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Statistical Characteristics of Stress Changes in Tunnel Surrounding Rock Based on Random Discrete Variability of Geotechnical Materials

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Abstract: In the construction of underground engineering, the determination of surrounding rock stresses has important theoretical significance for the design of tunnel structure reliability. In this paper, the nonlinear criterion is introduced to modify and improve the friction resistance in the calculation formula of Bierbaumer's loose surrounding rock stresses, and the improved application is carried out in view of the situation of composite formation. Due to the variability and discreteness of geotechnical parameters, combined with engineering examples, Monte Carlo random sampling is carried out for various geotechnical parameters in engineering, and the characteristics of loose surrounding rock stresses are analyzed from the perspective of statistical reliability by using the improved shallow tunnel formula. By using the standard regression coefficient, the weight influence of the parameters in various shallow surrounding rock stress formulas is analyzed, and the confidence degree of the calculated results of various formulas is analyzed from a statistical point of view, which verifies the rationality of the improved formula in the statistical sense.

Keywords: surrounding rock stress; nonlinear criterion; statistical characteristics; Monte Carlo method; sensitivity analysis

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1. Introduction

In the construction of underground engineering, the determination and distribution of surrounding rock stresses have always comprised an important basis for tunnel construction schemes and structural reliability designs. At present, there are various formulas to calculate the stresses of the loose rock surrounding shallow tunnels; the premise of these formulas is to make corresponding assumptions, so there are some limitations to their application. Because the geological environments of tunnels are each different, complex and changeable, there is a certain randomness and variability in the geotechnical materials. When using various formulas to calculate the stresses of loose surrounding rock, it is necessary to make an analysis specific to the geological environment. It is necessary to improve calculation formulas in order to better determine the results of the loose surrounding rock's stresses. Furthermore, it is essential to study how to obtain surrounding rock stresses from the perspective of statistics and according to the complexity of geological environments.

At present, there are various theoretical, empirical and numerical calculation methods used in the calculation of the stresses of loose rock surrounding shallow tunnels. These methods each have their own characteristics and applicability in calculations [1–7].

Calculation theories for the pressure of loose surrounding rock in shallow buried tunnels mainly include the total soil column formula, the Bierbaumer formula, the Terzaghi formula and the Xie Jiajie formula [8–10]. Keawawasvong and Ukritchon [11] studied the stability of shallow unlined circular tunnels in rock masses based on the Hoek–Brown criterion and provided design equations.

Li et al. [12] conducted a comprehensive analysis of the influence of various parameters in various tunnel calculation formulas on surrounding rock stresses and made corresponding evaluations on the advantages, disadvantages and applicability of various methods. Based on Xie Jiajie's theory, in order to overcome the shortcomings of the original formula, Cheng [13] improved the calculation formula for the surrounding rock stresses of shallow tunnels by using the limit equilibrium method, starting by modifying the width of the rock column, and discussed the influence of various parameters on the results. Liu et al. [14] deduced a formula for the stresses of loose rocks surrounding tunnels under the action of earthquake forces and studied the fracture angle, the failure modes of tunnels and the distribution of stresses of loose surrounding rocks. Liu and Fang [15] deduced a formula for calculating the surrounding rock stresses of tunnels with variable slope eccentric stresses. Yu et al. [16] deduced a stress formula for loose rocks surrounding tunnels on the basis of assuming the failure modes of the Loess tunnel. Li et al. [17] and Chao et al. [18] deduced the Terzaghi formula for homogeneous and composite strata under the nonlinear criterion. Zhu [19] combined this with the classical calculation theory for the surrounding rock stresses of shallow tunnels and revised and established the calculation method of surrounding rock stresses of double-arch shallow tunnels. Qu and Li [20] provided suitable cave-type conditions for the formulas of Terzaghi, Xie Jiajie and Bierbaumer under the conditions of existing surrounding rock classification and defined the division of deep and shallow buried tunnels. According to the failure modes of indoor model tests, Liu et al. [21] used the strength reduction method to verify said model tests. Through analysis on model tests and numerical simulation results, a modified algorithm was deduced based on rock column theory. Gao et al. [22] analyzed the calculation method for loose rock pressures in surrounding rocks and discussed the impact of rock conditions and cavern size on the range of loose pressure zones by combining a numerical analysis and an orthogonal test. Hu et al. [23] studied the surrounding rock pressure characteristics of the Loess tunnel based on statistical analysis. Chen et al. [24] studied the influence of support parameters and excavation methods on statistical distribution characteristics of surrounding rock pressures in shallow buried metro tunnels. Ding et al. [25] studied surrounding rock pressure calculations based on the time functions and stress release rate determination of deep soft-rock tunnels.

Through the review of previous studies, it was found that there is less improvement in the calculation of friction resistance using the Bierbaumer formula and that the nonlinear M-C criterion is more in line with the actual situation of geotechnical materials [26–28]. When using the Bierbaumer formula to calculate the surrounding rock stresses in composite strata, the weighted average method is often used for the selection of rock mass parameters. However, there are few reports on the application of the modified Bierbaumer formula in composite strata, and there are few reports on the analysis of the distribution characteristics of surrounding rock stresses.

In view of this, in order to overcome the shortcomings of traditional methods, this paper deduces and establishes an improved Bierbaumer formula suitable for composite formation, based on the nonlinear M-C criterion, that improves the shortcomings of the original formula when the friction angle exceeds 30 degrees. Considering the randomness and uncertainty of geotechnical materials, the Monte Carlo random sampling method is used to analyze the statistical characteristics of the results to verify the rationality of the improved formula in the statistical sense. Combined with sensitivity analysis, the applicability of the various stresses of loose, surrounding, shallow-buried rock is discussed, which lays the foundation for future structural reliability design and calculation.

2. The Establishment of the Improved Bierbaumer Formula

The nonlinear M-C criterion can be expressed in the plane $\tau - \sigma_n$ as follows [29]:

$$\tau = c_o(1 + \frac{\sigma_n}{\sigma_t})^{\frac{1}{m}} \tag{1}$$

where m is the nonlinear coefficient whose value is greater than or equal to 1; τ is the shear stress at yield; σ_n is the normal stress; σ_t is the uniaxial tensile strength of the rock–soil interface; and c_o is the initial cohesion of the rock–soil interface.

When m = 1, the above formula degenerates into the linear M-C criterion.

$$\tau = c + \sigma_n \tan\varphi \tag{2}$$

Therefore, the internal friction angle of the rock–soil interface is:

$$\tan\varphi = \frac{c}{\sigma_t} \tag{3}$$

In this paper, the tangent method is used for equivalence, as shown in Figure 1.

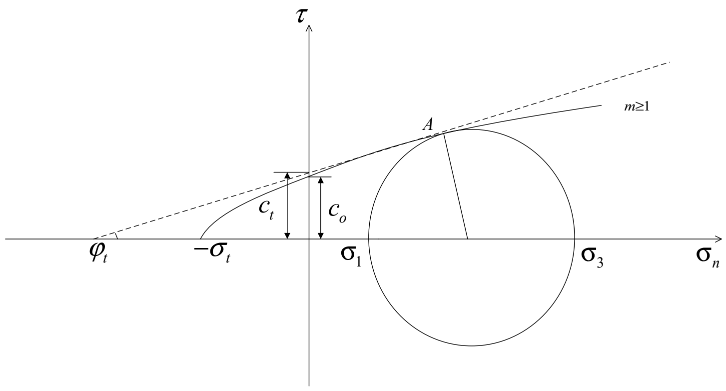


Figure 1. Tangent equivalent nonlinear M-C criterion.

In Figure 1, the tangent equation of point A can be expressed as:

$$\tau = c_t + \sigma_n \tan\varphi_t \tag{4}$$

where φ_t is the angle between the tangent and the normal stress axis; c_t is the intercept on the shear stress axis.

The slope of each point can be obtained by deriving Equation (1) as:

$$\tan\varphi_t = \frac{\mathrm{d}\tau}{\mathrm{d}\sigma_n} = \frac{c_o}{m\sigma_t}\left(1 + \frac{\sigma_n}{\sigma_t}\right)^{\frac{1-m}{m}} \tag{5}$$

The simultaneous Equations (1) and (4) [29] can be obtained as follows:

$$c_t = \frac{m-1}{m}c_o\left(\frac{m\sigma_t\tan\varphi_t}{c_o}\right)^{\frac{1}{1-m}} + \sigma_t\tan\varphi_t \tag{6}$$

According to Equations (3) and (6), it can be concluded that:

$$c_t = \frac{m-1}{m}c_o\left(\frac{m\tan\varphi_t}{\tan\varphi}\right)^{\frac{1}{1-m}} + \frac{c_o\tan\varphi_t}{\tan\varphi} \tag{7}$$

In consideration of the change in $\tan\varphi_t$ with σ_n in the calculation process, the initial tangent method with $\sigma_n = 0$ is used for equivalence as:

$$\tan\varphi_t = \frac{c_0}{m\sigma_t} = \frac{\tan\varphi}{m} \tag{8}$$

According to the conclusions of reference [30], when the m value is large, the error is large, so this paper studies when the m value is 1.0–1.5.

According to the actual tunnel failure, Bierbaumer revised the whole soil column theory and put forward the Bierbaumer calculation formula, as shown in Figure 2.

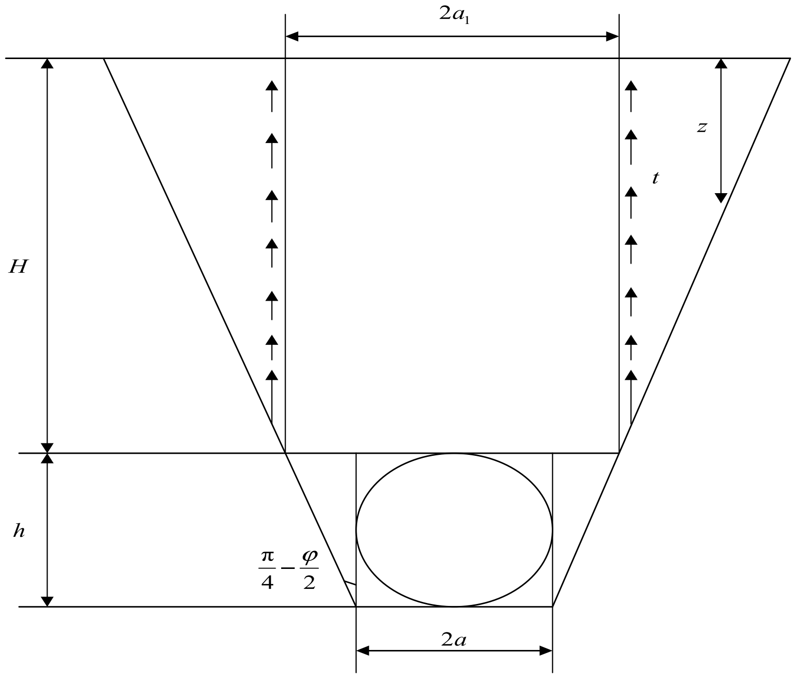


Figure 2. Theoretical calculation diagram of the soil column considering the friction and cohesion.

Considering the tangent equivalent internal friction angle, the formula is given as follows:

$$a_1 = a + h\tan(\frac{\pi}{4} - \frac{\varphi_t}{2}) \tag{9}$$

where a in Equation (9) is half of the tunnel span.

According to the loose body theory, the friction in the original formula is replaced by the tangent equivalent friction as:

$$t = c_t + \sigma_z \tan\varphi_t \tag{10}$$

In Equation (10), σ_z is the Rankine active soil stress:

$$\sigma_z = \gamma z\ \tan^2(\frac{\pi}{4} - \frac{\varphi_t}{2}) - 2c\tan(\frac{\pi}{4} - \frac{\varphi_t}{2}) \tag{11}$$

Integrating Equation (10) along the direction of depth, the lateral resistance of the rock column side is:

$$T = \int_0^H (c_t + \sigma_z \tan\varphi_t)dz = \frac{1}{2}\gamma H^2 k_1 + c_t H(1 - 2k_2) \tag{12}$$

where

$$k_1 = \tan^2(\frac{\pi}{4} - \frac{\varphi_t}{2})\tan\varphi_t \tag{13}$$

$$k_2 = \tan(\frac{\pi}{4} - \frac{\varphi_t}{2}) \tan \varphi_t \quad (14)$$

Therefore, according to the rock column theory, the stress of surrounding rock acting on the top of the tunnel is:

$$q = \frac{G - 2T}{2a_1} = \gamma H[1 - \frac{H}{2a_1}k_1 - \frac{c_t}{a_1\gamma}(1 - 2k_2)] = k\gamma H \quad (15)$$

where

$$k = [1 - \frac{H}{2a_1}k_1 - \frac{c_t}{a_1\gamma}(1 - 2k_2)] \quad (16)$$

When the nonlinear coefficient $m = 1$, the improved theoretical formula for the rock column degenerates to the original one.

In practical engineering, composite strata are common, which are simplified as horizontal strata, as shown in Figure 3.

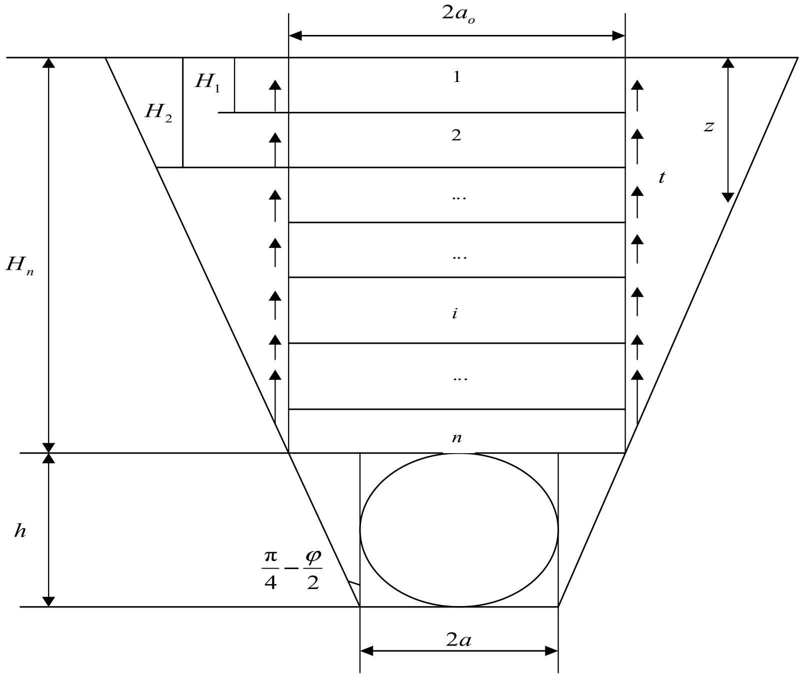


Figure 3. Calculation diagram of the Bierbaumer formula for composite formation.

In calculating the surrounding rock stresses of the composite horizontal strata, the Bierbaumer formula often makes a weighted average for the physical and mechanical parameters of the rock–soil interface, but there is a certain deviation associated with the calculation of the frictional resistance. Due to the discrete variability of physical and mechanical parameters of the rock–soil interface, different strata should be calculated separately.

$$\sigma_{iz} = \gamma_i z \ \tan^2(\frac{\pi}{4} - \frac{\varphi_{it}}{2}) - 2c_t \tan(\frac{\pi}{4} - \frac{\varphi_{it}}{2}),\ i = 1, 2, 3 \ldots n \quad (17)$$

where: i is the stratum number.

Therefore, the friction resistances of the first layer, the second layer and the nth layer are obtained as:

$$T_1 = \int_0^{H_1} (c_t + \sigma_z \tan \varphi_t) dz = \frac{1}{2}\gamma_1 H_1^2 k_{11} + c_t H_1(1 - 2k_{12}) \quad (18)$$

$$T_2 = \int_{H_1}^{H_2} (c_t + \sigma_z \tan\varphi_t)dz = \frac{1}{2}\gamma_2({H_2}^2 - {H_1}^2)k_{21} + c_{1t}(H_2 - H_1)k_{22} \quad (19)$$

$$T_n = \int_{H_{n-1}}^{H_n} (c_t + \sigma_z \tan\varphi_t)dz = \frac{1}{2}\gamma_2(H_n^2 - H_{n-1}^2)k_{n1} + c_{1t}(H_n - H_{n-1})k_{n2} \quad (20)$$

where:

$$k_{11} = \tan^2(\frac{\pi}{4} - \frac{\varphi_{1t}}{2})\tan\varphi_{1t}, k_{12} = \tan(\frac{\pi}{4} - \frac{\varphi_{1t}}{2})\tan\varphi_{1t} \quad (21)$$

$$k_{21} = \tan^2(\frac{\pi}{4} - \frac{\varphi_{2t}}{2})\tan\varphi_{2t}, k_{22} = \tan(\frac{\pi}{4} - \frac{\varphi_{2t}}{2})\tan\varphi_{2t} \quad (22)$$

$$k_{n1} = \tan^2(\frac{\pi}{4} - \frac{\varphi_{nt}}{2})\tan\varphi_{nt}, k_{22} = \tan(\frac{\pi}{4} - \frac{\varphi_{nt}}{2})\tan\varphi_{nt} \quad (23)$$

The total friction resistance can be formed by superposition of the friction resistances of individual layers:

$$T = \sum_i^n T = T_1 + T_2 + \ldots + T_n \quad (24)$$

The total self-weight of the soil masses in different strata can be calculated by the formula:

$$G = \sum_i^n G = G_1 + G_2 + \ldots\ldots + G_n = 2\gamma_1 H_1 a + 2\gamma_2 H_2 a + \ldots\ldots + 2\gamma_n H_n a \quad (25)$$

$$P = \frac{\sum_i^n G - 2\sum_i^n T}{2a_1} = k_1\gamma_1 H_1 + k_2\gamma_2(H_2 - H_1) + k_n\gamma_n(H_n - H_{n-1}) \quad (26)$$

where:

$$k_1 = [1 - \frac{H_1}{2a_1}k_{11} - \frac{c_{1t}}{a_0\gamma_1}(1 - 2k_{12})] \quad (27)$$

$$k_2 = [1 - \frac{H_2 + H_1}{2a_1}k_{11} - \frac{c_{1t}}{a_0}(1 - 2k_{12})] \quad (28)$$

$$k_n = [1 - \frac{H_n + H_{n-1}}{2a_1}k_{n1} - \frac{c_{nt}}{a_0}(1 - 2k_{n2})] \quad (29)$$

In composite strata, due to the different parameters of different strata, the width of the rock column in different strata is different. Therefore, the width of the rock column in the first layer, the second layer and the nth layer is:

$$a_1 = a + h\tan(\frac{\pi}{4} - \frac{\varphi_{1t}}{2}) \quad (30)$$

$$a_2 = a + h\tan(\frac{\pi}{4} - \frac{\varphi_{2t}}{2}) \quad (31)$$

$$a_n = a + h\tan(\frac{\pi}{4} - \frac{\varphi_{nt}}{2}) \quad (32)$$

In order to facilitate the calculation and the conservatism of the results, the method of reference [18] is taken as $a_0 = \max(a_1, a_2, \ldots, a_n)$.

When the tunnel depth reaches a certain value, the surrounding rock stresses decrease; the surrounding rock stresses can even be negative. This situation is not consistent with engineering practice, so the formula derived in this paper can overcome existing defects.

3. Example Analysis and Verification

In order to verify the rationality of the derived formula, the reference [18] data is cited for verification. For a metro shield tunnel project, stratum information is shown in Table 1, and the geological sections and instrument layout are shown in Figure 4. The horizontal surrounding rock stresses at the crown can be obtained from the earth pressure cells on the left and right sides.

Table 1. Ground parameters.

Stratum Information	Density ρ/g/cm^3	c/kPa	φ/°	Lateral Stress Coefficient k
Mixed backfill soil	1.90	50	15	0.45
Strong weathered slate	2.17	65	27	0.30
Moderately weathered slate	2.33	100	36	0.28
Slightly weathered slate	2.65	122	32	0.25

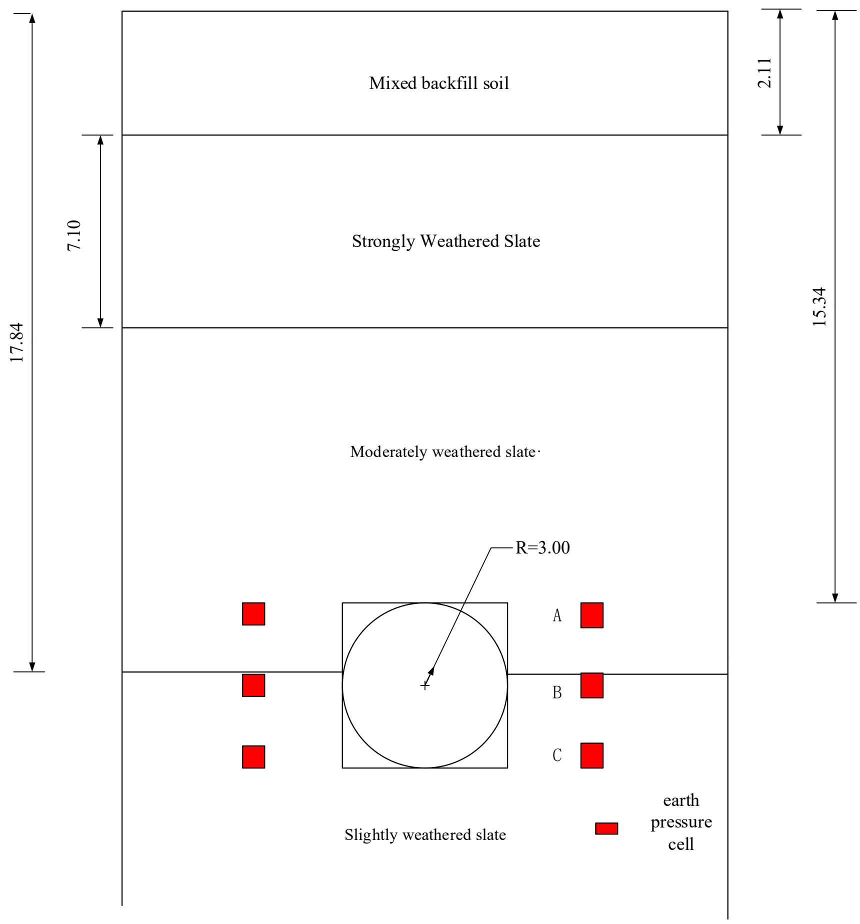


Figure 4. Geological sections and instrument layout (Unit: m).

It can be seen from Table 2 that the improved Bierbaumer formula can more closely predict surrounding rock stress values obtained from monitoring than other formulas. With an increase in the nonlinear coefficient m, the calculated value is closer to the measured value, which shows the rationality of the improved formula to some extent. Because m needs to be tested and the nonlinear coefficient m values of different types of strata should be different, the improved Bierbaumer formula is more adaptable.

Table 2. Comparison with the monitoring data.

Position	Calculation Results/kPa						Monitoring Data/kPa		
	m = 1.0	m = 1.2	m = 1.5	Terzaghi	Bierbaumer	Xie Jiajie	Left Side	Right Side	Average
A	74.57	73.40	68.20	73.78	76.58	80.38	61.87	65.50	63.685
B	80.86	83.00	81.34	83.75	86.25	89.65	75.40	79.49	77.445
C	96.31	94.14	92.15	103.63	106.13	109.53	96.28	101.09	98.685

3.1. Comparative Analysis on Statistical Characteristics of Stresses of Loose Class V and VI Surrounding Rocks

In order to further verify the rationality of the calculation method and considering the discreteness of geotechnical materials and the variations in spatial distributions, the statistical distribution characteristics of the calculated stress results of various rocks surrounding shallow tunnels are explored, which lays the foundation for the reliable design of tunnel lining. In this paper, based on the Monte Carlo method, the statistical laws of stresses of loose surrounding rock obtained by the improved Bierbaumer formula and other shallow tunnel calculation methods are compared. Refer to reference [31] and code TB-10003-2016 for the design of railway tunnels [32]; the physical and mechanical parameters of rock masses corresponding to classes V and VI are shown in Table 3.

Table 3. Statistical distribution of the physical and mechanical parameters of class V and VI surrounding rocks.

Rock Mass Classification	Physical and Mechanical Parameters	Statistical Distribution Type	Mean	Standard Deviation	Coefficient of Variation
V	Unit weight/kN/m^3	Normal	18	0.9	0.05
	Cohesive force/kPa	Normal	46.30	19.45	0.42
	tan φ	Normal	0.57	0.12	0.21
	tan φ_c	Normal	1	0.25	0.25
VI	Unit weight/kN/m^3	Normal	16	0.8	0.05
	Cohesive force/kPa	Normal	12.45	5.60	0.45
	tan φ	Lognormal	0.54	0.11	0.20
	tan φ_c	Normal	0.7	0.25	0.175

Due to the lack of statistical data of the nonlinear coefficients of class V and VI rocks, the values of the nonlinear coefficient m is taken as 1.0–1.5 in this paper. When m is taken as 1.0, 1.2 and 1.5, different tan φ values can be obtained. When m = 1.0, it is the control group, and it is the linear M-C criterion.

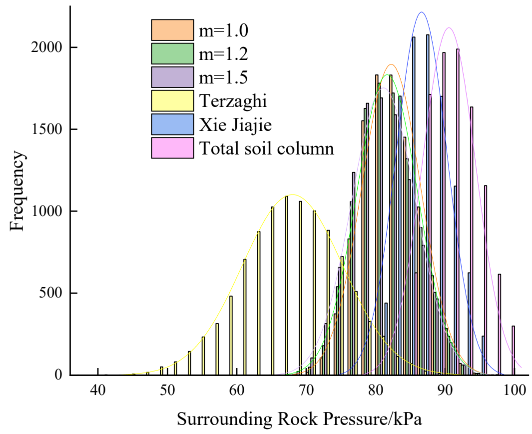
Assuming the span of the rectangular tunnel is 10 m, the burial depth and span of the rectangular tunnel in different numerical test groups are shown in Table 4.

Table 4. Buried depths and spans of tunnels under different coverage ratios.

Group	Thickness–Span Ratio	Depth (m)	Span (m)
1	0.5	5	10
2	1.5	15	10
3	2.5	25	10

According to the statistical distribution shown in Table 3, a certain random number is generated by MATLAB, and the calculation results of different burial depths (5 m, 15 m and 25 m) are obtained. After K-S tests, the calculation results obey the normal distribution, as shown in the Figures 5 and 6, and the sensitivity analysis results are shown in Figure 7.

Through variance analysis, the mean values and standard deviations of the surrounding rock stresses calculated by individual formulas are significantly different. As shown in Figure 5, in class V surrounding rock, when the burial depth of the tunnel is 5 m, the average values of the surrounding rock stresses calculated by individual formulas follow the order: total soil column formula > Xie Jiajie formula > m = 1.5 > m = 1.2 > m = 1.0 > Terzaghi formula. When the burial depth of the tunnel is 15 m, the mean values of the surrounding rock stresses calculated by individual formulas follow order: total soil column formula > Xie Jiajie formula > m = 1.0 > m = 1.5 > m = 1.2 > Terzaghi formula. When the burial depth of the tunnel is 25 m, the mean values of the surrounding rock stresses calculated by individual formulas follow the order: total soil column formula > Xie Jiajie formula > m = 1.5 > m = 1.2 > m = 1.0 > Terzaghi formula.



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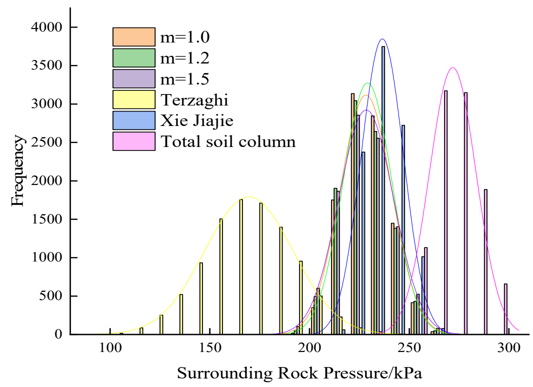

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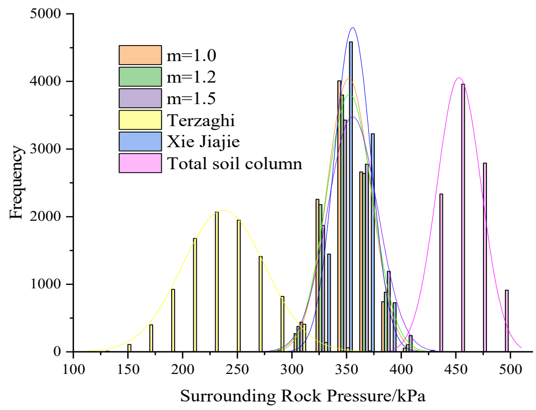

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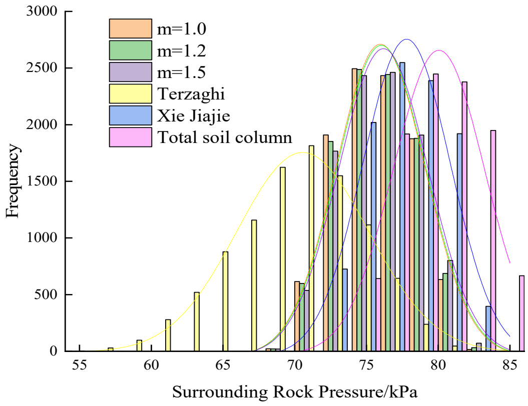
Figure 5. *Cont.*



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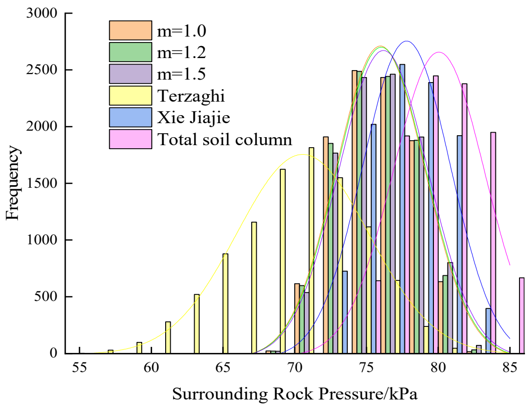

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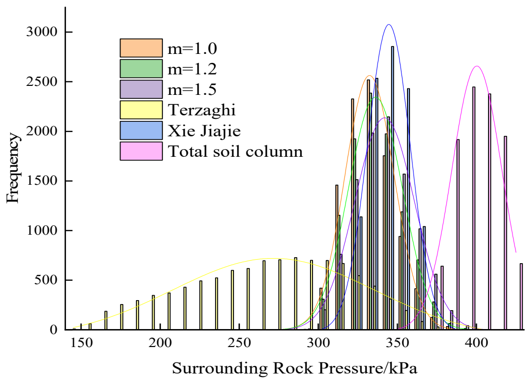


(**f**)

Figure 5. Statistical characteristics of class V and VI surrounding rocks at different burial depths. (**a**) Statistical characteristics of the surrounding rock stresses at 5 m burial depth for class V surrounding rock. (**b**) Statistical characteristics of the surrounding rock stresses at 15 m burial depth for class V surrounding rock. (**c**) Statistical characteristics of the surrounding rock stresses at 25 m burial depth for class V surrounding rock. (**d**) Statistical characteristics of the surrounding rock stresses at 5 m burial depth for class VI surrounding rock. (**e**) Statistical characteristics of the surrounding rock stresses at 15 m burial depth for class VI surrounding rock. (**f**) Statistical characteristics of the surrounding rock stresses at 25 m burial depth for class VI surrounding rock.

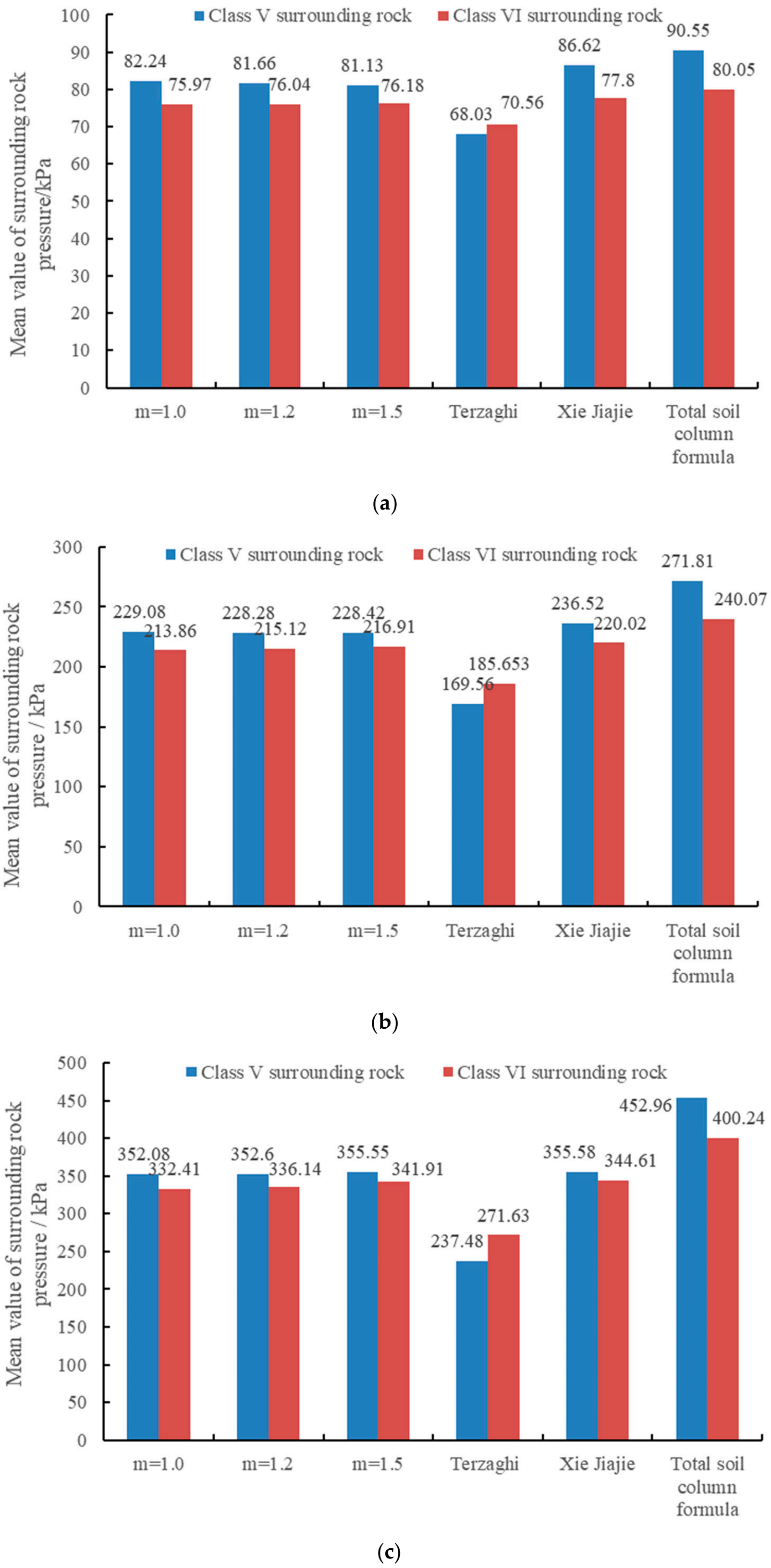


Figure 6. Mean values of the surrounding rock stresses at different burial depths. (**a**) Mean values of the surrounding rock stresses using individual formulas at 5 m burial depth. (**b**) Mean values of the surrounding rock stresses using individual formulas at 15 m burial depth. (**c**) Mean values of the surrounding rock stresses using individual formulas at 25 m burial depth.

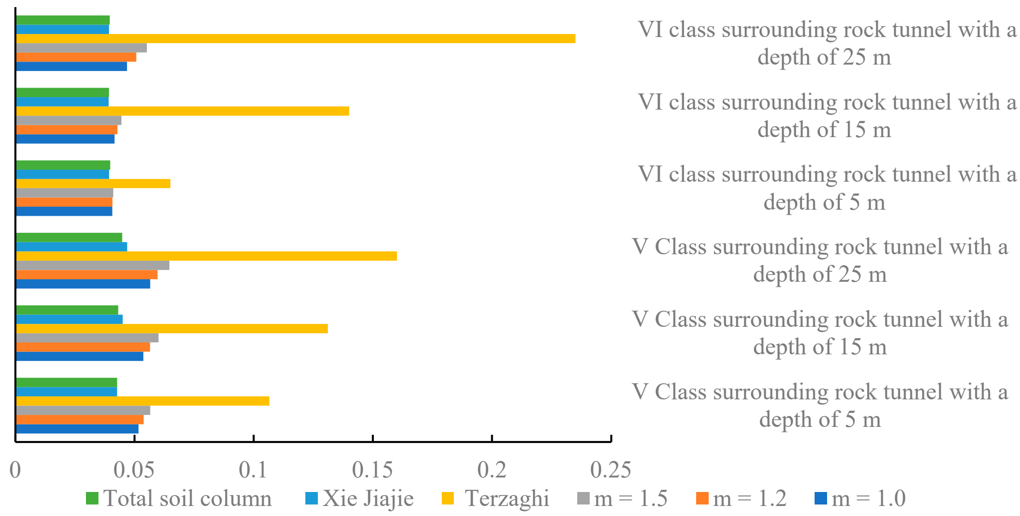


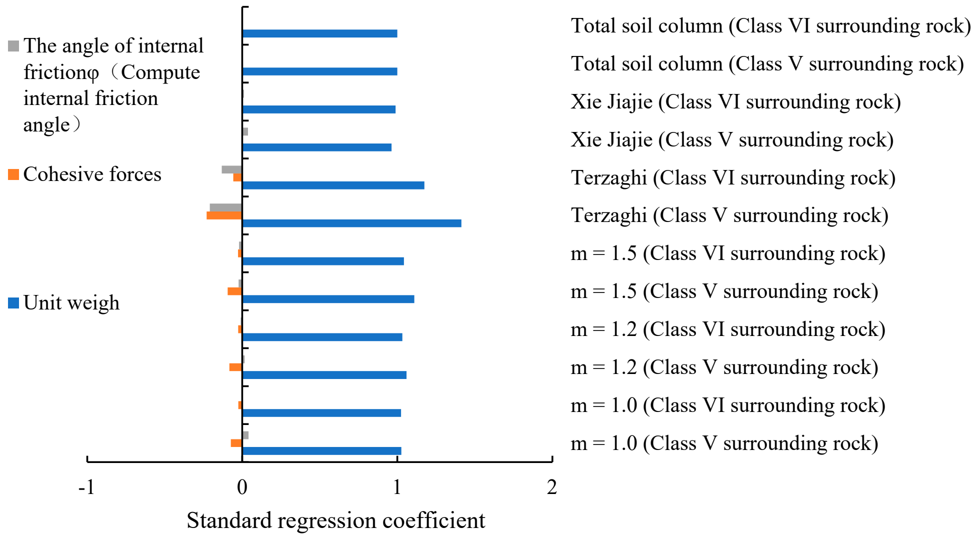
Figure 7. Analysis of coefficient of variation.

As shown in Figure 6, for class VI surrounding rock, when the burial depth of the tunnel is 5 m, the mean values of the surrounding rock stresses calculated by individual formulas follow the order: total soil column formula > Xie Jiajie formula > m = 1.5 > m = 1.2 > m = 1.0 > Terzaghi formula. When the burial depth of the tunnel is 15 m, the mean values of the surrounding rock stresses calculated by individual formulas follow the order: total soil column formula > Xie Jiajie formula > m = 1.5 > m = 1.2 > m = 1.0 > Terzaghi formula. When the burial depth of the tunnel is 25 m, the mean values of the surrounding rock stresses calculated by individual formulas follow the order: total soil column formula > Xie Jiajie formula > m = 1.5 > m = 1.2 > m = 1.0 > Terzaghi formula. As for the dispersion degree of data, through the comparison of the coefficients of variation in Figure 7, it can be concluded that the dispersion degree of the improved Bierbaumer formula is between that of the Xie Jiajie formula and the Terzaghi formula, and the dispersion degree increases with an increase in the burial depth.

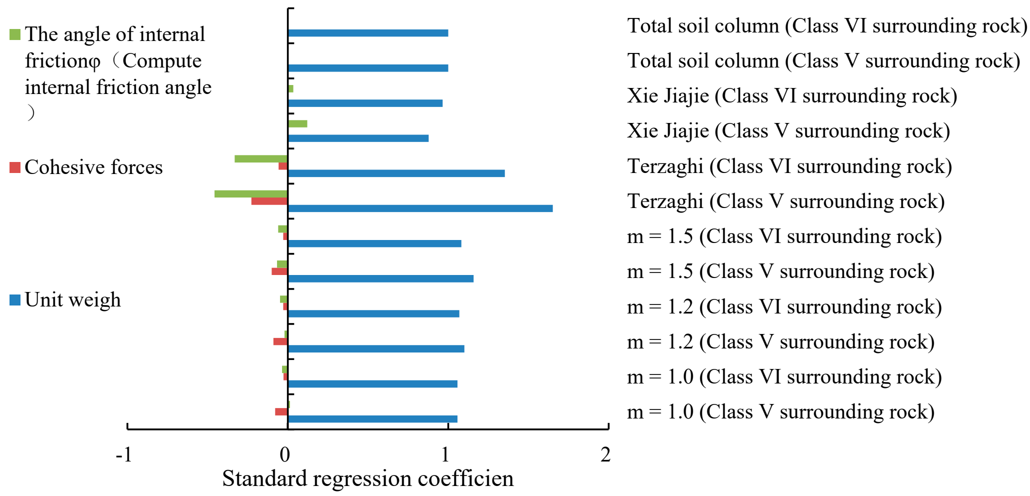
When the burial depth reaches a certain value, with an increases in the stress and surrounding rock level (the geological engineering conditions become worse), the calculated results of the improved Bierbaumer formula are larger than those for m = 1.0, which is due to the adoption of the nonlinear criterion correction result.

For further discussion, the formulas' parameter sensitivities in different cases are analyzed with standardized regression coefficient beta as shown in Figure 8.

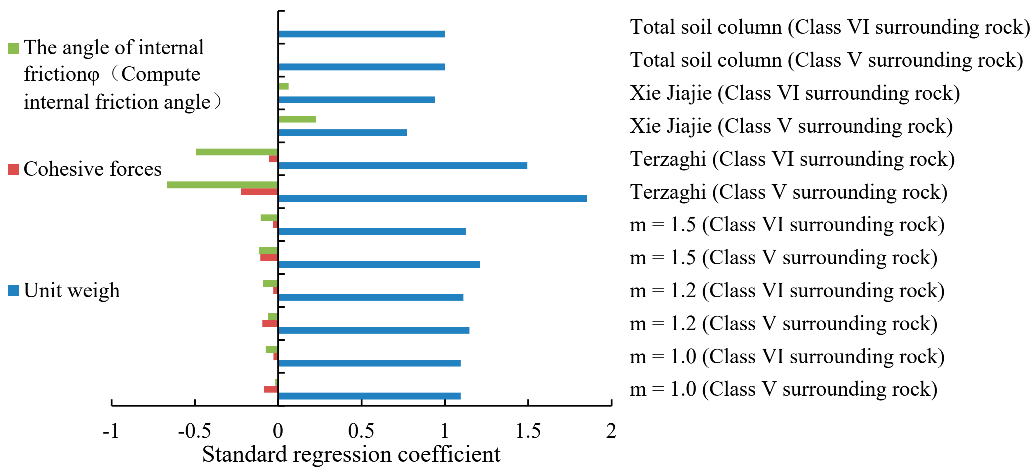
In the case of the same burial depth, the calculation results of the surrounding rock stresses of class VI surrounding rocks are less than those of class V surrounding rocks by the improved Bierbaumer formula, the Xie Jiajie formula and the total soil column formula, which shows that the surrounding rock gravity is an important index affecting the surrounding rock stresses of shallow tunnels with the greatest impact. For the Terzaghi formula, mechanical parameters, such as cohesion and friction angle, may affect the calculation results more than the above formulas, and more attention should be paid to the self-bearing capacities of surrounding rocks in soft and shallow-buried strata. The Terzaghi formula is more suitable for shallow tunnels with better geological engineering conditions. The improved Bierbaumer formula (including the theory of principle), the Xie Jiajie formula and the total soil column formula are suitable for shallow tunnels with poor geological engineering conditions. This can be seen from the parameters' sensitivity in the improved Bierbaumer formula corresponding to different m values, i.e., the improved Bierbaumer formula improves the deficiency of the original Bierbaumer formula when the friction angle exceeds 30°.



(**a**)



(**b**)



(**c**)

Figure 8. Standard regression coefficients using different formulas for different burial depths. (**a**) Standard regression coefficients at 5 m burial depth. (**b**) Standard regression coefficients at 15 m burial depth. (**c**) Standard regression coefficients at 25 m burial depth.

According to the results of the confidence analysis in Table 5, the confidence interval widths of different formulas for class V and VI surrounding rocks are ranked in the following order: Terzaghi formula > m = 1.5 > m = 1.2 > m = 1.0 > total soil column formula > Xie Jiajie formula. The confidence interval width of the improved Bierbaumer formula is between the commonly used formulas and increases with an increase in the burial depth, which shows that its statistical law is the same as other formulas.

Table 5. Bilateral 95% confidence analysis.

Depth	m = 1.0	m = 1.2	m = 1.5	Terzaghi	Xie Jiajie	Total Soil Column
5 m (Class V surrounding rock)	[82.16, 82.32]	[81.57, 81.75]	[81.04, 81.22]	[67.89, 68.17]	[86.55, 86.69]	[90.47, 90.63]
15 m (Class V surrounding rock)	[228.84, 229.32]	[228.03, 228.53]	[228.15, 228.69]	[169.12, 170.00]	[236.31, 236.73]	[271.58, 272.04]
25 m (Class V surrounding rock)	[351.69, 352.47]	[352.19, 353.01]	[355.10, 356.00]	[236.73, 238.23]	[355.25, 355.91]	[452.56, 453.36]
5 m (Class VI surrounding rock)	[75.91, 76.03]	[75.98, 76.10]	[76.12, 76.24]	[70.47, 70.65]	[77.74, 77.86]	[79.99, 80.11]
15 m (Class VI surrounding rock)	[213.69, 214.03]	[214.94, 215.30]	[216.72, 217.10]	[185.14, 186.16]	[219.85, 220.19]	[239.88, 240.26]
25 m (Class VI surrounding rock)	[332.10, 332.72]	[335.81, 336.47]	[341.54, 342.28]	[270.38, 272.88]	[344.34, 344.88]	[399.93, 400.55]

3.2. Comparative Analysis of Statistical Characteristics of Surrounding Rock Stress in Composite Strata

The validity and rationality of the improved Bierbaumer formula in composite strata are verified. However, considering the variability and discreteness of the geotechnical materials, the statistical characteristics of the surrounding rock stresses in composite strata are calculated by the improved Bierbaumer method, and the other formulas are used to analyze the sensitivities of the soil parameters. It is assumed that the burial depth of the tunnel is 10 m and the span is 10 m. The combination of class VI surrounding rock in the upper layer and class V surrounding rock in the lower layer is discussed. The thickness of class VI surrounding rock is set at 2.5 m, 5 m and 7.5 m, respectively, as shown in Figure 9. In the same way, the Monte Carlo method is used, and the improved Bierbaumer Formula with m = 1.5 is used to obtain the statistical characteristics of different calculation results of surrounding rock stresses, as shown in Figures 10–12.

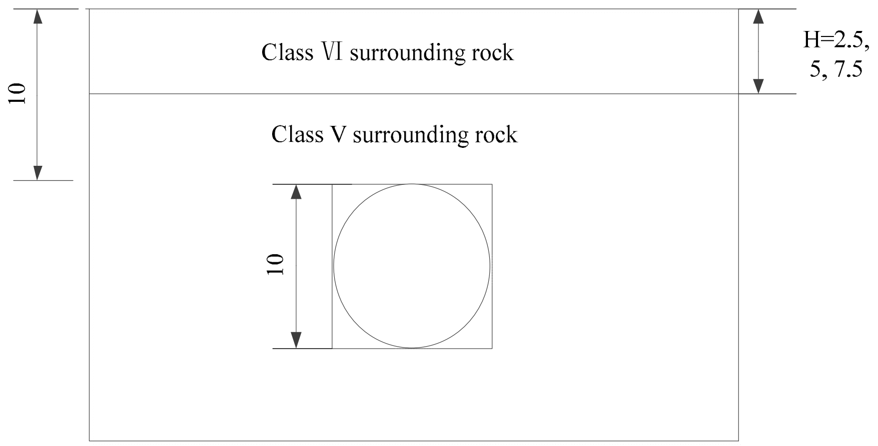
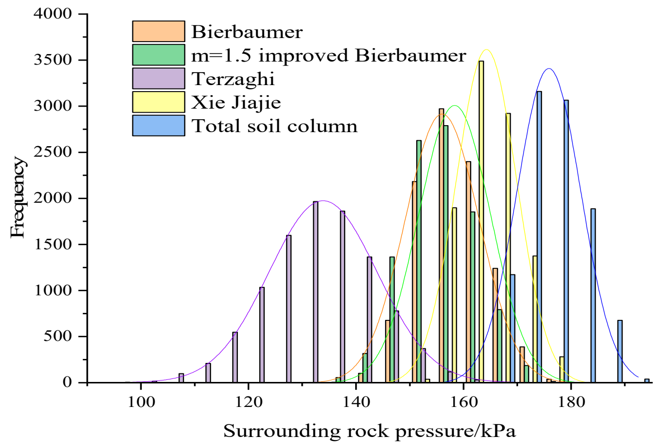
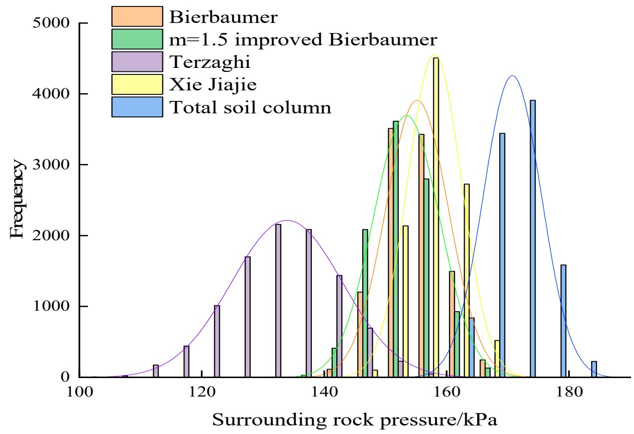


Figure 9. Composite surrounding rock strata (Unit: m).



(**a**)



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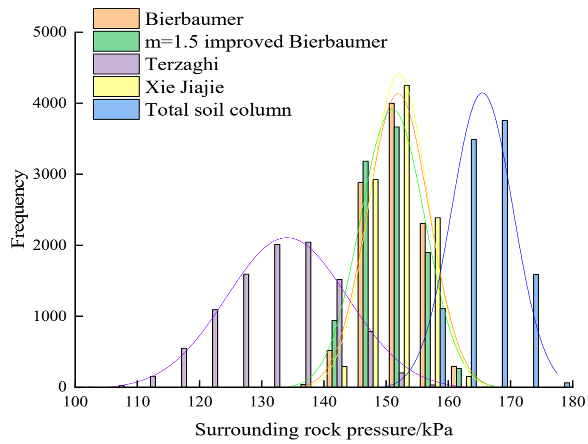

(**c**)

Figure 10. Statistical characteristics of surrounding rock stress in different layers. (**a**) Statistical characteristics of the surrounding rock stresses for 2.5 m thick class VI surrounding rock. (**b**) Statistical characteristics of the surrounding rock stresses for 5 m thick class VI surrounding rock. (**c**) Statistical characteristics of the surrounding rock stresses for 7.5 m thick class VI surrounding rock.

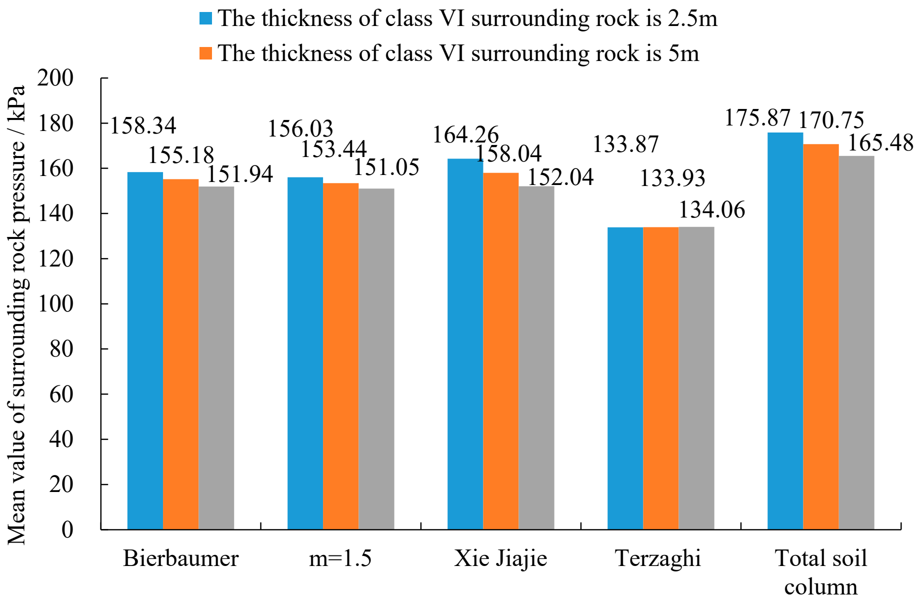

Figure 11. Mean values of the surrounding rock stresses in different layers.

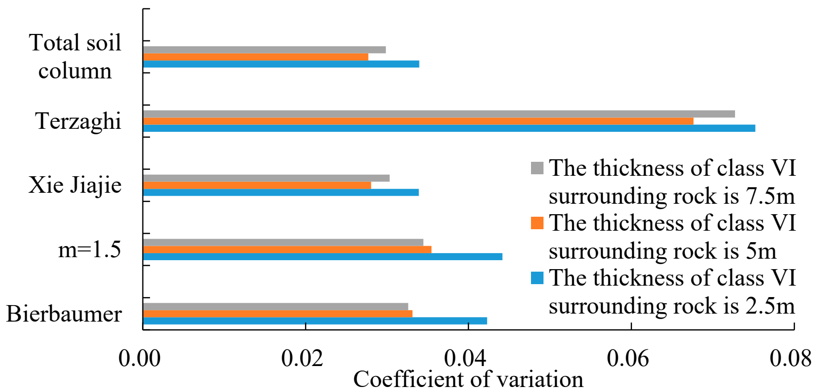

Figure 12. Coefficients of variation using individual formulas in different layers.

Through variance analysis, the mean values and standard deviations of the surrounding rock stresses calculated by individual formulas are significantly different. In terms of the mean values of the surrounding rock stresses, total soil column formula > Xie Jiajie formula > Bierbaumer formula > improved Bierbaumer formula > Terzaghi formula. With an increase in the layer thickness of class VI surrounding rock, the mean values of the surrounding rock stresses using the improved Bierbaumer formula, Bierbaumer formula, Xie Jiajie formula and the total soil column formula with m = 1.5 decrease, and those of the Terzaghi law are contrary, which is consistent with the above conclusion. The mean values of the improved Bierbaumer formula's surrounding rock stresses are between those of the Terzaghi formula and the Bierbaumer formula, which shows that the formula is statistically reasonable.

From Table 6, the bilateral 95% confidence intervals of individual formulas are studied. The confidence interval width of the improved Bierbaumer formula decreases with increases in the class VI surrounding rock thickness, which is the same as those obtained by Bierbaumer, Terzaghi, Xie Jiajie and total soil column formulas. The widths of the confidence intervals follow the order: Terzaghi > improved Bierbaumer formula > Bierbaumer formula > total soil column formula > Xie Jiajie formula. That is to say, the width of the confidence interval using the improved Bierbaumer formula is between the values obtained by the Bierbaumer formula and the Terzaghi formula, which also shows the rationality of the formula in statistical significance.

Table 6. Bilateral 95% confidence interval analysis.

Thickness (Class VI Surrounding Rock)	Bierbaumer Confidence Interval	m = 1.5 Confidence Interval	Xie Jiajie Confidence Interval	Terzaghi Confidence Interval	Total soil Column Confidence Interval
2.5 m	[158.21, 158.48]	[155.89, 156.16]	[164.15, 164.37]	[133.67, 134.07]	[175.75, 175.99]
5 m	[155.08, 155.28]	[153.33, 153.55]	[157.95, 158.12]	[133.75, 134.11]	[170.66, 170.84]
7.5 m	[151.85, 152.04]	[150.95, 151.15]	[151.95, 152.14]	[133.87, 134.26]	[165.38, 165.58]

4. Conclusions

(1) Based on the nonlinear M-C criterion, a nonlinear improvement was made to the original Bierbaumer formula, and a formula for calculating the surrounding rock stress in composite strata was provided, and its rationality was verified. The influence of nonlinear coefficient m on the results of the surrounding rock stress was revealed through Monte Carlo simulation statistics and weight analysis;

(2) We analyzed the calculated results of the formula proposed in this paper using the mean, coefficient of variation and confidence and obtained statistical rationality. Finally, it is concluded that the formula proposed in this paper can solve the shortcomings of the original Bierbaumer formula when the internal friction angle is large;

(3) By analyzing the mean and sensitivity of the calculated results of various surrounding rock stress formulas modeling shallow-buried tunnels, it can be concluded that the improved Bierbaumer formula in this paper is more suitable for calculating the surrounding rock stress of shallow-buried tunnels.

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Data Availability Statement: The data presented in this study are available upon reasonable request from the corresponding author. The data are not publicly available due to privacy.

Conflicts of Interest: Author Rong Jiang is employed by the Shanxi Geological Group Co., Ltd. Author Zhao Yang is employed by the Guangdong Hualu Transport Technology Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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