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# Sustainable Maritime Transportation Operations with Emission Trading

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**Abstract:** The European Union (EU) has recently approved the inclusion of shipping in its Emissions Trading System, aiming to foster sustainable development within the shipping industry. While this new policy represents a significant step towards reducing carbon emissions, it also poses challenges for shipping companies, particularly in terms of operation costs. To assist shipping companies in devising optimal strategies under the new policy, this study proposes new techniques to determine the optimal solutions for sailing speed and the number of ships on the route, covering both EU and non-EU areas. Additionally, we demonstrate how to adjust these optimal decisions in response to changes in charged fees, fuel prices, and weekly operational costs of ships. This research offers innovative insights into the optimal decision-making process for shipping companies under the new EU policy and serves as a valuable decision-making tool to minimize total costs.

**Keywords:** maritime transport; green shipping; sailing speed; Emissions Trading System



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## 1. Introduction

Maritime transportation has progressively gained recognition as the principal mode of conveyance for international trade [1,2]. Nevertheless, the shipping industry has faced mounting criticism in recent years, primarily due to the heightened focus on carbon emissions. This criticism stems from the industry's heavy reliance on the combustion of fossil fuels, leading to substantial emissions of carbon dioxide ($CO_2$) and contributing to the exacerbation of global warming and climate change. The Fourth International Maritime Organization (IMO) greenhouse gas (GHG) study provides notable insights into this issue [3]. It reveals that the cumulative $CO_2$ emissions originating from maritime shipping increased from 962 million tonnes in 2012 to 1056 million tonnes in 2018, signifying approximately 3% of the total global anthropogenic $CO_2$ emissions during the period spanning from 2012 to 2018. Under this challenging circumstance, many countries and international organizations have put forward carbon-emission-reduction policies to promote the sustainable development of the shipping industry, such as double carbon goals in China (carbon peaking and carbon neutrality) [4], and the "Fit for 55" plan launched by the European Union (EU) [5]. These policies have been strategically formulated to effectively attain substantial emissions reductions, with the primary objective of achieving a minimum 40% reduction in emissions by the year 2030 and a subsequent aim to curtail total annual GHG emissions by no less than 50% by 2050, as cited in IMO [6]. Moreover, the overarching goal of these policies is to actively encourage the widespread implementation and adoption of cutting-edge technologies, fuels, and alternative energy sources that possess zero or near-zero GHG emissions profiles, as stated in the same source [6].

On 18 April 2023, the EU Parliament approved the inclusion of shipping in its Emissions Trading System (ETS) [7]. This decision represents a significant step toward addressing carbon emissions from ships and promoting sustainability within the shipping industry. Specifically, shipping companies need to buy 100% of emissions for voyages within the EU and 50% of emissions for voyages into or out of the EU starting in 2026. This decision represents a significant step towards addressing carbon emissions from ships and promoting sustainability within the shipping industry. Suppose that there is a container shipping route starting from Shanghai (SHA) port, passing through Singapore (SIN) port, Rotterdam (RTM) port, Hamburg (HH) port, and SIN port, and finally returning to SHA port (see Figure 1 for an illustration):

$$\text{SHA} \rightarrow \text{SIN} \rightarrow \text{RTM} \rightarrow \text{HH} \rightarrow \text{SIN} \rightarrow \text{SHA}. \quad (1)$$

The reported emissions are 100% charged between the RTM port and the HH port, 50% charged between SIN port and RTM port and between HH port and SIN port, and 0% charged in the remaining legs. Obviously, this policy aims to make shipping companies consider methods to reduce emissions within the EU area because more emissions generate more costs.

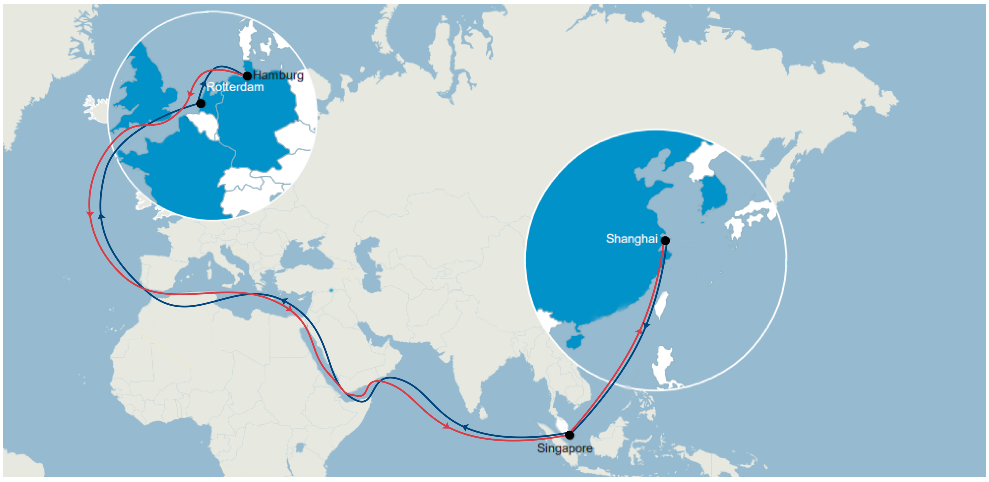


**Figure 1.** An example of a shipping route.

In this study, we propose mathematical models to facilitate optimal decision-making that seeks to minimize shipping companies' costs while adhering to the newly introduced EU policy on emissions. To the best of our knowledge, our research stands as the pioneering study that takes into account this recently promulgated policy by the EU. Specifically, our study answers the following research questions:

1. What are the optimal sailing speeds within the EU and non-EU areas that minimize the shipping company's total costs under the new EU policy on emissions?
2. What is the optimal number of ships to be equipped in the shipping route that leads to the lowest total costs while adhering to the emissions reduction requirements set by the EU's policy?
3. How do the optimal sailing speeds within EU and non-EU areas, as well as the optimal number of ships equipped in the shipping route, vary with changes in the charged fee for emissions, fuel price, and weekly operational costs of ships?

To address the three aforementioned research questions, we first propose a nonlinear optimization model, which presents challenges in terms of its complexity and solving difficulty. Leveraging the structural characteristics of the model, we establish two propositions that allow us to reduce its scale significantly. Then, we transform the nonlinear optimization model into an integer programming (IP) model by discretizing decision variables. This IP model can be solved using off-the-shelf optimization solvers. Finally, we conduct experiments and sensitivity analysis to examine the model performance regarding the changes in parameters.

### *1.1. Literature Review*

#### 1.1.1. Carbon Emission Reduction Policies in Shipping

From the perspective of policies' content, carbon emission reduction policies mainly focus on carbon emission allowance and tax. The implementation of the cap and trade (C&T) system within the shipping industry has garnered extensive attention from various stakeholders [8]. This system establishes a fixed emission target that is coupled with market flexibility. It allocates a specific number of carbon emission allowances (CEAs) to each participant within the system. In order to effectively curb carbon emissions in maritime transport, Zhu et al. [9] conducted an investigation into the strategies and performance of CEAs among shipping companies under the C&T mechanism. Their research findings serve as valuable guidance for multiple stakeholders, aiding them in formulating their own carbon-emission-reduction strategies, including determining the optimal carbon price and the overall carbon emissions targets. In addition, carbon allowance allocation in the shipping industry under the Energy Efficiency Design Index (EEDI) and the non-EEDI is explored. Chang and Huang [10] compared the carbon allowance and cost difference for shipping vessels that follow or ignore the guidelines of the EEDI. It has been proven that carbon tax exerts an important influence on carbon emission reduction. Based on previous scholarly investigations, the efficacy of a carbon tax primarily hinges on its extensive scope, inherent simplicity, and reduced uncertainty pertaining to future carbon pricing [11]. Moreover, its regulatory alignment with the existing governance framework and adherence to the "polluter pays principle" bolster its standing as a more suitable instrument for curtailing carbon emissions [12]. In the study conducted by Heine and Gäde [13], a novel hybrid mechanism was introduced, encompassing a cargo-based tax levied on international shipping emissions alongside a bunker levy targeting domestic shipping emissions. Notably, this approach incorporates the establishment of a default ship efficiency benchmark while incentivizing ship owners to operate energy-efficient vessels through subsidies. Consequently, such a hybrid tax regime facilitates the attainment of a global consensus. Additionally, various incentive-based carbon tax policies, such as the comparison between tonnage tax and conventional profit tax regimes [14], serve as catalysts for investment promotion and the advancement of green transformation within the maritime industry. For instance, existing tonnage tax regimes heavily subsidize international shipping activities [15]. Moreover, decarbonization continues to dominate the medium-term agenda of the maritime sector, exerting a profound influence on pivotal investments and strategic deliberations in the shipping industry [16]. So far, there have been plenty of options for decarbonizing the shipping sector, such as"slow steaming" [17] and measures that enhance energy efficiency [18].

At the regional level, the European Union (EU) is at the forefront of implementing effective measures to address carbon emissions in maritime shipping. On 18 April 2023, the legislative bodies of the EU reached a significant consensus by incorporating shipping into the Emission Trading System (ETS) [19]. Pending the EU's final approval, vessels above 5000 gross tonnages (GTs) that are engaged in the commercial transportation of cargo or passengers within the EU will be obligated to obtain and surrender emission allowances for their $CO_2$ emissions starting from 2024. Furthermore, by 2034, these ships will need to ensure that at least 2% of their fuel mix consists of specific renewable fuels [19]. This development will inevitably impose increasing compliance costs on the shipping industry. Moreover, in line with EU members' commitment to promoting renewable energy, the European Commission has introduced the Inducement Prize, aimed at encouraging the adoption of renewable fuels in retrofitted container ships [20].

The trend of carbon emissions reduction in the shipping industry is gaining momentum as green and sustainable development practices are widely recognized and promoted. Countries, regions, and international organizations are formulating policies to regulate shipping companies' development and encourage them to take measures to reduce carbon emissions. This growing emphasis on environmental responsibility reflects the global commitment to combat climate change and foster a more sustainable future for the shipping industry.

#### 1.1.2. Optimal Decisions in Shipping

Against the backdrop of carbon emission reduction, a range of policies profoundly impact optimal decisions in shipping. The management of ship operations has been a focal point of prior research, primarily focusing on ship routing, ship deployment, and ship sailing speed. With regard to shipping routing, Lin and Tsai [21] delved into the intricate problem of ship routing and freight assignment in daily liner shipping operations, introducing a Lagrangian relaxation technique. Moreover, Lin and Chang [22] employed a decomposition algorithm that incorporates the Lagrangian factor to optimize route selection and freight-allocation decisions. Exploring the optimization of the shipping network with respect to carbon dioxide emissions charges on the Asia–Europe route, Dai et al. [23] conducted an in-depth analysis. In terms of ship deployment, Zhen et al. [24] devised a nonlinear mixed-integer programming model to facilitate strategic ship deployment, taking into account the stochastic nature of the ships' weight distribution. Additionally, Gu et al. [12] explored the maritime carbon-trading mechanism within the context of conventional green ship deployment, discovering its limited effectiveness in reducing short-term carbon dioxide emissions. Notably, there exist studies that consider shipping demand uncertainty, with researchers proposing stochastic optimization models and robust optimization models [25] to address uncertainties in decision-making. Furthermore, the optimization of ship sailing speed has garnered attention. For instance, Wang and Xu [26] tackled the optimization problem of sailing speed during voyages, accounting for distinct carbon-emission-taxation regimes. A mixed-integer programming model was established by Sheng et al. [27] to explore optimal ship speed and size when traversing emission control areas such as the EU region. Moreover, several research efforts have pursued holistic decision-making approaches, considering the interplay of multiple factors. Wang et al. [28] advanced a sophisticated mathematical programming model, aiming to concurrently optimize ship routes and the interconnected cargo-allocation schemes. The model itself is effectively solved by transforming it into an equivalent mixed-integer linear program, allowing for efficient computational analysis. Furthermore, researchers such as Koza [29] and Ozcan and Eliiyi [30] developed distinct algorithms to streamline the optimization of service scheduling, encompassing crucial elements like transit time and container volume, alongside cargo allocation strategies within the realm of liner shipping. In addition, Giovannini and Psaraftis [31] undertook the optimization of various factors, including shipping speed, the number of ships deployed, and service frequency, to maximize the average daily profit of liner shipping companies, illustrating a holistic approach to decision-making in the industry.

Substantially, the optimal decisions mainly focus on the deployed ships, sailing speeds, and sailing routes, as well as the connections between them. To obtain optimal decisions, the IP model and other derived models are proposed to provide decision information for stakeholders, especially for shipping companies' operators. In general, carbon-emission-reduction policies are stricter and diversified all over the world as the goal of green transformation and sustainable development in maritime shipping, especially in the EU area. These policies have a significant influence on ship deployment and sailing speed, which means the optimal decisions will change with newly introduced policies. Our study takes into account a recently published policy by the EU, proposing an IP model to obtain the optimal sailing speed and deployed ships in different routes.

### *1.2. Research Contributions*

The theoretical and practical contributions of our research are summarized as follows.

1. Theoretical contributions. This study addresses a research gap, as existing literature has not focused on the optimal decisions of sailing speed and the number of ships under the newly proposed EU policy. To the best of our knowledge, this is the

first study to establish mathematical models aimed at minimizing the total costs of shipping companies while considering the implications of the new EU policy. The proposed approach involves a nonlinear optimization model to determine the shipping company's optimal decisions. By leveraging the unique structure of the optimization problem under the new EU policy, two propositions are proven. We further transform the nonlinear model into a solvable IP model. Through experiments and sensitivity analyses, specific solutions are obtained, and the impacts of different parameters are tested.

2. Practical contributions. This study contributes valuable insights into optimal strategies for shipping companies to minimize costs and comply with the new EU emissions policy. The results have practical implications for the sustainable development of the shipping industry and its adherence to environmental regulations. The proposed mathematical model can serve as a decision tool for shipping companies facing the new EU emissions policy.

The rest of the paper is organized as follows. Section 2 describes the research problem in detail and develops the mathematical model. Section 3 proposes solution methods for addressing the initial proposed model. Section 4 presents the experiments and sensitivity analysis that were conducted. Finally, conclusions are drawn in Section 5.

The main notations used in this study are summarized in Table 1.

**Table 1.** Notations.

| **Sets** | |
|---|---|
| $I$ | Set of ports of call in a shipping route, $i \in I$ |
| $I^{EU}$ | Set of ports in the EU area, $i \in I^{EU}$ |
| $I^{NEU}$ | Set of ports outside the EU area, $i \in I^{NEU}$ |
| $I^0$ | Set of the legs on which emissions are 0 charged, $i \in I^0$ |
| $I^1$ | Set of the legs on which emissions are 50% charged, $i \in I^1$ |
| $I^2$ | Set of the legs on which emissions are 100% charged, $i \in I^2$ |
| **Parameters** | |
| $c$ | The fixed cost for each ship per week |
| $\mu$ | The fuel price per tonne |
| $\beta$ | The charged fee of emissions per tonne |
| $\gamma$ | The conversion rate of fuel consumption and emissions |
| $Q$ | The emission per hour during berthing |
| $L_i$ | The distance of leg $i$ |
| $t_i$ | The berthing time at port $i$ |
| $\hat{v}_i$ | The sailing speed on leg $i$ |
| $v_{\min}$ | The minimum sailing speed |
| $v_{\max}$ | The maximum sailing speed |
| $X$ | The integer used to discretize sailing speed, $x = 0, 1, ...., X$ |
| $v_k^x$ | The discretized sailing speed, $k = 0, 1, 2$ |
| **Function** | |
| $f(\hat{v}_i^3)$ | Fuel consumption at the sailing speed of $\hat{v}_i$ |
| **Decision variables** | |
| $z$ | The number of deployed ships in a route |
| $y_k^x$ | Binary decision variable that equals 1 if ships sail with speed $v_k^x$ and 0 otherwise |

## 2. Problem Description and Model Development

We use set $I$ to denote the set of ports of call in one shipping route covering both EU and non-EU areas, and $i \in I$ denotes port $i$ and also denotes leg $i$. Since ships need to return to the original port, there is no need to denote the final port. For example, $I = \{1, 2, 3, 4, 5\}$ in (1). $i = 1$ indicates the SH port; $i = 2$ and $i = 5$ all represent the SIN port because the ship calls at the SIN port twice in this route. $i = 1$ also indicates the leg between the SHA port and the SIN port, and $i = 5$ represents the leg between the SIN port and the SHA port.

For the shipping companies operating container ships on this route, their decision involves the number of ships used on this route (denoted by $z$) and the sailing speed of these ships in each leg $i$ (denoted by $\hat{v}_i$). The distance of leg $i$ is denoted by $L_i$. The berthing time at each port $i$ is denoted by $t_i$. Certainly, the emissions generated during the berthing at EU ports are subject to a full charge, as mandated by the new policy. According to widely recognized domain knowledge, speed and fuel consumption are cubically related [32]. That is, fuel consumption equals $f(\hat{v}_i^3)$, where function $f(\hat{v}_i^3)$ maps sailing speed to fuel consumption and $f(\hat{v}_i^3) = a\hat{v}_i^3$. And we assume the emissions during the berthing are $Qt_i$, where $Q$ is a constant representing the emission per hour during the berthing. To facilitate model construction, we define set $I^{EU}$ and set $I^{NEU}$. The set $I^{EU}$ comprises ports within the EU area, while the set $I^{NEU}$ includes ports outside the EU area. Furthermore, we define sets $I^0$, $I^1$, and $I^2$, where $I^0$ denotes the legs on which emissions are 0% charged, $I^1$ denotes the legs on which emissions are 50% charged, and $I^2$ denotes the legs on which emissions are 100% charged. Taking the route in Figure 1 as an example, the set $I^{EU}$ includes RTM and HH, the set $I^{NEU}$ consists of SHA and SIN, the set $I^0$ comprises the leg from SHA to SIN and the leg from SIN to SHA, the set $I^1$ includes the leg from SIN to RTM and the leg form HH to SIN, and the set $I^2$ comprises the leg from RTM to HH. Obviously, we have $I = I^{EU} \cup I^{NEU} = I^0 \cup I^1 \cup I^2$. To minimize the total costs, the shipping company's optimal decision problem can be formulated as follows:

[M1]

$$\min cz + \mu\left(\sum_{i\in I} Qt_i + \sum_{i\in I} \frac{L_i}{\hat{v}_i} f(\hat{v}_i^3)\right) + \beta\gamma\left(\sum_{i\in I^{EU}} Qt_i + 0.5\sum_{i\in I^1} \frac{L_i}{\hat{v}_i} f(\hat{v}_i^3) + \sum_{i\in I^2} \frac{L_i}{\hat{v}_i} f(\hat{v}_i^3)\right) \quad (2)$$

subject to

$$\sum_{i\in I}\left(\frac{L_i}{\hat{v}_i} + t_i\right) \leq 168z \quad (3)$$

$$v_{\min} \leq \hat{v}_i \leq v_{\max}, i \in I \quad (4)$$

$$z \in Z_+. \quad (5)$$

The objective function (2) involves three parts. Firstly, $cz$ calculates the total fixed costs of ships. Secondly, $\mu(\sum_{i\in I} Qt_i + \sum_{i\in I} \frac{L_i}{\hat{v}_i} f(\hat{v}_i{}^3))$ represents the total fuel costs, where $\mu$ is the fuel price. Thirdly, $\beta\gamma(\sum_{i\in I^{EU}} Qt_i + 0.5\sum_{i\in I^1} \frac{L_i}{\hat{v}_i} f(\hat{v}_i{}^3) + \sum_{i\in I^2} \frac{L_i}{\hat{v}_i} f(\hat{v}_i{}^3))$ calculates the emission tax, where $\gamma$ represents the conversion rate of fuel consumption and emissions and $\beta$ represents the charged fee. Constraint (3) restricts the weekly service frequency. Constraint (4) gives the domain of $\hat{v}_i$, indicating the maximum and minimum of $\hat{v}_i$. Constraint (5) requires that the number of deployed ships in a route should be a positive integer.

## 3. Solution Methods

Model [M1] is complex to solve due to different $\hat{v}_i$ values in different legs, which means a significant number of decision variables and sophisticated algorithms. In addition, the term $\frac{L_i}{\hat{v}_i}$ is nonlinear, making it harder to obtain the optimal solution in this model. Jointly considering the model characteristics and sailing speeds in different areas, we put forward Proposition 1 to reduce the number of decision variables and further discretize sailing speed to linearize the proposed [M1].

**Proposition 1.** *In the same type of leg, the sailing speed remains consistent. Specifically, $\hat{v}_i$ for $i \in I^0$ is the same, $\hat{v}_i$ for $i \in I^1$ is the same, and $\hat{v}_i$ for $i \in I^2$ is the same.*

**Proof.** To simplify the notation, we define $\hat{v} = (\hat{v}_1, \hat{v}_2, ..., \hat{v}_{|I|})$, where $|I|$ denotes the total number of legs in set $I$. We further use $F(z, \hat{v})$ to denote the objective function (2). The objective function (2) is actually a monotonically increasing function of $z$ and $\hat{v}_i$ because

$$\frac{\partial F(z,\hat{v})}{\partial z} = c > 0 \tag{6}$$

$$\frac{\partial F(z,\hat{v})}{\partial \hat{v}_i} = 2\mu L_i \hat{v}_i > 0,\ i \in I^0 \tag{7}$$

$$\frac{\partial F(z,\hat{v})}{\partial \hat{v}_i} = 2\mu L_i \hat{v}_i + \beta\gamma L_i > 0,\ i \in I^1 \tag{8}$$

$$\frac{\partial F(z,\hat{v})}{\partial \hat{v}_i} = 2\mu L_i \hat{v}_i + 2\beta\gamma L_i > 0,\ i \in I^2. \tag{9}$$

Suppose that there are two decision variables $\hat{v}_{i^\#}$ and $\hat{v}_{i'}$, $i^\# \in I^0$, $i' \in I^0$, and $\hat{v}_{i^\#} \neq \hat{v}_{i'}$ satisfies Constraints (3) and (4). We use $L_{i^\#}$ and $L_{i'}$ to denote the corresponding lengths of legs of $\hat{v}_{i^\#}$ and $\hat{v}_{i'}$, respectively. We can always find optimal solutions $\overline{v}_{i^\#}$ and $\overline{v}_{i'}$ equaling $\frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}}$ that satisfy Constraints (3) and (4) and generate a smaller value of $F(z, \hat{v}_i)$.

To facilitate the proof process, we define $t = \frac{L_{i^\#}}{\hat{v}_{i^\#}} + \frac{L_{i'}}{\hat{v}_{i'}}$. Therefore, we have $\hat{v}_{i'} = \frac{L_{i'}}{t-\frac{L_{i^\#}}{\hat{v}_{i^\#}}}$. The objective function aims to minimize the following formula because $i^\# \in I^0$ and $i' \in I^0$:

$$L_{i^\#}\hat{v}_{i^\#}^2 + L_{i'}\hat{v}_{i'}^2, \tag{10}$$

which is

$$L_{i^\#}\hat{v}_{i^\#}^2 + L_{i'}\left(\frac{L_{i'}}{t-\frac{L_{i^\#}}{\hat{v}_{i^\#}}}\right)^2. \tag{11}$$

We take the derivative with respect to $\hat{v}_{i^\#}$:

$$\frac{d\left[L_{i^\#}\hat{v}_{i^\#}^2 + L_{i'}\left(\frac{L_{i'}}{t-\frac{L_{i^\#}}{\hat{v}_{i^\#}}}\right)^2\right]}{d\hat{v}_{i^\#}} = L_{i^\#}\hat{v}_{i^\#} - L_{i'}^3\left[(t-\frac{L_{i^\#}}{\hat{v}_{i^\#}})^{-3}\frac{L_{i^\#}}{\hat{v}_{i^\#}}\right] = 0. \tag{12}$$

The $\overline{v}_{i^\#}$ satisfying the above equation should be

$$\overline{v}_{i^\#} = \frac{L_{i^\#}+L_{i'}}{t} = \frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}} \tag{13}$$

and thus

$$\overline{v}_{i'} = \frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}}. \tag{14}$$

Therefore, $\overline{v}_{i^\#} = \overline{v}_{i'} = \frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}}$ should be the optimal solutions. Next, we analyze how $\overline{v}_{i^\#}$ and $\overline{v}_{i'}$ satisfy Constraints (3) and (4).

First, we have

$$\frac{L_{i^\#}}{\hat{v}_{i^\#}} + t_{i^\#} + \frac{L_{i'}}{\hat{v}_{i'}} + t_{i'} + \sum_{i\in I\setminus\{i^\#,i'\}}(\frac{L_i}{\hat{v}_i} + t_i) \le 168z. \tag{15}$$

Because $\overline{v}_{i^\#} = \overline{v}_{i'} = \frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}}$, we have the following relationship:

$$\begin{aligned}\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}-\left(\frac{L_{i^\#}}{\overline{v}_{i^\#}}+\frac{L_{i'}}{\overline{v}_{i'}}\right) &= \frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}-\left(\frac{L_{i^\#}}{\frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}}}+\frac{L_{i'}}{\frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}}}\right)\\ &= \frac{L_{i'}\hat{v}_{i^\#}+L_{i^\#}\hat{v}_{i'}}{\hat{v}_{i^\#}\hat{v}_{i'}}-\frac{L_{i'}\hat{v}_{i^\#}+L_{i^\#}\hat{v}_{i'}}{\hat{v}_{i^\#}\hat{v}_{i'}}\\ &= 0,\end{aligned} \tag{16}$$

which indicates that $\overline{v}_{i^\#}$ and $\overline{v}_{i'}$ satisfy Constraint (3).

In terms of Constraints (4), we have:

$$v_{\min} \le \hat{v}_{i^\#} \le v_{\max} \tag{17}$$

$$v_{\min} \le \hat{v}_{i'} \le v_{\max}. \tag{18}$$

With loss of generality, we suppose that $\hat{v}_{i^\#} > \hat{v}_{i'}$. Therefore, we have the following relationship:

$$\overline{v}_{i^\#} = \overline{v}_{i'} = \frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}} = \frac{\hat{v}_{i'}\hat{v}_{i^\#}(L_{i^\#}+L_{i'})}{L_{i'}\hat{v}_{i^\#}+L_{i^\#}\hat{v}_{i'}} < \frac{\hat{v}_{i'}\hat{v}_{i^\#}(L_{i^\#}+L_{i'})}{(L_{i'}+L_{i^\#})\hat{v}_{i'}} \le v_{\max}, \tag{19}$$

and

$$\overline{v}_{i^\#} = \overline{v}_{i'} = \frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}} = \frac{\hat{v}_{i'}\hat{v}_{i^\#}(L_{i^\#}+L_{i'})}{L_{i'}\hat{v}_{i^\#}+L_{i^\#}\hat{v}_{i'}} > \frac{\hat{v}_{i'}\hat{v}_{i^\#}(L_{i^\#}+L_{i'})}{(L_{i'}+L_{i^\#})\hat{v}_{i^\#}} \ge v_{\min}. \tag{20}$$

Therefore, we prove that $\overline{v}_{i^\#} = \overline{v}_{i'} = \frac{L_{i^\#}+L_{i'}}{\frac{L_{i^\#}}{\hat{v}_{i^\#}}+\frac{L_{i'}}{\hat{v}_{i'}}}$ should be the optimal solutions when $i \in I^0$. By the same logic, the optimal values of $\hat{v}_i$, $i \in I^1$ must be the same, and the optimal values of $\hat{v}_i$, $i \in I^1$ must be the same. □

Taking advantage of Proposition 1, we can reduce the number of decision variables in Model [M1]. That is, for each type of leg, we only need to decide on one optimal sailing speed. We use $v_0$, $v_1$, and $v_2$ to denote the sailing speed on each type of leg. Model [M1] can be converted to the following model.

[M2]

$$\min cz + \mu\left(\sum_{i\in I^0}\frac{L_i}{v_0}f(v_0^3)+\sum_{i\in I^1}\frac{L_i}{v_1}f(v_1^3)+\sum_{i\in I^2}\frac{L_i}{v_2}f(v_2^3)\right)+\beta\gamma\left(0.5\sum_{i\in I^1}\frac{L_i}{v_1}f(v_1^3)+\sum_{i\in I^2}\frac{L_i}{v_2}f(v_2^3)\right)+C \tag{21}$$

subject to

$$\sum_{i\in I^0}(\frac{L_i}{v_0}+t_i)+\sum_{i\in I^1}(\frac{L_i}{v_1}+t_i)+\sum_{i\in I^2}(\frac{L_i}{v_2}+t_i) \le 168z \tag{22}$$

$$v_{\min} \le v_0 \le v_{\max} \tag{23}$$

$$v_{\min} \le v_1 \le v_{\max} \tag{24}$$

$$v_{\min} \le v_2 \le v_{\max} \tag{25}$$

$$z \in Z_+, \tag{26}$$

where $C = \mu \sum_{i \in I} Qt_i + \beta\gamma \sum_{i \in I^{EU}} Qt_i$, which is a constant and does not affect the optimal solutions. However, Model [M2] is still difficult to solve because of the nonlinear terms $\frac{L_i}{v_0}$, $\frac{L_i}{v_1}$, and $\frac{L_i}{v_2}$. Referring to [33], we discretize sailing speed with 0.1 knots. We define

$$X = \lfloor \frac{v_{\max} - v_{\min}}{0.1} \rfloor + 1. \tag{27}$$

We set $x = 0, 1, ..., X$. Therefore, the sailing speed $v_k$, $k = 0, 1, 2$ can be discretized to $v_k^0 = v_{\min}$, $v_k^1 = v_{\min} + 0.1 \times 1$, $v_k^2 = v_{\min} + 0.1 \times 2$, ..., $v_k^X = \max\{v_{\max}, v_{\min} + 0.1 \times X\}$. We introduce binary decision variables $y_k^x$, $k = 0, 1, 2$, and $x = 0, ..., X$ to indicate which discretized sailing speed is chosen. Specifically, $y_k^x = 1$ means the corresponding sailing speed $v_k^x$ is selected and 0 otherwise. With this newly introduced binary decision variable, we can transform Model [M2] to the following IP model:

[M3]

$$\begin{aligned} \min cz + \mu \left( \sum_{i \in I^0} \sum_{x=0}^{X} y_0^x \frac{L_i}{v_0^x} f(v_0^{x3}) + \sum_{i \in I^1} \sum_{x=0}^{X} y_1^x \frac{L_i}{v_1^x} f(v_1^{x3}) + \sum_{i \in I^2} \sum_{x=0}^{X} y_2^x \frac{L_i}{v_2^x} f(v_2^{x3}) \right) \\ + \beta\gamma \left( 0.5 \sum_{i \in I^1} \sum_{x=0}^{X} y_1^x \frac{L_i}{v_1^x} f(v_1^{x3}) + \sum_{i \in I^2} \sum_{x=0}^{X} y_2^x \frac{L_i}{v_2^x} f(v_2^{x3}) \right) + C \end{aligned} \tag{28}$$

subject to

$$\sum_{i \in I^0} (t_i + \sum_{x=0}^{X} y_0^x \frac{L_i}{v_0^x}) + \sum_{i \in I^1} (t_i + \sum_{x=1}^{X} y_1^x \frac{L_i}{v_1^x}) + \sum_{i \in I^2} (t_i + \sum_{x=2}^{X} y_2^x \frac{L_i}{v_2^x}) \leq 168z \tag{29}$$

$$\sum_{x=0}^{X} y_k^x = 1, k = 0, 1, 2 \tag{30}$$

$$y_k^x \in \{0, 1\}, k = 0, 1, 2, \ x = 0, ..., X \tag{31}$$

$$z \in Z_+. \tag{32}$$

Model [M3] has two types of decision variables: the first one is the integer decision variable $z$, which means how many ships should be used in a route; the second one is the binary decision variable $y_k^x$ and we have a total of $3X$ binary decision variables. If $y_k^x = 1$, the corresponding sailing speed $v_k^x$ is selected, and the fuel consumption is decided. Therefore, the objective function and the constraints are linear and the decision variables are all integers, which means that we transform Model [M2] to IP model [M3].

We can prove that the optimal solutions of Model [M3] satisfy Proposition 2.

**Proposition 2.** *We use $v_0^*$, $v_1^*$, and $v_2^*$ to denote the optimal values of sailing speed within non-EU areas, linking non-EU and EU areas, and within EU areas, respectively. We must have $v_0^* \geq v_1^* \geq v_2^*$.*

**Proof.** We use $z^*$ to denote the optimal value of the decision variable $z$. Based on [M3], the value of the objective function (28) is

$$cz^* + \mu a \left( \sum_{i \in I^0} L_i v_0^{*2} + \sum_{i \in I^1} L_i v_1^{*2} + \sum_{i \in I^2} L_i v_2^{*2} \right) + \beta\gamma a \left( 0.5 \sum_{i \in I^1} L_i v_1^{*2} + \sum_{i \in I^2} L_i v_2^{*2} \right) + C. \tag{33}$$

And we also have

$$\sum_{i \in I^0} (t_i + \frac{L_i}{v_0^*}) + \sum_{i \in I^1} (t_i + \frac{L_i}{v_1^*}) + \sum_{i \in I^2} (t_i + \frac{L_i}{v_2^*}) \leq 168z^* \tag{34}$$

$$v_{\min} \leq v_0^* \leq v_{\max} \tag{35}$$

$$v_{\min} \leq v_1^* \leq v_{\max} \tag{36}$$

$$v_{\min} \leq v_2^* \leq v_{\max}. \tag{37}$$

There is a trade-off between the objective function (33) and Constraint (34). To be more specific, Constraint (34) tends to generate greater values of $v_0^*$, $v_1^*$, and $v_2^*$. However, minimizing the objective function (33) tends to generate smaller values of $v_0^*$, $v_1^*$, and $v_2^*$. Therefore, there are two cases in the optimal solutions. The first case is that Constraint (34) is binding. That is,

$$\sum_{i \in I^0} (t_i + \frac{L_i}{v_0^*}) + \sum_{i \in I^1} (t_i + \frac{L_i}{v_1^*}) + \sum_{i \in I^2} (t_i + \frac{L_i}{v_2^*}) = 168z^*. \tag{38}$$

And the second case is that the optimal solutions equal the minimum sailing speed, i.e.,

$$v_0^* = v_1^* = v_2^* = v_{\min}. \tag{39}$$

This is because the coefficients of $v_0^*$, $v_1^*$, and $v_2^*$ are $\mu a$, $\mu a + 0.5\beta\gamma a$, and $\mu a + \beta\gamma a$, respectively. Under the condition of satisfying Equation (38), the optimal solutions that satisfy $v_0^* \geq v_1^* \geq v_2^*$ can achieve the minimum value of the objective function (33) in the first case. Moreover, the optimal solutions in the second case also satisfy $v_0^* \geq v_1^* \geq v_2^*$. $\square$

## 4. Experiments

### *4.1. Experiment Settings*

With the help of Proposition 1 and discretization, we transform the original optimization model into an IP programming model with the minimum number of decision variables, which can be solved using the off-of-shelf optimization solvers, such as CPLEX and Gurobi. We here introduce the selected container shipping routes for the experiment and how to set the parameters, e.g., $c$, $\mu$, $\beta$, $\gamma$, and $Q$ according to practice.

The experiments were run on a laptop computer equipped with 2.60 GHz of Intel Core i7 CPU and 16 GB of RAM, and Model [M3] was solved using the Gurobi Optimizer 10.0.2 via Python API.

#### 4.1.1. Selected Shipping Routes

We select two routes from Asia to northern Europe[1] to test the performance of Model [M3]. These two routes play a pivotal role in fostering communication between Asia and Europe. Remarkably, certain ports within these routes occupy a paramount position within the realm of international transportation of goods, including notable ones like Singapore and Rotterdam. Moreover, for comprehensive insights into the distances of various segments within the routes, we relied on some authoritative websites[2]. Details are shown in Table 2, and the names of EU ports are bolded. The travel distances (in nautical miles) of these two routes, i.e., $L_i$, are shown in Figure 2 and Figure 3, respectively.

**Table 2.** Summary of shipping routes.

| Route ID | Port Rotation (City) |
|---|---|
| 1 | Busan → Ningbo → Shanghai → Yantian → Singapore → **Algeciras** → **Dunkerque** → **Le Havre** → **Hamburg** → **Wilhelmshaven** →**Rotterdam** → Port Klang → Busan |
| 2 | Tianjin → Dalian → Qingdao → Shanghai → Ningbo → Singapore→ **Piraeus** → **Rotterdam** → **Hamburg** → **Antwerp** → Shanghai → Tianjin |

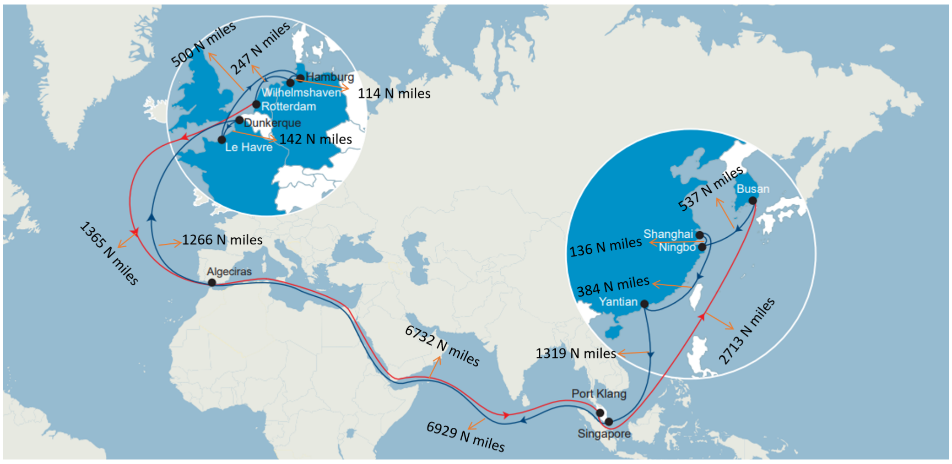


**Figure 2.** Shipping route 1.

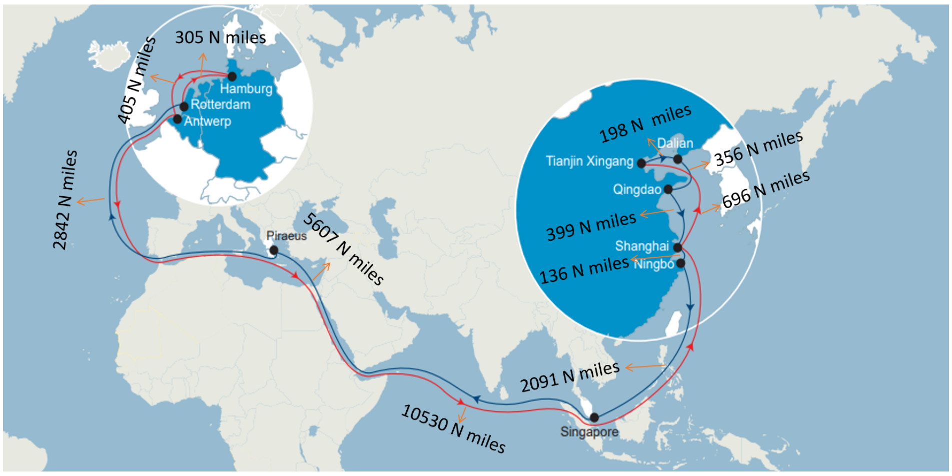


**Figure 3.** Shipping route 2.

4.1.2. Parameter Settings

We first set the values of parameters for drawing the basic results, and we conduct sensitivity analysis to examine the impacts of these parameters.

1. The fixed cost $c$. Referring to [34], we first set $c$ = 180,000 per week for a 5000-TEU (twenty-foot equivalent unit) container ship.
2. The fuel price $\mu$. Referring to [35], we set $\mu$ to be an average value of 600 (USD/tonne).
3. The charged fee of emissions $\beta$. EU ETS allowance prices closed at USD 102 per tonne on April 17, according to Ice Exchange data [7].
4. The conversion rate of fuel consumption and emissions $\gamma$ is set to 3.15 [36].
5. Referring to [32], we set $f(v^3) = 0.00043 \times v^3$.
6. The berthing time at port $i$ $t_i$[3]: Busan—1.1 days; Ningbo—1.5 days; Shanghai—1.0 day; Yantian—0.6 day; Singapore—1.0 day; Algeciras—0.7 day; Dunkerque—1.6 days; Le Havre—0.8 day; Hamburg—1.4 days; Wilhelmshaven—1.1 days; Rotterdam—1.3 days; Tianji—1.2 days; Dalian—1.5 days; Qingdao—1.5 days; Antwerp—1.3 days.
7. We set the emissions per hour during the berthing to be 2 tonnes; i.e., $Q = 2$.
8. We set $v_{\max} = 18$ knots and $v_{\min} = 10$ knots.

*4.2. Basic Results*

Using the routes in Table 2, we conducted numerical experiments and report the results in Table 3. As defined in Section 2, the legs within the EU area are represented by $I^2$, the legs into or out of the EU areas are represented by $I^1$, and the remaining legs are represented by $I^0$. The optimal value of the objective function of [M3] is represented by "OBJ". From Table 2, Route 1 is equipped with more ships compared to Route 2, which indicates that more ships are needed for a longer route to maintain the weekly service frequency (the total distance of Route 1 is 22,258 nautical miles, and the total distance of Route 2 is 23,565 nautical miles). The ship's sailing speed is the highest during the non-EU legs, followed by the legs linking the EU and non-EU areas, and finally, it is the lowest within the EU area, which validates Proposition 2. For instance, the sailing speed in EU legs on Route 1 is 11.6 knots, while in the legs linking the EU and non-EU areas, it reaches 12.1 knots, and in non-EU legs, it reaches its peak at 13.0 knots. Indeed, the rationale behind varying sailing speeds is to manage emissions effectively with the aim of minimizing costs. As ships increase their sailing speed, they also generate higher emissions, which subsequently results in a higher charged fee. The policy of charging fees based on emissions aims to encourage shipping companies to adopt more environmentally friendly practices and optimize their sailing speeds to minimize their carbon footprint. By aligning charging with emissions, the policy incentivizes the adoption of sustainable measures.

We next use Route 2 as a computational instance for the following sensitivity analysis.

**Table 3.** Basic results.

| Route ID | Set of Legs | Legs | Total Distance (Nautical Mile) | Sailing Speed (knot) | Number of Ships | OBJ (USD) |
|---|---|---|---|---|---|---|
| 1 | $I^0$ | Busan → Ningbo → Shanghai → Yantian → Singapore; Port Klang → Busan | 3901 | 13.0 | 13 | 3,924,499.1 |
| | $I^1$ | Singapore → Algeciras; Rotterdam → Port Klang | 15,020 | 12.1 | | |
| | $I^2$ | Algeciras → Dunkerque → Le Havre → Hamburg → Wilhelmshaven → Rotterdam | 2269 | 11.6 | | |
| 2 | $I^0$ | Tianjin → Dalian → Qingdao → Shanghai → Ningbo → Singapore | 3876 | 12.8 | 14 | 4,166,763.3 |
| | $I^1$ | Singapore→ Piraeus; Antwerp → Shanghai | 16,137 | 12.0 | | |
| | $I^2$ | Piraeus→Rotterdam → Hamburg → Antwerp | 3552 | 11.1 | | |

### 4.3. Sensitivity Analysis

As concerns about carbon emissions intensify, the levied fees associated with emissions are subject to potential changes as more countries prioritize addressing the issue of carbon emissions. Moreover, in the fundamental analysis, certain critical parameters, such as the unit fuel price and the weekly fixed cost per ship, are assumed to be deterministic. Nevertheless, these parameters actually often experience fluctuations. Consequently, sensitivity analyses are undertaken to examine the impacts of these parameters on operational decisions, considering their dynamic nature in real-world scenarios.

#### 4.3.1. Impact of the Charged Fee of Emissions

This study first investigates the impact of the charged fee of emissions $\beta$ on the operation decisions. With more emphasis on the carbon-emission problems and sustainable development recently, the charged fee of emissions has become an important tool to control carbon emissions and exerts a significant influence on shipping companies' decisions. This means the value of $\beta$ may change in reality. So the sensitivity analysis of $\beta$ is necessary. In this experiment, we set the fee of the emissions range from 80 to 170 USD per tonne. Given current trends, it is anticipated that the EU area will indeed implement stricter policies for carbon emissions control in the future. As a result, a higher emissions fee is expected to be imposed on shipping companies operating within the EU region.

Computational results are summarized in Table 4, where we can find that the incremental surge in the charged emissions fee engenders an upward trajectory in the objective value, primarily driven by the simultaneous amplification in the cumulative fee levied. In addition, when the fee charged for emissions increases, more ships are needed and the sailing speed and fuel consumption decreases, because the faster the ships sail, the more fuel they will consume, leading to more emissions, which means more costs. So shipping companies will decrease sailing speed to reduce emissions.

**Table 4.** Impact of the charged fee of emissions on the operation decision.

| $\beta$ (USD/ton) | Set of Legs | Sailing Speed (knot) | Fuel Consumption (ton) | Number of Ships | OBJ (USD) |
|---|---|---|---|---|---|
| 80 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>11.9<br>11.5 | 0.90<br>0.72<br>0.65 | 14 | 4,101,154.6 |
| 90 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 0.90<br>0.74<br>0.59 | 14 | 4,131,128.5 |
| 100 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 0.90<br>0.74<br>0.59 | 14 | 4,160,824.1 |
| 110 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 0.90<br>0.74<br>0.59 | 14 | 4,190,519.8 |
| 120 | $I^0$<br>$I^1$<br>$I^2$ | 13.3<br>11.9<br>11.1 | 1.01<br>0.72<br>0.59 | 14 | 4,220,180.5 |
| 130 | $I^0$<br>$I^1$<br>$I^2$ | 13.3<br>11.9<br>11.1 | 1.01<br>0.72<br>0.59 | 14 | 4,249,612.9 |
| 140 | $I^0$<br>$I^1$<br>$I^2$ | 13.3<br>11.9<br>11.1 | 1.01<br>0.72<br>0.59 | 14 | 4,279,045.4 |
| 150 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>11.1<br>10.2 | 0.76<br>0.59<br>0.46 | 15 | 4,305,595.6 |
| 160 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>11.0<br>10.2 | 0.76<br>0.57<br>0.46 | 15 | 4,331,828.7 |
| 170 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>11.0<br>10.2 | 0.76<br>0.57<br>0.46 | 15 | 4,358,061.8 |
| 180 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>11.0<br>10.2 | 0.76<br>0.57<br>0.46 | 15 | 4,384,294.9 |

#### 4.3.2. Impact of the Fuel Price

Within the framework of the fundamental analysis, the deterministic assumption sets the unit price of fuel ($\mu$) at 600 USD/ton. However, to account for the inherent volatility observed in real-life scenarios, this sensitivity analysis considers a range of values for $\mu$, spanning from 570 to 700 USD/ton. This range is determined based on the minimum and maximum fuel prices recorded between September 2020 and July 2023, amounting to 579.00 and 690.50 USD/ton, respectively [35]. The findings of this analysis are succinctly presented in Table 5.

Examining Table 5, it becomes evident that a direct relationship exists between the fuel price and the objective value, whereby an increase in fuel price leads to a corresponding increase in the objective value due to amplified total fuel costs. Additionally, as the fuel price increases, a higher number of ships are necessitated and, consequently, the sailing speed diminishes. This phenomenon arises from the cubic relationship between fuel consumption and sailing speed. Heightened fuel prices prompt shipping companies to explore methods of curbing fuel consumption, thereby resulting in a decrease in sailing speed. Furthermore, in the event that the unit price of fuel becomes excessively exorbitant, shipping companies may opt to employ more ships in order to fulfill the weekly service frequency requirement while operating the vessels at reduced speeds.

**Table 5.** Impact of the fuel price on the operation decisions.

| $\mu$ (USD/ton) | Set of Legs | Sailing Speed (knot) | Number of Ships | OBJ (USD) |
|---|---|---|---|---|
| 570 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,099,569.9 |
| 580 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,121,967.7 |
| 590 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,144,365.5 |
| 600 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,166,763.3 |
| 610 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,189,161.1 |
| 620 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,211,558.85 |
| 630 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,233,956.7 |
| 640 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,256,354.4 |
| 650 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,278,752.2 |
| 660 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>10.9<br>10.6 | 15 | 4,300,885.9 |
| 670 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>10.9<br>10.6 | 15 | 4,321,062.5 |
| 680 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>10.9<br>10.6 | 15 | 4,341,239.1 |

**Table 5.** *Cont.*

| $\mu$ (USD/ton) | Set of Legs | Sailing Speed (knot) | Number of Ships | OBJ (USD) |
|---|---|---|---|---|
| 690 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>10.9<br>10.6 | 15 | 4,361,415.6 |
| 700 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>10.9<br>10.6 | 15 | 4,381,592.2 |

#### 4.3.3. Impact of the Weekly Fixed Cost per Ship

In this study, the investigation focuses on exploring the influence of the weekly fixed cost per ship on operational decisions. The predetermined value for the weekly fixed cost per ship is set at 180,000 USD, which aligns with the configuration employed in [34]. However, it is worth noting that the value of $c$ can significantly vary due to various factors such as the impact of epidemics or other unforeseen circumstances [37], or it can even experience substantial reductions as a result of technological advancements. Consequently, the value range for $c$ is established between 60,000 and 300,000 USD, and the corresponding outcomes are documented in Table 6.

Based on the findings presented in Table 6, it becomes apparent that the objective value exhibits an upward trend as the weekly fixed cost per ship increases. This outcome can be attributed to the fact that with a higher weekly fixed operating cost, fewer ships are deployed within the route networks to mitigate the overall operational expenses. Notably, due to the fixed service frequency, liner ships are compelled to navigate toward their destinations at swifter speeds.

**Table 6.** Impact of the weekly fixed cost per ship.

| $c$ (USD/week) | Set of Legs | Sailing Speed (knot) | Number of Ships | OBJ (USD) |
|---|---|---|---|---|
| 60,000 | $I^0$<br>$I^1$<br>$I^2$ | 10.6<br>10.2<br>10.0 | 16 | 2,309,423.4 |
| 80,000 | $I^0$<br>$I^1$<br>$I^2$ | 10.6<br>10.2<br>10.0 | 16 | 2,629,423.4 |
| 100,000 | $I^0$<br>$I^1$<br>$I^2$ | 10.6<br>10.2<br>10.0 | 16 | 2,949,423.4 |
| 120,000 | $I^0$<br>$I^1$<br>$I^2$ | 10.6<br>10.2<br>10.0 | 16 | 3,269,423.4 |
| 140,000 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>11.0<br>10.2 | 15 | 3,579,676.8 |
| 160,000 | $I^0$<br>$I^1$<br>$I^2$ | 12.1<br>11.0<br>10.2 | 15 | 3,879,676.8 |
| 180,000 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,166,763.3 |

**Table 6.** *Cont.*

| $c$ (USD/week) | Set of Legs | Sailing Speed (knot) | Number of Ships | OBJ(USD) |
|---|---|---|---|---|
| 200,000 | $I^0$<br>$I^1$<br>$I^2$ | 12.8<br>12.0<br>11.1 | 14 | 4,446,763.3 |
| 220,000 | $I^0$<br>$I^1$<br>$I^2$ | 14.0<br>13.1<br>12.2 | 13 | 4,722,375.4 |
| 240,000 | $I^0$<br>$I^1$<br>$I^2$ | 14.0<br>13.1<br>12.2 | 13 | 4,982,375.3 |
| 260,000 | $I^0$<br>$I^1$<br>$I^2$ | 14.0<br>13.1<br>12.2 | 13 | 5,242,375.4 |
| 280,000 | $I^0$<br>$I^1$<br>$I^2$ | 14.0<br>13.1<br>12.2 | 13 | 5,502,375.4 |
| 300,000 | $I^0$<br>$I^1$<br>$I^2$ | 15.5<br>14.4<br>13.6 | 12 | 5,748,343.4 |

## 5. Conclusions

This study delves into the optimal strategies of shipping companies regarding sailing speeds and the number of ships on shipping routes, taking into account the policy recently proposed by the European Union—mandating the purchase of 100% of emissions for voyages within the EU and 50% of emissions for voyages to and from the EU commencing in 2026. Initially, we develop a non-linear optimization model, which we subsequently transform into an efficient IP model by introducing two propositions and discretizing decision variables. This transformation facilitates the utilization of effective solution methods. Through comprehensive experimentation, we demonstrate the efficacy of our proposed model and solution techniques. Moreover, we conduct an in-depth analysis of the effects ensuing from changes in the charged fee of emissions, fuel price, and weekly operational costs of ships on optimal decision making. In general, the total cost experiences an increase when the charged fee of emissions, fuel price, and weekly operational costs arise. Specifically, as the charged fee of emissions or fuel price escalates, shipping companies tend to reduce sailing speeds to curtail fuel consumption, while simultaneously augmenting the deployment of ships on routes to ensure compliance with weekly service frequency requirements. Conversely, in the event of an increase in weekly operational costs, fewer vessels are deployed to mitigate overall expenses.

Overall, our study significantly adds to the comprehension of how shipping enterprises can strategically formulate well-informed decisions as a proactive response to the EU's innovative policy landscape. By offering novel solution methodologies and conducting meticulous analyses on the susceptibility of optimal choices to diverse influencing factors, our research offers practical guidance for navigating the challenges and opportunities presented by the policy.

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## Notes

[1] https://www.cma-cgm.com/products-services/flyers, accessed on 4 August 2023
[2] http://port.sol.com.cn/licheng.asp, accessed on 4 August 2023
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