

Article

Research and Design of the RF Cavity for an 11 MeV Superconducting Cyclotron

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Featured Application: The focus of this paper is on the design of the RF (Radio Frequency) resonant cavities required for small modular superconducting cyclotrons. The design of these RF cavities aims to match the smaller extraction radius of superconducting cyclotrons and make easier of installation and maintenance. To this purpose, one has to adopt a design that allows for easy assembly and disassembly with the magnet system that has been adopted. Through the study of the circuitry of such RF cavities, along with calculations of electromagnetic fields and methods of frequency tuning, we provide a set of solutions that can serve as a reference for the design of smaller superconducting accelerators in the future. This not only helps in advancing the technology of superconducting cyclotrons but also can provide a method reference for related technical fields.

Abstract: In contrast to the room temperature cyclotron, the superconducting cyclotron's high operational magnetic field and small extraction radius lead to a magnet design with a reduced radius. This limits the space available for the RF cavity in the 11 MeV superconducting cyclotron, necessitating a more compact RF cavity design. By using the transmission line theory, the complex structure of the quarter-wavelength coaxial cavity can be represented as a microwave circuit. Through relevant theoretical analytical formulas, equivalent circuit parameters can be derived. The resonant frequency of the RF cavity is then determined using the equivalent circuit method. The optimization of the RF cavity structure was achieved by creating a numerical model and conducting finite element numerical calculations on the high-frequency resonant system. The comparative results between the equivalent circuit and numerical calculations indicate that the frequency error remains within 0.1%, validating the compact RF cavity design. A multiple linear regression analysis facilitates the prediction of resonance frequency across various parameter variables. By analyzing the fitting formula, RF cavity machining error requirements are established, ensuring a prediction error within 1%, thus meeting engineering design criteria.

Keywords: superconducting cyclotron; compact resonant cavity; microwave equivalent circuit; multiple linear regression



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1. Introduction

Cyclotrons are pivotal tools for the production of medical radioisotopes due to their lower construction and operational costs compared to reactors [1–3]. With the growing prominence of positron emission tomography (PET) in biomedical imaging, the demand for positron-emitting isotopes has surged. Notably, ¹¹C and ¹⁸F stand out as the most effective PET isotopes, characterized by their short half-lives of 20 and 108 min, respectively. Currently, the predominant method of producing ¹¹C and ¹⁸F involves the use of low-energy cyclotrons (with energies below 15 MeV). A prevailing trend in the development of these cyclotrons is the emphasis on creating more compact, lightweight, and cost-efficient

models [4–6]. Furthermore, there is a growing preference for cyclotrons that can operate in smaller settings and produce isotopes in proximity to patients. This approach aims to mitigate transportation challenges, minimize isotope activity loss, substantially reduce production costs, decrease the demand for associated infrastructure, and curtail expenses related to subsequent auxiliary equipment [7].

In the overall design of the cyclotron, a modularized assembly approach is adopted, allowing for the independent design of its key systems, including the particle source system, magnet system, and RF system [8,9]. During the preliminary design phase, it is essential to determine the specific parameters for each system and finalize the design of each independent module. Assembly is then carried out upon the completion of the research and design for each system. This modular design strategy not only curtails the research costs but also offers flexibility during maintenance, enabling the replacement of only the malfunctioning system. Such an approach enhances the development efficiency of the cyclotron, simplifies its debugging process, and reduces both operational and maintenance expenses.

For low-energy superconducting cyclotrons, the heightened magnetic field necessitates a reduced magnet radius for particle acceleration, leading to a diminished radial dimension for the RF cavity situated between the upper and lower magnets. Employing a conventional quarter-wavelength coaxial cavity would result in a length that significantly surpasses the radius of both the magnet and the vacuum chamber. This underscores the need for research into a more compact RF cavity design. To address the impedance matching challenge, an SIR (Stepped Impedance Resonator) structure is introduced. This resonator comprises a combination of two or more transmission lines, each with distinct characteristic impedances. The design permits an adjustment of the impedance ratio based on the radial size requirement, facilitating the resonator's miniaturization. Furthermore, the quarter-wavelength coaxial SIR structure employed in this design is straightforward and amenable to fabrication [10].

The RF cavity is a pivotal component of the cyclotron primarily responsible for harnessing the accelerating electric field produced by high-frequency power coupling to propel charged particles. The RF cavity's performance is intricately tied to its structural dimensions. Precise modeling and simulations are imperative to ascertain the design dimensions that align with the desired frequency. Cyclotron RF cavities, given their intricate structures, are typically analyzed and designed through both numerical and equivalent circuit calculations. For numerical evaluations, the common numerical method of high-frequency electromagnetic field calculation is the Finite Element Method (FEM), so the Finite Element Method is predominantly employed in the article [11]. For complex cavities, a unified computer modeling and calculation method can be used, which greatly simplifies the process and difficulty of resonant cavity analysis. At the same time, it can also be compared and verified with the equivalent circuit method. The equivalent circuit calculation leans on transmission line theory, which translates a complex coaxial cavity into multiple transmission lines, effectively rendering the resonant cavity as a standard microwave circuit model. This model streamlines the assessment of how each segment's structural parameters influence the cavity's characteristic parameters and offers a more direct route to design parameters. During this analytical process, pertinent circuit parameters are extracted, with the RF cavity's resonant frequency ascertained using the equivalent circuit [12,13]. Both methodologies are concurrently employed in the design phase, with their results juxtaposed to bolster the precision and credibility of the resonant cavity analytical calculations.

2. Miniaturization of RF Cavity for an 11 MeV Superconducting Cyclotron

2.1. Structure of the Cyclotron's RF Cavity Using the Stepped Impedance Resonator (SIR) Approach

Figure 1 illustrates the design of a quarter-wavelength coaxial cavity of a classical cyclotron. The short-circuit (S.C.) end is located at the Stem's end, while the open-circuit (O.C.) end is at the Dee's head, where the Dee is the accelerating electrode, and the Stem is the component that supports and fixes the Dee, and senses and balances the electric capacity

of the Dee during resonance. This entire cavity can be conceptualized as an SIR structure, composed of two transmission line sections with distinct characteristic impedances. The characteristic impedances at the short-circuit and open-circuit ends of the transmission line are Z_1 and Z_2 , respectively. Their electrical lengths are represented by θ_1 and θ_2 , where $\theta = \beta l$. In this equation, β stands for the propagation constant, and l signifies the length of the transmission line. For simplicity in analysis, the characteristic impedances Z_1 and Z_2 of the structure are assumed to be uniformly distributed [10,14,15].

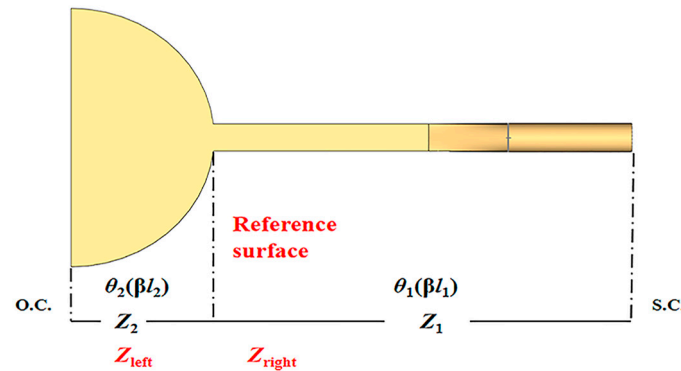


Figure 1. The electrical parameters of the cyclotron Dee's SIR structure.

Figure 1 can be regarded as the basic structure of the SIR, which includes an open-circuit end, a short-circuit end, and a step-junction surface. Here, O.C. denotes the open-circuit end, S.C. represents the short-circuit end, Z_{left} is the input impedance on the left side of the reference surface, and Z_{right} is the input impedance on the right side of the reference surface. According to the input impedance formula [16]:

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \theta}{Z_0 + jZ_L \tan \theta} \quad (1)$$

where Z_0 is the characteristic impedance of the transmission line and Z_L is the characteristic impedance of the end load. The left and right input impedances can be calculated by selecting the step junction surface as the input reference surface. When the step junction discontinuity and the edge capacitance effect at the open-circuit end are neglected, the total susceptance at the reference surface is zero, and the impedance ratio R_{Z0} is calculated as:

$$R_{Z0} = \frac{Z_2}{Z_1} = \tan \theta_1 \tan \theta_2 \quad (2)$$

The resonance conditions of the structure depend on θ_1 , θ_2 , and the impedance ratio R_{Z0} . Before the introduction of the SIR structure, the resonance condition of a typical quarter-wavelength coaxial cavity depended only on the length of the transmission line, which had one less degree of design freedom compared to the SIR structure.

After normalizing the total length L of the cavity, L obtains a minimal value of L_n at $0 < R_{Z0} < 1$:

$$L_n = \frac{4 \arctan \sqrt{R_{Z0}}}{\pi} \quad (3)$$

According to this formula, the impedance ratio R_{Z0} critically affects the length of the normalized resonator. Thus, theoretically, a smaller impedance ratio can be utilized to reduce the cavity's length. The design strategy involves increasing the transmission line's transverse dimension to compensate for a reduction in its longitudinal dimension. This adjustment aligns with the design needs of a superconducting cyclotron that requires a smaller radial dimension. Consequently, cavity miniaturization can be achieved by optimizing the shape and structural parameters of the RF cavity's Dee and Stem.

2.2. Effects of Discontinuity

The above results are achieved by neglecting discontinuities in the structure shown in Figure 1. In order to obtain a more accurate resonant frequency formulation, transmission line discontinuities, including open-end capacitance and step junction effects, must be considered. The open-end capacitance can be directly replaced by an additional length of transmission line during the actual design process. The step junction effects are mainly the discontinuity capacitance generated by the step junction in the transmission line [10]. The reason for this is that the connection of two transmission lines with different characteristic impedances excites higher TM modes due to impedance discontinuities, but these higher TM modes are cut off, and the resulting effect can be replaced by an equivalent capacitance. In the calculation, this equivalent capacitance can be obtained either by numerical analysis of the electromagnetic field distribution at the step junction surface or by using the theoretical formulas in the conclusions of existing studies [12,13]. After considering the above discontinuities, a more accurate expression for the resonant frequency can be further derived.

The step junction surface is selected as the reference surface, and the input admittance at the open-circuit end ($Y_{O.C}$), at the short-circuit end ($Y_{S.C}$), and the admittance due to the impedance discontinuity (Y_S) are, respectively:

$$\begin{aligned} Y_{O.C} &= j \frac{1}{Z_2} \tan \theta_2 \\ Y_{S.C} &= -j \frac{1}{Z_1} \cot \theta_1 \\ Y_S &= j\omega C_d \end{aligned} \quad (4)$$

where ω is the angular resonant frequency and C_d is the step capacitance. The resonance condition can be obtained according to the susceptance method as:

$$B = -\frac{1}{Z_1} \cot \theta_1 + \frac{1}{Z_2} \tan \theta_2 + \omega C_d = 0 \quad (5)$$

Therefore, the SIR resonance condition, which accounts for the discontinuity capacitance of the transmission line step junction, can be obtained from the derivation as follows:

$$R_Z = \frac{Z_2}{Z_1} = \frac{\tan \theta_1 \tan \theta_2}{1 - \omega C_d Z_1 \tan \theta_1} \quad (6)$$

where R_Z is the impedance ratio after considering the effects of discontinuity.

According to the above equation, the resonant frequency of the cavity can be calculated after determining the characteristic impedance of the different transmission lines and the step junction capacitance.

The above calculations are mainly for the division of the cavity into two transmission lines. When the cavity structure is more complex or to achieve greater accuracy, it becomes necessary to perform a more detailed equivalence of the cavity. This analysis includes determining the location of the step junction surface and dividing the cavity into multiple segments of transmission lines, each with different characteristic impedances. Subsequently, all these equivalent transmission lines are integrated, ultimately yielding the equivalent circuits of the complex cavity. As a result, the transcendental equations, including the cavity's resonant frequency derived from an equivalent circuit model with a more intricate structure, will be more elaborate. However, the computational methods and principles remain consistent with the previously described approach.

3. Equivalent Circuit of the RF Cavity

3.1. Equivalent Circuit Model of RF Cavity

According to the principle of equivalent transmission line, the waveguide-based RF cavity of the cyclotron can be decomposed and equated into multiple transmission lines. In this way, the boundary-value problem of the three-dimensional electromagnetic field can be

simplified into the boundary-value problem of the two-dimensional Helmholtz equation, which is satisfied by the transverse-mode function and the problem of the one-dimensional coupled transmission line equation, which is satisfied by the mode voltages and the mode currents. Since only the longitudinal component of the propagation constant of the TEM mode exists in the coaxial cavity of the cyclotron, the longitudinal propagation constant is equal to the propagation constant k_0 of a plane wave in free space, and the transverse component is zero [12,13].

The 11 MeV superconducting cyclotron RF cavity can be viewed as consisting of the inner wall of the cavity, the Stem, the Dee, and the Dummy Dee, excluding the frequency tuning structure and so on. One side of the Stem is connected to the end of the chamber's inner wall, and the other side is connected to the Dee. The Dee and the Dummy Dee act as the accelerating electrodes, which are electrically connected to each other.

According to the transmission line theory, the cavity wall and Stem can be equated to a section of the transmission line, and the main mode of the transmission line is the TEM mode. Similarly, the cavity wall and Dee can be equated to a section of the transmission line. Due to the characteristics of the TEM mode, when propagating inside the coaxial cable, the phase constants of the two different segments are equal to the phase constants of the free space [17].

According to the above theory, the equivalent of the Stem and the cavity wall can be divided into three segments: a rectangular coaxial segment, a transitional coaxial segment, and a round coaxial segment. As a result, the RF cavity can be equated to four coaxial transmission line segments. Because the difference in characteristic impedance across transmission lines produces discontinuities and consequently higher modes, it can be replaced by the equivalent capacitance C_d . In addition, the distributed capacitance C_a between Dee and Dummy Dee needs to be considered. The equivalent circuit of the cavity is shown in Figure 2.

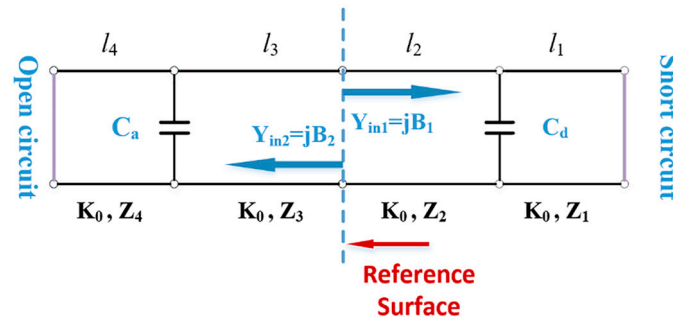


Figure 2. The equivalent circuit of an 11 MeV superconducting cyclotron's RF cavity.

The microwave equivalent circuit of the RF cavity can be expressed using the coaxial transmission line with distributed parameters and the lumped parameter capacitance based on the preceding theory. According to the transverse resonance method, it is known that the sum of the total susceptance at any reference plane of the equivalent circuit at resonance is zero. As a result, a transcendental equation containing the resonant frequency can be obtained, and the resonant frequency of the cavity can be obtained by solving this transcendental equation using the iterative method after the relevant circuit parameters of the microwave equivalent circuit have been determined [18].

For the convenience of calculation, the contact surface between the rectangular coaxial transmission line and the transitional coaxial transmission line is selected as the reference surface. The input admittance is then calculated according to [16],

$$Y_{in} = Y_0 \frac{Y_L + jY_0 \tan \beta l}{Y_0 + jY_L \tan \beta l} \quad (7)$$

where Y_{in} is the input admittance, and Y_0 and Y_L are the characteristic admittance and load admittance of the transmission line, respectively. Thus, for a two-segment transmission line, when the load end is the short-circuit end, its input admittance is [19],

$$Y_{in1} = Y_2 \frac{-jY_1 \cot \beta l_1 + jY_2 \tan \beta l_2}{Y_2 - Y_1 \cot \beta l_1 \tan \beta l_2} \quad (8)$$

where Y_1 and Y_2 are the characteristic admittance of transmission line 1 and transmission line 2, respectively. When the load end is the open-circuit end, its input admittance is,

$$Y_{in2} = Y_3 \frac{jY_4 \tan \beta l_4 + jY_3 \tan \beta l_3}{Y_3 - Y_4 \tan \beta l_4 \tan \beta l_3} \quad (9)$$

where Y_3 and Y_4 are the characteristic admittance of transmission line 3 and transmission line 4, respectively.

3.2. Determination of Equivalent Circuit Parameters

After selecting the reference surface, the transmission line model can be divided into four segments: the Dee segment, the rectangular coaxial transmission line, the transitional coaxial transmission line, and the round coaxial transmission line. Using the input admittance formula and taking into account the step junction capacitance and the Dee–Dummy Dee local capacitance, a transcendental equation including the resonance frequency can be derived. For calculating the resonant frequency, the characteristic impedance, total step junction capacitance, and local capacitance of each section of the equivalent transmission line must be determined. The characteristic impedance of each part of the coaxial transmission line can be calculated using the following formula, which includes the round coaxial transmission line (Z_{c0}), the rectangular coaxial transmission line (Z_{r0}) [20], and the transition coaxial transmission line (Z_{cr}):

Where an approximate method is used to calculate the characteristic impedance of the transition coaxial transmission line. The characteristic impedances of the coaxial transmission line with rectangular and circular inner conductors are obtained numerically and analytically, respectively. Then, the ratio of the average value of the two to the minimum value between them is taken as the approximation coefficient of Z_{cr} .

$$\begin{aligned} Z_{c0} &= 60 \ln \left(\frac{D}{d} \right) \Omega \\ Z_{r0} &= \frac{\eta_0}{4\sqrt{\epsilon_r}} \left[\frac{w/b + \frac{2}{\pi} \ln \left(\frac{1}{1-t/b} + \coth \frac{\pi a}{2b} \right)}{1} \right] \Omega \\ Z_{cr} &= n_{ac} \cdot 60 \ln \left(\frac{D}{d} \right) \Omega \end{aligned} \quad (10)$$

where D and d are the diameters of the outer and inner conductors of the round coaxial transmission line, respectively. In addition, $\eta_0 = (\mu_0/\epsilon_0)^{1/2} = 376.73 \Omega$, ϵ_r is the relative dielectric constant, w and t are the length and width of the inner conductor of the rectangular coaxial transmission line, a and b are the length and width of the outer conductor of the rectangular coaxial transmission line, and n_{ac} is the approximation coefficient of Z_{cr} .

The step capacitance distribution per unit length can be obtained by utilizing the step capacitance formula as [21]:

$$C(\alpha) = \frac{\epsilon}{\pi} \left(\frac{\alpha^2 + 1}{\alpha} \ln \frac{1 + \alpha}{1 - \alpha} - 2 \ln \frac{4\alpha^2}{1 - \alpha^2} \right) \quad (11)$$

where $\alpha = A/B$, where A is the minimum clearance distance between the inner and outer conductors and B is the maximum clearance distance between the inner and outer conduc-

tors. Therefore, the distributed capacitance due to discontinuities in the vertical (C_v) and horizontal (C_h) directions can be calculated separately:

$$\begin{aligned} C_v &= 2mC(\alpha) \\ C_h &= 2nC(\alpha) \end{aligned} \quad (12)$$

where m is the length of the inner conductor in the vertical direction and n is the width of the inner conductor in the horizontal direction.

For the Dee–Dummy Dee local capacitance part of the calculation, which can be approximated as a plate capacitor, an approximation can be obtained by using the plate capacitance calculation equation.

4. Design and Simulation of the RF Cavity

4.1. Design of the RF Cavity

In a particle accelerator, the RF cavity can generate a sinusoidal electric field with a desired resonant frequency. There are two types of RF cavities in cyclotrons: the half-wavelength resonator (HWR) and quarter-wavelength resonator (QWR). The resonance frequencies of HWR and QWR depend mainly on the length of the resonator, and QWR is more compact than HWR for the same resonance frequency. The design of the RF cavity depends on various parameters such as the resonant frequency, Dee number, extraction radius, and number of accelerated harmonics. The RF cavity of a classical cyclotron uses a quarter-wavelength resonant cavity [22,23].

Table 1 outlines the technical parameters of the 11 MeV cyclotron's RF cavity. A classical cyclotron is used, with a nonhomogeneous magnetic pole gap height. The design of high-frequency cavities is challenging due to the small overall size of the superconducting cyclotron used to accommodate the cavity. The design of the RF cavity is mainly based on the classical 180° Dee, and the inner conductor part of the cavity mainly consists of the Dee, Stem, and Dummy Dee. A quarter-wavelength coaxial line resonant cavity was utilized. The main part of the cavity consists of an external rectangular box and an internal resonator.

Table 1. Conceptual design parameters for an 11 MeV superconducting cyclotron's RF cavity.

Cyclotron Parameters	Value
Energy	11 MeV
Resonant Frequency	45.026 MHz
Harmonic No.	1
Extraction Radius	161 mm
Dee Voltage	30 kV
Coupling Type	Inductive

4.2. Structural Analysis of the RF Cavities

Varied geometries for RF cavities result in cavities with varied characteristic parameters. In general, it is necessary to consider the parametric margin of the cavity at the beginning of the design to ensure the flexible design of related structures (including tuning capacitors, shorting plates, couplers, and others). In addition, the compact design requires that the overall dimensions and losses be minimized while maintaining the main design requirements. Therefore, the results of the different shapes of resonant cavities are compared under this design premise.

According to the previous analysis, it can be known that after changing the shape of the cavity, the equivalent characteristic impedance of the structural part in which it is located will be changed, and thus different characteristic parameters of the resonant cavity will be obtained. Referring to the design of the RF cavity of the KIRAMS-13 cyclotron [24,25], the rectangular inner conductor of the Stem was replaced by a cylindrical inner conductor.

As shown in Figure 3, the radius of the conductor inside the cylindrical Stem should not be too large in order to prevent the connection between the Dee and the Stem from

creating additional impedance discontinuities. The calculation ensures that the length of the Stem's inner conductor remains constant while only the diameter changes. From Figure 4, the maximum resonant frequency of the RF cavity with the circular Stem structure does not exceed 46 MHz, while the resonant frequency of the RF cavity with the rectangular Stem structure can easily reach 46 MHz for the same length. In addition, when the resonant frequency of the RF cavity with the rectangular Stem structure is 45.321 MHz, the resonant frequency of the RF cavity with the circular Stem structure under the equivalent radius is 44.952 MHz. Because the characteristic impedance of a circular Stem is greater than that of a rectangular Stem, the resonance frequency of the cavity is reduced while the length remains constant. According to the calculation formula for the characteristic impedance of the circular coaxial transmission line whose filling medium is air, $Z_c = 138 \lg(D/d) \Omega$, where d is the diameter of the inner conductor and D is the diameter of the outer conductor. When the diameter of the circular Stem increases, the radius ratio of the circular coaxial line decreases, the characteristic impedance becomes smaller, and the resonant frequency of the cavity increases. Therefore, the designed target resonant frequency value can be achieved when the diameter of the circular Stem inner conductor is adequate. However, increasing the diameter leads to an increase in the impedance discontinuity at the connection between the Dee and the Stem inner conductor, resulting in an increase in the step capacitance, which leads to a decrease in the resonant frequency of the RF cavity. It is calculated that the RF cavity resonant frequency decreases from 45.224 MHz to 42.001 MHz when the step capacitance is increased from 1 pF to 10 pF. However, when the step capacitance is increased from 10 pF to 100 pF, the RF cavity resonant frequency decreases rapidly to 26.145 MHz. In addition, the increase in the diameter of the conductor inside the circular Stem increases the step junction surface loss and the RF cavity surface loss. After comprehensive consideration, a rectangular Stem is used to connect to the Dee, making the step capacitance at the contact surface as small as possible.

In addition, since the RF cavity utilizes a classical 180° Dee, tip effects need to be avoided at the edges, and therefore the corners of the Dee are suitably chamfered. From Figure 5, it can be observed that the size of the Dee's edge fillet radius has a limited effect on the resonant frequency of the RF cavity. In addition, the electric field strength at the edge of the Dee can be significantly reduced by increasing the radius of the edge fillet. Therefore, it is sufficient to ensure that the size of the Dee's edge fillet radius meets the requirement of the maximum electric field strength during design.

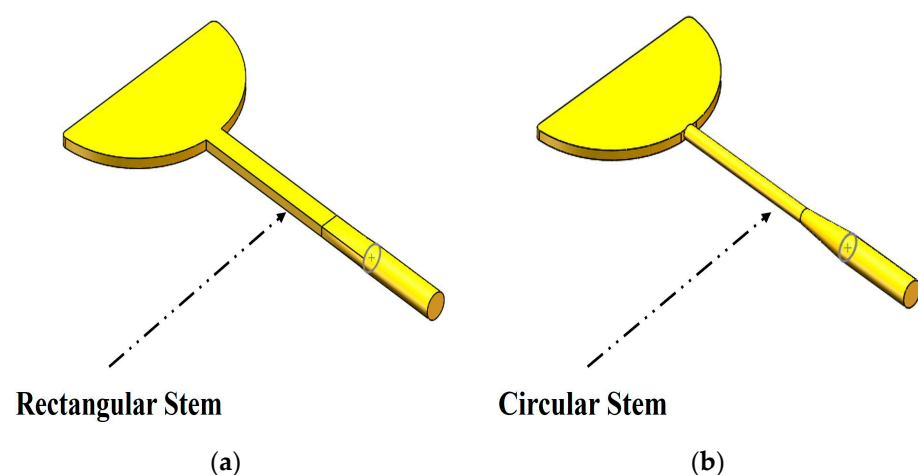


Figure 3. The RF cavity with different shapes of Dee and Stem structures: (a) rectangular Stem and (b) cylindrical Stem.

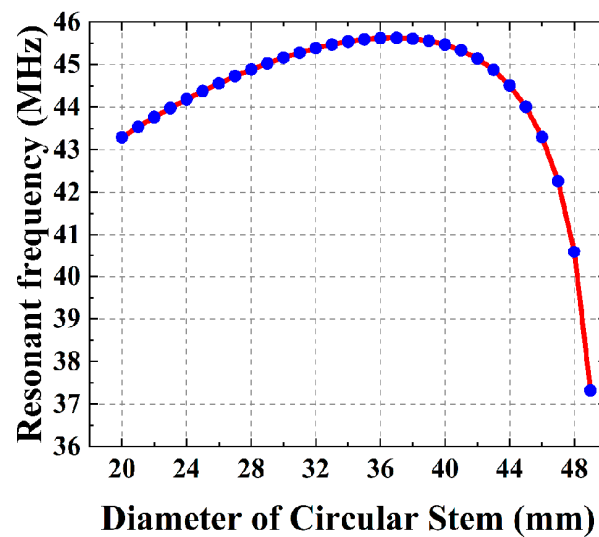


Figure 4. Circular Stem diameter versus resonant frequency.

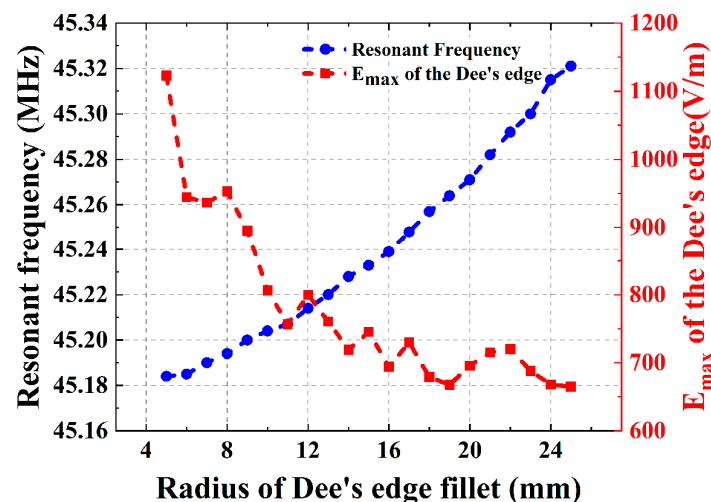


Figure 5. Chamfer radius at the edge of Dee versus resonant frequency.

4.3. Optimized Design of the RF Cavity

The RF cavity designed in this paper is partially placed in the magnet system, and this part of the RF cavity structure should not be changed significantly after the size is determined. The other part of the RF cavity (mainly the round coaxial resonator part in the back half of the cavity structure) is placed outside the magnet system so that the relevant structure parameters can be adjusted according to the design requirement.

The main structural parameters that can be adjusted in the round coaxial resonant cavity part are the inner conductor radius r_{ic} , the outer conductor radius r_{oc} , and the length L . The effect of these three structural factors on the resonant frequency of the cavity computed using 3D electromagnetic field numerical simulation software is shown in Figure 6. The RF cavity resonant frequency is smaller when r_{ic} is larger, r_{oc} is smaller, and L is shorter. When r_{ic} is certain, r_{oc} is roughly inversely proportional to L at the same resonant frequency. Therefore, it should be avoided that the length L of the round coaxial resonator is too short, which leads to a sharp increase in the radius r_{oc} of the outer conductor of the round coaxial resonator.

In order to further determine the selection of each structural parameter of the round coaxial resonator, the effect of these parameters on the unloaded quality factor Q_0 of the cavity is calculated. Figure 7 shows the effect of r_{oc} and L on the magnitude of the unloaded quality factor of the RF cavity for $r_{ic} = 20$ mm. It can be determined that the larger r_{oc} and L

are, the larger the Q_0 value of the RF cavity is, and different Q_0 values can be taken at the same resonant frequency.

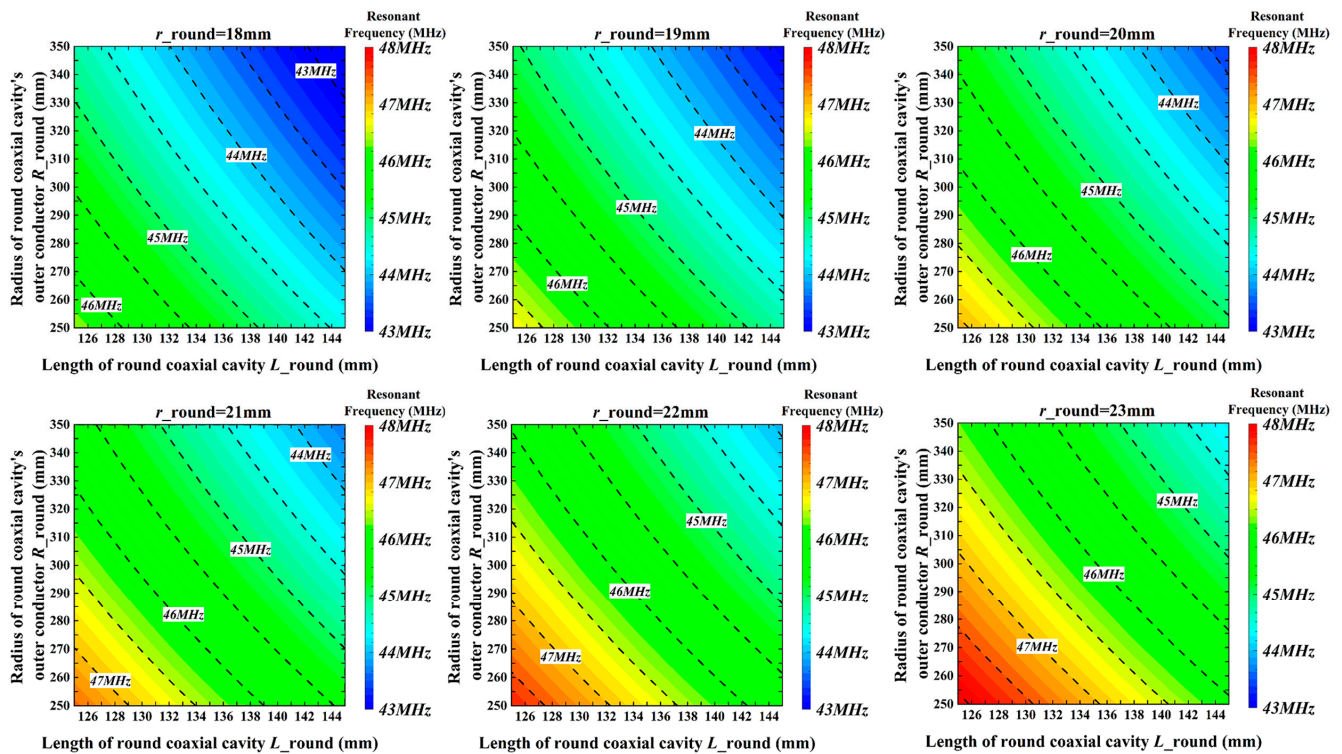


Figure 6. Influence of each parameter of a round coaxial resonator on the resonant frequency of the RF cavity.

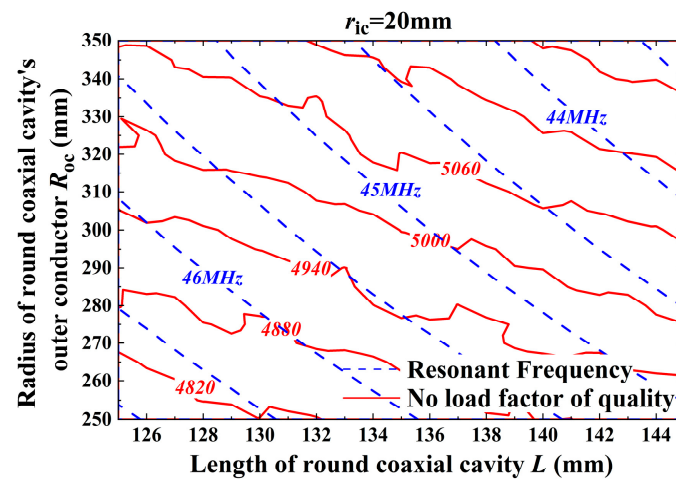


Figure 7. Influence of each parameter of the round coaxial resonator on the unloaded quality factor of the RF cavity.

The equivalent circuit method is used to find the L corresponding to different r_{oc} at $r_{ic} = 20$ mm and the same resonant frequency, and the surface area size of the round coaxial cavity corresponding to each set of data and the unloaded quality factor are calculated by using 3D electromagnetic field simulation software. According to Figure 8, it is observed that at the same resonant frequency, the smaller L is, the larger the surface area of the cavity, and the larger the unloaded quality factor Q_0 . The surface area of the round coaxial resonator is related to the vacuum degree of the cavity; thus, the cavity design needs to follow the principle of minimizing the surface area of the cavity. This requirement works

in the opposite way to the selection of the quality factor. Furthermore, it is established that L is inversely related to r_{oc} . In summary, choosing a smaller L will result in a larger Q_0 but also lead to an overly large r_{oc} and surface area of the cavity. Conversely, selecting a larger L , although it minimizes the surface area of the cavity as much as possible and achieves a smaller r_{oc} , necessitates a reduction in the Q_0 of the cavity. Consequently, the above influencing factors are considered in the actual design, and an appropriate value is chosen as a compromise based on demand.

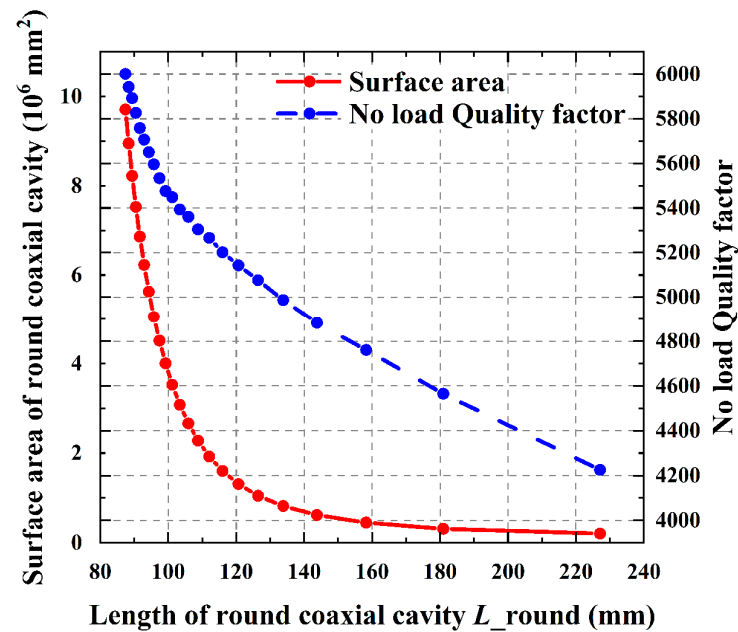


Figure 8. Effects of different round coaxial resonator lengths on the surface area of the round coaxial resonator and unloaded quality factor at the same resonant frequency.

4.4. Numerical Calculation of Electromagnetic Fields of the RF Cavity

The SIR structure is introduced to realize the miniaturized design of the RF cavity, and the 3D electromagnetic field numerical simulation software is used to compute the electromagnetic field distribution and resonant frequency of the RF cavity. After determining the boundary conditions of the cavity, the calculation results can be obtained by using the eigenmode solution method.

Since the RF cavity can be viewed as consisting of several segments of transmission lines with different characteristic impedances and a normalized resonator length of 0.42, the impedance ratio can be calculated to be 0.117. However, the characteristic impedance of the Dee and Stem is actually a variable, especially in the Dee region, where the characteristic impedance can be viewed as a function. So, shortening the quarter-wavelength resonator to the desired size is not purely dependent on the contribution of the discontinuity alone, and its impedance ratio R_Z will fluctuate compared to the theoretically calculated value 0.117.

From Figure 9, it can be seen that the electric field is mainly distributed at the Dee, which is called the capacitive region, while the magnetic field is mainly distributed at the Stem, which is called the inductive region. Where the electric field is the strongest at the edge of the Dee near the center area, here the magnetic field is the weakest, and where the magnetic field is the strongest at the end of the Stem, here the electric field is the weakest. The distribution of the electromagnetic field in the cavity can also be used as a reference for the design of subsequent water-cooled structures.

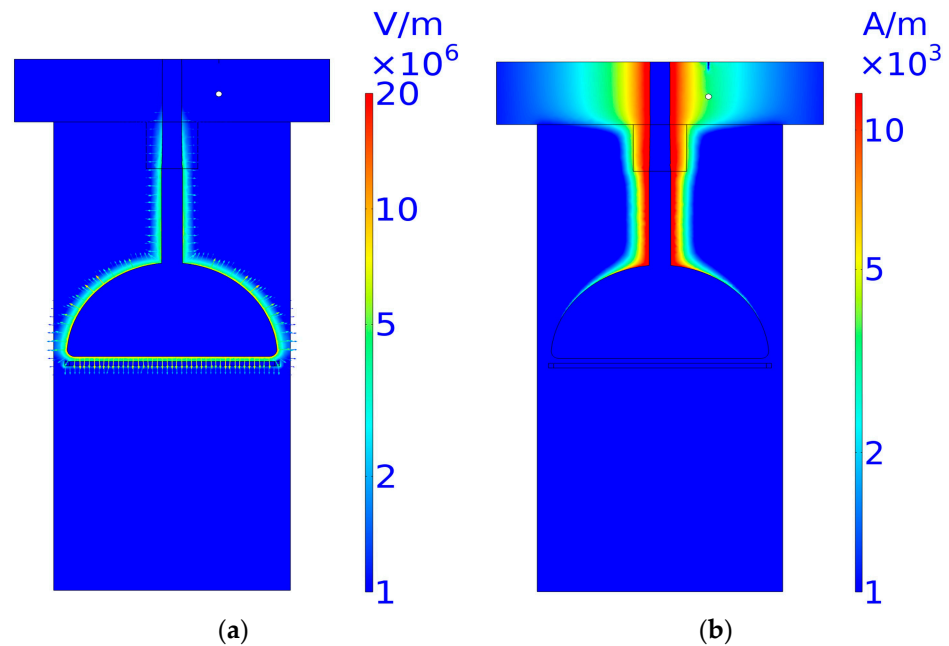


Figure 9. Field distribution of the RF cavity: (a) electric field and (b) magnetic field.

5. Calculation and Verification of the RF Cavity for 11 MeV Superconducting Cyclotrons

5.1. Calculation and Verification of the RF Cavity with Different Structural Parameters

After determining all the circuit parameters, the transcendental equation is substituted and the resonance frequency of the circuit is calculated by the iterative method and also utilizing the numerical simulation software to calculate the resonance frequency of the RF cavity. The results obtained by the two calculation methods are compared and verified, and the errors between them are within the acceptable range. At the same time, the influence trend of the structural parameters of the RF cavity on the resonant frequency is determined.

The effect of the structural parameters of the round coaxial resonator on the resonant frequency of the cavity is shown in Figure 10, where it can be seen that the longer the length, the larger the radius of the outer conductor, and the smaller the radius of the inner conductor, the smaller the resonant frequency of the RF cavity. At the same time, it can be seen that the numerical calculation results and the equivalent circuit calculation results are consistent and show good correlation; only the change in the radius of the inner conductor of the round coaxial will produce a relatively large error; the maximum error reaches about 2.7%. The main reason is that changing the radius of the inner conductor of a round coaxial resonator results in a change in the dimensions of the transition coaxial resonator part. When the dimensions of the rectangular part of the transition coaxial and the dimensions of the round part are similar, the error in the results of the characteristic impedance of the transition coaxial calculated by using the empirical formula is small. However, when there is a large difference between the dimensions of the rectangular part and the round part, the results obtained from the empirical formula tend to produce certain errors, which can eventually result in the generation of resonant frequency calculation errors. This has been taken into account in the design when modeling based on the target frequency, so that the error at the target frequency is small, about 0.1%.

The effects of changing the dimensions of some structure parameters of the transition coaxial transmission line and the rectangular coaxial transmission line on the resonance frequency can be observed in Figures 11 and 12. It can be seen that the results of the numerical simulation calculations are in good agreement with the results of the equivalent circuit model. When the transition coaxial length is longer and the outer conductor radius is larger, the resonant frequency of the cavity is smaller. For the rectangular coaxial transmission line, the longer the length and the lower the height, the lower the resonant

frequency of the RF cavity. The overall maximum error is around 1% and the error at the target frequency is about 0.13%.

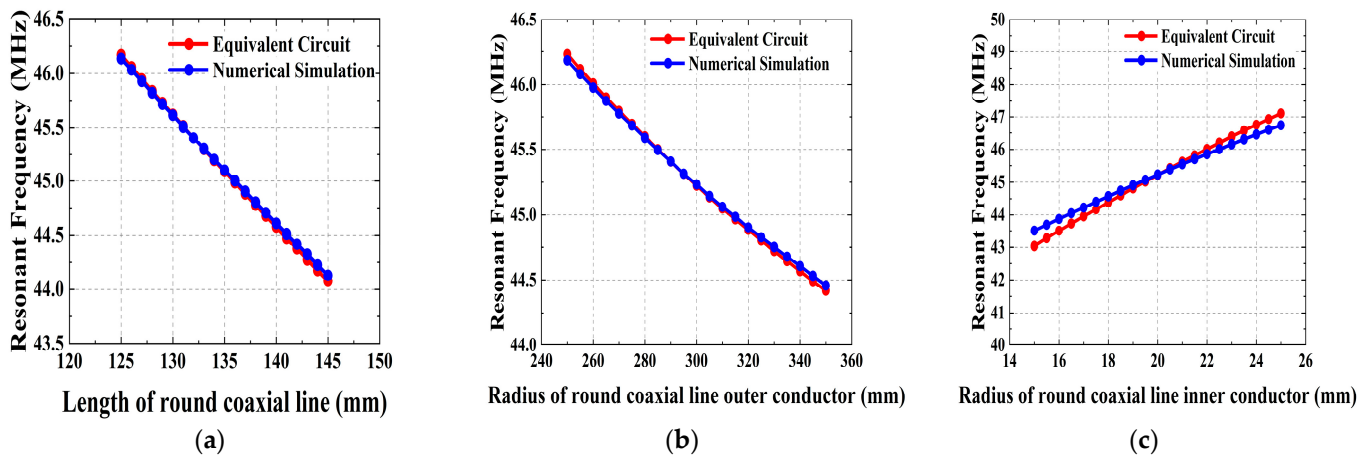


Figure 10. Round coaxial line parameters versus resonant frequency of cavity. (a) Line length, (b) radius of line outer conductor, (c) radius of line inner conductor radius.

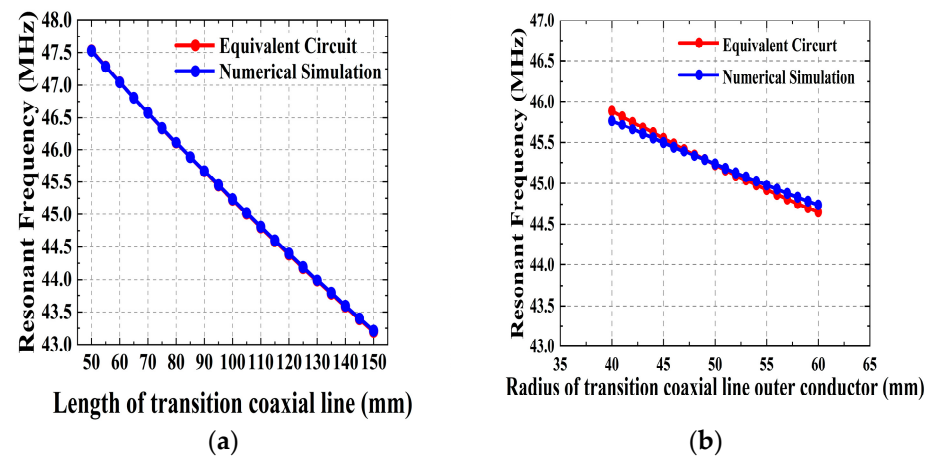


Figure 11. Impact of transition coaxial line parameters on the resonant frequency of the cavity. (a) Line length and (b) radius of line outer conductor.

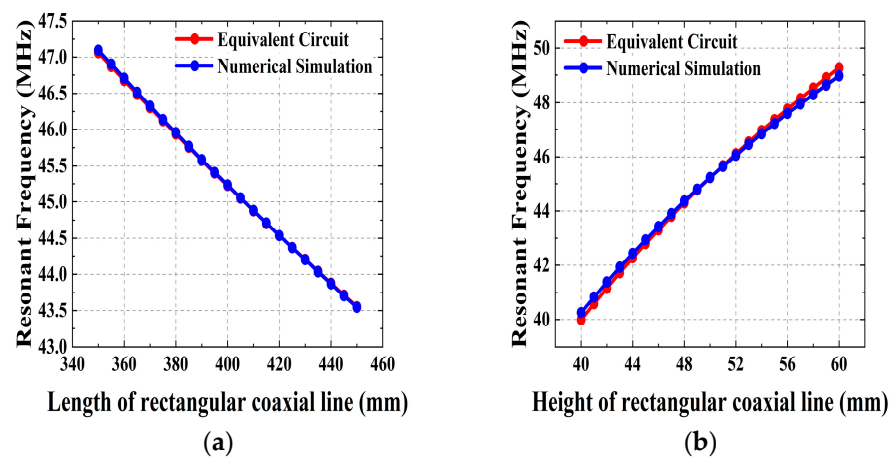


Figure 12. Impact of rectangular coaxial line parameters on the resonant frequency of cavity. (a) Line length and (b) line height.

5.2. Analysis of the RF Cavity Based on Multiple Linear Regression Models

The 11 MeV superconducting cyclotron adopts a modular design, and according to the foregoing, it is known that the only parameters available for extensive changes after the preliminary design is completed are the structural parameters of the round coaxial resonant cavity, including the inner conductor radius r_{ic} , the outer conductor inner radius r_{oc} , and the cavity length L . The effect of each of these three parameters on the resonant frequency of the RF cavity is shown in Figure 10. In order to determine the relationship between these three structural parameters and the cavity resonance frequency, and to obtain the importance of the influence of these three variables on the cavity resonance frequency, a multiple linear regression model can be introduced.

In multiple regression, the main goal is to relate the dependent variable y to more than one independent or predictor variable. Multiple linear regression analysis provides a good fit and prediction of the relationship between multiple sets of independent variables and the corresponding dependent variables. It also reduces the possibility of overfitting that occurs in simple linear regression analysis. The general multiple linear regression model equation is [26]:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \cdots + \hat{\beta}_k X_k + \varepsilon \quad (13)$$

where $\hat{Y}$ is the estimated value of observation Y ; $\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_k$ denote k regression coefficients; $X_1, X_2, \dots, X_k$ represent k explanatory variables; and ε is a random error term that explains the variation in the observation Y caused by the random variable, where $E(\varepsilon) = 0$ and $V(\varepsilon) = \sigma^2$ and $\varepsilon \sim N(0, \sigma^2)$. In this paper, there are three main structural parameter variables ($k = 3$), where Y is the RF cavity resonance frequency f , X_1 corresponds to r_{ic} , X_2 corresponds to r_{oc} , and X_3 corresponds to L .

In order to assess the effectiveness of the regression model in predicting the observation Y , the Root Mean Square Error (RMSE) and the Goodness of Fit, R^2 , were chosen to test and assess the effectiveness and accuracy of the model. Where RMSE is used to assess the deviation between actual and predicted values and R^2 is used to measure the degree of fit of the regression model, the respective expressions are [27]:

$$RMSE = \sqrt{\frac{1}{m} \sum_{i=1}^m (y_i - \hat{y}_i)^2} \quad (14)$$

$$R^2 = 1 - \frac{\sum_{i=1}^m (y_i - \hat{y}_i)^2}{\sum_{i=1}^m (y_i - \bar{y}_i)^2} \quad (15)$$

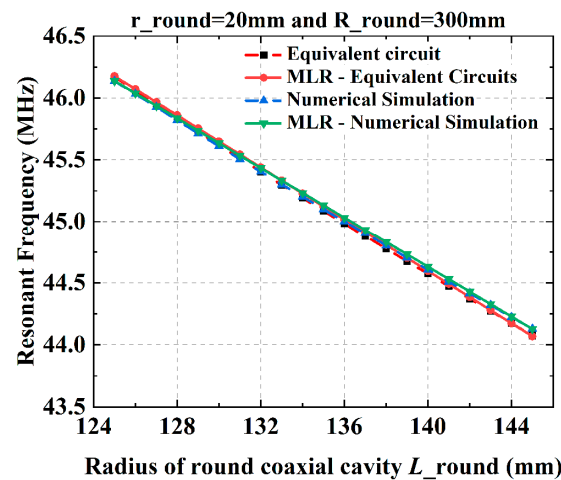
where y_i is the true value, $\hat{y}_i$ is the predicted value, and $\bar{y}_i$ is the sample mean.

Table 2 shows the results after fitting the data obtained by numerical calculation and equivalent circuit calculation using the multiple linear regression analysis method, including RMSE, R^2 , and regression equations, where the smaller the RMSE and the closer the R^2 is to 1, the better the model's predictive accuracy. The regression equation shows that r_{ic} and L have a significant impact on the resonance frequency. Conversely, r_{oc} 's effect is less pronounced. If r_{ic} increases significantly, widening the magnet yoke opening becomes necessary. Thus, changes to r_{ic} are kept within the rectangular cavity's height limits. By adjusting r_{ic} and L , the resonant frequency of the RF cavity can vary widely. Altering r_{oc} leads to a narrower frequency adjustment. Therefore, in further cavity design and optimization, the cavity resonant frequency can be roughly adjusted by adjusting the length of the round coaxial cavity L . After determining the resonant frequency adjustment range, the regression equation can be used to accurately calculate the available range for moving the short-circuit end. In addition, the resonant frequency fine-tuning structure design of the RF cavity can refer to the influence of the outer conductor inner radius r_{oc} on the resonant frequency.

Table 2. Performance of the RF cavity resonant frequency prediction based on multiple linear regression models.

Method	RMSE	R ²	Regression Equation
Numerical simulation	0.040	0.998	$f = 57.573 + 0.3172 \times r_{ic} - 0.0174 \times r_{oc} - 0.1005 \times L$
Equivalent circuit	0.052	0.998	$f = 56.856 + 0.3995 \times r_{ic} - 0.0183 \times r_{oc} - 0.1054 \times L$

Figure 13 shows the effects of the round cavity length L on the resonant frequency when $r_{ic} = 20$ mm and $r_{oc} = 240$ mm. It illustrates a high degree of fit between numerical simulation and equivalent circuit calculations. The errors in the prediction results at the target frequency for both models do not surpass 1%. They remain under 0.3%, satisfying the engineering design's predictive needs.

**Figure 13.** Performance of the RF cavity resonant frequency prediction based on multiple linear regression models.

6. Conclusions

The miniaturization of the resonator is achieved by introducing SIR into the RF cavity of a cyclotron, and the equivalent circuit method is used to simplify and equate the complex RF cavity so as to realize further analysis. The computational results show that the error of the equivalent circuit method and the numerical simulation calculation method can be up to 0.1% at the target frequency. In addition, multiple linear regression analysis can be used to predict the resonance frequency under multiple parameters, and the error of the results is basically kept within 1%. Therefore, a combination of the two methods can greatly simplify the design process and ease the design of the RF cavity of the cyclotron, save computational time, and be suitable for engineering applications.

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