



# Article Model-Predictive-Control-Based Centralized Disturbance Suppression Strategy for Distributed Drive Electric Vehicle

Aiping Tan <sup>1</sup>, Lixiao Gao <sup>2</sup> and Yanfeng Chen <sup>3,\*</sup>

- School of Cyber Science and Engineering, Liaoning University, Shenyang 110036, China; aipingtan@lnu.edu.cn
- <sup>2</sup> School of Electrical Engineering and Automation, Harbin Institute of Technology, Harbin 150001, China; 15b906015@hit.edu.cn
- <sup>3</sup> College of Computer Science and Engineering, Northeastern University, Shenyang 110167, China
- \* Correspondence: chenyanfeng@stumail.neu.edu.cn

**Abstract:** This paper presents a centralized disturbance suppression strategy for distributed drive electric vehicles which is based on model predictive direct motion control. This strategy is capable of addressing issues such as parameter uncertainties and external disturbances in vehicles. Firstly, the paper provides a brief introduction to model predictive direct motion control. Secondly, it analyzes the impact of vehicle parameter uncertainties and external disturbances on the mathematical model. Finally, a centralized disturbance suppression strategy based on a sliding mode observer is proposed. Simulation results demonstrate that this strategy exhibits excellent disturbance rejection capabilities.

**Keywords:** distributed drive electric vehicle; in-wheel motor; model predictive control; vehicle motion control

# 1. Introduction

Distributed drive electric vehicles (DDEVs) are garnering significant attention in vehicle research owing to their numerous benefits, including rapid dynamic response, streamlined chassis structure, and adaptable control systems. Unlike centralized drive setups, DDEVs provide greater controllability with additional degrees of freedom. However, their vehicle models display nonlinear characteristics, posing challenges for effective control. As a result, optimizing driving conditions has emerged as a pivotal focus in DDEV research endeavors.

Unlike centralized drive systems, DDEVs offer more controllable degrees of freedom, and their vehicle models exhibit nonlinear characteristics. Traditional control systems for DDEVs typically adopt a series-connected structure consisting of a vehicle controller and four wheel hub motor controllers. The vehicle controller, which takes the vehicle body state as the control object, designs overall control strategies to calculate torque expectations for the four wheel hub motors. Then, the wheel hub motor controllers utilize appropriate motor control strategies to calculate the input control signals for the inverters based on these torque expectations, thereby achieving vehicle state tracking. The vehicle controller used for driving and stability design often fully utilizes the independent controllability of each wheel of distributed drive electric vehicles to achieve motion control in longitudinal, lateral, and yaw directions. Firstly, reference [1] presents a lateral dynamic control system tailored for electric vehicles, focusing on customized objectives to improve performance. Then, reference [2] introduces a coordinated torque distribution method aimed at enhancing vehicle lateral stability and ride comfort by merging balanced torque vectoring distribution and differential braking methods. Moreover, reference [3] proposes a coordinated control strategy specifically designed for DDEVs integrating adaptive model predictive control and direct yaw moment control to ensure yaw stability. Additionally, reference [4] presents a hierarchical coordinated control strategy to address challenges in maintaining speed and



Citation: Tan, A.; Gao, L.; Chen, Y. Model-Predictive-Control-Based Centralized Disturbance Suppression Strategy for Distributed Drive Electric Vehicle. *Energies* **2024**, *17*, 2268. https://doi.org/10.3390/en17102268

Academic Editor: Atriya Biswas

Received: 8 April 2024 Revised: 30 April 2024 Accepted: 6 May 2024 Published: 8 May 2024



**Copyright:** © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). desired distance within vehicle platoons, particularly under difficult conditions such as low adhesion or curves. Furthermore, reference [5] introduces a robust framework merging active front-wheel steering with direct yaw moment control to bolster maneuverability and stability in DDEVs. Subsequently, reference [6] devises a supervisory control strategy for DDEVs, amalgamating various controllers to enhance vehicle handling, lateral stability, and energy efficiency. Lastly, reference [7] introduces an innovative motion control strategy utilizing model predictive control to handle longitudinal and lateral motion concurrently in DDEVs, resulting in cost reduction and improved handling performance through more efficient control mechanisms. Due to the advantages of the model predictive direct motion control strategy presented in reference [7], such as its simple structure, high integration, and fast dynamic response, this paper chooses this control strategy as the foundational control strategy for DDEVs. Based on this strategy, the paper conducts research on centralized disturbance suppression in vehicles.

Model predictive control (MPC) offers simplicity in structure, excellent dynamic performance, and ease in handling nonlinear systems, making it highly promising for DDEVs. However, its efficacy in DDEVs hinges on the accuracy of the vehicle's mathematical model for predicting future states. Variations in inertia parameters, nonlinear tire forces, and external disturbances can introduce inaccuracies. As a result, scholars globally have explored various disturbance suppression strategies for DDEVs, including centralized compensation methods, parameter identification techniques, and robust control strategies.

Implementing MPC in DDEVs presents several challenges. Firstly, the nonlinear characteristics of DDEVs escalate the complexity of required models for effective control. Secondly, the distributed drive system introduces additional control degrees of freedom, complicating predictive control design and execution. Moreover, DDEVs' rapid dynamic response may strain computational resources as MPC optimizes across multiple future time steps, potentially impeding real-time performance. Additionally, real-time acquisition of vehicle state and environmental data poses a challenge, given their critical role in MPC control. Thus, effectively applying MPC in DDEVs remains a pertinent technical hurdle requiring resolution.

#### (1) Centralized Disturbance Compensation Methods

Firstly, some researchers aim to suppress the impact of disturbances on vehicle motion control by observing and compensating for these disturbances. In the context of lateral motion disturbance suppression, reference [8] utilizes a second-order lateral dynamics model to consolidate parameter uncertainties and external disturbances into a disturbance term. They design an extended observer to estimate this disturbance and employ a robust nonlinear control strategy based on obstacle Lyapunov functions to compensate for the observed disturbance while achieving lateral vehicle control. In reference [9], a sliding mode control strategy based on obstacle Lyapunov functions is employed to address unknown lateral tire forces, road curvature angles, and parameter uncertainties, achieving robust lateral vehicle control. Addressing disturbances caused by lateral wind, reference [10] uses a combination of feedforward and feedback control strategies to achieve lateral vehicle control. They also use a disturbance observer to mitigate the effects of disturbances caused by lateral wind. To tackle the centralized disturbance issue resulting from model errors and complex driving environments, reference [11] employs a nonlinear disturbance observer for estimation and compensation, resolving the sliding mode control oscillation problem caused by centralized disturbances. Reference [12] adopts model predictive control as a vehicle stability control strategy, utilizing a disturbance observer based on extended Kalman filtering to estimate the equivalent additional yaw moment generated by external disturbances. This estimated moment is then incorporated into the linear vehicle model for compensation, but the article only considers model errors in the yaw direction. Reference [13] uses a disturbance observer based on sliding mode theory to address uncertainties and unknown relationships between tire-road friction coefficients and vehicle slip ratio. They propose a nonlinear sliding mode surface. Reference [14] employs an extended state observer to provide real-time estimation and compensation for unmodeled dynamics and unknown external disturbances.

(2) Parameter Identification Methods

Parameter identification methods are the primary approach for predicting and suppressing certain external disturbances in distributed drive electric vehicles, such as tire load, tire stiffness, vehicle mass, and center of gravity position. In reference [15], intelligent tires with built-in sensors and vehicle states are employed to rapidly and stably estimate tire load and vehicle parameters (total mass, center of gravity position). Reference [16] uses nonlinear particle filtering to estimate vehicle mass. Reference [17] employs the method of least squares with dual forgetting factors and an extended Kalman particle filter to perform real-time estimation of vehicle mass and road incline. Reference [18] utilizes a dual Kalman filter for real-time estimation of vehicle inertia parameters. In reference [19], a sliding mode observer is designed to predict longitudinal and lateral tire forces separately. Reference [20] adopts a cascaded observer approach to predict longitudinal and lateral tire forces. Reference [21] utilizes existing vehicle sensors and a dual Kalman filter to design a state parameter observer, enabling real-time estimation of vehicle mass. The article also discusses the impact of mass changes on the vehicle. Reference [22] employs Bayesian methods to predict tire stiffness.

# (3) Disturbance-Resistant Robust Control Strategies

In addition to the aforementioned methods, some scholars have focused on enhancing the disturbance resistance capabilities of the strategies themselves. Reference [23] proposes a robust controller with adaptive gain scheduling for stability control incorporating linear parameter-varying systems and H∞-optimized control. This controller utilizes linear matrix inequalities to suppress system uncertainties and sensor noise. Reference [24] designs a robust loop-shaping method and engineers weight functions according to engineering specifications. Through this control structure, the strategy can overcome issues related to model uncertainty, lateral wind disturbances, and parameter variations. Reference [25] establishes a time-varying vehicle model considering changes in the center of gravity position and uses sliding mode control to achieve lateral stability control. Reference [26] develops a vehicle lateral dynamics model incorporating combined slip tire forces considering tire force nonlinearity and combined slip friction effects. The authors use this model to design a model predictive controller that maintains the lateral response of the vehicle within a constrained stable region. To address uncertainties in parameters such as vehicle mass and longitudinal velocity, reference [27] employs a polytope uncertainty approach to construct a linear parameter-varying lateral model. This leads to the design of an H $\infty$ feedback control algorithm based on linear matrix inequalities. Given the presence of many uncertain disturbances during vehicle operation, expressed as a random additional yaw moment, reference [28] designs a state feedback elastic controller to ensure that yaw angle and yaw angular velocity meet performance criteria for  $H\infty$  and  $L2-L\infty$ . Reference [29] uses global sliding mode control theory to derive the discontinuous part of the control law, addressing issues related to changing vehicle parameters and model uncertainty. In response to external disturbances caused by lateral wind, reference [30] designs a disturbance suppression strategy based on Lyapunov theory, restricting control objectives to a reasonable range. The article presents a complex model, capturing the effects of lateral wind, with disturbance terms in both the yaw and lateral directions. Reference [31] uses a nonlinear model predictive control strategy to achieve stable control of vehicle lateral dynamics. To enhance controller performance, the authors propose a high-order neural network modeling method, enabling iterative optimization of the vehicle's yaw angular velocity and steady-state response of the center of gravity lateral deviation.

DDEVs are electric vehicles where each wheel is powered independently by its own motor. This design offers several advantages, including improved traction, enhanced maneuverability, and more efficient energy usage. Here are some examples of DDEVs and their preferred fields of use: In reference [32], Rivian's electric pickup truck (R1T) and

SUV (R1S) are equipped with four electric motors, one for each wheel. These vehicles are designed for off-road adventures and outdoor enthusiasts who require ruggedness, versatility, and all-terrain capabilities. In reference [33], the Lordstown Endurance is an allelectric pickup truck designed for commercial fleets. It features four in-wheel hub motors, providing power to each wheel individually. This configuration enhances traction and stability, making it suitable for utility companies, delivery services, and other fleet operators. These examples demonstrate how DDEVs are utilized across various sectors, including off-road exploration, high-performance driving, commercial fleet operations, and luxury transportation. The distributed drive system offers benefits such as improved control, efficiency, and performance, making DDEVs versatile options for a range of applications.

Scholars have conducted in-depth research on electric vehicles. In reference [34], a bilayer coordinated operation strategy is introduced for a multi-energy building microgrid (MEBM) to manage various uncertainties. This strategy involves a two-layer approach: first, employing a stochastic-weighted robust optimization-based day-ahead operation to dispatch energy storage assets and demand response followed by a second layer determining the operation of power-to-thermal conversion units, CCHP plants, and electricity transactions with the utility grid considering uncertainty realizations. In reference [35], researchers address challenges in vehicle-to-grid (V2G) technology, aiming to improve load peak shaving, valley filling, and PV self-consumption. They introduce FedPT-V2G, a federated transformer learning method, to handle data distribution disparities among charging stations, facilitating real-time decision-making while preserving local privacy. This approach integrates deep learning models trained on past and present data, incorporating the Proximal algorithm and Transformer model to synchronize local models and leverage feature diversity, enhancing V2G scheduling predictions. Unlike these existing studies, the paper focuses on the control problem under the complex disturbances of distributed drive electric vehicles and proposes a centralized disturbance suppression strategy for disturbance mitigation.

In summary, vehicles experience the influence of multiple disturbance factors during operation. In the case of distributed drive electric vehicles, the research on disturbance suppression methods is more flexible and complex due to their multiple power sources. Implementing multiple-disturbance suppression through a multi-motor drive system for distributed drive electric vehicles is expected to significantly enhance the vehicles' maneuvering performance under complex operating conditions.

# 2. Model Predictive Direct Motion Control

The mathematical model of a distributed drive electric vehicle is primarily divided into two parts: the vehicle mathematical model and the wheel hub motor mathematical model. The vehicle mathematics is depicted in Figure 1, and its dynamic equation is as follows:

$$m\frac{dv_x}{dt} = mv_y r + \cos\delta F_{xfl} + \cos\delta F_{xfr} + F_{xrl} + F_{xrr} - \sin\delta\left(F_{yfl} + F_{yfr}\right) \tag{1}$$

$$m\frac{dv_y}{dt} = -mv_x r + \sin\delta F_{xfl} + \sin\delta F_{xfr} + \cos\delta \left(F_{yfl} + F_{yfr}\right) + F_{yrl} + F_{yrr}$$
(2)

$$J_{z}\frac{dr}{dt} = \left(-d_{fl}\cos\delta + l_{f}\sin\delta\right)F_{xfl} + \left(d_{fr}\cos\delta + l_{f}\sin\delta\right)F_{xfr} - d_{rl}F_{xrl} + d_{rr}F_{xrr} + l_{f}\cos\delta\left(F_{yfl} + F_{yfr}\right) + d_{fl}\sin\delta F_{yfl} - d_{fr}\sin\delta F_{yfr} - l_{r}\left(F_{yrl} + F_{yrr}\right)$$
(3)

In the equations,  $v_x$ ,  $v_y$ , and r represent the longitudinal velocity, lateral velocity, and yaw angular velocity of the vehicle, respectively.  $F_{xfl}$ ,  $F_{xfr}$ ,  $F_{xrl}$ , and  $F_{xrr}$  denote the longitudinal forces experienced by the front left, front right, rear left, and rear right wheels, respectively.  $F_{yfl}$ ,  $F_{yfr}$ ,  $F_{yrl}$ , and  $F_{yrr}$  represent the lateral forces on the front left, front right, rear left, and rear right wheels, respectively.  $\sigma$  is the steering angle of the front wheels, mis the total vehicle mass,  $l_f$  and  $l_r$  represent the longitudinal distances from the vehicle's center of gravity to the front and rear axles, respectively.  $I_z$  is the vehicle's rotational inertia,  $d_{fl} + d_{fr}$  and  $d_{rl} + d_{rr}$  are the front and rear axle distances, respectively. Additionally, the dynamic equation for the wheels is as follows:

$$\frac{d\omega_{eij}}{dt} = \frac{T_{eij}}{J_w} - \frac{R}{J_w} F_{xij}$$
(4)

In the equations, i = f, r represents the front or rear wheels, j = l, r represents the left or right wheels,  $T_{eii}$  is the electromagnetic torque of the wheel hub motor,  $J_{\omega}$  is the rotational inertia of the wheel, *R* is the wheel radius, and  $\omega_{eij}$  is the wheel rotational speed.



(a) Vehicle dynamics model.

Figure 1. Models.

The proposed model predictive direct motion control strategy can directly obtain the optimal voltage vector combination applied to the wheel hub motors within a finite time. As illustrated in Figure 2, the model predictive control used in this paper for DDEVs can efficiently determine the optimal combination of voltage vectors applied to the motors within a finite time frame. Essentially, the control strategy operates by considering the discrete nature of motor inverters, resulting in a finite set of potential voltage vector combinations for the four motor systems. By utilizing a unified mathematical model, predicted values for a finite number of future vehicle states are obtained, followed by the design of an evaluation function to assess these predictions while considering constraints. The selected voltage vector combination that minimizes the evaluation function is then outputted to the motor inverter, and this process is iteratively repeated for rolling optimization of the entire system. The control objectives of the four motors are unified to represent core variables of the vehicle state, encompassing longitudinal velocity, centroid lateral deviation angle, and yaw angular velocity. By aligning motor control objectives with these core state variables, a cohesive description of vehicle motion is achieved, thereby preventing inconsistencies in objectives among the motors. This unified design ensures that all four motors work towards the same overarching objective throughout the control process, thereby improving system stability and coordination.

To achieve precise and stable operation of a DDEV, the vehicle control objectives are primarily divided into two aspects: maintaining longitudinal speed for vehicle movement along the longitudinal axis and controlling yaw rate and sideslip angle to ensure vehicle stability. The calculation of specific setpoints can be found in reference [7].



Figure 2. Model predictive direct motion control strategy.

#### 3. Vehicle Disturbance Analysis

The cooperative model predictive control (MPC) strategy for a multi-motor vehicle system utilizes the dynamic model of the vehicle and the mathematical model of the permanent-magnet synchronous motor (PMSM) to predict and control the states of both the vehicle and the motors. The accuracy of the mathematical models for both the vehicle and the motors is crucial for the control performance of this strategy. The design and effectiveness of the control system directly depend on the accuracy of the models. If there are errors or uncertainties in the mathematical models, it will directly impact the performance and stability of the control strategy. However, vehicles face even more complex and variable disturbance factors during actual operation, which are challenging to precisely estimate. Examples include variations in vehicle parameters, external disturbances, and the nonlinear characteristics of the tires. Therefore, for the composite disturbances that a vehicle may encounter during operation, adopting traditional methods of precise modeling and estimation may be impractical or not accurate enough. This paper establishes a vehicle dynamics model that incorporates disturbance factors. To effectively address these disturbances, the paper represents them as a composite disturbance term in the dynamic model and designs a sliding mode observer to estimate them.

In the motion of distributed drive electric vehicles, disturbance factors mainly include variations in vehicle parameters, external disturbances, and the nonlinear characteristics of the tire model, as illustrated in Figure 3.

During the operation of the vehicle, it may encounter different operating conditions, such as changes in the load of goods or the number of passengers. These factors can cause variations in the vehicle's center of gravity position and total mass, leading to changes in vehicle parameters. In Figure 3, the center of gravity position changes from  $CG_{old}$  to  $CG_{new}$ , resulting in errors  $\Delta$ ,  $\Delta l_f$ ,  $\Delta d_{lf}$ ,  $\Delta d_{rl}$ ,  $\alpha d_{rl}$  and  $\Delta d_{rr}$  in the distances from the center of gravity to the front and rear axles and the front and rear wheelbase. The vehicle's total mass error is  $\Delta m$ . Additionally, the vehicle may face external disturbances, w, such as crosswinds or lateral forces on the center of gravity, which cause the vehicle to deviate from the intended trajectory. Furthermore, the mechanical characteristics of the tire pose a complex nonlinear problem. Under different conditions, the tire's mechanical properties can change, leading to variations in the vehicle's longitudinal forces, i.e.,  $\Delta F_{xfl}$ ,  $\Delta F_{xrr}$ ,  $\Delta F_{xrrl}$ , and  $\Delta F_{xrrr}$ . These changes in parameters introduce uncertainty into the mathematical model of the vehicle, affecting the accuracy of the control system.



Figure 3. Vehicle dynamics model with error.

Therefore, incorporating these disturbance errors into Equations (5)–(7) yields the actual mathematical model of the vehicle dynamics as follows:

$$v_{x}(k+1) = v_{x}(k) + T_{s}v_{y}(k)r(k) + \frac{T_{s}\cos\delta}{m+\Delta m} \left(F_{xfl}(k) + \Delta F_{xfl}(k)\right) + \frac{T_{s}\cos\delta}{m+\Delta m} \left(F_{xfr}(k) + \Delta F_{xfr}(k)\right) + \frac{T_{s}}{m+\Delta m} \left(F_{xrl}(k) + \Delta F_{xrl}(k)\right)$$
(5)  
$$+ \frac{T_{s}}{m+\Delta m} \left(F_{xrr}(k) + \Delta F_{xrr}(k)\right) - \frac{T_{s}\sin\delta}{m+\Delta m} \left(F_{yfl} + F_{yfr}\right) v_{y}(k+1) = v_{y}(k) - T_{s}v_{x}(k)r(k) + \frac{T_{s}\sin\delta}{m+\Delta m} \left(F_{xfl}(k) + \Delta F_{xfl}(k)\right) + \frac{T_{s}}{m+\Delta m} \left(\cos\delta(F_{yfl} + F_{yfr}) + F_{yrl} + F_{yrr}\right)$$
(6)  
$$+ \frac{T_{s}\sin\delta}{m+\Delta m} \left(F_{xfr}(k) + \Delta F_{xfr}(k)\right)$$

From the above description, it can be concluded that the disturbance factors affecting the vehicle are highly complex, including variations in vehicle parameters, external disturbances, and the nonlinearities of the tire model. Unlike motor parameter errors, it is challenging to individually identify and compensate for these vehicle disturbance factors. Therefore, to effectively address the disturbance errors in the vehicle, all disturbance factors are comprehensively considered and consolidated into a composite disturbance term. By aggregating all disturbance errors into a composite disturbance term, it becomes more efficient to observe and compensate for the disturbance errors in the vehicle. This integrated approach is better equipped to handle various vehicle disturbance factors, thereby enhancing the robustness of the control system against complex disturbances.

$$r(k+1) = r(k) + \frac{-(d_{fl} + \Delta d_{fl}) \cos \delta}{J_z + \Delta J_z} (F_{xfl}(k) + \Delta F_{xfl}(k)) T_s$$

$$+ \frac{(l_f + \Delta l_f) \sin \delta}{J_z + \Delta J_z} (F_{xfl}(k) + \Delta F_{xfl}(k)) T_s$$

$$+ \frac{(d_{fr} + \Delta d_{fr}) \cos \delta}{J_z + \Delta J_z} (F_{xfr}(k) + \Delta F_{xfr}(k)) T_s$$

$$+ \frac{(l_r + \Delta l_r) \sin \delta}{J_z + \Delta J_z} (F_{xrl}(k) + \Delta F_{xrl}(k)) T_s$$

$$- \frac{d_{rl} + \Delta d_{rl}}{J_z + \Delta J_z} (F_{xrrl}(k) + \Delta F_{xrrl}(k)) T_s$$

$$+ \frac{d_{rr} + \Delta d_{rl}}{J_z + \Delta J_z} (F_{xrr}(k) + \Delta F_{xrr}(k)) T_s$$

$$+ \frac{l_f + \Delta l_f}{J_z + \Delta J_z} (F_{xrr}(k) + \Delta F_{xrr}(k)) T_s$$

$$+ \frac{l_f + \Delta l_f}{J_z + \Delta J_z} T_s (F_{yfl} + F_{yfr}) \cos \delta + \frac{d_{fl} + \Delta d_{fl}}{J_z + \Delta J_z} T_s F_{yfl} \sin \delta$$

$$- \frac{d_{fr} + \Delta d_{fr}}{J_z + \Delta J_z} T_s F_{yfr} \sin \delta - \frac{l_r + \Delta l_r}{J_z + \Delta J_z} T_s (F_{yrl} + F_{yrr}) + \frac{T_s}{J_z + \Delta J_z} w(k)$$

In the centralized disturbance suppression strategy designed in this paper, only centralized disturbances in three directions exist. By employing a sliding mode observer to observe and compensate for these three centralized disturbances, different disturbances to the vehicle can be suppressed. The nonlinearity of the tire model is only a small part of these three centralized disturbances and can indeed be ignored.

# 4. Observation and Suppression of Composite Disturbances Based on Sliding Mode Observer

The composite disturbance suppression strategy based on the sliding mode observer is employed to address the impact of parameter variations, external disturbances, and nonlinearities in the tire model on the operation of distributed drive electric vehicles. This strategy involves designing a sliding mode observer to centrally observe the composite disturbance errors in the vehicle and achieve their suppression through online compensation. The goal is to enhance the stability and control performance of the vehicle.

By consolidating the disturbance errors caused by parameter variations, external disturbances, and the nonlinearity of the tire model into a composite disturbance term, the actual dynamics model of the vehicle can be rewritten as:

In the equation,  $f_{vx}$ ,  $f_{vy}$ , and  $f_r$  represent the concentrated disturbances caused by parameter variations, external disturbances, and the nonlinearity of the tire model. Additionally,  $F_{vx}$ ,  $F_{vy}$ , and  $F_r$  denote the disturbance rates of  $f_{vx}$ ,  $f_{vy}$ , and  $f_r$ , respectively. The composite disturbance terms  $f_{vx}$ ,  $f_{vy}$ , and  $f_r$  encapsulate the disturbance errors caused by parameter variations, external disturbances, and the nonlinearity of the tire model. Their specific expressions are as follows:

$$f_{vx}(k) = \frac{\Delta m}{T_s} (v_x(k+1) - v_x(k)) - \Delta m v_y(k) r(k) - \cos \delta \left( \Delta F_{xfl}(k) + \Delta F_{xfr}(k) \right) - \Delta F_{xrl}(k) + \Delta F_{xrr}(k)$$
(10)

$$f_{vy}(k) = \frac{\Delta m}{T_s} \left( v_y(k+1) - v_y(k) \right) + \Delta m v_x(k) r(k) - \sin \delta \left( \Delta F_{xfl}(k) + \Delta F_{xfr}(k) \right)$$
(11)

$$f_{r}(k) = \frac{\Delta J_{z}}{T_{s}} (r(k+1) - r(k)) + \left( \Delta d_{fl} F_{xfl}(k) + d_{fl} \Delta F_{xfl}(k) + \Delta d_{fl} \Delta F_{xfl}(k) \right) \cos \delta$$

$$- \left( \Delta l_{f} F_{xfl}(k) + l_{f} \Delta F_{xfl}(k) + \Delta l_{f} \Delta F_{xfl}(k) \right) \sin \delta$$

$$- \left( \Delta d_{fr} F_{xfr}(k) + d_{fr} \Delta F_{xfr}(k) + \Delta d_{fr} \Delta F_{xfr}(k) \right) \cos \delta$$

$$- \left( \Delta l_{f} F_{xfr}(k) + l_{f} \Delta F_{xfr}(k) + \Delta l_{f} \Delta F_{xfr}(k) \right) \sin \delta$$

$$+ \left( \Delta d_{rl} F_{xrl}(k) + d_{rl} \Delta F_{xrl}(k) + \Delta d_{rl} \Delta F_{xrl}(k) \right)$$

$$- \left( \Delta d_{rr} F_{xrr}(k) + d_{rr} \Delta F_{xrr}(k) + \Delta d_{rr} \Delta F_{xrr}(k) \right) - \Delta l_{f} \left( F_{yfl} + F_{yfr} \right) \cos \delta$$

$$- \Delta d_{fl} F_{yfl} \sin \delta - \Delta d_{fr} F_{yfr} \sin \delta + \Delta l_{r} \left( F_{yrl} + F_{yrr} \right) + w(k)$$

$$(12)$$

To design the corresponding sliding mode observers, consider the following:

$$\begin{cases} \frac{m}{T_s} (\vartheta_x(k+1) - \vartheta_x(k)) = mv_y(k)r(k) + \cos \delta(F_{xfl}(k) + F_{xfr}(k)) - \hat{f}_{vx}(k) \\ + F_{xrl}(k) + F_{xrr}(k) - \sin \delta(F_{yfl} + F_{yfr}) - U_{vxsmo} \\ \frac{m}{T_s} (\vartheta_y(k+1) - \vartheta_y(k)) = -mv_x(k)r(k) + \sin \delta(F_{xfl}(k) + F_{xfr}(k)) \\ + \cos \delta(F_{yfl} + F_{yfr}) + F_{yrl} + F_{yrr} - \hat{f}_{vy}(k) - U_{vysmo} \\ \frac{J_z}{T_s} (\hat{r}(k+1) - \hat{r}(k)) = (-d_{fl} \cos \delta + l_f \sin \delta)F_{xfl}(k) \\ + (d_{fr} \cos \delta + l_f \sin \delta)F_{xfr}(k) \\ - d_{rl}F_{xrl}(k) + d_{rr}F_{xrr}(k) + l_f(F_{yfl} + F_{yfr}) \cos \delta \\ + d_{fl}F_{yfl} \sin \delta - d_{fr}F_{yfr} \sin \delta \\ - l_r(F_{yrl} + F_{yrr}) - \hat{f}_r(k) - U_{rsmo} \end{cases}$$

$$\begin{cases} \frac{\hat{f}_{vx}(k+1) - \hat{f}_{vx}(k)}{T_s} = \lambda_{vy}U_{vysmo} \\ \frac{\hat{f}_{vy}(k+1) - \hat{f}_{vx}(k)}{T_s} = \lambda_{vy}U_{vysmo} \\ \frac{\hat{f}_{r}(k+1) - \hat{f}_{r}(k)}{T_s} = \lambda_{r}U_{rsmo} \end{cases}$$
(14)

In the equation,  $\hat{v}_x$ ,  $\hat{v}_y$ , and  $\hat{r}$  are the predicted values of the longitudinal velocity, lateral velocity, and yaw rate, respectively. Similarly,  $\hat{f}_{vx}$ ,  $\hat{f}_{vy}$ , and  $\hat{f}_r$  are the predicted values of the rates of change for  $f_{vx}$ ,  $f_{vy}$ , and  $f_r$ . The parameters  $\lambda_{vx}$ ,  $\lambda_{vy}$ , and  $\lambda_r$  are sliding mode parameters, and  $U_{vxsmo}$ ,  $U_{vysmo}$ , and  $U_{rsmo}$  are sliding mode functions.

In this paper, the estimation errors of longitudinal velocity, lateral velocity, and yaw rate are chosen as the linear sliding mode surface as follows:

$$\begin{cases} s_{vx} = \hat{v}_x(k) - v_x(k) \\ s_{vy} = \hat{v}_y(k) - v_y(k) \\ s_r = \hat{r}(k) - r(k) \end{cases}$$
(15)

To suppress the chattering issue, the following sliding mode reaching law is chosen, where  $\beta$  is the sliding mode reaching law parameter:

$$\frac{ds}{dt} = -\beta |s| sign(s) \tag{16}$$

Reference [36] examines the observation effects of different sliding mode surfaces, which could be referenced in future research to further optimize the sliding mode observer proposed in this paper.

The discrete equation for the sliding mode reaching law is:

$$\begin{cases} \frac{s_{vx}(k+1) - s_{vx}(k)}{T_s} = -\beta_{vx}|s_{vx}(k)|sign(s_{vx}(k)) \\ \frac{s_{vy}(k+1) - s_{vy}(k)}{T_s} = -\beta_{vy}|s_{vy}(k)|sign(s_{vy}(k)) \\ \frac{s_r(k+1) - s_r(k)}{T_s} = -\beta_r|s_r(k)|sign(s_r(k)) \end{cases}$$
(17)

The error equation for the sliding mode observer is:

$$\begin{cases} \frac{s_{vx}(k+1) - s_{vx}(k)}{T_s} = -\frac{1}{m} e_{fvx}(k) - \frac{1}{m} U_{vxsmo} \\ \frac{e_{fvx}(k+1) - e_{fvx}(k)}{T_s} = \lambda_{vx} U_{vxsmo} - F_{vx} \\ \frac{s_{vy}(k+1) - s_{vy}(k)}{T_s} = -\frac{1}{m} e_{fvy}(k) - \frac{1}{m} U_{vysmo} \\ \frac{e_{fvy}(k+1) - e_{fvy}(k)}{T_s} = \lambda_{vy} U_{vysmo} - F_{vy} \\ \frac{s_r(k+1) - s_r(k)}{T_s} = -\frac{1}{J_z} e_{fr}(k) - \frac{1}{J_z} U_{rsmo} \\ \frac{e_{fr}(k+1) - e_{fr}(k)}{T_s} = \lambda_r U_{rsmo} - F_r \end{cases}$$
(18)

In the equations,  $e_{fvx}$  is the predicted error for longitudinal disturbance,  $e_{fvy}$  is the predicted error for lateral disturbance, and  $e_{fr}$  is the predicted error for yaw disturbance. Thus, the sliding mode control functions are given by:

$$\begin{cases}
U_{vxsmo} = m\beta_{vx}|s_{vx}(k)|sign(s_{vx}(k)) \\
U_{vysmo} = m\beta_{vy}|s_{vy}(k)|sign(s_{vy}(k)) \\
U_{rsmo} = J_z\beta_r|s_r(k)|sign(s_r(k))
\end{cases}$$
(19)

Finally, the sliding mode observer can be written in the following discrete form:

$$\begin{cases} \hat{v}_{x}(k+1) = \hat{v}_{x}(k) + T_{s}v_{y}(k)r(k) + \frac{T_{s}}{m}\cos\delta(F_{xfl}(k) + F_{xfr}(k)) \\ + \frac{T_{s}}{m}(F_{xrl}(k) + F_{xrr}(k)) \\ + \frac{T_{s}}{m}\left(-\sin\delta(F_{yfl} + F_{yfr}) - \hat{f}_{vx}(k) - U_{vxsmo}\right) \\ \hat{v}_{y}(k+1) = \hat{v}_{y}(k) - T_{s}v_{x}(k)r(k) + \frac{T_{s}}{m}\sin\delta(F_{xfl}(k) + F_{xfr}(k)) \\ + \frac{T_{s}}{m}(\cos\delta(F_{yfl} + F_{yfr}) + F_{yrl} + F_{yrr}) \\ + \frac{T_{s}}{m}\left(-\hat{f}_{vy}(k) - U_{vysmo}\right) \\ \hat{r}(k+1) = \hat{r}(k) + \frac{T_{s}}{J_{z}}\left(-d_{fl}\cos\delta + l_{f}\sin\delta\right)F_{xfl}(k) \\ + \frac{T_{s}}{J_{z}}\left(d_{fr}\cos\delta + l_{f}\sin\delta\right)F_{xfr}(k) \\ + \frac{T_{s}}{J_{z}}\left(-d_{rl}F_{xrl}(k) + d_{rr}F_{xrr}(k)\right) \\ + \frac{T_{s}}{J_{z}}\left(d_{fl}F_{yfl} + F_{yfr}\right)\cos\delta \\ + \frac{T_{s}}{J_{z}}\left(d_{fl}F_{yfl}\sin\delta - d_{fr}F_{yfr}\sin\delta\right) \\ + \frac{T_{s}}{J_{z}}\left(-l_{r}(F_{yrl} + F_{yrr}) - \hat{f}_{r}(k) - U_{rsmo}\right) \end{cases}$$
(20)

$$\begin{cases} \hat{f}_{vx}(k+1) = \hat{f}_{vx}(k) + T_s \lambda_{vx} U_{vxsmo} \\ \hat{f}_{vy}(k+1) = \hat{f}_{vy}(k) + T_s \lambda_{vy} U_{vysmo} \\ \hat{f}_r(k+1) = \hat{f}_r(k) + T_s \lambda_r U_{rsmo} \end{cases}$$
(21)

The block diagram of the proposed FPLO is shown in Figure 4.

To ensure the stability of the sliding mode observer, it is necessary to reasonably select the sliding mode parameters for the longitudinal, lateral, and yaw directions. According to the sliding mode stability condition, the derivative of the Lyapunov function  $V = 1/2s^2$  needs to be negative, i.e.,  $\dot{V} = sds/dt < 0$ . Therefore, the following inequality must be satisfied:

$$s_{vx} \frac{ds_{vx}}{dt} = s_{vx}(k) \left( -\frac{1}{m} e_{fvx}(k) - \frac{1}{m} U_{vxsmo} \right) \\ = s_{vx}(k) \left( -\frac{1}{m} e_{fvx}(k) - \beta_{vx} |s_{vx}(k)| sign(s_{vx}(k)) \right) < 0 \\ = \begin{cases} s_{vx}(k) \left( -\frac{1}{m} e_{fvx}(k) - \beta_{vx} |s_{vx}(k)| \right) < 0(s_{vx}(k) > 0) \\ s_{vx}(k) \left( -\frac{1}{m} e_{fvx}(k) + \beta_{vx} |s_{vx}(k)| \right) < 0(s_{vx}(k) < 0) \end{cases}$$

$$s_{vy} \frac{ds_{vy}}{dt} = s_{vy}(k) \left( -\frac{1}{m} e_{fvy}(k) - \frac{1}{m} U_{vysmo} \right) \\ = s_{vy}(k) \left( -\frac{1}{m} e_{fvy}(k) - \beta_{vy} |s_{vy}(k)| sign(s_{vy}(k)) \right) < 0 \\ s_{vy}(k) \left( -\frac{1}{m} e_{fvy}(k) - \beta_{vy} |s_{vy}(k)| \right) < 0(s_{vy}(k) > 0) \\ s_{vy}(k) \left( -\frac{1}{m} e_{fvy}(k) + \beta_{vy} |s_{vy}(k)| \right) < 0(s_{vy}(k) < 0) \end{cases}$$

$$s_{r} \frac{ds_{r}}{dt} = s_{r}(k) \left( -\frac{1}{J_{z}} e_{fr}(k) - \frac{1}{J_{z}} U_{rsmo} \right) \\ = s_{r}(k) \left( -\frac{1}{J_{z}} e_{fr}(k) - \beta_{r} |s_{r}(k)| sign(s_{r}(k)) \right) < 0 \\ s_{r}(k) \left( -\frac{1}{J_{z}} e_{fr}(k) - \beta_{r} |s_{r}(k)| \right) < 0(s_{r}(k) > 0) \\ s_{r}(k) \left( -\frac{1}{J_{z}} e_{fr}(k) + \beta_{r} |s_{r}(k)| \right) < 0(s_{r}(k) < 0) \end{cases}$$

$$(24)$$

Therefore, the admissible ranges for the reaching law parameters  $\beta_{vx}$ ,  $\beta_{vy}$ , and  $\beta_r$  are:

$$\beta_{vx} > \left| \frac{e_{fvx}}{ms_{vx}} \right|$$

$$\beta_{vy} > \left| \frac{e_{fvy}}{ms_{vy}} \right|$$

$$\beta_r > \left| \frac{e_{fr}}{J_z s_r} \right|$$
(25)

When the sliding mode observer converges, the sliding mode surface and its derivative satisfy s = ds/dt = 0. Equation (17) can be simplified to:

$$\begin{cases} \frac{e_{fvx}(k+1) - e_{fvx}(k)}{T_s} + \lambda_{vx}e_{fvx}(k) + F_{vx} = 0\\ \frac{e_{fvy}(k+1) - e_{fvy}(k)}{T_s} + \lambda_{vy}e_{fvy}(k) + F_{vy} = 0\\ \frac{e_{fr}(k+1) - e_{fr}(k)}{T_s} + \lambda_r e_{fr}(k) + F_r = 0 \end{cases}$$
(26)

The solutions for the predicted errors of the compound disturbances in the longitudinal, lateral, and yaw directions are as follows: Here,  $K_{vx}$ ,  $K_{vy}$ , and  $K_r$  are constants. According to the above equations, the sliding mode parameters  $\lambda_{vx}$ ,  $\lambda_{vy}$ , and  $\lambda_r$  need to be positive to ensure that the disturbance prediction errors  $e_{fvx}$ ,  $e_{fvy}$ , and  $e_{fr}$  converge to zero.



(a) Sliding mode observer of  $v_x$ .

(b) Sliding mode observer of  $v_y$ .

$$\begin{array}{c} r(k) & - & & \\ &$$

(c) Sliding mode observer of r.

Figure 4. Block diagram of sliding mode observer.

# 5. Simulation Verification

To validate the composite disturbance suppression strategy for the vehicle's multimotor system, the MCMPC with disturbance compensation is tested in response to disturbances such as changes in vehicle parameters, external perturbations, and nonlinearities in the tire model. In the simulation, the standard D-series SUV from CarSim is chosen as the control object, as shown in Figure 5. The key parameters for the vehicle and hub motors are listed in Table 1. The algorithm validation platform consists of MATLAB and CarSim. MATLAB is primarily used for algorithm implementation (including MPDMC and the disturbance compensation proposed in this paper) and motor simulation. The high-fidelity CarSim is mainly used for vehicle simulation and scenario simulation. The combined simulation of MATLAB and CarSim enables effective validation of the algorithms proposed in this paper.





Figure 5. Block diagram of MATLAB and CarSim co-simulation.

Table 1. Main parameters for vehicle model.

Symbol	Description	Value
т	Vehicle total mass	1430 (kg)
$l_f$	Center of gravity to front axle distance	1.05 (m)
ĺr	Center of gravity to rear axle distance	1.61 (m)
$d_f$	Distance between front left and right wheels	1.565 (m)
$d_r$	Distance between rear left and right wheels	1.565 (m)
R	Tire radius	0.364 (m)
$J_z$	Yaw moment of inertia of vehicle	2059 (kg⋅m <sup>2</sup> )
Jω	Rotational moment of inertia of each wheel	0.9 (kg⋅m²)
$C_{f}$	Equivalent nominal front-tie cornering stiffness	79,240 (N/rad)
Čr	Equivalent nominal rear-tie cornering stiffness	87,002 (N/rad)
L	d- and q-axis inductances	0.000124 (H)
$R_s$	Stator resistance	0.0286 (Ω)
Ψ	Flux linkage	0.164 (Wb)
р	Pole pairs	4

# 5.1. Pure Acceleration Scenario

The results shown in Figure 6 indicate that the impact on longitudinal velocity is small, mainly due to the high weighting coefficient of longitudinal velocity, which ensures unaffected tracking performance. However, under disturbances, fluctuations occur in the



yaw rate and lateral deviation angle of the center of mass in the lateral direction. Therefore, the stability of the vehicle is not as good without compensation as it is with compensation.

(c) Sideslip angle.

Figure 6. Responses of DDEV under pure acceleration scenario.

### 5.2. Acceleration and Steering Scenario

The operating conditions are set to comprehensively verify the vehicle's longitudinal, lateral, and yaw motion acceleration under a double-line moving condition. Specifically, the configuration is as follows: double-line moving condition,  $\mu = 0.85$ , the vehicle starts with an initial speed of 50 km/h, and, at 4 s, the reference speed Vxref changes to 80 km/h. The vehicle's total mass error is  $\Delta m = 0.5$  m, including errors in the distance from the center of gravity to the front and rear axles and front and rear track errors  $\Delta l_f = 0.5l_f$ ,  $\Delta l_r = 0.5l_r$ ,  $\Delta d_{fl} = 0.5d_{fl}$ ,  $\Delta d_{fr} = 0.5d_{fr}$ ,  $\Delta d_{rl} = 0.5d_{rl}$ , and  $\Delta d_{rr} = 0.5d_{rr}$ . The additional yaw torque caused by external disturbances is 0.3 times the original yaw torque and random term,  $\omega = 0.3M_z + M_{zran}$ , and the vehicle's longitudinal force error caused by the nonlinearity of the tire model is  $\Delta F_{xfl} = 0.4\Delta F_{xfl}$ ,  $\Delta F_{xfr} = 0.4\Delta F_{xrr}$ ,  $\Delta F_{xrrl} = 0.4\Delta F_{xrrl}$ ,  $\Delta F_{xrrr} = 0.4\Delta F_{xrrr}$ .

The waveforms before and after disturbance compensation are shown in Figure 7. Figure 7e shows the response of the vehicle's longitudinal speed, where the speed response before compensation fluctuates about 1% more than after compensation. Figure 7b displays the lateral trajectory error waveform, indicating that the lateral trajectory error of the vehicle before compensation ranges from -0.88 m to 1.00 m, while, after compensation, the lateral trajectory error ranges from -0.86 m to 0.80 m, showing a 20% improvement. This optimization in trajectory tracking is attributed to the enhanced performance of the composite disturbance suppression strategy. Figure 7c,d represent the response waveforms of the yaw angular velocity and center of gravity lateral deviation angle, respectively. In the presence of a disturbance, the yaw angular velocity exhibits a maximum fluctuation of 0.044 rad/s. However, after compensating for the composite disturbance term using

the disturbance suppression strategy, the disturbance-induced fluctuations in yaw angular velocity are effectively eliminated.

In conclusion, this section validates the effectiveness of the vehicle compound disturbance suppression strategy based on the sliding mode observer in the control of multi-motor systems in simulated analyses. The designed sliding mode observer can systematically observe and compensate for compound disturbances experienced by the vehicle during operation. This strategy is proven to significantly enhance the tracking performance of distributed drive electric vehicles and reduce fluctuations in the multi-motor system caused by disturbances.



Figure 7. Responses of DDEV under acceleration and steering scenario.

# 5.3. Double-Lane-Change Scenario

To verify the influence of longitudinal vehicle disturbances on control and the effectiveness of suppression strategies, a double-lane-change scenario is simulated. The results shown in Figure 8 indicate that disturbances have a significant impact on the lateral stability of the vehicle. The uncompensated vehicle exhibits significant deviations from the trajectory and high-frequency fluctuations in yaw rate. After compensating for disturbances, the stability and maneuverability of the vehicle are greatly improved.



Figure 8. Responses of DDEV during double-lane-change scenario.

# 5.4. Comparison with Nonlinear Model Predictive Control

In this section, the proposed method is compared with the nonlinear MPC from reference [37], with a contrast condition differing from that in reference [7]. In contrast to the previous conditions, this study incorporates vehicle disturbance factors with specific settings, consistent with previous conditions.

This situation mainly verifies the effectiveness of the proposed MPC in the longitudinal motion drive control of a DDEV. The simulation results are shown in Figure 9. Figure 9a shows the longitudinal speed responses. The vehicle accelerates from a standstill to 120 km/h in a straight line and then decelerates to 0 after running stably for 5 s. It can be seen that the dynamic response of the longitudinal motion under the proposed MPDMC is faster than that of the nonlinear MPC.

The effectiveness of the proposed MPC in the lateral and yaw motion stability control of the DDEV is verified in this part. The vehicle completed a double-lane-change maneuver at a longitudinal speed of 90 km/h. Figure 9b shows that the trajectory of the vehicle using the proposed MPC is closer to the reference trajectory, so this strategy should be more effective for the lateral and yaw motion control of DDEVs.



Figure 9. Responses of DDEV with nonlinear MPC.

# 5.5. Validation of Effectiveness

To validate the effectiveness of the method proposed in this paper, a four-motor control platform is designed. The motor experiment platform is shown in Figure 10. The experimental platform includes two LAUNCHXL-F28379D control boards from TI, four BOOSTXL-3PhGaNInv drive boards from TI, and four M-2310P permanent-magnet synchronous motors from Teknic. The main control chip used by the LAUNCHXL-F28379D control board is TI's TMS320F28379D. The chip's main frequency is 200 MHz, and there are two 32-bit CPUs inside. Its peripheral configuration can meet the control requirements of four motors. However, because a LAUNCHXL-F28379D board only leads out half of the chip's pins, this article uses two LAUNCHXL-F28379D control boards to realize the function of a complete TMS320F28379D chip, and each board uses only one CPU to control two motors. The experimental program is based on MATLAB's Motor Control Blockset, and the experimental data are transmitted to the PC through the SCI serial communication interface and then read with MATLAB, thus omitting the use of the oscilloscope. The mechanical equation of the motor is used to replace the dynamic equations of the vehicle to implement the proposed MPC strategy.

The experimental results are shown in Figure 11. The four motor drive systems can realize speed control of 500 rpm, 1000 rpm, 1500 rpm, and 2000 rpm, respectively, which proves that the proposed MPC strategy is feasible.



Figure 10. Four-motor experimental platform.



Figure 11. Speed response of four motors.

# 6. Conclusions

This paper establishes a composite disturbance model that incorporates disturbances such as vehicle parameter errors, external disturbances, and tire nonlinearity. It proposes a vehicle composite disturbance suppression strategy based on a sliding mode observer, enhancing the robustness of the multi-motor cooperative model predictive control strategy. Simulation results demonstrate that this strategy effectively reduces system disturbances.

This paper aims to conduct related research on model predictive control with DDEVs. In addition to the research described in the paper, further exploration and discussion can be extended to the following issues: (1) To better achieve motion control of DDEVs, targeted optimization is needed for various special application scenarios such as rainy and snowy weather, climbing, rapid turning, etc., to achieve more stable and faster maneuverability; (2) considering the different driving habits of different drivers, the designed control algorithm should be extended to the vehicle's optimal operating mode, summarizing the vehicle's operating status combined with driving habits for targeted optimization.

**Author Contributions:** Software, L.G.; Writing—original draft, A.T.; Writing—review & editing, Y.C. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by Open Project of National Engineering Research Center of Advanced Network Technologies OF FUNDER grant number No.ANT2023003.

**Data Availability Statement:** Data is contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

#### Nomenclature

$v_x, v_y$ , and r	the longitudinal velocity, lateral velocity, and yaw angular velocity
<i>x,</i> , <i>y</i> ,	of the vehicle.
Freque Freque Freque and Freque	the longitudinal forces experienced by the front left, front right, rear left,
- x);; - x);; - x;;; - : x;;	and rear right wheels.
E a E a E and E	the lateral forces on the front left, front right, rear left, and
<i>1 y f l, 1 y f r, 1 y f l, and 1 y f r</i>	rear right wheels.
$\sigma$	the steering angle of the front wheels.
m	the total vehicle mass.
1.1	the longitudinal distances from the vehicle's center of gravity to
<i>l</i> f, <i>l</i> r	the front and rear axles.
$I_Z$	the vehicle's rotational inertia.
$d_{fl} + d_{fr}$ and $d_{rl} + d_{rr}$	the front and rear axle distances.
i = f, r	the front or rear wheels.
j = l, r	the left or right wheels.
T <sub>eij</sub>	the electromagnetic torque of the wheel hub motor.
Jω	the rotational inertia of the wheel.
R	the wheel radius.
$\omega_{eij}$	the wheel rotational speed.
f f and f	the concentrated disturbances caused by parameter variations,
Jvx, $Jvy$ , and $Jr$	external disturbances, and the nonlinearity of the tire model.
$F_{vx}$ , $F_{vy}$ , and $F_r$	the disturbance rates of $f_{vx}$ , $f_{vy}$ , and $f_r$ .
	the predicted values of the longitudinal velocity, lateral velocity,
$\sigma_x, \sigma_y$ , and r	and yaw rate.
$\hat{f}_{vx}, \hat{f}_{vy}, \text{and } \hat{f}_r$	the predicted values of the rates of change for $f_{vx}$ , $f_{vy}$ , and $f_r$ .
$\lambda_{vx}, \lambda_{vy}, \text{ and } \lambda_r$	sliding mode parameters.
$U_{vxsmo}, U_{vysmo}, and U_{rsmo}$	sliding mode functions.
e <sub>fvx</sub>	the predicted error for longitudinal disturbance.
$e_{f_{774}}$	the predicted error for lateral disturbance.
e <sub>fr</sub>	the predicted error for yaw disturbance.
$K_{vir}$ , $K_{vir}$ , and $K_r$	constants.

# References

- Liao, Z.; Cai, L.; Yang, Q.; Zhang, Y. Design of lateral dynamic control objectives for multi-wheeled distributed drive electric vehicles. *Eng. Sci. Technol. Int. J.* 2024, 50, 101629. [CrossRef]
- 2. Zhang, Z.; Yu, J.; Huang, C.; Du, R. Coordinated torque distribution method of distributed drive electric vehicle to reduce control intervention sense. *Veh. Syst. Dyn.* 2024, *62*, 198–221. [CrossRef]
- 3. Wu, D.; Guan, Y.; Xia, X.; Du, C.; Yan, F.; Li, Y.; Hua, M.; Liu, W. Coordinated control of path tracking and yaw stability for distributed drive electric vehicle based on AMPC and DYC. *Proc. Inst. Mech. Eng. Part D J. Automob. Eng.* **2024**. [CrossRef]
- Fang, T.; Wang, Q.; Zhao, L.; Chen, W.; Yan, M.; Cai, B. Coordinated control of longitudinal and lateral movements considering dynamics for distributed drive electric vehicle platoon on curved roads. *Trans. Inst. Meas. Control* 2024, 46, 592–607. [CrossRef]
- 5. Fang, Z.; Liang, J.; Tan, C.; Tian, Q.; Pi, D.; Yin, G. Enhancing Robust Driver Assistance Control in Distributed Drive Electric Vehicles through Integrated AFS and DYC Technology. *IEEE Trans. Intell. Veh.* **2024**, 1–14. [CrossRef]
- 6. Guo, N.; Zhang, X.; Zou, Y.; Lenzo, B.; Du, G.; Zhang, T. A supervisory control strategy of distributed drive electric vehicles for coordinating handling, lateral stability, and energy efficiency. *IEEE Trans. Transp. Electrif.* **2021**, *7*, 2488–2504. [CrossRef]
- Gao, L.; Chai, F. Model Predictive Direct Motion Control for Distributed Drive Electric Vehicles. *IEEE Access* 2023, 11, 67443–67459. [CrossRef]
- 8. Hwang, Y.; Kang, C.M.; Kim, W. Robust nonlinear control using barrier Lyapunov function under lateral offset error constraint for lateral control of autonomous vehicles. *IEEE Trans. Intell. Transp. Syst.* **2020**, *23*, 1565–1571. [CrossRef]
- Kang, C.M.; Kim, W.; Baek, H. Cascade backstepping control with augmented observer for lateral control of vehicle. *IEEE Access* 2021, 9, 45367–45376. [CrossRef]
- Yu, S.; Wang, J.; Wang, Y.; Chen, H. Disturbance observer based control for four wheel steering vehicles with model reference. *IEEE/CAA J. Autom. Sin.* 2016, 5, 1121–1127. [CrossRef]

- 11. Wang, W.; Ma, T.; Yang, C.; Zhang, Y.; Li, Y.; Qie, T. A path following lateral control scheme for four-wheel independent drive autonomous vehicle using sliding mode prediction control. *IEEE Trans. Transp. Electrif.* **2022**, *8*, 3192–3207. [CrossRef]
- 12. Xu, T.; Ji, X.; Liu, Y.; Liu, Y. Differential drive based yaw stabilization using MPC for distributed-drive articulated heavy vehicle. *IEEE Access* **2020**, *8*, 104052–104062. [CrossRef]
- 13. Hu, C.; Wang, R.; Yan, F.; Huang, Y.; Wang, H.; Wei, C. Differential steering based yaw stabilization using ISMC for independently actuated electric vehicles. *IEEE Trans. Intell. Transp. Syst.* **2017**, *19*, 627–638. [CrossRef]
- 14. Wu, Y.; Wang, L.; Zhang, J.; Li, F. Path following control of autonomous ground vehicle based on nonsingular terminal sliding mode and active disturbance rejection control. *IEEE Trans. Veh. Technol.* **2019**, *68*, 6379–6390. [CrossRef]
- 15. Jeong, D.; Kim, S.; Lee, J.; Choi, S.B.; Kim, M.; Lee, H. Estimation of tire load and vehicle parameters using intelligent tires combined with vehicle dynamics. *IEEE Trans. Instrum. Meas.* **2020**, *70*, 1–12. [CrossRef]
- 16. Sun, S.; Zhang, N.; Walker, P.; Lin, C. Intelligent estimation for electric vehicle mass with unknown uncertainties based on particle filter. *IET Intell. Transp. Syst.* 2020, 14, 463–467. [CrossRef]
- 17. Li, X.; Ma, J.; Zhao, X.; Wang, L. Intelligent two-step estimation approach for vehicle mass and road grade. *IEEE Access* 2020, *8*, 218853–218862. [CrossRef]
- Jin, X.; Yang, J.; Zhu, T.; Wang, J.; Yin, G. Real-Time Estimation of Inertial Parameter for Lightweight Electric Vehicle Using Dual Kalman Filter. In Proceedings of the 2019 3rd Conference on Vehicle Control and Intelligence (CVCI), Hefei, China, 21–22 September 2019; pp. 1–5.
- 19. Guo, H.; Liu, H.; Yin, Z.; Wang, Y.; Chen, H.; Ma, Y. Modular scheme for four-wheel-drive electric vehicle tire-road force and velocity estimation. *IET Intell. Transp. Syst.* **2019**, *13*, 551–562. [CrossRef]
- Cordeiro, R.A.; Victorino, A.C.; Azinheira, J.R.; Ferreira, P.A.; de Paiva, E.C.; Bueno, S.S. Estimation of vertical, lateral, and longitudinal tire forces in four-wheel vehicles using a delayed interconnected cascade-observer structure. *IEEE/ASME Trans. Mechatron.* 2019, 24, 561–571. [CrossRef]
- 21. Boada, B.L.; Boada, M.J.L.; Zhang, H. Sensor fusion based on a dual Kalman filter for estimation of road irregularities and vehicle mass under static and dynamic conditions. *IEEE/ASME Trans. Mechatron.* **2019**, *24*, 1075–1086. [CrossRef]
- Berntorp, K.; Di Cairano, S. Tire-stiffness and vehicle-state estimation based on noise-adaptive particle filtering. *IEEE Trans. Control. Syst. Technol.* 2018, 27, 1100–1114. [CrossRef]
- 23. Hang, P.; Xia, X.; Chen, X. Handling stability advancement with 4WS and DYC coordinated control: A gain-scheduled robust control approach. *IEEE Trans. Veh. Technol.* **2021**, *70*, 3164–3174. [CrossRef]
- Hu, J.S.; Wang, Y.; Fujimoto, H.; Hori, Y. Robust yaw stability control for in-wheel motor electric vehicles. *IEEE/ASME Trans. Mechatron.* 2017, 22, 1360–1370. [CrossRef]
- Zhang, Z.; Xiao, B. The influence of cargo moving and sliding mode control strategy for forklift. *IEEE Access* 2020, *8*, 16637–16646. [CrossRef]
- 26. Hashemi, E.; Jalali, M.; Khajepour, A.; Kasaiezadeh, A.; Chen, S.k. Vehicle stability control: Model predictive approach and combined-slip effect. *IEEE/ASME Trans. Mechatron.* **2020**, *25*, 2789–2800. [CrossRef]
- Cheng, S.; Li, L.; Liu, C.Z.; Wu, X.; Fang, S.N.; Yong, J.W. Robust LMI-based H-infinite controller integrating AFS and DYC of autonomous vehicles with parametric uncertainties. *IEEE Trans. Syst. Man Cybern. Syst.* 2020, 51, 6901–6910. [CrossRef]
- Chang, X.H.; Liu, Y.; Shen, M. Resilient control design for lateral motion regulation of intelligent vehicle. *IEEE/ASME Trans. Mechatron.* 2019, 24, 2488–2497. [CrossRef]
- 29. Zhang, L.; Wang, Y.; Wang, Z. Robust lateral motion control for in-wheel-motor-drive electric vehicles with network induced delays. *IEEE Trans. Veh. Technol.* 2019, *68*, 10585–10593. [CrossRef]
- 30. Wang, D.; Yang, Q.; Ge, P. Diffeomorphism transformation based disturbance rejection control and its allocation of vehicle dynamic stable systems. *IEEE Access* **2019**, *7*, 105734–105745. [CrossRef]
- Chen, H.; Zhang, J.; Lv, C. Rhonn modelling-enabled nonlinear predictive control for lateral dynamics stabilization of an in-wheel motor driven vehicle. *IEEE Trans. Veh. Technol.* 2022, 71, 8296–8308. [CrossRef]
- 32. Nicholas, M.;Hall, D.; Lutsey, N. Quantifying the electric vehicle charging infrastructure gap across US markets. *Int. Counc. Clean Transp* **2019**, *20*, 1–39.
- Weitzman, M.L.; Nakamoto, S.; ZXhang, Y.X.; Haxo, Y.M.; Mat, Y.X. Lordstown Motors Stock: An Analysis of Opportunities and Challenges. AC Invest. Res. J. 2023, 220, 44–50.
- Li, Z.; Wu, L.; Xu, Y.; Zheng, X. Stochastic-weighted robust optimization based bilayer operation of a multi-energy building microgrid considering practical thermal loads and battery degradation. *IEEE Trans. Sustain. Energy* 2021, 13, 668–682. [CrossRef]
- Shang, Y.; Li, S. FedPT-V2G: Security enhanced federated transformer learning for real-time V2G dispatch with non-IID data. *Appl. Energy* 2024, 358, 122626. [CrossRef]

- 36. Kyslan, K.; Petro, V.; Bober, P.; Šlapák, V.; Ďurovskỳ, F.; Dybkowski, M.; Hric, M. A comparative study and optimization of switching functions for sliding-mode observer in sensorless control of pmsm. *Energies* **2022**, *15*, 2689. [CrossRef]
- 37. Zhou, H.; Jia, F.; Jing, H.; Liu, Z.; Güvenç, L. Coordinated longitudinal and lateral motion control for four wheel independent motor-drive electric vehicle. *IEEE Trans. Veh. Technol.* **2018**, *67*, 3782–3790. [CrossRef]

**Disclaimer/Publisher's Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.