

# Miniaturized and high volumetric energy density power supply device based on a broad-frequency vibration driven triboelectric nanogenerator

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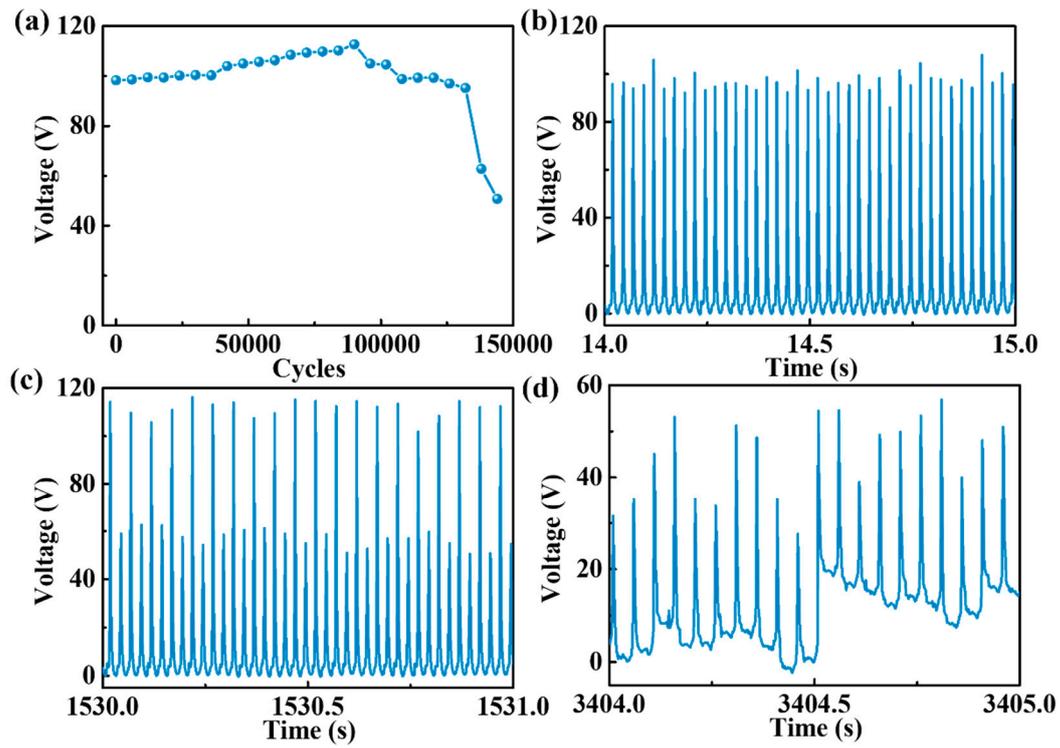
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**Figure S1.** (a) Cycling stability testing of the TENG in MPS at a frequency of 40 Hz; (b-d) the output  $V_{oc}$  of the TENG in MPS during cyclic stability testing process.

## Supporting Notes

The electrostatic force between the two friction layers is much smaller than the elastic force and that can be ignored. According to Figure 3b, the formulas can be rearranged as follows [24,26,27]:

$$\ddot{y}_1 + 2\xi_1\omega_1\dot{y}_1 + \omega_1^2y_1 = -\ddot{y}_0, (y_1 > -d) \quad (S1)$$

$$\ddot{y}_1 + (2\xi_0\omega_0 + 2\xi_1\omega_1)\dot{y}_1 + (\omega_0^2 + \omega_1^2)y_1 + \omega_0^2d = -\ddot{y}_0, (y_1 \leq -d) \quad (S2)$$

where  $y_0(t) = Y\sin(\omega t)$ ,  $Y$  is the excitation amplitude at the base,  $\omega$  is the excitation frequency,  $y_0$  is the excitation of the base,  $y_1$  is its relative motion,  $d$  is the initial distance,  $\xi_1$  and  $\omega_1$  are the suspension damping and angular frequency of Al friction layer, and  $\xi_0$  and  $\omega_0$  are the suspension damping and angular frequency of FEP friction layer, which can be further defined as  $2\xi_1\omega_1 = \frac{c_1}{m}$ ,  $2\xi_0\omega_0 = \frac{c_0}{m}$ ,  $\omega_1^2 = \frac{k_1}{m}$ ,  $\omega_0^2 = \frac{k_0}{m}$ . To study the frequency response of the TENG, a dimensionless variable is used:

$$\tau = \omega_1 t, \rho = \frac{\omega}{\omega_1}, \rho_1 = \frac{\omega_0}{\omega_1}, u = \frac{y_1}{Y}, v = \frac{y_0}{Y} = \sin(\rho\tau), \dot{u} = \frac{du}{d\tau}, \dot{v} = \frac{dv}{d\tau}, \delta = \frac{d}{Y} \quad (S3)$$

We can obtain

$$\dot{y}_1 = \omega_1 Y \dot{u}, \dot{y}_0 = \omega_1 Y \dot{v} \quad (S4)$$

$$\ddot{u} + 2\xi\dot{u} + u = \rho^2 \sin(\rho\tau) + f(u, \dot{u}) \quad (S5)$$

where

$$f(u, \dot{u}) = \begin{cases} 0, & u > -\delta \\ -2\xi_1\rho_1\dot{u} - \rho_1^2u - \rho_1^2\delta, & u \leq -\delta \end{cases} \quad (S6)$$

By using the averaging method, the first-order approximate solution of (S5) is assumed to be:

$$u = a(\tau) \sin(\varphi(\tau)) \quad (S7)$$

$$\dot{u} = a(\tau)\rho \cos(\varphi(\tau)) \quad (S8)$$

$$\varphi(\tau) = \rho\tau + \beta(\tau) \quad (S9)$$

where  $a(\tau)$  is a slowly varying amplitude, and  $\beta(\tau)$  is a slowly varying phase difference between the base excitation and the amplitude response. A conclusion can be obtained by Equations (S7) and (S8).

$$\dot{a} \sin \varphi + a\dot{\beta} \cos \varphi = 0 \quad (S10)$$

Substituting Equations (S7) and (S8) into (S5) obtains

$$\dot{a}\rho \cos \varphi - a\dot{\beta}\rho \sin \varphi = a(\rho^2 - 1) \sin \varphi + \rho^2 \sin(\varphi - \beta) - 2\xi a\rho \cos \varphi + f(u, \dot{u}) \quad (S11)$$

Solving Equations (S10) and (S11) for  $\dot{a}$  and  $\dot{\beta}$ , it has

$$\dot{a}\rho = [a(\rho^2 - 1) + \rho^2 \cos \beta] \sin \varphi \cos \varphi - (2\xi a\rho + \rho^2 \sin \beta) \cos^2 \varphi + f(u, \dot{u}) \cos \varphi \quad (S12)$$

$$a\dot{\beta}\rho = -[a(\rho^2 - 1) + \rho^2 \cos \beta] \sin^2 \varphi + (2\xi a\rho + \rho^2 \sin \beta) \cos \varphi \sin \varphi - f(u, \dot{u}) \sin \varphi \quad (S13)$$

Since the variables  $\dot{a}$  and  $\dot{\beta}$  vary slowly, it may suppose that their average values remain constant over a cycle period of  $2\pi$ :

$$2\pi\dot{a}\rho = \int_0^{2\pi} [a(\rho^2 - 1) + \rho^2 \cos \beta] \sin \varphi \cos \varphi d\varphi + \int_0^{2\pi} [-(2\xi a\rho + \rho^2 \sin \beta) \cos^2 \varphi + f(u, \dot{u}) \cos \varphi] d\varphi \quad (S14)$$

$$2\pi a\dot{\beta}\rho = \int_0^{2\pi} [-a(\rho^2 - 1) - \rho^2 \cos \beta] \sin^2 \varphi d\varphi + \int_0^{2\pi} (2\xi a\rho + \rho^2 \sin \beta) \cos \varphi \sin \varphi d\varphi - \int_0^{2\pi} f(u, \dot{u}) \sin \varphi d\varphi \quad (S15)$$

where

$$f(u, \dot{u}) = \begin{cases} 0, & (0 < \varphi < \pi + \varphi_1, 2\pi - \varphi_1 < \varphi < 2\pi) \\ -2\xi_1\rho_1 a\rho \cos \varphi - \rho_1^2 a \sin \varphi - \rho_1^2 \delta, & (\pi + \varphi_1 < \varphi < 2\pi - \varphi_1) \end{cases} \quad (S16)$$

$\varphi_1 = \sin^{-1}(\delta/a)$  is the phase values when the proof mass engages the up-plate.

For the steady-state response solution of the system, the time derivatives on the left-hand sides of Equations (S14) and (S15) are considered into zero. Hence, integration of Equations (S14) and (S15) gives:

$$\pi\rho^2 \sin \beta = -2\xi_1 a\rho\pi - \rho_0\xi_0 a\rho(\pi - 2\varphi_1 - \sin 2\varphi_1) \quad (S17)$$

$$\pi\rho^2 \cos \beta = \pi a(1 - \rho^2) - \left[ -\frac{1}{2}\rho_0^2 a(\pi - 2\varphi_1 + \sin 2\varphi_1) \right] \quad (S18)$$

$$+ 2\rho_0^2 \delta \cos \varphi_1$$

Combining Equations (S17) and (S18), the implicit Equation for the amplitude  $a$  as a function of the excitation frequency  $\rho$  is given by

$$\pi^2 \rho^4 = Z_1^2 + Z_2^2 \quad (S19)$$

where  $Z_1$  and  $Z_2$  are in the right-hand sides of Equations (S17) and (S18), respectively. Based on the frequency response function (S19), the dimensionless amplitude  $a$  with respect to frequency  $\rho$  can be obtained accordingly.

**References:**

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