



# Article Habitat Destruction and Restoration in Relation to Extinction and Survival of Species in Competitive Communities

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**Abstract:** We studied the N-species competitive coexistence model with direct effect on habitat destruction to analyze the behaviors of abundant and extinct species in the system caused by habitat loss. The nontrivial equilibrium points of the system are determined for a general habitat destruction function. For the trivial equilibrium, species that survived the habitat destruction are identified using eigenvalues of the Jacobian matrix. Solutions of the system are also presented using the recursive method. Three special cases of habitat destruction functions are addressed: continuous destruction, which is a typical habitat destruction; sudden habitat destruction with aftershocks. The proportional abundances of 50 species are numerically portrayed in each case. We found that the survival of a species is guaranteed if its corresponding eigenvalue is positive. However, the fact that a species has negative corresponding eigenvalue does not guarantee its extinction, as this also depends on the initial number of that species.

**Keywords:** mathematical model; eigenvalue; equilibrium analysis; recursive method; habitat destruction; species extinction

# 1. Introduction

Habitat destruction is a known cause of species extinction over the past billion years. Dominant species are not endangered if their habitat is not completely destroyed, as their territory might be randomly spread across the whole habitat and, hence, they can survive in undisturbed areas. Consequently, extinctions of rare species are assumed to be biased. Early work in this area by Nee and May [1] analyzed the effect of habitat removal of two species that are regionally abundant. Later, Tilman et al. [2] developed the coexistence model, which became widely applied to competitive multi-species' habitat loss. In the Tilman study, extinction in abundant species may be due to habitat loss with time delay. In a subsequent work by Tilman et al. [3], an analytical model of competitive coexistence in spatial habitats is introduced. Here, the extinct species with the poorest distribution are the best competitors. However, since the model assumptions are on dispersal ability and mortality rates, this extinction model is considerably biased.

Tilman's models have been widely studied over the past three decades. A wider range of parameters is investigated by Klausmeier [4], and a critical habitat size for a single species was also examined using a spatially explicit model. A prey–predator metapopulation model in Bascompte and Solé [5] indicates that dispersal rate is crucial for habitat destruction. The extension of this model by Swihart et al. [6] shows that the effect of habitat destruction on the coexistence of prey and predator depends on the invasion or colonization rate of prey



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**Copyright:** © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). and predator. More works related to space utilization on prey-predator interaction model can be found in [7–9].

Accounting for spatial structure and heterogeneity, the work of Prakash and de Roos [10] demonstrates that a predator–prey metapopulation is affected by habitat destruction. Their model is similar to the NMH model of Nee et al. [11]. However, in their model, prey are dispersed in inhabited patches of both predator and prey. The single-species argument NMH model is generally not enough to explain the effects of predator–prey interaction since it allowed the feedback loop into the model. In a later work by Prakash and de Roos [12] on metacommunities of mutualistic species, they vary the strength of mutualistic interaction and fraction of suitable habitat.

In Ovaskainen [13], an eigenvalue–eigenvector relationship was analytically studied to find the effects of habitat destruction and restoration in a metapopulation model using simple models. Earlier, Mean-field theory (MFT) had been used by Nakagiri et al. [14] to develop a model of ecosystems containing two or three species and study the relationships between habitat destruction and species extinction. A nonlinear n-population metapopulation model was also improved from Tilman's model by Lin et al. [15], whose work showed that the extinction number of superior competitors is related to the disharmony exponent of metapopulation. Lin and Zhen-Shan [16] demonstrated the odd-even evolution rule to show that the extinctions of some species are biased. Nakagiri et al. [17] used MFT to simulate and showed that a parity law reveals even-odd species different behaviors.

Species–area and endemics–area relationships (SAR and EAR) models have also been used to predict extinctions. Rybicki et al. [18] showed that EAR underestimated extinctions due to habitat loss while SAR gave a good approximation of short-term extinction unless the remaining habitat is so highly fragmented that SAR also underestimated the extinctions. Rompre et al. [19] used SAR model to predict the rates of bird species loss in fragmented forests and the results showed that SAR underestimated actual losses in the Panama Canal region due to nonrandom patterns of species distributions. Due to a lack of temporal data, long-term losses have not been widely investigated compared to short-terms losses. However, a recent study by Dri [20] showed that habitat losses reduced bird species diversity in urban South America over a time span of only 10 years.

More works on metapopulation model have been undertaken by Xu et al. [21] and Morozov and Li [22]. Through existence and equilibrium stability, the work by Xu et al., showed that the condition of extinction depended on habitat destruction and creation. Based on the original deterministic model, their model is a stochastic differential equations model. Another extension to Tilman's model was carried out by Morozov and Li [22]. In their work, three scenarios on the abundant species are studied: the most abundant species being the best competitors; equally abundant species; and the most abundant species being the poorest competitors. Explicit expressions for species equilibrium in these different scenarios are derived in their study.

Tilman's model was further modified by Liu et al. [23]. In this work, the rates of temporal heterogeneity are introduced via H(t) function. The parameters used are similar to Tilman's input parameters and three types of destruction were simulated: instantaneous destruction, continuous-complete destruction, and continuous-partial destruction. With a slight modification to the model in Tilman et al. [2], the direct effects of habitat destruction on species abundance are captured. Weak and strong Al-lee-like effects are studied in Chen et al. [24] whose work is also based on Tilman's model with a drastically shortened time delay of the species extinction.

In Pongvuthithum and Likasiri [25], the metapopulation model was improved by approximating the direct effect rate of habitat destruction using differentiable function. This approximation allowed analytical discussion on the equilibrium and stability of the model. Various habitat destruction functions were also studied and discussed including cyclical habitat destruction. Due to the superiority of Pongvuthithum and Likasiri's model to Liu et al., and Tilman's model, we are interested in extending this model and predicting the survival species. We also aim to analyze the equilibrium and stability of the model.

Further, due to the model's specific form, we are able to present a recursive method to find the model solution. Several habitat-loss behaviors will also be studied numerically.

#### 2. The Models and Analyses

The model in Pongvuthithum and Likasiri [25] describing the habitat proportion occupied by species i,  $p_i$ , can be written as follows:

$$F_{1} = \frac{dp_{1}}{dt} = c_{1}p_{1}(h(t) - p_{1}) - m_{1}p_{1} + f(t)p_{1}$$

$$F_{i} = \frac{dp_{i}}{dt} = c_{i}p_{i}\left(h(t) - \sum_{j=1}^{i} p_{j}\right) - m_{i}p_{i} - \sum_{j=1}^{i-1} c_{j}p_{i}p_{j} + f(t)p_{i}, i = 2, 3, \dots, n$$
(1)

where  $c_i$  is the colonization or invasion rate of *i*th species. In this study, the  $(i + 1)^{th}$  species is a better colonization or invasion species compared to the *i*th species,

- $m_i$  is the mortality rate of *ith* species,
- h(t) represents the remaining habitat,

 $f(t)p_i$  is the direct effect of habitat destruction where  $f(t) = \frac{h'(t)}{h(t)}$  is the ratio between the rate of change of the remaining habitat and the remaining habitat; in other words, f(t) is the balance between the two figures. If f(t) is a decreasing function, the direct effect of the habitat loss will be decreasing. Otherwise, it will be increasing over time.

The Jacobian of the system (1) can be expressed as a lower triangular matrix:

$$J = \begin{bmatrix} c_1h - 2c_1p_1 - m_1 + f & 0 & \cdots & \cdots & 0 \\ -(c_2 + c_1)p_2 & c_2h - (c_2 + c_1)p_1 - 2c_2p_2 - m_2 + f & 0 & \cdots & 0 \\ -(c_3 + c_1)p_3 & -(c_3 + c_2)p_3 & \frac{\partial F_3}{\partial p_3} & \cdots & 0 \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ -(c_n + c_1)p_n & -(c_n + c_2)p_n & \cdots & -c_np_n - c_{n-1}p_n & \frac{\partial F_n}{\partial p_n} \end{bmatrix}$$

where  $\frac{\partial F_j}{\partial P_j}$ , the *j*th diagonal element is  $c_jh - c_j\sum_{i=1}^{j-1} p_i - \sum_{i=1}^{j-1} c_ip_i - 2c_jp_j - m_j + f$ .

System (1) has the trivial equilibrium point at  $p_i = 0, \forall i$  with the Jacobian at the equilibrium:

$$J_0 = \begin{bmatrix} c_1h - m_1 + f & 0 & \cdots & 0 \\ 0 & c_2h - m_2 + f & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & c_nh - m_n + f \end{bmatrix}$$

From the structure of  $J_o$ , the linearized system is decoupled. Therefore, to guarantee the survival of the *ith* species, it is sufficient to show that there exists a time  $T \ge 0$  in which the *ith* term of the diagonal of  $J_o$  (or the *ith* eigenvalue of  $J_o$ ) is positive for all  $t \ge T$ . However, it does not mean that the species having negative eigenvalues for all  $t \ge 0$  will be extinct since the associated solution of (1) might not converge to the origin depending on the initial condition. There are several special cases worth exploiting explicitly for the trivial equilibrium case:

Case 1:  $c_i h - m_i + f(t) = \lambda_i$  case when  $\lambda_i$  are constant and h(t) is a solution to the equation:

$$h' + k_i h = c_i h^2$$

where  $k_i = \lambda_i + m_i$  are constant.

Solving the above differential equation yields

$$h = k_i / \left( c_i + \hat{c} k_i e^{-k_i t} \right),$$

where  $\hat{c} = \frac{1}{h(0)} - \frac{c_i}{k_i}$ . Hence, in order for  $h = \frac{k_i}{c_i + \hat{c}k_i e^{-k_i t}}$ ,  $\forall i$ , we must have  $k_i = k$  and  $c_i = c$ for some constants k and c. Thus, in this case the habitat is equally distributed among species and  $\lambda_i = k - m_i$ . Hence, for any *i*, if  $\lambda_i > 0$ , the *i*th species survives the habitat destruction.

**Remark 1.** For constant h, i.e., f(t) = 0, we have that  $\lambda_i = c_i h - m_i$  is constant and is a special case to this case.

Case 2: A more general  $c_i h - m_i + f(t)$  case. In order for the *i*th species to survive the habitat loss,  $c_i h - m_i + f(t)$  needs to be positive,  $\forall t > T$ , for some large enough T.

**Remark 2.** We assume in this study that  $c_{i+1} > c_i$ ,  $\forall i \in \{1, 2, ..., n\}$ . Consider the ith and  $(i+1)^{th}$  diagonal elements in the Jacobian J<sub>0</sub>, corresponding to the two adjacent species ith and  $(i+1)^{th}$ ,

$$c_{i+1}h - m_{i+1} + f(t) > c_ih - m_i + f(t)$$
 iff  $h > \frac{m_{i+1} - m_i}{c_{i+1} - c_i}$ 

Hence, if  $c_i h - m_i + f(t) > 0$ ,  $\forall t > T$  and  $h > \frac{m_{i+1} - m_i}{c_{i+1} - c_i}$ , the (i+1)th species will survive as well as the *ith* species.

A special case when  $m_i = m$ ,  $\forall i \in \{1, 2, ..., n\}$  studied in Tilman et al. [2,3] and Liu et al. [23], if the *ith* species survives, all other higher ranked species will also survive the habitat destruction.

Case 3: h(t) is a monotone decreasing function. If the *i*th diagonal element of the Jacobian matrix  $J_0$ ,  $c_ih - m_i - f > 0$ , the *i*th species survive the habitat loss. Since h(t) > 0, we have  $h' < m_i h - c_i h^2$ . While *h* is decreasing as the habitat is being destroyed, we have that h' < 0. As a consequence, the *i*th species will survive the habitat loss if  $h < \frac{c_i}{m_i}, \forall t > T$ , for some large enough T. In particular, if  $\lim_{t\to\infty} h' = 0$  and  $\lim_{t\to\infty} h = h_{\infty}$  we have that the *i*th species will survive so long as  $h_{\infty} < \lim_{t\to\infty} \frac{c_i}{m_i}$ . We will illustrate this case in the simulation.

### 2.1. Finding a Nontrivial Equilibrium Point

For Complete Survival Equilibrium:  $p_i^* \neq 0$ ,  $\forall i$  with f(t) = 0, the abundance equilibrium: rium in a recursive form shown in Tilman et al., (1994) [2] is

$$p_i^* = h - \frac{m_i}{c_i} - \sum_{j=1}^{i-1} p_j^* \cdot \left(1 + \frac{c_j}{c_i}\right), i = 2, \dots, n,$$

where  $p_1^* = h - \frac{m_1}{c_1}$ , In general, if  $f(t) \neq 0$ , the equilibrium,  $p_{fi}^*$ , can be written as

$$p_{fi}^* = p_i^* + (-1)^i f(t) \prod_{j=1}^{\left\lceil \frac{i}{2} \right\rceil} c_{2j-2} / \prod_{j=1}^{\left\lceil \frac{i}{2} \right\rceil} c_{2j-1}, \quad i = 1, \dots, n$$
(2)

where  $p_i^*$  is the equilibrium when f(t) = 0, written explicitly in Pongvuthithum and Likasiri [25], are governed by the following equations:

$$p_{2i}^{*} = (-1)^{2i+1} h \frac{\prod_{j=1}^{2i-1} c_{j}}{\prod_{j=2}^{2i} c_{j}} + \sum_{k=1}^{i} (m_{2k-1} - m_{2k}) \frac{\prod_{j=2k+1}^{2i-1} c_{j}}{\prod_{j=2k}^{2i} c_{j}} + \sum_{k=1}^{i} (m_{2k-1} - m_{2k}) \frac{\prod_{j=2k-1}^{2i-2} c_{j}}{\prod_{j=2k-1}^{2i-1} c_{j}}, \quad (3)$$

and

$$p_{2i+1}^{*} = h \frac{\prod\limits_{j=1}^{2i-1} c_{j}}{\prod\limits_{j=2}^{2i} c_{j}} + \sum\limits_{k=1}^{i} (-m_{2k-1} + m_{2k}) \frac{\prod\limits_{j=2k+1}^{2i-1} c_{j}}{\prod\limits_{j=2k}^{2i} c_{j}} + \sum\limits_{k=1}^{i} (-m_{2k-1} + m_{2k}) \frac{\prod\limits_{j=2k}^{2i} c_{j}}{\prod\limits_{j=2k-1}^{2i+1} c_{j}}, \quad (4)$$

where  $p_1^* = h - \frac{m_1}{c_1}$ . This equilibrium point is useful in determining habitat abundance. It has been known that the equilibrium is locally stable if the eigenvalues of the Jacobian matrix  $\lambda_i = c_i h - c_i h$  $c_i \sum_{j=1}^{i-1} p_{fj}^* - \sum_{j=1}^{i-1} c_j p_{fj}^* - 2c_i p_{fi}^* - m_i + f < 0, \forall i.$  All species will converge to the equilibrium  $p_{fi}^* \neq 0$ , if the initial conditions are close enough to  $p_{fi}^*$ . Unfortunately, finding the region of attraction of an equilibrium point for a nonlinear system is very difficult and usually impossible. As a result, we generally cannot conclude where the solution will converge analytically. However, due to the special form of the model, an analytic solution can be written explicitly, as in the next section.

### 2.2. Finding a Solution

The system (1) can be rewritten as

$$\frac{dp_1}{dt} = (c_1h(t) - m_1 + f)p_1 - c_1p_1^2,$$
  
$$\frac{dp_2}{dt} = (c_2h(t) - (c_1 + c_2)p_1 - m_2 + f)p_2 - c_2p_2^2,$$
  
$$\frac{dp_3}{dt} = (c_3h(t) - (c_1 + c_3)p_1 - (c_2 + c_3)p_2 - m_3 + f)p_3 - c_3p_3^2, \dots,$$
  
$$\frac{dp_n}{dt} = \left(c_nh(t) - \sum_{i=1}^{n-1} (c_i + c_n)p_i - m_n + f\right)p_n - c_np_n^2$$

Since  $A_n(t) = c_n h(t) - \sum_{i=1}^{n-1} (c_i + c_n) p_i - m_n + f$  is a function of *t*, it can be viewed as a special case of the Bernoulli equation in the following form:

$$\frac{dp_n}{dt} = A_n(t)p_n - c_n p_n^2, \forall n$$

where  $A_1(t) = c_1 h(t) - m_1 + f$  and  $A_n(t) = c_n h(t) - \sum_{i=1}^{n-1} (c_i + c_n) p_i - m_n + f$ ,  $\forall n = 2, ...,$ whose solution can be solved analytically and recursively as

$$\frac{1}{P_n(t)} = \frac{1}{P_n(0)} \exp\left(-\int_0^t A_n(\tau)d\tau\right) + c_n \int_0^t \exp\left(\int_t^{\hat{\tau}} A_n(\tau)d\tau\right) d\hat{\tau}, \forall n$$
(5)

### 3. Varieties of Habitat Destruction and Their Simulation Results

Three special cases of h(t) are considered in this study.

# 3.1. Basic Continuous Destruction

To illustrate the case when h(t) is a decreasing function or the habitat is continuously being destroyed, we use the exponentially decay function for the remaining habitat, which is defined as  $h(t) = h_{\infty} - (h_{\infty} - 1) \cdot e^{-t}$ . Shown in Figure 1, the final ratio of habitat destruction,  $h_{\infty} = \lim_{t\to\infty} h(t)$ , is equal to 0.20, which is to say 20% of the habitat remains after the destruction.



**Figure 1.** (a) shows the habitat destruction function when  $h(t) = e^{-t}$  and  $h_{\infty} = 0.20$ . (b) compares  $h_{\infty} = \lim_{t \to \infty} h(t)$  and the ratio  $\frac{c_i}{m_i}$  of species *i*. In this particular example, the ratio  $\frac{c_i}{m_i} > h_{\infty}$  for all  $i \ge 27$ . The ratio (c) shows the proportional abundance of all 50 species. The first 26 species become extinct whereas the remaining 24 species survive the habitat loss.

## 3.2. Sudden Habitat Destruction

Habitat destruction such as flooding or earthquake can be described using a unimodal function where the remaining habitat shown in Figure 2 can be written as  $h(t) = 1 - 0.94(4)[e^{-t} - e^{-2t}] - 0.1[1 - e^{-t}]$ . We assume that the habitat is destroyed by a natural disaster until there is only 1% of the habitat remaining, and then it is restored back to 90% of the original level.



**Figure 2.** The function of the remaining habitat governed by  $h(t) = 1 - 0.94(4)[e^{-t} - e^{-2t}] - 0.1[1 - e^{-t}]$  when  $h_{\infty} = 0.90$ .

Using the parameters given in Tilman et al., (1994) [2], we consider the case where all species experience the same loss, i.e.,  $m_i = m = 0.02$ ,  $\forall i$ , and  $c_i = m/(1-q)^{2i-1}$ , where q = 0.03. These parameters will ensure the competitiveness assumed in this study, that is, the (i + 1)th species has a better ratio of colonization compared to the *ith* species.

In this case,  $\lim_{t\to\infty} h = h_{\infty} = 0.90$ . The analysis shows that the *ith* species will survive so long as  $h^* > \lim_{t\to\infty} \frac{c_i}{m_i}$ . To capture this case, we then simulate the proportional abundance of species 1–50

To capture this case, we then simulate the proportional abundance of species 1–50 using a unimodal function giving  $h(t) = 1 - 0.94(4) [e^{-t} - e^{-2t}] - 0.1 [1 - e^{-t}]$  as shown in Figure 2 and  $h_{\infty} = 0.90$  or 10% of the habitat has been completely destroyed. Simulations of Equation (1) in this case are shown in the Figure 3a–d.



Figure 3. Cont.



**Figure 3.** Proportional abundance of 50 species when  $h(t) = 1 - 0.94(4) [e^{-t} - e^{-2t}] - 0.1[1 - e^{-t}]$ , 10% of the habitat is completely destroyed. The simulation shows 48 out of 50 species converging to the second equilibrium (2.2) in (a). Species 1 and 2 converge to the trivial equilibrium; therefore, they both will become extinct while the remaining 48 species will survive the habitat loss. (b–d) shows the proportional abundance of species 1–10 and 41–50 of the first 10 years, 100 years, and 1000 years, respectively. Better competitors approach their equilibria faster and they oscillate closer around the equilibria. The simulation closely shows how the proportional abundances of species 41–50 oscillate around their equilibriums.

### 3.3. Sudden Habitat Destruction with Aftershock

Habitat destruction with aftershock such as flooding or earthquake with a smaller amplitude aftershock can be described using a bimodal function. Here, the remaining habitat shown in Figure 4 is governed by  $h(t) = 1 - 0.935(4)(e^{-t} - e^{-2t}) - \frac{0.375}{(1+5(t-5)^2)} - 0.1(1 - e^{-t})$  where we assume that the habitat is first reduced to only 1% and then naturally restored. There is one aftershock, which destroys the habitat to 50%, which happens as the habitat is being restored to 80%. Eventually, only 10% of the habitat is completely destroyed.



**Figure 4.** The remaining habitat function of sudden habitat destruction with one after shock governed by  $h(t) = 1 - 0.935(4) \left(e^{-t} - e^{-2t}\right) - \frac{0.375}{(1+5(t-5)^2)} - 0.1(1-e^{-t})$  where  $h_{\infty} = 0.90$ .

To capture this case, we simulate the proportional abundance of species 1–50 using h(t) as shown in Figure 4 and  $h_{\infty} = 0.90$  or 10% of the habitat having been completely destroyed. Simulations of Equation (1) in this case are shown in Figures 5 and 6. While Figure 5a–d show 10,000-year simulations of specific groups of species, Figure 6a–d capture the first 10, 100, 1000 and 2000 years of species 1–10 and 41–50, respectively. Note that all other parameters are also taken from Tilman et al., (1994) [2].



**Figure 5.** (a) shows the simulation of the proportion abundance of species 1–50 when  $h(t) = 1 - 0.935(4)(e^{-t} - e^{-2t}) - \frac{0.375}{(1+5(t-5)^2)} - 0.1(1 - e^{-t})$  and  $h_{\infty} = 0.90$ . Only species 1 and 2 are eventually extinct. The other 48 species converge to the second equilibrium in (2.2). The species with lower colonization rate approach the equilibrium faster than the ones with higher colonization rate. (b) shows the details on species 1–10 and 41–50, the higher ranked groups of species oscillate closer to the equilibrium. (c) compares the group of 1–10 species and the group of 31–40 species while (d) compares the groups of 11–20 and 41–50 species.



**Figure 6.** Proportional Abundance of 50 species when  $h(t) = 1 - 0.935(4)(e^{-t} - e^{-2t}) - \frac{0.375}{(1+5(t-5)^2)} - 0.1(1 - e^{-t})$  and  $h_{\infty} = 0.90$ , where 10% of the habitat cannot be restored. The 10-year simulation is shown in (**a**), where we can see species 1 has the maximum occupancy rate. (**b**) shows a 100-year simulation, where lower ranked species still show high occupancy rate, especially species 1 and 2, appearing in the top two. (**c**) shows 1000-year simulation where we start to see species 1 and 2 dropping from the top two and species 1 eventually becoming minimal near the end of the simulation. (**d**) shows 2000 years simulation; here species 1 and 2 are approaching their zero equilibrium.

In the analyses, we show in Remark 2 that the survival of the *ith* species can be guaranteed by the sign of its corresponding eigenvalue. That is, it is sufficient for a species to survive if there exists a time  $T \ge 0$  in which its corresponding eigenvalue is positive for all  $t \ge T$ . We have plotted the eigenvalues of the system in the previous simulation to illustrate this. Figure 7a shows the corresponding eigenvalues of all 50 species over 10 years. A closer look at Figure 7b shows that the eigenvalues of species 1–3 are negative for some time  $T \ge 0$ . However, species 3 does not become extinct. As mentioned earlier in the analysis, having a negative eigenvalue for all  $t \ge 0$  does not necessarily mean that the species will become extinct since the associated solution of (1) might not converge to the origin depending on the initial condition.



**Figure 7.** (a) shows corresponding eigenvalues for all 50 species over 10 years. (b) shows the eigenvalues of species 1-4 over 100 years. We can see that species 1-3 have negative eigenvalues for all  $t \ge 0$ .

# 4. Summary and Discussion

We study the generalized N-species competitive coexistence model proposed in Pongvuthithum and Likasiri (2010) [25]. Due to the specific form of the model, each equation can be reduced to the Bernoulli equation where we show how to construct a solution in integral form recursively. The model considered is a smooth direct effect on habitat destruction on the abundance and extinction of species in the system. We analyze the behaviors of survival and extinction species and predict the survival species by using the corresponding eigenvalues of the Jacobian matrix, and the equilibrium and stability of the model.

Special cases of continuous habitat destruction and sudden habitat destruction, such as flooding and earthquakes with and without aftershocks, are discussed numerically. In the simulations, we can see that we can predict the survival of a species by its corresponding eigenvalue. If there exists a time  $T \ge 0$  in which its corresponding eigenvalue is positive for all  $t \ge T$ , that species' survival of the habitat loss is abundant. However, corresponding negative eigenvalues for all  $t \ge 0$  of a specific species does not predict the extinction of a species since the solution of the system may or may not converge to its equilibrium depending on the initial condition.

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