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Simple Estimate of the Impact of M^2 and Strehl Ratio on the Effective Focusable Spot Size

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Abstract: A simple method (first-order approximation) to determine the impact of M^2 and the Strehl Ratio on the effective focusable spot size avoiding complex propagations of the beam wavefront is proposed. The model is based upon previous models and the definition of M^2 and the Strehl Ratio in a simple manner. This work provides qualitative and quantitative estimates for the interplay of M^2 and the Strehl Ratio on the effectively focusable spot size.

Keywords: M^2; Strehl ratio; focus; spot size; beam wavefront

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1. Introduction

The spot size of a laser beam close to the focus is an important parameter for many laser applications [1]. The spot size depends on numerous factors, including, among others, the laser wavelength, the beam profile, the numerical aperture (NA) of the system, and, in general, the caustic of the beam (i.e., the propagation of the beam's wavefront towards and close to the focus) [2].

Often, simple numerical values are provided as a compact expression of the complex effects of the beam's wavefront close to the focus. Among them, M^2 and the Strehl Ratio are commonly reported values for the beam quality and the optical quality of laser systems.

M^2 is simply a heuristic parameter of the laser beam [3], whereas the Strehl Ratio (SR) is a metric of the point spread function (PSF) relative to the equivalent diffraction-limited PSF [4]. A thorough presentation of beam quality metrics can be found in a seminal work from 2006 [5].

M^2 is a heuristic parameter of the laser beam related to the spot size in focus [6], whereas the SR is a measure of the quality of optical image formation, originally proposed by Karl Strehl [7]. The SR is frequently defined as the ratio of the peak aberrated image intensity from a point source compared to the maximum attainable intensity using an ideal optical system limited only by diffraction over the system's aperture [8]. Usually, M^2 values of 1.25 or below are considered excellent (and non-rigorously as "close to the diffraction-limit") [9]. Similarly, optical systems with SR values of 0.8 or better are considered excellent (and non-rigorously as "close to the diffraction-limit") [7].

It has been previously shown that M^2 and the SR affect different laser–tissue interaction processes including laser-induced optical breakdown (LIOB) [10] and laser-induced refractive index change (LIRIC) [11].

In this work, a simple method (first order approximation) to determine the impact of M^2 and the SR on the effective focusable spot size avoiding complex propagations of the beam wavefront is proposed. The model is based upon previous models and the definition of M^2 and the SR in a simple manner. This work provides qualitative and quantitative estimates for the interplay of M^2 and the SR on the effectively focusable spot size. The novelty of this work is providing analytical (yet simple) closed-form equations to account for M^2 and the SR effects in photochemical and laser–tissue interaction models.

2. Materials and Methods

M^2 has a value > 1, with a hypothetical, perfectly unaberrated beam having an M^2 of 1. M^2 is define as

$$M^2 = \frac{\theta \pi \omega}{\lambda} \tag{1}$$

where θ is the beam divergence, ω is the measured beam waist, and λ is the wavelength of light.

In other words,

$$\omega_{actual} = \omega_0 M \tag{2}$$

where ω_{actual} is the actual beam waist and ω_0 is the theoretical minimum beam waist at focus under ideal conditions, and

$$\theta_{actual} = \theta_0 M \tag{3}$$

where θ_{actual} is the actual beam divergence and θ_0 is the theoretical minimum beam divergence under ideal conditions.

The SR has a value between 0 and 1, with a hypothetical, perfectly unaberrated optical system having a Strehl Ratio of 1.

$$SR = \frac{max(PSF_{actual})}{max\left(PSF_{diffraction-limited}\right)} \tag{4}$$

Both M^2 and the SR are thus related to a "redistribution" of the (constant) energy in a different (wider) region (the spread). For a super-Gaussian profile, the following equation applies for the intensity [12]:

$$I_0 = \frac{2^{\frac{1}{N}} N E_{Pulse}}{\pi R_0^2 \Gamma\left(\frac{1}{N}\right)} \tag{5}$$

where N is the super-Gaussian order of the beam profile (where N = 1 represents a simple Gaussian beam profile, and $N = \infty$ represents a flat-top beam profile), E_{Pulse} is the energy of the laser pulse, R_0 is the spot size when the radiant exposure falls to $1/e^2$ its peak value, and Γ is the gamma function (the general factorial function for non-integer arguments).

In first-order approximation for not strongly aberrated beams, the intensity (related to the PSF and the SR) is approximately inversely proportional to the squared spot size (the waist at focus) (i.e., inversely proportional to the area of the spot).

From simple geometrical considerations, this relationship holds true also for any arbitrary shape other than super-Gaussian. We could scale the intensity by s_z (affecting SR) and the radial distance of the spot by $s_{x/y}$ (along x-y-plane). To keep the volume constant (energy is redistributed) $1 = s_{vol} = s_x s_y s_z$, we have to define the scaling factors as $s_z = 1/(s_x s_y)$. For a rotational symmetric scaling, $s_x = s_y$ holds. Therefore, $s_z = 1/{s_r}^2$.

In this work, the calculations have been made by "pen and paper", and the figures have been simply generated using Microsoft Excel (Office 365, Redmond, WA, USA).

3. Results

3.1. Simple Estimate of the Impact of Strehl Ratio on the Effective Focusable Spot Size

From the methods ($s_z = 1/{s_r}^2$) and from the explicit expression of the intensity for super-Gaussian beams, it follows that

$$\omega_{actual} = \frac{\omega_0}{\sqrt{SR}} \tag{6}$$

i.e., in a first-order approximation, the SR (reduction in the peak PSF relative to the diffraction limited PSF) produces an increase in the spot size inversely proportional to the square root of the SR.

3.2. Quantitative Estimates

Usually, M^2 values of 1.25 or below are considered excellent (and non-rigorously as "close to the diffraction-limit") [9]. According to the results presented above, the M^2 of 1.25 would increase the spot diameter by +12% (1.12×); the spot area by +25% (1.25×); and reduce the PSF intensity by −20% (0.80×). Figure 1 presents the simple estimate of the impact of M^2 on the effective focusable spot size.

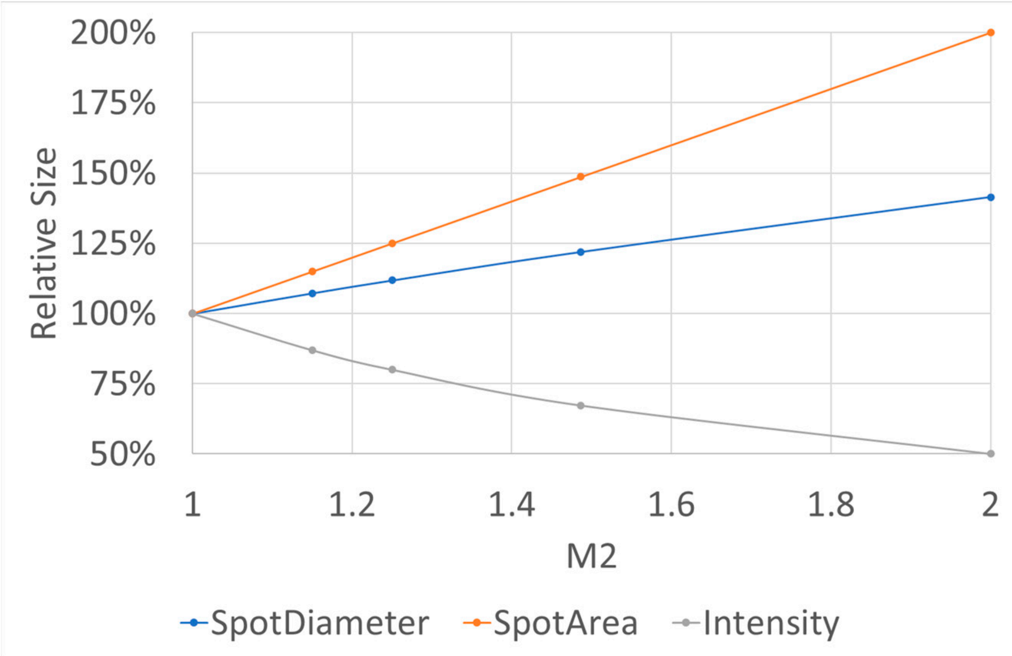

Figure 1. M^2 is a heuristic parameter of the laser beam related to the spot size in focus. M^2 has a value > 1, with a hypothetical, perfectly unaberrated beam having an M^2 of 1. M^2 is defined as $M^2 = \theta\pi\omega/\lambda$, where θ is the beam divergence, ω is the measured beam waist, and λ is the wavelength of light. In other words, $\omega_{actual1} = \omega_0 M$, where ω_{actual} is the actual beam waist and ω_0 is the theoretical minimum beam waist at focus under ideal conditions; $\theta_{actual} = \theta_0 M$, where θ_{actual} is the actual beam divergence and θ_0 is the theoretical minimum beam divergence under ideal conditions. M^2 is related to a "redistribution" of the (constant) energy in a different (wider) region (the spread). Usually, M^2 values of 1.25 or below are considered excellent (and non-rigorously as "close to the diffraction-limit"). M^2 of 1.25 would increase spot diameter by +12% (1.12×); spot area by +25% (1.25×); and reduce the PSF intensity by −20% (0.80×).

Similarly, optical systems with SR values of 0.8 or better are considered excellent (and non-rigorously as "close to the diffraction-limit") [7]. According to the results presented above, an SR of 0.8 would increase spot diameter by +12% (1.12×); the spot area by +25% (1.25×); and reduce the PSF intensity by −20% (0.80×). Figure 2 presents the simple estimate of the impact of Strehl Ratio on the effective focusable spot size.

This suggests "equivalence" for both metric values considered excellent, since the focusability impact of an M^2 of 1.25 approaches that of an SR of 0.8.

Combining a laser with M^2 values of 1.25 or below into an optical system with SR values of 0.8 or better is considered excellent (and non-rigorously as "close to the diffraction-limit"). According to the results presented above, an M^2 of 1.25 combined with an SR of the optical train of 0.8 would increase the spot diameter by +25% (1.25×); the spot area by +56% (1.56×); and reduce the PSF intensity by −36% (0.64×).

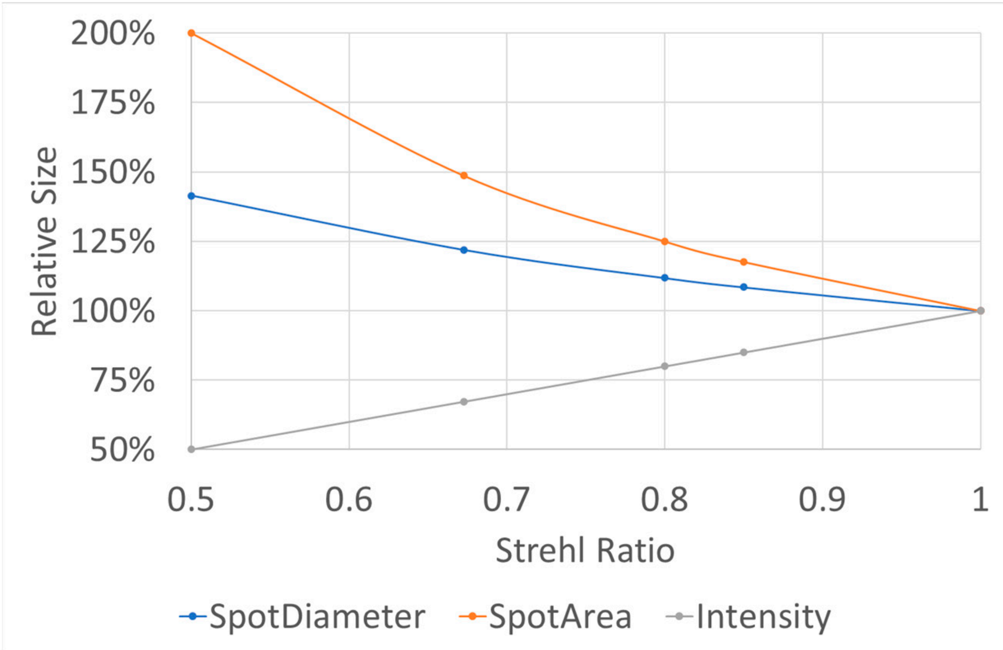

Figure 2. The SR is a measure of the quality of optical image formation. The SR has a value between 0 and 1, with a hypothetical, perfectly unaberrated optical system having a Strehl Ratio of 1. The SR is frequently defined as the ratio of the peak aberrated image intensity from a point source compared to the maximum attainable intensity using an ideal optical system limited only by diffraction over the system's aperture: SR = $max(\mathrm{PSF}_{actual})/max(\mathrm{PSF}_{diffraction\text{-}limited})$. SR is related to a "redistribution" of the (constant) energy in a different (wider) region (the spread). It follows that $\omega_{actual} = \omega_0/\sqrt{\mathrm{SR}}$, i.e., in a first-order approximation, the SR (reduction in the peak PSF relative to the diffraction limited PSF) produces an increase in the spot size inversely proportional to the square root of SR. Optical systems with SR values of 0.8 or better are considered excellent (and non-rigorously as "close to the diffraction-limit"). SR of 0.8 would increase spot diameter by +12% (1.12×); spot area by +25% (1.25×); and reduce PSF intensity by −20% (0.80×).

In a more typical engineering situation, combining a laser with M^2 values of 1.15 into an optical system with SR values of 0.85 would increase the spot diameter by +16% (1.16×); the spot area by +35% (1.35×); and reduce the PSF intensity by −26% (0.74×).

4. Discussion

In this simple work, we have provided some useful explorations of the effects of M^2 and SR on intensity. The laser intensity (close to the focus) is the driving force for many laser–tissue interaction processes including laser-induced optical breakdown (LIOB) [10] and laser-induced refractive index change (LIRIC) [11].

In a sense, this model provides an analogy between M^2, SR, and NA, since, ultimately, (for the same pulse energy) all three affect both the spot size and laser intensity. A priori, a relationship between M^2, SR, and NA does not seem intuitive. M^2 is simply a property of the laser beam, and the NA of the focusing lens is determined by the lens and input beam size. The M^2 of the laser beam affects its divergence and would affect the size of the beam going into the focusing lens; the effects may be evaluated by a full aberrated Gaussian beam analysis instead of simplified to a single-value M^2 property.

For some cases, M^2 may be "high" despite good focusing properties. A clear example would be a uniform beam (top-hat or flat-top) [13]. An exact top-hat has an angular intensity distribution that follows a sinc-squared function and thus has a $\Delta\theta$ that diverges and is not well defined. It has been theoretically found that $M^2 = N\sqrt{(\Gamma(3/N)\Gamma(2-1/N))/\Gamma(1/N)} \approx \sqrt{(N/3)}$, where the approximation is valid for a large N. A smooth top-hat can be approximated for a super-Gaussian order $N > 10$, leading to a theoretical $M^2 > 2$. This may well represent a sort of soft limit for the application of this simple model.

The effects of M^2 are subtle and not easily accounted for with precision, but the general idea at least captures (perhaps only to first order) the effects of imperfect beams. Therefore, to simplify things, the case of diffraction-limited beams (i.e., removing M^2 and the SR from all the equations) is generally considered. Most of the literature on laser–tissue interaction processes assumes an ideal scenario for both parameters ($M^2 = 1$; SR = 1). This clearly overestimates the laser intensity in focus, and is then re-normalized to empirical values using different sorts of gain factors [14]. From an engineering point of view, a "realistic ideal"/"proper optimum" would be more in the range of $M^2 < 1.25$ (~1.15); SR > 0.8 (~0.85). As shown in the results, this means a spot size +16% larger than ideal, a spot area +35% larger than ideal, and an intensity in the focus only 74% from ideal.

Interestingly, in the proposed model, the isolated effects derived from both "close to diffraction-limited" values ($M^2 < 1.25$ and SR > 0.8) are equivalent to one another (although, rigorously, this is strongly dependent on the mix of modes included in the laser beam).

The M^2, SR vs. NA analogy is based on the idea that the effects depend on the number of photons combined in the material in space and time [15]. The beam waist in the focus is proportional to M and inversely proportional to the NA. This is the reason why the parameters are considered in the same manner. It may be (and it is likely) that the provided equations are oversimplified, and the cross effects of M^2 and the SR are much more complex.

Importantly, it is debatable whether the SR and M^2 (both related to the focusing abilities) shall be combined, since it is likely that the SR may already include the effect of M^2 (and vice versa). A second limitation is the use of a first-order approximation. The presented approximation is consistent with Gaussian beam propagation (of "quasi-perfect" beams) [16]. M^2 and the SR are both convenient single-figure parameters to represent generic aberrated beam propagation and focusing. However, the actual PSF in general may not preserve the shape (in a xyz scaled version) of the diffraction-limited PSF. It may be that the assumption $1/\sqrt{\text{SR}}$ is actually "punishing" the spot size too little.

Yet, there are other approximations in optical aberrations theory which provide a reasonable estimate of the SR based simply on the root mean square (RMS) of the beam aberrations, regardless of the specific waveform of the aberration [8]. These approximations (including the presented model) eliminate the complexity of a formal beam caustic measurement for generic aberrated beams (both from the laser source and from the optical system).

Multiple different combinations of aberrated beams (from the source and from the optical system) may lead to the same M^2 and SR figures, and yet to different behaviours in focus. The presented first-order approximation, although it cannot replace a rigorous formal approach, shall provide better estimates than ignoring the effects of M^2 and the SR (in other words, assuming all beams are perfect, and all optical systems are ideal, meaning diffraction-limited focusing conditions are assumed to apply).

Probably, the proposed equations hold for "small" levels of aberrations and break down for larger magnitudes. An interesting question is "where the transition from small aberration to large aberration lies", what determines this transition point, how it depends on spatial frequency, the type of aberration, phase reversals, and so on.

The RMS to SR approximation holds valid for an SR as low as ~0.3 [8], and yet, as mentioned before, an SR > 0.8 is considered reasonably diffraction limited [7]. The proposed approximation model is not meant for a low SR (<0.5) or for a large M^2 (>2), and probably works best limited to an SR > 0.67 (~0.8^2; ~$0.5^{0.5}$) or $M^2 < 1.5$ (~1.25^2; ~$2^{0.5}$).

The relation between M^2, the SR, and beam aberrations causing a different spot size would be a study on its own. However, it made sense to assume a simple linear contribution instead of taking the beam quality as not influencing the beam size at all (confirmed in light of the relevant reductions in intensity and increases in the spot size obtained even for excellent beam and optical qualities, $M^2 < 1.25$ and SR > 0.8).

In principle, M^2 and the SR could be ignored as an approximation, and an aberrated wavefront on the focused beam should be incorporated instead, and (photochemical) models with an arbitrarily aberrated wavefront beam should be developed.

For a variety of laser–tissue interaction processes, it is not only the laser pulse intensity which comes from the focus size, but the pulse overlapping as well [17]. Thus, the focus size and the overlapping factor are both influenced by the NA as well as the beam quality (M^2 and the SR in our model).

In the limiting case for which the laser–tissue interaction process is only driven by the applied dose (in the presence of tight pulse overlapping, average energy per unit of area) [18], the effects of M^2 and the SR vanish (since the reduction in intensity is compensated by the cumulative overlap effect) and the results are identical to those obtained under the diffraction-limited assumption.

At the other end of the limiting case for which the laser–tissue interaction process is a single pulse effect (i.e., no pulse overlapping or cumulative effects play a relevant role), the effects of M^2 and the SR are maximized compared to the results obtained under the diffraction-limited assumption [10].

In between, there is a corridor of increasing/decreasing importance for the effects of M^2 and the SR. For multiphoton absorption (non-linear) processes for which both the single pulse intensity I (suprathreshold process) and pulse overlapping are involved, a formal estimate can be derived [19].

$$Effect \propto E\, I^{m-1} \frac{\omega_x}{\Delta x} \frac{\omega_y}{\Delta y} \tag{7}$$

where m is the multiphoton order, $\omega_{x/y}$ the spot size, and Δx and Δy the interspot distance. Since

$$I = \frac{I_0 \mathrm{SR}}{M_x M_y} \tag{8}$$

$$\omega_{x/y} = \frac{\omega_{0,x/y} M_{x/y}}{\sqrt{\mathrm{SR}}} \tag{9}$$

The effect can be rewritten as

$$Effect \propto E\, I^{m-1} \frac{\omega_x}{\Delta x} \frac{\omega_y}{\Delta y} = E \left(\frac{I_0 \mathrm{SR}}{M_x M_y} \right)^{m-1} \frac{\omega_{0,x} M_x}{\Delta x \sqrt{\mathrm{SR}}} \frac{\omega_{0,y} M_y}{\Delta y \sqrt{\mathrm{SR}}} \tag{10}$$

$$Effect \propto E\, {I_0}^{m-1} \frac{\omega_{0,x}}{\Delta x} \frac{\omega_{0,y}}{\Delta y} \left(\frac{\mathrm{SR}}{M_x M_y} \right)^{m-2} \tag{11}$$

$$Effect \propto Effect_0 \left(\frac{\mathrm{SR}}{M_x M_y} \right)^{m-2} \tag{12}$$

As it could be expected for two-photon processes, the reduction in the intensity due to M^2 and the SR is fully compensated by the increase in pulse overlapping. But for increasing orders of the multiphoton process, the reduction due to M^2 and the SR dominates over the increase in the pulse overlapping. For the engineering optimum described before (M^2 ~1.15 and SR ~0.85), the actual effect would be 74% of the diffraction-limited effect for a three-photon process; but further reduce to 55%, 40%, and 30% of the diffraction-limited effect for four-photon, five-photon, and six-photon processes, respectively. Figure 3 presents the simple estimate of the impact of M^2 and Strehl Ratio for different multiphoton orders on the effect of photochemical models for laser tissue interaction accounting for spot overlapping.

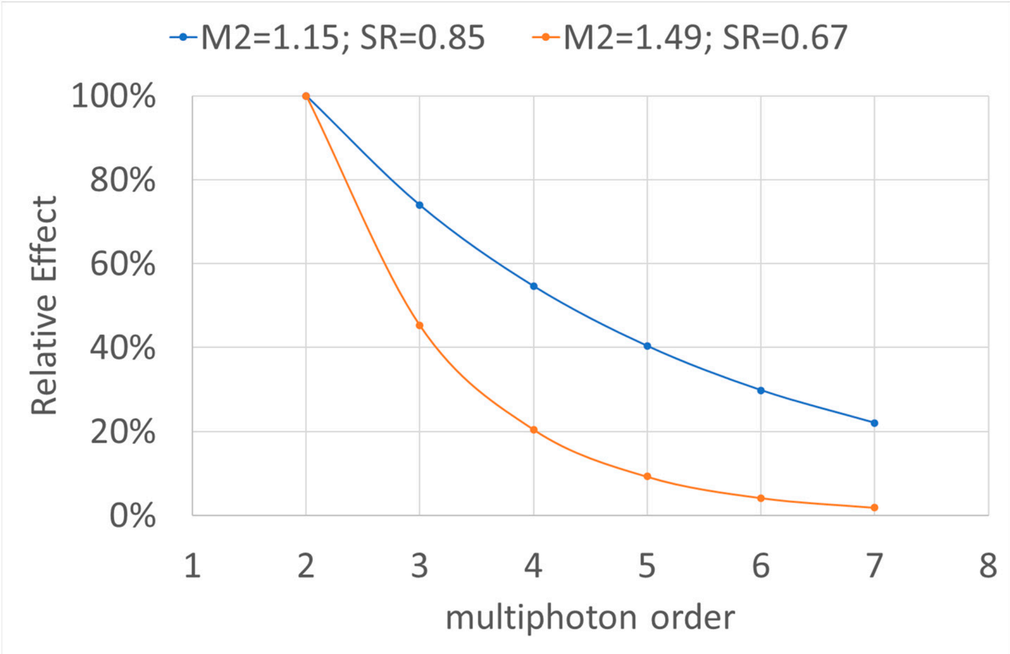

Figure 3. For 2-photon processes, the reduction in intensity due to M^2 and SR is fully compensated for by the increase in pulse overlapping. But for increasing orders of the multiphoton process, the reduction due to M^2 and SR dominates over the increase in pulse overlapping. For the engineering optimum described before (M^2 ~1.15 and SR ~0.85), the actual effect would be 74% of the diffraction-limited effect for a 3-photon process, but further reduce to 55%, 40%, and 30% of the diffraction-limited effect for 4-photon, 5-photon, and 6-photon processes, respectively.

These values hint that numerical modelling under the assumption of diffraction-limited setups may largely miss the empirical findings. However, it would be of value to establish a convenient "rule of thumb", like the one proposed in this model.

Further, this model may help one to understand differences among the units of the same design (as a plausible explanation), when clearly distinct behaviours are observed at different units which should be otherwise identical. The use of "semi-empirical" gain factors [14] may work (and seems to work, since this is the conventional approach) to describe the process for a particular model setup (provided that model setup can be reasonably replicated); yet static "semi-empirical" gain factors fail when it comes to explaining subtle differences in the behaviours for units which were identical in design, but different within the spread of the normal optical quality ranges.

Table 1 summarizes all the findings.

Table 1. Summary of all findings in relative terms.

M^2	SR	m	Spot Diameter	Spot Area	Intensity	Effect
1	1	4	100%	100%	100%	100%
1.15	1	4	107%	115%	87%	76%
1	0.85	4	108%	118%	85%	72%
1.15	0.85	4	116%	135%	74%	55%
1.49	1	4	122%	149%	67%	45%
1	0.67	4	122%	149%	67%	45%
1.15	0.85	2	116%	135%	74%	100%
1.15	0.85	6	116%	135%	74%	30%

This work emphasizes the need for a conscious and driven selection of the adequate wavelength to drive the aimed for process. In general, the shortest feasible wavelength to drive the process is advisable. This has at least two positive implications: on the one hand, the efficiency of the process will be increased (due to the lower multiphoton order), and on the other hand, the detrimental effects of M^2 and the SR may be strongly reduced.

For corneal work (the expertise domain of the authors), most intracorneal laser–tissue interactions are currently performed with ~1 μm wavelengths (representing a five-photon process). Increasing the wavelength to ~1.2–1.3 μm (representing a six-photon process) would be detrimental since the statistical likelihood of six photons reaching the effect volume will be strongly reduced (compared to five photons) and is more sensitive to optical imperfections in terms of M^2, the SR, or aberrations. On the contrary, shortening the wavelength to ~760 nm (representing a four-photon processes) would enhance the effect since the statistical likelihood of four photons reaching the effect volume will be higher (compared to five photons) and less sensitive to optical imperfections in terms of M^2, the SR, or aberrations. Further shortening the wavelength to ~570 nm (representing a three-photon process) would enhance it even further, yet it may not be clinically feasible due to the strong irradiation falling in the visible spectrum. Finally, shortening the wavelength to ~370 nm (representing a two-photon process) would "optimize" and "stabilize" the effect, since just two photons are required in the effect volume and the process may be insensitive to M^2 and the SR.

Corneal processes which may benefit from a proper selection of the wavelength include the LIOB [10] and LIRIC [11] processes. For LIOB [10], a reduction from ~1035 nm (five-photon process) down to ~345 nm (two-photon process) suggests an almost ~5-fold decrease in the LIOB threshold (but the same sensitivity to M^2 and the SR, due to the single pulse nature of the effect). For LIRIC [11], the same reduction suggests an over ~1000-fold increase (over three orders of magnitude) in efficiency (plus improved stability against M^2 and the SR, as shown in this work).

Since the NA and focusability are very important for many applications, in terms of other possible factors affecting the beam quality, i.e., the spot size, in this case, M^2 and the SR have been considered in the model. The main idea behind the model is to explain and support actual experimental data which are produced using real-life systems which do not have perfect beams.

Previous works have considered the effects of or provided refined methods to determine M^2 or the SR. For the single-pulse machining of glass with ultrashort pulses at near-infrared wavelength, a theoretical calculation and corrections based on M^2 was provided for the ablation of glass inferred from experimental results [20]. Beam quality measurements with a Shack–Hartmann wavefront sensor and M^2 sensor were compared in another work. With a Shack–Hartmann wavefront sensor, the wavefront of the beam can be easily measured and M^2 can be determined from the wavefront [21]. In another work, for spatio-temporal coupling distortion cases, the Strehl Ratios showed a trend of improvement as the F-number decreases [22]. A previous publication corrected the wavefront distortion induced during the post-compression of a 100-TW Ti:Sapphire laser. By compensating for the wavefront aberrations with an adaptive optics system, the Strehl Ratio of the post-compressed beam was improved from 0.37 to 0.52 and the focused intensity of the post-compressed beam could be enhanced by a factor of 1.5 [23].

How the NA, SR, and M^2 crosstalk in different systems is a different story, because it depends on both the M^2 of the laser source and the SR modifications along the specific optical train.

Of note, the combined use of M^2 and SR corrections is only granted if the SR is calculated for an ideal beam, and it does not intrinsically/implicitly incorporate the effects of the M^2 of the actual beam. Otherwise, the provided corrections shall only consider the SR (of the system) and ignore M^2 (since M^2 is "embedded" into the system's SR).

5. Conclusions

Adding M^2 and the SR as parameters into photochemical models to consciously deviate from the diffraction-limited assumptions could explain the empirical findings observed/determined with real-world experiences, in particular, using commercially available systems. This work provides analytical (yet simple) close-form equations to account for the M^2 and SR effects in photochemical and laser–tissue interaction models.

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