracy of the models used to assess traffic performance within the framework of environmental considerations, such as estimates of CO_2 emissions and noise pollution. The use of DTA is not limited to traffic prediction; it can also be leveraged for traffic optimization by comparing various infrastructures or traffic regulation scenarios.



Figure 3. Examples of microscopic phenomena: (**A**) Deadlock situation; (**B**) Comparison of scenarios (**B1,B2**): Increased travel time due to the extension of green light time.

3.2.1. Time-Dependent User Equilibrium

In the context of *DTA*, the definition of the first principle of Wardrop's equilibrium (see Definition 2) is adapted to consider users' departure schedules along with the travel times they experience. In this context, we refer to it as dynamic user equilibrium, defined as follows:

Definition 3 (Dynamic User Equilibrium (*DUE*)). *The DUE refers to a situation in a transportation network consisting of multiple origin–destination (o–d) pairs over a specific time period, where the following conditions are satisfied:*

- For each o-d pair and for each departure time interval, the routes taken by users exhibit a
 generalized cost (experienced travel time) that is both equal and minimized to the extent possible.
- No user can unilaterally reduce their experienced generalized cost (travel time).

Definition (3) outlines the concept of DUE in a transportation network, considering departure time intervals and the minimization of experienced travel times for users across various o-d pairs.

Like Wardrop's equilibrium defined for static assignment, DUE implies that different routes or paths available to connect o-d zones are used so that the perceived travel time for users is balanced and minimized. Moreover, dynamic equilibrium requires that no user can unilaterally reduce their experienced travel time. This means that users cannot individually choose to change their route to reduce their personal travel time without considering other users and the consequences on the overall system equilibrium. However, this equilibrium applies to each departure time interval, where no route should be significantly faster or slower than others for a given *o*–*d* pair. Indeed, it is difficult to measure the travel time of all alternative roads at a given instant (at the departure time of a given vehicle). The problem of estimating travel time makes the search for DUE mathematically intractable [75,76]. To overcome this problem, other definitions of the equilibrium, such as Boston traffic equilibrium [60] and integral equilibrium [77] of the trajectory, have been proposed. However, these approaches require verifying the generally accepted assumption of monotonic increasing delay as a flow function. Even if this assumption is widely admitted in macroscopic and mesoscopic traffic models, experience and microscopic models can sometimes invalidate this assumption, mainly because intersection management is not accurately considered in models with low resolution. Let us take the example of adaptive traffic lights that manage the intersection of two alternative paths. Reducing the number of vehicles on one path will increase the travel time of the remaining vehicles (see Figure 3B1,B2).

The relative gap measure is commonly employed for each o-d pair at a given departure time interval (o-d, T) to assess solution quality. This gap quantifies the difference between the total cost of paths used by vehicles and the total cost of the shortest path used by

all vehicles [78]. The relative gap can be defined and expressed as follows to gauge the proximity of an equilibrium solution [79]:

$$Rgap = \frac{\sum_{T} \sum_{(o,d) \in \delta} \sum_{k \in K_{od}(T)} r_k^{od}(T) \left(c(k^{od}, T) - \mu^{od}(T) \right)}{\sum_{t} \sum_{od} q^{od}(T) \mu^{od}(T)},$$
(13)

where $\mu^{od}(T)$ represents the generalized cost of the shortest path connecting origin–destination (o-d) pair for time interval, T. Note that, in Equation (13), the expression $(c(k^{od}, T) - \mu^{od}(T))$ quantifies the difference between the cost experienced by the $r_k^{od}(T)$ vehicles following path k^{od} and the minimal cost, $\mu^{od}(T)$. If the conditions of dynamic equilibrium were met at each time interval T, this difference would be null. Conversely, the greater the variation in costs experienced by vehicles connecting the same o-d pair, the higher the value of Rgap becomes. In practice, achieving identical travel times for a given o-d pair, especially when measuring travel times based on a microscopic model, is challenging. Hence, the solution of DTA is deemed valid when the relative gap is below an accepted threshold. At that point, it is considered that the network in question has reached an equilibrium state.

In addition to the relative gap, other criteria can be taken into account, particularly when travel demand is high. Microscopic simulation can reveal situations of waiting for access to the network or of deadlock where no vehicle can move [80–82], as shown in Figure 3A. The percentage of vehicles served is also an interesting indicator.

3.2.2. Resolving Approaches

The algorithmic procedures for static assignment and dynamic assignment share comparable structures. The structure is outlined in Algorithm 1. The differences primarily lie in the following elements:

- Initialization: In the majority of studies, the network is initialized using the all-ornothing assignment based on the computation of the shortest path. In the context of *STA*, the shortest path is computed with an empty network. Consequently, the all-or-nothing assignment associates a route with each origin–destination (*o*–*d*) pair throughout the entire temporal horizon of the study. In the case of dynamic assignment, the travel time (generalized cost) for a given route, *k*^{od}, varies based on the departure time intervals of vehicles. Indeed, with each new time interval, it is necessary to update the generalized costs induced by vehicles already assigned in previous time intervals. Thus, the search for the shortest path occurs progressively as the network fills up.
- Iteration: Recall that, at each iteration, a flow of vehicles is shifted from a costly route to a less costly one until a convergence criterion is met. In the context of *DTA*, this flow shift occurs for all time intervals, *T*, within the study interval at each iteration. Several approaches exist in the literature for determining the direction and quantity of the traffic shift. The most classical approach is based on the Frank–Wolfe algorithm [30,83]. Other more efficient algorithms have been proposed since, including the gradient projection algorithm [41] and the method of successive averages (*MSA*) [79,84,85] to compute the generalized cost. Approaches based on meta-heuristics have also been proposed.
- Evaluation of Travel Times: The time it takes to travel a road segment can vary from when a vehicle starts its journey to when it is moving on that segment. Drivers base their route decisions on the time spent actively driving on the segment. Thus, it is crucial to predict this travel time accurately. In advanced methods, travel times are assessed by adding up the times for each segment, forming a travel time chain. The estimated travel time for each segment is based on when the vehicle is expected to reach that specific segment [78].
- Stopping Criterion: Recall that, in *STA*, there is a unique equilibrium solution in terms of traffic flow for each segment. This solution is reached when Algorithm 1

converges. Unlike *STA*, the convergence of vehicle routes in *DTA* does not imply that the network has reached a dynamic equilibrium. The convergence problem becomes more complex when the granularity of the traffic model is high. It is widely reported that microscopic models are intractable.

Similar to *STA*, classical approaches for solving the dynamic traffic assignment problem are based on iterative algorithms that involve route permutation in the direction of gradient descent. The outcome of these approaches and their convergence depends not only on the initial solution (initialization phase) but also on intermediate solutions [86] and the network saturation state [87–89] (e.g., deadlock and shock waves). Additionally, classical *DTA* approaches are susceptible to the risk of converging to a local minimum. Furthermore, the solution space exploration is limited [87]. Consequently, several authors propose the use of heuristics and meta-heuristics for dynamic assignment problem resolution, such as simulated annealing [86,90,91], population-based search (e.g., genetic algorithm and teaching–learning-based optimization) [86,92–96], and ant colonies [97,98]. A noticeable work presented in [99] introduces a new modeling paradigm for DTA and uses multi-agent reinforcement learning to solve DTA.

Algorithm 2 presents an overview of population-based search, where multiple solutions are generated, evaluated, and enhanced, enabling a comprehensive exploration of the solution space.

Algorithm 2 Population-Based Search

1. Initialization: Generate a collection of candidate solutions for a given problem.

- 2. Repeat the following operations multiple times:
- Evaluation: Assess the "score" of each candidate solution.
- Selection: Reduce the number of "poor" solutions (there are various methods to achieve this).
- Construction of new solutions: Build new solutions and add them to the collection of candidate solutions.

4. Extended Traffic Assignment

Traffic assignment has been widely implemented in the context of road infrastructure design. It has been extensively used to assess various road investment scenarios and lane directions. Thus, the previous section specifically focused on travel times, vehicle traffic, and trip balance. In this section, we will discuss the expansion of the application of traffic assignment, particularly to enhance environmental conditions, likely a reduction in pollution or an increase in environmental sustainability. This section explores contributions related to alternative modes of transportation and road toll policies. In addition, it shows how agent-based modeling allowed a paradigm shift.

4.1. Traffic Assignment for Alternative Modes

Alternative modes of transportation are of particular interest to transportation authorities, aiming not only to provide transport options suitable for all socioeconomic backgrounds of inhabitants but also to enhance environmental conditions and the quality of services for tourists and impaired people. Traffic assignment for alternative modes is vital for facilitating economic evaluations that assess the sustainability of projects related to innovative transportation systems [100–103]. Following the formulation of the traffic assignment problem, there has been a specific focus on the transit assignment. However, assigning public transport users requires some adjustments to the initial traffic assignment formulations. The first pertains to the criterion motivating users. Travel time cannot be considered the predominant criterion. Users' perceived quality of service is not limited to the travel time. It depends on vehicle and transport line capacity as well as frequency [104–109]. These factors determine access queue lengths and transfer times [110,111]. Additionally, the number of connections may discourage certain route choices, even if the travel time is shorter. From the supply modeling standpoint, there are two approaches to modeling transit. The first one is based on line frequencies [112–114], where specific waiting time models have been proposed, either with or without consideration of the vehicle capacities. The second one considers the transit timetables (schedule) [115–120]. From the demand modeling standpoint, the notion of equilibrium [104,106,107,121–123], including the stochastic one [120,124–127], is widely deployed in the context of transit assignment.

However, the assumption of a strongly monotonic generalized cost function increasing as a flow function is not necessarily verified. First, travel time is no longer the most important criterion. Second, even if it were, the function does not benefit from the same characteristics as the functions used in road traffic. More precisely, the traveling time is not necessarily monotone, and "an increase in flow does not deteriorate the performance of the system" [123].

For the reasons presented previously, the static assignment of transit users has also benefited from its theoretical developments, not only for formulating the problem but also for calculating the solution. As far as the formulation of the problem is concerned, a hyperpath model is used and assumptions have also been relaxed in [128] to ensure the uniqueness of the solution. For the calculation of the solution, the contribution of [128] reformulates the problem in the form of a nonlinear complementary problem, and [129] proposes an approach for its resolution. In the latter paper, the authors highlight the convergence issue of classical gradient-based approaches. It uses a heuristic to minimize the gap (similar to the relative gap given in Equation (13) but adapted to the transit travel time and there is no denominator) function, coupled with *MSA* to estimate the travel time. Advancements in computing capabilities, coupled with the utilization of heuristics and iterative approaches, have created the opportunity to address two challenges simultaneously—namely, planning the public transport network while considering its impact on users. This is exemplified by the approach presented in [130].

Similar to the dynamic assignment of road traffic, transit assignment has benefited from microscopic traffic simulators. These simulators incorporate departure times, account for delays caused by congestion, and consider the time users spend boarding and alighting at stops. Given the complexity of the problem, agent-based approaches seem inevitable. In this approach, agents learn based on their criteria through numerous iterations of decision-making. Such approaches have been successfully employed in refs. [15,131]. An agent-based approach has also proven beneficial in addressing the complexity of the carpooling assignment problem [132]. Carpooling introduces the challenge of matching routes and determining the balance between itinerary extension and the reward, influencing the decision on the number of individuals sharing the same itineraries.

4.2. Environmental Concerns and Traffic Control

The results of both *STA* [133] and *DTA* [10,11] were used for the evaluation of the resulting pollution. This was applied not only to road traffic but also to transit [131]. Apart from the use of *STA* and *DTA* for environmental assessment, the problem of traffic assignment can be extended to control the traffic. Recall that the travel time is the central user's criteria in traffic assignment models. The equilibrium assignment results from a selfish choice that is not in the community's best interest (see the illustrative example presented in Section 2.4). Therefore, tolling is the lever for controlling the distribution of flows.

The control was addressed in several papers to mitigate the cost of anarchy. However, the studied problems remain limited [4]. In contrast to the problem of having a known cost function for roads and distributing the flow accordingly, the control problem is based on a centralized optimization function for which the travel costs of roads are adjusted. More precisely, in [10] the authors suggest an approach where the dynamic system optimum with free flow is solved first. Then, the dynamic pricing is optimized after several iterations. More recently, Ref. [134] shows the impact of the design of incentive policy to minimize

the price of anarchy in a congested network. This work was extended in [12] to compare between subsidies and tolls.

Another control lever is the traffic light. A traffic-light plan can be used to either increase or decrease the travel time for a given road. However, such an approach poses the problem of modeling the travel time according to traffic-light plans. Moreover, it is widely admitted that this makes the problem more complicated to solve, requiring simplifications and artificial-intelligence (*AI*) techniques. One approach is to carry out several iterations in which the two problems, i.e., traffic assignment and traffic-light optimization, are dealt with in sequence. In the work presented in [135], the authors use several heuristic strategies [136] to optimize traffic lights while using MatSim [93] for traffic assignment. The authors of [137] suggest a navigation rule based on gene expression programming. The navigation rule allows vehicles to select the neighboring road. The traffic-light plan is optimized to maximize the throughput. In both papers, the traffic-light optimization aims to improve traffic flow according to the assignment result.

4.3. Paradigm Shift through Multiagent-Based Approaches

The traffic assignment problem, as defined in Section 2.1, requires the o-d matrix as input. However, experience has shown that travel time can modify the entries in the o-dmatrix. A user may be discouraged from going to a given supermarket if there is congestion. Therefore, the o-d matrix is calculated iteratively. In the transportation investment project context, the traffic assignment problem is included in a four-step model [138] that follows several iterations until the convergence. In this model, the first three steps, which are trip generation, trip distribution, and mode choice, calculate the o-d matrix based on the socioeconomic data of the zones and the distance matrix between the zones. The last step is the traffic, which provides the new distance matrix. This matrix is used again for the first three stages, and so on. Agent-based models of users allow an activity-based approach [139]. Rather than assign the traffic according to the o-d matrix, each transportation user is an agent that aims to increase its utility according to a collection of activities. Time spent on transport is a decreasing function that reduces the agent's utility. The agent-based model enables agents either to learn [140] or to co-evolve in parallel [93], not only to optimize their routes and approach a kind of equilibrium but also to adapt their departure times according to traffic conditions. A noticeable description in [93] details the utility function and the co-evolution principle used in MatSim software version 0.8. Moreover, other optimizations can be launched, such as the location of stop stations and traffic-light plans [136]. However, to reach the convergence, the multi-agent learning algorithms are limited to a specific equilibrium, such as Nash, correlated, or coarse equilibrium [141,142].

5. Discussion

The traffic assignment problem has been extensively addressed in the literature. It has benefited from traffic supply models and demand models. Several resolution methods have been proposed for this problem. They generally share a common algorithmic structure, starting from an initial traffic situation and iterating until a stopping criterion is satisfied. At each iteration, a portion of the traffic is reassigned to less costly routes. In general, resolution methods are designed for transportation planning scenarios where relatively long computation times are acceptable.

While traffic assignment approaches and associated models have been extensively tested through studies and real-world applications, the advancements in intelligent transportation systems reveal their limitations and call for new contributions. To illustrate the claim, this section is organized into two parts. The first one reviews the major progression in intelligent transportation systems and how emergent technologies question the theories behind traffic assignment. The second part provides promising research directions.

5.1. Intelligent Transportation Systems

To illustrate the claim of limitations of the classical approaches, let us begin by examining current intelligent transportation systems. Many road users rely on navigation applications [143–145] to choose their routes in major cities. These applications provide optimal routes based solely on real-time traffic information without necessarily seeking equilibrium for obvious computational time reasons [146,147].

Another widely adopted intelligent transportation system is adaptive traffic signals [148]. Signal regulation is based on measured flows, favoring the most significant flows by extending the duration of the green light and providing sequences of green waves. This could challenge the assumption of increasing travel time based on demand. Additionally, these optimizations raise questions about the multitude of itineraries, as it may be more beneficial to define efficient corridors and reduce conflicts that lead to congestion [149]. Furthermore, other adaptive control systems exist to streamline traffic outside conflict zones, such as speed limit control systems in smart segments, aiming to prevent the formation of traffic congestion.

Whether it is the increasing use of navigation systems in major cities or adaptive traffic control systems, current intelligent transportation systems challenge the models used in static assignment. Specifically, it is crucial to leverage microscopic simulation to incorporate the new traffic supply and demand behaviors.

We can expect the future widespread integration of connected and autonomous vehicles (*CAV*), with technological shifts and new transportation behaviors across various aspects. *CAV* is anticipated to streamline the utilization of road infrastructure, leading to a reduction in the need for parking spaces and lanes. Consequently, the challenge of traffic assignment becomes pivotal in realizing urban space efficiencies and designing autonomous shuttle services. Hence, traffic assignment approaches must effectively tackle this upcoming traffic.

These vehicles would require new modes of driving and regulation. For instance, leveraging their communication capabilities and trajectory control, they can self-organize in conflict zones without the need for traffic lights or priority signs [150,151]. They determine the sequence of passage (who goes first, second, etc.) and adjust their speed accordingly. A highly active scientific community is working on this mode of regulation, commonly referred to as an 'autonomous intersection management system'. This technology is also set to be applied to ordinary vehicles equipped with connectivity, thanks in particular to greenlight optimal speed advice (GLOSA) [152–155]. In addition to autonomous intersections, other driving systems are conceivable, such as cooperative platooning [156], connected merging systems [157–160], dynamic lane reversal [82,161–163], etc.

Intelligent transport systems are not limited to vehicle and infrastructure equipment. The domain of smart cities opens up new perspectives. Cities have access to the cloud, where real travel data can be analyzed. These data come from the GPS tracks of mobile applications, increasingly precise roadside sensors (cameras with number-plate detection), WiFi hot spots on public transportation tracking cellphones' MAC addresses, and GPS-equipped vehicles (privacy preservation needs to be considered carefully in this context.). The data can feed traffic assignment approaches not only to calculate o-d matrices but also to calibrate both supply and demand models. With these technologies, one can expect that the city's traffic models, including traffic assignment models, would be improved to become increasingly accurate. Thanks to the progress in *AI* techniques and data science, the data may also be sufficient alone to carry out traffic assessments and traffic projections [164–168].

5.2. Promising Directions

In light of the aforementioned developments in intelligent transport systems, we will now outline two key directions that show promise for traffic assignment:

• **Mixed traffic digital twin**: The cohabitation of a multitude of systems would require either new macroscopic and mesoscopic traffic models or the use of microscopic mod-

els coupled with an agent-based approach [76,169]. The latter option appears better suited to accommodate the diverse objectives of road users, especially in the context of mixed traffic: human drivers with navigation systems, human drivers familiar with the road, autonomous shuttles, optimized freight transport, etc. Furthermore, employing microscopic models could enable us to address the challenge of finding an equilibrium between diverse equitable routes and the costs induced by conflicts. However, it is worth noting that the use of microscopic traffic models for computing the travel times is resource-intensive in terms of computation resources and time. Another option is to investigate the implementation of digital twins [170,171] with different levels of granularity of the traffic entities. This digital twin should be capable of assimilating data from the cloud, ranging from macroscopic quantities such as road flow and occupancy rates to more detailed user profiles at the vehicle level, including departure times and itineraries. Harnessing AI techniques is crucial [172] for generating a comprehensible map of road usage for decision-makers (e.g., identifying saturated intersections, assessing environmental impact, improving safety, and offering the forecasts needed by cities). More importantly, AI can also be used to propose solutions (e.g., the necessity for new transit services and a new toll policy). As emphasized throughout this article, the problem is undeniably intricate. Nevertheless, the substantial advancements in machine learning technologies, data analysis techniques, and cloud computing hold the promise of highly innovative approaches. The data has previously been employed for a range of tasks related to traffic assignment, including estimating the origin–destination (o-d) matrix [173–175], evaluating travel times [176], and calibrating models and traffic assignment results [174,177–180]. This utilization encompasses the data of cell phones, information systems, and magnetic loops. The development of the mixed traffic digital twin aims to transcend these contributions, representing a significant stride towards exploring advanced artificialintelligence (AI) techniques for traffic management. An illustrative foundation for data-driven traffic assignment is established in the study presented in [181]. Instead of assuming user behavior, the authors showcased the feasibility of precisely estimating road demand by directly learning flow patterns from the available data. This innovative approach lays the groundwork for incorporating AI into traffic forecasting. The extension introduces additional parameters, such as traffic control strategies and new transportation scenarios. Moreover, the mixed traffic digital twin can be designed to be proactive, not only highlighting problems, such as those proposed in ref. [182], but also proposing solutions to the decision-makers. To this end, the mixed traffic digital twin may be trained by using "classical traffic" assignment approaches.

• New generation of navigation systems: The evolving uses of *CAV* and associated regulatory systems will likely challenge current traffic assignment approaches. These vehicles will need to plan their routes in real time based on received demands and communicate their estimated arrival times as accurately as possible. Although Wardrop's equilibrium assumption does not allow for the perfect rationalization of vehicle usage, it remains desirable for fairness reasons. Indeed, it seems evident that no user would want to take a shuttle service that significantly takes longer than other shuttles without financial compensation. However, current traffic assignment techniques do not allow for real-time route planning. Computation times increase when considering vehicle behavior and traffic regulation details with high resolution. Currently, traditional traffic assignment approaches fall short in delivering real-time itineraries. Simultaneously, there exists a lack of consensus in the literature regarding the impact of contemporary navigation systems. Several authors have underscored adverse effects, as indicated in refs. [183–186].

This issue pertains to the challenge of efficiently assigning *CAV* on a constantly evolving road network, taking into consideration real-time traffic conditions. One crucial aspect of the problem is determining optimal routes for vehicles based on real-time traffic information. The commonly used shortest path search method, which

calculates the fastest routes between two points on a road network, does not effectively solve the real-time traffic assignment problem. The main drawback of this method is its lack of responsiveness to constantly changing traffic conditions. Figures 4 and 5 depict the adverse effects of such an approach on route selection. The first figure illustrates how vehicles can become trapped when new vehicles are rerouted to an alternative path with smoother traffic, highlighting the necessity for a thorough evaluation of intersection times. Meanwhile, the second figure demonstrates how a vehicle may be misdirected due to the absence of traffic that is not yet present, emphasizing the critical importance of accurately estimating upcoming traffic conditions.

New strategies with simple rules must be defined to guide vehicles in real time. These rules should be capable of providing both efficient and fair routes while fostering smooth traffic flow. Some studies already address these issues by proposing itinerary reservations [187,188]. Among these studies, some focus on road booking in order to not exceed their capacity [189–192]. Others are inclined towards intersection reservations [149,193–195] to alleviate costs associated with conflicts arising from diversified routes. However, these approaches are relatively recent and deserve to garner broader attention within the community to receive more feedback on microscopic models of large cities incorporating innovative strategies for sharing road infrastructures with more transparency.



Figure 4. Risk of congestion due to inefficient permutation.



Figure 5. Example of a wrong choice due to unexpected traffic.

6. Conclusions

This article offers a comprehensive review of traffic assignments. It begins by illustrating the issue through an example that underscores the differences between centralized and decentralized assignment methods. It then delves into static and dynamic approaches to traffic assignment, detailing the *UE* principles linked with each approach. Additionally, this paper sheds light on how integrating agent-based models could significantly impact optimizing public transport. It then looks at emerging techniques and the challenges facing traffic assignment in the years ahead.

With the increase in city sizes and the urgent need for energy conservation and environmental quality improvement, transportation forecasting based on proven scientific techniques is becoming increasingly necessary in order to successfully carry out urban development projects. Traffic assignment is one of the central means to assess different scenarios. However, technological advancements lead to the emergence of new needs. On the one hand, the abundance of data sources and advancements in calculation methods and means open the prospect of developing new approaches to support decision-makers. On the other hand, the evolution of vehicles and equipment used by transportation users allows the introduction of new infrastructure-sharing principles that need to assign traffic efficiently and equitably in real time.

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Abbreviations

The following abbreviations are used in this manuscript:

- AI Artificial Intelligence
- BPR Bureau of Public Roads
- CAV Connected and Autonomous Vehicle
- DTA Dynamic Traffic Assignment
- DUE Dynamic User Equilibrium
- MSA Method of Successive Averages
- STA Static Traffic Assignment
- SUE Stochastic User Equilibrium
- *UE* User Equilibrium
- vph vehicle per hour

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