

Article

Resource Allocation of Cooperative Alternatives Using the Analytic Hierarchy Process and Analytic Network Process with Shapley Values

Jih-Jeng Huang ¹  and Chin-Yi Chen ^{2,*}

¹ Department of Computer Science & Information Management, Soochow University, No. 56 Kueiyang Street, Section 1, Taipei 100, Taiwan; jjhuang@scu.edu.tw

² Department of Business Administration, Chung Yuan Christian University, No. 200, Zhongbei Rd., Zhongli District, Taoyuan City 320, Taiwan

* Correspondence: iris@cycu.edu.tw

Abstract: Cooperative alternatives need complex multi-criteria decision-making (MCDM) consideration, especially in resource allocation, where the alternatives exhibit interdependent relationships. Traditional MCDM methods like the Analytic Hierarchy Process (AHP) and Analytic Network Process (ANP) often overlook the synergistic potential of cooperative alternatives. This study introduces a novel method integrating AHP/ANP with Shapley values, specifically designed to address this gap by evaluating alternatives on individual merits and their contributions within coalitions. Our methodology begins with defining problem structures and applying AHP/ANP to determine the criteria weights and alternatives' scores. Subsequently, we compute Shapley values based on coalition values, synthesizing these findings to inform resource allocation decisions more equitably. A numerical example of budget allocation illustrates the method's efficacy, revealing significant insights into resource distribution when cooperative dynamics are considered. Our results demonstrate the proposed method's superiority in capturing the nuanced interplay between criteria and alternatives, leading to more informed urban planning decisions. This approach marks a significant advancement in MCDM, offering a comprehensive framework that incorporates both the analytical rigor of AHP/ANP and the equitable considerations of cooperative game theory through Shapley values.



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1. Introduction

Resource allocation problems have long been a central concern within the field of multi-criteria decision-making (MCDM) [1,2], as they require the careful consideration of multiple conflicting criteria and the evaluation of potential alternatives. The importance of effectively addressing resource allocation problems cannot be overstated, as the choices made in such scenarios have far-reaching implications for various stakeholders, including governments, organizations, and individuals alike. One aspect of resource allocation problems that warrants particular attention is the cooperative nature of alternatives, where interdependence between criteria and alternatives plays a critical role in the decision-making process. In the MCDM literature, although numerous methods have been proposed to handle complicated decision problems [1–6], strategies to adequately address the cooperative nature of alternatives remain unavailable.

Owing to the intricacy of decision-making problems, recent years have witnessed the development of various MCDM methodologies, such as the extension of the Analytic Hierarchy Process (AHP) [7], Analytic Network Process (ANP) [8], and Decision-Making Trial and Evaluation Laboratory (DEMATEL) [9] to encompass group decisions or integrate these

techniques with others. These methods take into account both independent and interdependent criteria and alternatives in MCDM quandaries. Furthermore, the integration of other methodologies, including fuzzy set theory and grey theory, has broadened the applicability of MCDM methods to tackle more convoluted decision-making problems [10–12].

Despite these advancements, the majority of papers have concentrated on the interdependence between criteria and have overlooked the scenario of cooperative alternatives, which may exhibit synergy or hindered effects and create a research gap by not considering the coalition concept [13]. Cooperative game theory, a branch of game theory, focuses on forming and analyzing coalitions among players (or decision-makers) [14]. In a cooperative game, players collaborate to achieve a common goal, and the payoff for each player depends on the coalition they join. The central concept in cooperative game theory is the characteristic function, which assigns a value to each coalition, representing the total payoff that the coalition can generate by cooperating.

Within cooperative games, the formation of coalitions is crucial for measuring how players collaborate to achieve shared objectives. In MCDM, the coalition concept can be applied to cooperative alternatives, and their performance hinges on forming coalitions with other alternatives. This information proves significant for some MCDM problems, such as when a decision-maker seeks to determine the resource allocation of alternatives. In order to reflect this information, the Shapley value is a prominent solution concept in cooperative game theory, quantifying each player's contribution to a coalition [14]. The Shapley value is a unique way to distribute the total gains of a cooperative game among the players, considering their marginal contributions to all possible coalitions. It satisfies essential properties such as efficiency, symmetry, dummy player, and additivity. Here, we employed the Shapley value to gauge the level of a coalition of alternatives, allowing for more intricate decision-making problems to be addressed.

This study aimed to address the following research questions:

1. How can Shapley values be integrated with AHP/ANP to better handle cooperative alternatives in MCDM?
2. What are the implications of this integrated approach for resource allocation decisions?

The main objectives of this research were as follows:

1. To develop a comprehensive MCDM framework that accounts for both individual and cooperative contributions of alternatives by integrating AHP/ANP with Shapley values.
2. To demonstrate the applicability and efficacy of the proposed method through a numerical example of budget allocation.
3. To provide insights into the implications of considering cooperative alternatives in resource allocation decisions and compare the results with traditional AHP/ANP methods.

In this paper, we put forth a novel method that amalgamates the AHP/ANP with Shapley values to tackle the cooperative alternatives of a problem. This proposed method first discerns the problem structure, e.g., independent or interdependent, and utilizes the AHP/ANP to obtain weights for the criteria and scores of the alternatives. Subsequently, we compute the Shapley values of the alternatives based on the coalition values of the alternatives. Finally, we can synthesize the scores from the results of the AHP/ANP and Shapley values to achieve the ultimate resource allocation decision. The innovation of this study lies in integrating AHP/ANP with Shapley values, offering a comprehensive approach to tackling the nuances of cooperative alternatives. The integration with Shapley values, which calculates the contributions of alternatives based on coalition values, allows for a more nuanced and equitable resource allocation. To illustrate the effectiveness of our approach, we present a numerical example of budget allocation for transportation facilities, showcasing significant insights when Shapley values are considered. This paper contributes to the MCDM field by presenting a novel framework that simplifies complex decision-making and ensures fairer resource distribution.

The structure of our paper is organized as follows. Section 2 offers a literature review of the recent developments in addressing the interdependence between criteria and alternatives in MCDM. Section 3 presents our proposed method that combines the AHP/ANP with Shapley values, outlining the theoretical foundation and the step-by-step process for implementing this approach. Section 4 demonstrates the proposed method by applying it to a budget allocation problem for transportation facilities as a numerical example and compares the result with the conventional AHP and ANP. Section 5 discusses the implications of our findings and accentuates the benefits of incorporating Shapley values. Finally, in Section 6, we conclude our study and delineate potential future research directions to further explore the applicability of our proposed method in various fields and scenarios.

2. Literature Review

This section provides a comprehensive overview of the existing research on MCDM methods, their extensions, and their applications in various fields. We organized the literature review into subsections focusing on specific aspects, such as MCDM methods and their extensions, integration of MCDM methods with other techniques, interdependence between criteria and alternatives in MCDM, and cooperative game theory and its applications in decision-making.

2.1. MCDM Methods and Their Extensions

The AHP, introduced in [6], has been widely used for structuring complex decision problems and determining the relative importance of criteria and alternatives. Ref. [7] proposed a consistency ratio to assess the consistency of pairwise comparisons in the AHP. Ref. [15] provided a comprehensive review of the main developments and applications of the AHP. The ANP, an extension of the AHP, was developed in [8] to handle decision problems with interdependent criteria and alternatives. DEMATEL, another MCDM method, was introduced in [9] to analyze the causal relationships among criteria.

Recent studies have focused on extending these methods to incorporate group decision-making and integrate them with other techniques. For example, Ref. [3] presented a supplier selection framework using fuzzy DEMATEL, the ANP, and DEA, considering both efficiency and responsiveness criteria. Ref. [4] proposed a green supplier selection model using the ANP and modified GRA, addressing the limitations of previous models in the automotive industry. Ref. [10] introduced the IR'DANP-MAIRCA model, which combines interval rough numbers with DEMATEL, the ANP, and MAIRCA to handle uncertainties in MCDM.

2.2. Integration of MCDM Methods with Other Techniques

Researchers have integrated MCDM methods with various techniques to enhance their applicability and address complex decision-making problems. Fuzzy set theory has been incorporated into MCDM methods to deal with the vagueness and imprecision of human judgments [10,11]. Grey theory has also been combined with MCDM methods to handle incomplete and uncertain information [5]. These integrations have expanded the capabilities of MCDM methods to tackle real-world decision problems more effectively.

2.3. Interdependence between Criteria and Alternatives in MCDM

The interdependence between criteria and alternatives has received increasing attention in MCDM research. Several studies have addressed this aspect in various contexts, such as supplier selection [16], green supply chain management [4], and [11]. These studies have emphasized the importance of considering the interrelationships among criteria and alternatives to make more informed and realistic decisions.

However, most of these studies have focused on the interdependence between criteria, while the interdependence between alternatives, particularly in the context of cooperative decision-making, has been largely overlooked. This limitation highlights the need for further research on incorporating cooperative game theory concepts into MCDM frameworks.

2.4. Cooperative Game Theory and Its Applications in Decision-Making

Cooperative game theory, a branch of game theory, deals with the formation and analysis of coalitions among players [17]. The Shapley value, a fundamental concept in cooperative game theory, measures the marginal contribution of each player to the coalitions they belong to [14]. However, the integration of cooperative game theory concepts, particularly the Shapley value, into MCDM methods remains limited. Ref. [18] used the Shapley value to determine the weights of the criteria in MCDM problems, but they did not consider the cooperative nature of alternatives. This research gap motivated our study, which aimed to integrate AHP/ANP with Shapley values to address the cooperative alternatives in resource allocation problems.

2.5. Authors' Contribution Table

To highlight the novelty of our study compared to the existing literature, we present an authors' contribution table (Table 1). This table summarizes the key aspects of our research and compares them with relevant studies in the field. Compared to [3], which presented a supplier selection framework using fuzzy DEMATEL, ANP, and DEA, our proposed method differs by integrating Shapley values to address cooperative alternatives, an aspect not considered in their work. While [4] introduced a green supplier selection model using ANP and modified GRA, our method goes beyond their approach by incorporating cooperative game theory concepts to capture the interdependence between alternatives. Similarly, Ref. [10] proposed the IR'DANP-MAIRCA model, combining interval rough numbers with DEMATEL, the ANP, and MAIRCA to handle uncertainties in MCDM. However, our study extends their work by considering the cooperative nature of alternatives and their implications for resource allocation decisions.

Table 1. Authors' contribution table.

Study	MCDM Method	Integration with Other Techniques	Interdependence Criteria	Interdependence Alternatives	Cooperative Game
[3]	Fuzzy DEMATEL, ANP, DEA	Fuzzy set theory	Yes	No	No
[4]	ANP, Modified GRA	No	Yes	No	No
[10]	IR'DANP-MAIRCA	Interval rough numbers	Yes	No	No
[16]	AHP	No	No	No	Shapley value (criteria weights)
[19]	CRITIC-TOPSIS	No	Yes	No	No
[20]	Fuzzy TOPSIS	Fuzzy set theory	Yes	No	No
Ours	AHP/ANP	Shapley value	Yes	Yes	Shapley value (cooperative alternatives)

Next, Ref. [16] focused on evaluating and prioritizing suppliers and customers in the supply chain using sustainability metrics. While their approach integrates economic, environmental, and social factors using the AHP and fuzzy inference, our study differs by addressing the cooperative nature of alternatives in resource allocation problems using Shapley values. Then, Ref. [19] determined the most suitable very light business jet using multi-criteria decision-making with the CRITIC and TOPSIS methods. In contrast, our study integrates the AHP/ANP with Shapley values to handle the interdependence between alternatives and incorporate cooperative game theory concepts, which are not addressed in their work. Finally, Ref. [20] developed an algorithm for selecting the optimal hybrid energy power plant using fuzzy multi-criteria decision-making. While their research focused on evaluating single and hybrid energy resources, our study distinguishes itself by considering the cooperative nature of alternatives and integrating Shapley values with the AHP/ANP for resource allocation decisions.

As shown in Table 1, our study distinguishes itself from the existing literature by integrating the AHP/ANP with Shapley values to address the cooperative nature of alternatives in resource allocation problems. While previous studies have focused on the interdependence between criteria and the integration of MCDM methods with other techniques, our research fills the gap by considering the interdependence between alternatives and incorporating cooperative game theory concepts. In summary, this literature review highlights the research gaps in addressing the cooperative nature of alternatives in MCDM and the limited integration of cooperative game theory concepts into MCDM frameworks. Our study aimed to fill these gaps by proposing a novel method that combines the AHP/ANP with Shapley values to make more informed and equitable resource allocation decisions.

3. AHP/ANP with Shapley Values

This section introduces the innovative framework combining the AHP/ANP with Shapley values for resource allocation, particularly focusing on the cooperative nature of alternatives in MCDM.

3.1. The Framework of the Proposed Method

The framework, as depicted in Figure 1, begins with defining the problem, including the relationship between criteria and alternatives (independence or interdependence), and understanding the coalition values of alternatives. Experts are consulted to quantify the PCMs of criteria and alternatives, using the AHP or ANP to calculate the weights of criteria and scores of alternatives. Simultaneously, we employ Shapley values, calculated from the coalition values of alternatives, to ensure a fair distribution of resources based on contribution and interdependence.

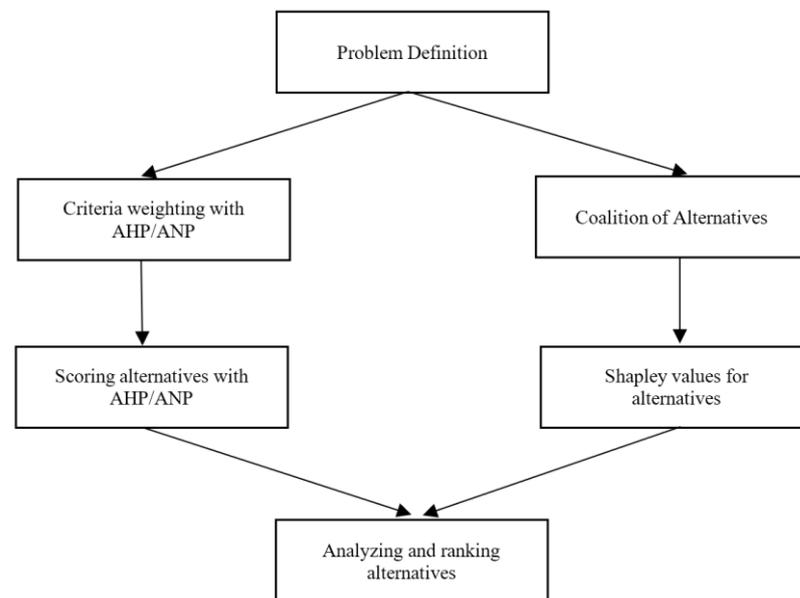


Figure 1. The framework of the proposed method.

3.2. Methods for AHP and ANP

The AHP, introduced in [21], serves as a decision-making methodology for intricate, multi-criteria problems. The AHP hinges on pairwise comparisons within a hierarchical structure, thereby streamlining decision-making by deconstructing it into smaller, more manageable components [15]. The mathematical model of the AHP encompasses the construction of the PCM, the calculation of criteria or alternative weights, and the computation of the consistency ratio [18,20]. Since its inception, the AHP has witnessed significant advancements in both theory and application. Researchers have concentrated on enhancing

the mathematical foundations, broadening their applicability, and amalgamating them with other methods to address various decision-making problems.

To quantify the PCMs, Ref. [21] introduced a scale spanning from 1 to 9, where 1 signifies equal importance, and 9 denotes extreme importance. Let $A = [a_{ij}]$ be a PCM matrix, where a_{ij} represents the importance of element i relative to element j . The decision-maker allocates values from this scale to convey the relative significance of one element over another. This culminates in a reciprocal PCM that is employed to calculate the eigenvector corresponding to the largest eigenvalue. This eigenvector embodies the normalized weights of the criteria or alternatives [21], which can be represented as solving the subsequent eigenvalue problem:

$$Aw = \lambda_{max} \tag{1}$$

where A is the PCM, w denotes the weight vector of criteria, and λ_{max} denotes the maximum eigenvalue.

The methodology of the AHP also accommodates the consistency of the decision-maker’s assessments. Ref. [21] proposed the consistency ratio (CR), which juxtaposes the consistency index (CI) with the random index (RI)—an average index derived from randomly generated matrices. A CR less than or equal to 0.10 is generally deemed acceptable, signifying that the decision-makers’ judgments exhibit a reasonable degree of consistency. The formulas for CR and CI are as follows:

$$CR = \frac{CI}{RI} = \frac{\lambda_{max}}{n - 1}, \tag{2}$$

where n denotes the number of criteria, and RI is used to estimate the average consistency of a randomly generated pairwise comparison matrix, which can be obtained from Table 2 [21].

Table 2. RI table [21].

Number of Criteria	Random Index
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41

Concerning theoretical foundations, consistency and inconsistency measures have been proposed to appraise the reliability of pairwise comparisons, such as the CR in [21,22] and the geometric consistency index (GCI) in [23]. Alternative weight derivation methods have been devised, including the row geometric mean method (RGMM) in [24], the logarithmic least squares method (LLSM) in [25], and the least square priorities method (LSM) in [26]. Furthermore, group decision-making has been integrated into the AHP through aggregation approaches like the weighted arithmetic mean (WAM) in [27] and the geometric aggregation operator (GAO) in [28]. With respect to practical applications, AHP has been employed in supply chain management for supplier selection and evaluation, incorporating green supplier evaluation [16] and assessing supply chain resilience [29]. In environmental decision-making, the AHP has been applied to prioritize climate change adaptation strategies and renewable energy source prioritization in Turkey [30].

The ANP was proposed by Saaty [8] to generalize the AHP for considering interdependencies and feedback between criteria. Initially, we must depict the relationship between the criteria to form the structured graph of a problem. It is noteworthy that the graph should satisfy the property of the irreducible Markov chain, as previously explained, to ensure that the steady state exists. Subsequently, we derive the relative weights of criteria through the AHP to indicate how much a column criterion is more important than the row

criteria and form a supermatrix. Ultimately, we can compute the supermatrix’s limit to obtain the steady-state weights of the criteria, as described in [8]:

$$\Phi = \lim_{k \rightarrow \infty} \mathbf{W}^{(k)}, \tag{3}$$

where (k) denotes the power operator and the condition for convergence of Equation (3) is $\mathbf{W}^{(k+1)} = \mathbf{W}^{(k)}$.

3.3. Method for the AHP/ANP with Shapley Values

We detail the mathematical formulation of Shapley values, a concept from cooperative game theory, to distribute resources equitably among alternatives in MCDM. This integration offers a novel approach to account for alternatives’ collective and interconnected nature, enhancing the decision-making process. The proposed method comprises the following steps:

1. **Problem Definition and Initial Assessment with the AHP/ANP:** Initially, the problem is defined, including identifying the criteria and alternatives. Depending on the nature of these criteria and alternatives (independent or interdependent), the AHP or ANP is employed to assess the weights of the criteria and scores of the alternatives. This process involves constructing PCM’s and calculating criteria weights and alternative scores through eigenvector extraction. Consistency ratios are evaluated to ensure the reliability of judgments.
2. **Quantification of Coalition Values:** Parallel to the AHP/ANP assessment, we determine the coalition values among the alternatives. This involves consulting domain experts to understand possible coalitions among the alternatives and quantifying the value each coalition brings. These coalition values are essential for computing Shapley values, as they represent the worth of each alternative’s contribution to potential cooperative groupings.
3. **Calculating Shapley Values:** With the coalition values at hand, we calculate the Shapley values for each alternative. The Shapley value, $\phi_i(v)$, for alternative i is determined using the formula that incorporates the sum of marginal contributions of alternative i across all possible coalitions, weighted by the factorial of the sizes of the coalitions and the set of all alternatives. This step is crucial for acknowledging the contributions of individual alternatives to the collective utility of coalitions they are a part of. The Shapley value is a mathematical concept employed in cooperative game theory to distribute resources among players based on their contributions. In multi-criteria decision-making, the Shapley value aids in allocating resources among alternatives by considering their interdependence. The mathematical formulation of the Shapley value for an alternative i is given by

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(N - |S| - 1)!}{N!} (v(S \cup \{i\}) - v(S)) \tag{4}$$

where $\phi_i(v)$ is the Shapley value of alternative i ; N is the set of all alternatives; S is a subset of N that does not include alternative i ; $v(S)$ is the coalition value function, which assigns a value to each coalition S ; and $|S|$ and $|N|$ are the cardinalities of the sets S and N , respectively. The Shapley value is calculated by summing the marginal contributions of alternative i across all possible coalitions S that do not include i . The marginal contribution of i to a coalition S is the difference between the coalition value with i , i.e., $v(S \cup \{i\})$, and the coalition value without i , i.e., $v(S)$. The binomial coefficient then weights the sum to account for the different ways that the coalition can be formed.

4. **Synthesizing AHP/ANP and Shapley Values:** After obtaining the scores from the AHP/ANP and the Shapley values, these are synthesized to achieve a final ranking of the alternatives. This synthesis is performed using normalized geometric averages, allowing for a balanced integration of the criteria-based evaluation and the cooperative

contributions captured by the Shapley values. Such a holistic synthesis provides a comprehensive viewpoint that accounts for both the intrinsic merits of alternatives and their cooperative dynamics.

5. Final Decision-Making: By leveraging the synthesized scores, decision-makers can make a more informed and equitable resource allocation decision. This final step benefits from a methodology that considers the criteria and alternatives in isolation and appreciates the value of cooperative interactions among alternatives, thereby enhancing the quality of decisions in complex MCDM scenarios.

By combining the AHP/ANP with Shapley values, our method facilitates a comprehensive evaluation that respects alternatives' individual contributions and collective efficacy within coalitions. This approach enriches the decision-making toolkit available for MCDM problems and promotes fairness and inclusivity in resource allocation decisions, reflecting both individual and cooperative merits.

3.4. Modeling the Decision-Making Problem

In this subsection, we present the modeling of the decision-making problem, focusing on the variables and parameters used in our proposed AHP/ANP with the Shapley value method.

3.4.1. Decision-Making Variables

The main decision-making variables in our model are as follows:

1. Criteria weights (w_i): The relative importance of each criterion i in the decision-making problem, derived from the AHP/ANP.
2. Alternative scores (s_{ij}): The performance score of alternative j with respect to criterion i , obtained from the AHP/ANP.
3. Coalition values ($v(S)$): The value or utility of each coalition S of alternatives, representing the cooperative contribution of the alternatives in the coalition.
4. Shapley values (ϕ_j): The Shapley value of each alternative j , quantifying its average marginal contribution to all possible coalitions.

3.4.2. Parameters

The parameters used in our model include the following:

1. Number of criteria (n): The total number of criteria considered in the decision-making problem.
2. Number of alternatives (m): The total number of alternatives being evaluated.
3. Pairwise comparison matrices (A_i): The matrix containing pairwise comparisons of the alternatives with respect to criterion i , used in the AHP/ANP.
4. Consistency index (CI): A measure of the consistency of the pairwise comparison judgments, calculated using the eigenvalues of the pairwise comparison matrices.
5. Random index (RI): An index used to normalize the consistency index based on the size of the pairwise comparison matrix.
6. Consistency ratio (CR): The ratio of the consistency index to the random index used to determine the acceptability of the pairwise comparison judgments.

3.4.3. Modeling the Shapley Value

By incorporating these decision-making variables and parameters into our model, we aimed to provide a clear and comprehensive framework for integrating the AHP/ANP with Shapley values to address the cooperative nature of alternatives in resource allocation problems.

Next, we will use a numerical example to demonstrate our method's practical application and efficacy. This example showcases how our integrated approach can address complex decision-making scenarios, emphasizing the value of considering cooperative alternatives in MCDM.

4. A Numerical Example

This numerical example assumes the scenario of a city planner’s decision-making process in choosing three transportation options: road, subway, and bike lanes. The planner considers five criteria: cost, accessibility, environmental impact (EI), safety, and economic development (ED). The case highlights the interdependent nature of these alternatives and their evaluation as a cooperative game.

First, let an expert quantify the PCM of the criteria, leading to the calculation of the maximum eigenvalue, priority vector, CI, and CR. These calculations ensure the consistency of the criteria evaluation. First, assume the PCM of the criteria can be quantified by an expert as follows:

PCM	Cost	Accessibility	EI	Safety	ED
Cost	1	1/2	1	1/3	1
Accessibility	2	1	2	1	3
EI	1	1/2	1	1/3	1
Safety	3	1	3	1	4
ED	1	1/3	1	1/4	1

Using the eigenvector method, we can derive the maximum eigenvalue $\lambda_{max} = 5.0364$ and the corresponding eigenvector, i.e., priority vector, as [0.1223, 0.2895, 0.1223, 0.3592, 0.1067]. Then, we can derive CI = 0.0091 and CR = 0.0081, where RI = 1.12, which is less than 0.1, justifying the consistency of the criteria. Next, we can use the AHP to derive the scores of alternatives concerning each criterion. We assume the PCMs of the alternatives for each criterion are as follows:

Cost	Road	Subway	Bike Lanes	Accessibility	Road	Subway	Bike Lanes
Road	1	1/3	1/5	Road	1	1/3	1/2
Subway	3	1	1/2	Subway	3	1	2
Bike Lanes	5	2	1	Bike Lanes	2	1/2	1
EI	Road	Subway	Bike Lanes	Safety	Road	Subway	Bike Lanes
Road	1	1/2	1/3	Road	1	1/3	1/4
Subway	2	1	1/2	Subway	3	1	1/2
Bike Lanes	3	2	1	Bike Lanes	4	2	1
ED	Road	Subway	Bike Lanes				
Road	1	1/2	1/3				
Subway	2	1	3/4				
Bike Lanes	3	4/3	1				

We can now calculate the eigenvectors for each PCM, which represent the scores of the alternatives for each criterion, and obtain the scores of the alternatives for each criterion as shown in Table 3.

Table 3. Scores of alternatives for each criterion.

Alternatives	Cost	Accessibility	EI	Safety	ED
Road	0.1095	0.1634	0.1634	0.1220	0.1677
Subway	0.3090	0.5396	0.2970	0.3196	0.3487
Bike Lanes	0.5816	0.2970	0.5396	0.5584	0.4836
Weight	0.1223	0.2895	0.1223	0.3592	0.1067

Next, to account for the interdependence between the alternatives and evaluate them in the context of cooperative game theory, we calculated the Shapley values using the worth of each coalition, as shown in Table 4

Table 4. The worth of each coalition.

Coalition	Worth
Road	100
Subway	150
Bike Lanes	75
(Road, Subway)	260
(Road, Bike Lanes)	185
(Subway, Bike Lanes)	240
(Road, Subway, Bike Lanes)	400

Furthermore, we calculated the marginal distribution of each alternative, taking into account all possible permutations of the alternatives, as shown in Table 5.

Table 5. Marginal distribution of the alternatives.

Permutation	Road	Subway	Bike Lanes
(1, 2, 3)	100	160	140
(1, 3, 2)	100	85	215
(2, 1, 3)	150	110	140
(2, 3, 1)	150	90	160
(3, 1, 2)	75	110	215
(3, 2, 1)	75	165	160

Based on these calculations, the Shapley values for the three transportation alternatives are shown in Table 6.

Table 6. Shapley values of the alternatives.

Alternative	Shapley Value	Normalized Shapley Value
Road	123.333	0.3083
Subway	175.833	0.4396
Bike Lanes	100.833	0.2521

The city government can use these Shapley values with the AHP method to evaluate the alternatives based on multiple criteria while accounting for their interdependent nature. This will provide a more comprehensive evaluation of the transportation options, helping the city make better-informed decisions for urban planning. Since the AHP and Shapley’s scores play a critical role in the final budget allocation, we used normalized geometric averages (NGAs) to combine the two scores, as shown in Table 7.

Table 7. The final result of the budget allocation.

Alternatives	AHP Score	Shapley Score	NGA	Rank (AHP)	Rank (Our)
Road	0.1424	0.3083	0.2170	3	2
Subway	0.3823	0.4396	0.4245	2	1
Bike Lanes	0.4753	0.2521	0.3585	1	3

In conclusion, by combining the AHP and Shapley values, the city government can evaluate the transportation alternatives based on multiple criteria while accounting for their interdependent nature. The calculated normalized geometric averages show that subway (0.4245) is the preferred option, followed by bike lanes (0.3585) and road (0.2170), where the scores of the alternatives indicate the decision of the budget allocation. Compared to the AHP’s result, bike lanes should be the most important alternative, and our method displays different perspectives on the problem.

Compared to the conventional AHP, the proposed method integrating the AHP/ANP with Shapley values provides a more comprehensive and equitable evaluation of alternatives by considering their cooperative potential and contributions to coalitions, which can lead to a rank reversal and a more informed decision-making process in resource allocation problems.

Next, using the above example, we considered an interdependence between criteria and cooperative alternatives. First, we needed to create our problem structure, which consisted of two clusters: Criteria and Alternatives. The Criteria cluster contains five criteria: cost, accessibility, EI, safety, and ED. The Alternatives cluster contains three transportation options: road, subway, and bike lanes. The interdependence between the two clusters and goal, given by an expert, can be depicted as shown in Figure 2.

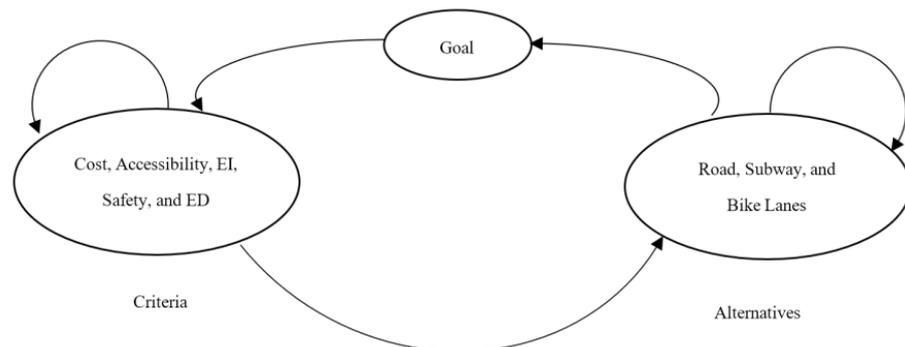


Figure 2. Interdependence between criteria and alternatives.

According to Figure 2, we can design our supermatrix as follows:

$$W = \begin{matrix} & \text{Goal} & \text{Criteria} & \text{Alternatives} \\ \text{Goal} & 0 & \mathbf{0} & \mathbf{I}_{3 \times 3} \\ \text{Criteria} & \mathbf{1} & \mathbf{B}_{11} & \mathbf{B}_{12} \\ \text{Alternatives} & \mathbf{0} & \mathbf{B}_{21} & \mathbf{B}_{22} \end{matrix}$$

$$\mathbf{B}_{11} = \begin{bmatrix} 0.1223 & 0 & 0 & 0 & 0 \\ 0 & 0.2895 & 0 & 0 & 0 \\ 0 & 0 & 0.1223 & 0 & 0 \\ 0 & 0 & 0 & 0.3592 & 0 \\ 0 & 0 & 0 & 0 & 0.1067 \end{bmatrix}$$

where, $\mathbf{B}_{22} = \mathbf{I}_{3 \times 3}$ and

$$\mathbf{B}_{21} = \begin{bmatrix} 0.1095 & 0.1634 & 0.1634 & 0.1220 & 0.1677 \\ 0.3090 & 0.5396 & 0.2970 & 0.3196 & 0.3487 \\ 0.5816 & 0.2970 & 0.5396 & 0.5584 & 0.4836 \end{bmatrix}$$

Then, we normalized the supermatrix to a transition matrix, in which, each column sum equals one, and we calculated the normalized supermatrix’s limiting power to derive the alternatives’ ANP scores, as shown in Table 8.

Table 8. ANP and Shapley scores.

Alternative	ANP Score	Shapley Score	NGA	Rank (ANP)	Rank (Our)
Road	0.1452	0.3083	0.2197	3	3
Subway	0.3628	0.4396	0.4147	2	1
Bike Lanes	0.4920	0.2521	0.3657	1	2

The comparison reveals that the ranking of the alternatives differed slightly between the combined AHP and ANP and the Shapley value methods. In the previous situation, the preferred option was subway (0.4245), followed by bike lanes (0.3585) and road (0.2170). On the other hand, the ANP and Shapley method also indicated subway as the preferred option (0.4147), followed by bike lanes (0.3657) and road (0.2197). Both methods suggested that subway is the most suitable transportation alternative. However, the ANP and Shapley method provided a slightly different perspective on the problem by accounting for the interdependence between the criteria and alternatives, which was not captured by the AHP method alone. In conclusion, the proposed method showed different rankings with respect to the AHP or ANP and incorporates the information of coalition between the alternatives to adjust the final decision of the budget allocation.

5. Discussion

This study's key insight is the significant advantage of integrating the AHP/ANP with Shapley values in complex decision-making, particularly where the criteria and alternatives are interdependent. The numerical example of urban planning demonstrates this method's practicality, highlighting how different rankings between the AHP/ANP and Shapley value methods emphasize the need to consider cooperative alternatives. The Shapley values enhance decision-making by providing a holistic assessment and acknowledging each alternative's contribution in a cooperative scenario. This method's versatility extends its application to various contexts, offering a comprehensive tool for complex decision-making challenges.

In both the AHP and ANP cases, we can observe the rank reversal situation, which occurs when the introduction of a new factor (in this case, the Shapley values) alters the relative preferences among the alternatives. The rank reversal in our example highlights the importance of considering the cooperative nature of the alternatives. While the AHP/ANP focuses on the individual performance of each alternative based on the criteria, our proposed method takes into account the potential synergies and complementarities that may arise when the alternatives form coalitions. The integration of Shapley values reveals that the subway alternative contributed more to the overall performance of the transportation system when considering its cooperative potential. This insight led to a higher ranking for subway compared to the conventional AHP, where it was ranked second. This example demonstrates the value of our proposed method in addressing the limitations of the conventional AHP and providing decision-makers with a more realistic and equitable assessment of the alternatives. The rank reversal problem, in this case, is not a flaw but rather a strength of our method, as it incorporates relevant factors that were previously overlooked, leading to a more informed and socially responsible decision-making process.

Compared to traditional MCDM algorithms, such as the AHP, the ANP, TOPSIS, and DEMATEL, our proposed approach offers several distinct advantages:

1. **Consideration of cooperative alternatives:** While most MCDM algorithms focus on evaluating alternatives based on their individual merits, our method integrates Shapley values to capture the cooperative dynamics among the alternatives. This allows decision-makers to account for the synergistic or hindering effects that may arise when the alternatives form coalitions, providing a more realistic and comprehensive assessment of their performance.
2. **Equitable resource allocation:** By incorporating Shapley values, our approach ensures a fair distribution of resources among the alternatives based on their marginal contributions to coalitions. This is particularly important in resource allocation problems, where the goal is to optimize the overall performance of the system while ensuring that each alternative receives its due share of resources.
3. **Flexibility in handling interdependence:** Our method is capable of addressing both independent and interdependent criteria and alternatives, thanks to the integration of the AHP/ANP with Shapley values. This flexibility allows decision-makers to model com-

- plex decision problems more accurately, capturing the intricate relationships among the criteria and alternatives that may be overlooked by other MCDM algorithms.
4. **Robustness in decision-making:** The combination of the AHP/ANP and Shapley values provides a robust framework for decision-making, as it leverages the strengths of both approaches. The AHP/ANP helps in structuring the problem and eliciting expert judgments, while Shapley values introduce the concept of fairness and cooperation into the evaluation process. This dual perspective enhances the reliability and acceptability of the decision outcomes.
 5. **Adaptability to various domains:** Our proposed method is not limited to a specific application area but can be adapted to various domains, such as supply chain management, environmental decision-making, and project portfolio selection. This versatility is a significant advantage over some specialized MCDM algorithms that may be tailored to specific contexts.

However, it is essential to acknowledge that our approach also has some limitations compared to other MCDM algorithms. The complexity of the method, both in terms of conceptual understanding and computational requirements, may be higher than simpler techniques like the AHP or TOPSIS. This complexity may require additional effort in data collection, expert elicitation, and result interpretation. Moreover, the subjectivity involved in determining the coalition values and the sensitivity of the results to these inputs may be a concern in some applications. Decision-makers should be aware of these limitations and take appropriate measures to validate the inputs and test the robustness of the findings. Despite these challenges, we believe that the benefits of our approach outweigh its limitations, particularly in decision problems where cooperative dynamics and equitable resource allocation are critical concerns. By providing a more comprehensive and nuanced evaluation of alternatives, our method can lead to better-informed and more defensible decisions.

6. Conclusions

In this study, we have proposed a novel method that integrates the AHP/ANP with Shapley values for complex decision-making problems involving independent or interdependent criteria and cooperative alternatives. Our approach demonstrates the benefits of combining these techniques to comprehensively evaluate resource allocation problems, and leading to improved decision-making outcomes. The numerical example showed that integrating the AHP/ANP with Shapley values allows decision-makers to account for cooperative alternatives, resulting in more informed and equitable resource allocation decisions.

The practical implications of our work are significant, as the proposed method can be applied to various real-world decision-making scenarios. In the context of urban planning, as demonstrated in our numerical example, city governments can use this approach to evaluate transportation alternatives based on multiple criteria while accounting for the interdependence between the criteria and alternatives. By considering the cooperative nature of the alternatives and their contributions to coalitions, decision-makers can make more informed choices regarding resource allocation, ultimately leading to improved urban mobility and overall quality of life for residents.

Moreover, the versatility of our method extends its potential applications to a wide range of fields, such as supply chain management, environmental decision-making, and project portfolio selection. In supply chain management, the integration of the AHP/ANP with Shapley values can help decision-makers evaluate and select suppliers based on multiple criteria while considering the cooperative dynamics among suppliers. This approach can lead to more resilient and efficient supply chains, as it accounts for the synergistic effects of supplier coalitions.

To facilitate the adoption and application of our method in practice, we encourage the development of user-friendly software tools that integrate the AHP/ANP with Shapley values. These tools should provide a clear and intuitive interface for decision-makers to input criteria, alternatives, and coalition values, as well as allowing for the visualization the

results and sensitivity analyses. Additionally, we recommend the creation of case studies and training materials to help practitioners understand and apply the method in their specific contexts.

In conclusion, the integration of the AHP/ANP with Shapley values offers a powerful and comprehensive approach to complex decision-making problems involving cooperative alternatives. The practical implications of our work span across various domains, from urban planning to supply chain management and beyond. By emphasizing the real-world applications and potential impact of our method, we hope to encourage its adoption and further research in this area. Future studies can explore the integration of our approach with other MCDM techniques, as well as its application in diverse decision-making contexts, ultimately contributing to more informed, equitable, and effective decision-making processes.

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