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The Annual Difference: How a Four-Century Debate between Confucian, Daoist, and Buddhist Thinkers on a Problem of Classical Exegesis Shaped a Predictive Mathematical Construct

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Abstract: In China, the precession of the equinoxes was conceptualized as an "annual difference" (*suicha* 歲差) between the tropical and sidereal year. The idea was introduced in the fourth century, it saw universal acceptance from the eighth century on, and it was in the four centuries in between that it was tried, debated, and spread whilst a Mediterranean-origin concept thereof arrived from India. Its four-century journey from fringe idea to universal truth is a well-studied point of interest in the history of astronomy. In this article, we will shift focus to the idea's polymathic protagonists and epistemic foundations to explore how a scientific idea was born, debated, transmitted, and taught in Confucian commentary and, no less important, how politics, geography, regional schools, foreign transmission, and the Buddhist and Daoist religions shaped how thinkers engaged with it as individuals and as communities. Inspired by the work of Chen Kanli and Randall Collins, the goal is to show that there is nothing simple or inexorable about how even an empirically useful tool of predictive astronomical modeling is received in the relevant expert community, as it is but one element in a complex network of people and ideas aligning and opposing in ever-evolving strategies to assert relevance.

Keywords: history of astronomy; precession of the equinoxes; Confucianism; commentary; Li Chunfeng 李淳風 (602–670); Kong Yingda 孔穎達 (574–648); Liu Zhuo 劉焯 (543–610)

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1. Introduction

Where science was once equated with "Taoists" in Chinese history (Sivin 1978), the last two decades have seen a shift towards thinking in terms of polymaths. Among the many recent studies on the topic,[1] two stand out for how they apply this, using the polymathy of their subjects to tell a more complex story. One is Goodman (2010), which explores how Xun Xu 荀勗 (*ca.* 220–289) wove a common philosophy, aesthetic, and political strategy into his various intellectual engagements, from music and cataloging to metrology. The other is K. Chen (2015), which traces how individual personalities shaped the triad of omenology, Classical studies, and court politics over the *longue durée* in a process of endless alignment and opposition. Reminiscent of the "hybrids" described in Latour's (1993) *We Have Never Been Modern*, the results are a reminder of the complexities of the "purification" of one continuous social, intellectual, and political fabric into tidy labels like "Taoists", let alone "science" and "religion". In this vein, and somewhat contrary to the theme of "the interaction of Daoist, Buddhist and Confucian thinkers", this article explores how a discrete problem in the history of astronomy was both the site of such an interaction and of more ephemeral yet pertinent distinctions.

The problem in question is the precession of the equinoxes, the timeline of which, for a historian of astronomy, begs for explanation. Introduced in the fourth century, the analogous concept would be alternately developed and ignored for two centuries, winning universal acceptance only after two further centuries of debate—all while a Mediterranean-

origin analogue would arrive from India to be completely ignored. Fortunately, the groundwork for this choice of subject is well laid. As concerns technical matters and the history of debate, Miaofu He (1978) and Anjing Qu (2008) leave little left to say. As concerns the lives and works of the key protagonists, the same is true of M. Chen (2003) and, to an extent, Martin and Chaussende (2020). As concerns the human geography, social networks, and transmission lines between protagonists, Morgan (2021) provides the tools for a finer contextual analysis of the ensemble than previously possible. Lastly, Yutong Wang (2022) has helped make further sense of the timeline by returning what has been long treated as a neat chapter in the history of astronomy to its context in Classical (i.e., "Confucian") exegesis and chronology. I will simply lean on the work of He, Qu, Chen, and Wang to flesh out the exploratory social and geographic treatment of precession in Morgan (2021), bringing additional texts and biographical details to bear particularly as the subject intersects with the "Three Teachings".

At its core, this article provides an overview of a technical concept, its epistemic bases, and its history up to the exegetical work of Kong Yingda 孔穎達 (574–648) and the mathematical work of his contemporary and ally, Li Chunfeng 李淳風 (602–670), in the early Tang (618–907). Following Yutong Wang (2022), I have structured this overview around the Classical evidence for precession and the commentarial tradition surrounding it, since the concept's history as a scientific construct only makes sense when we recognize that the "scientists" and "Confucians" were in dialogue, if not the same people.[2] Where this article differs is in its focus on the people: on the individuals, groups, affiliations, and the question of how a seemingly neutral matter of mathematical modeling further brought these "scientists" and "Confucians" into dialogue with "Buddhists" and "Daoists". As we will see, the matter was *not neutral*, it involved some of the most important and divisive figures in the history of the "Three Teachings", and yet this had no obvious relation with how individuals lined up for, against, or in silence concerning precession. There were, I argue, greater structural forces at play. One was the articulation of the contradictory epistemologies of mathematical astronomy and Classical exegesis (Wang 2022). One was the fracture of mathematical astronomy into distinct regional networks and traditions in the Period of Disunion (220–589) (Morgan 2021). Another, inspired by the aforementioned approach of K. Chen (2015) as read through the lens of Collins' (1998) *The Sociology of Philosophies*, is that the dividing line seems to have been in constant flux, as each generation redrew it in a bid to distinguish itself amid the realignment of various forces.

The (non-)treatment of precession in Kong Yingda and Li Chunfeng's respective canons captures a particular position at a particular moment in time, and attention to these structural forces provides, in my opinion, the most comprehensible account as to how they got there.

For readers unfamiliar with astronomy, titles ending in *li* 曆, like the *Huangji li* 皇極曆 (604), are mathematical procedure texts for calculating both the civil calendar and the astronomical phenomena that go into ephemerides, eclipse reports, etc. The majority of authors composed *li* to instigate a "reform" (*gaili* 改曆) that would see the court "implement" (*shi* 施) and "operate" (*xing* 行) their new *li* for official purposes (so-called "calendar reform"). These procedure texts are principally preserved in the "Li zhi" 曆志 and "Lüli zhi" 律曆志 monographs in the dynastic histories, and most of those mentioned here are translated and studied in Hongtao Liu (2003) and Cullen (2017). What matters for our purposes here is simply whether an individual *li* includes a model for precession.

As concerns people, it is difficult to establish an individual's polymathy and affiliations, let alone how those affiliations may have informed a position, without mention of a large number of names, biographical details, and events. My overview synthesizes information covered in greater detail in the aforementioned scholarship, and to help the reader, I end each people-heavy section with a summary of the pertinent features and dynamics of the communities described. Lastly, to argue that, compared to his other affiliations outside this debate, a subject's identity as a "Buddhist", "Daoist", or "Confucian" is ambiguous and irrelevant involves, ironically, establishing said identity. I have attempted to

do this with maximum disinterest, highlighting a subject's concrete engagements, how our sources label him, and how he is discussed in relevant scholarship.

2. Scripture, Science, and the Measure of the Invisible

As illustrated in Figure 1, the equinoxes are where the ecliptic (the sun's path) intersects the equator (the great circle midway between and perpendicular to the celestial poles). The solstices are the midway points between the equinoxes, where the ecliptic is furthest north and south of the equator. And all of these points shift westwards relative to the stars at a rate of approximately 1° every 71.6 years. This is called the "precession of the equinoxes", and it is due to a slow—roughly 26,000-year—gravity-induced wobble in the earth's rotational axis, the effects of which include the migration of the pole and, thus, a gyration of the polar–equatorial coordinate grid.

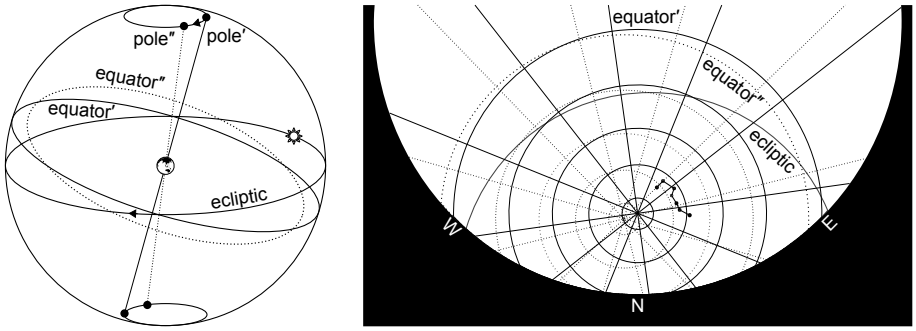

Figure 1. The Earth's axial precession.

In the West, Hipparchus (*ca*. 190–*ca*. 120 B.C.) was the first to discover and model the precession of the equinoxes. This was refined (Swerdlow 2010), and the Greek spherical understanding filtered into India by at least the time of Bhāskara I (fl. A.D. 629) (Pingree 1972). In China, the discovery was piecemeal, unfolding at its own pace until the introduction of the Islamic and Jesuit extensions of the Greek tradition. For the purposes of this article, it suffices to reduce this history, studied in He (1978) and Qu (2008), to three points. First, the opening step was to recognize an "annual difference" (*suicha* 歲差) between the time it took the sun to go from one winter solstice to the next (the tropical year) and to return to the same position among the stars (the sidereal year). Second, it was starting from Liu Zhuo's 劉焯 (N. Qi–Sui, 543–610) *Huangji li* 皇極曆 of 604 that experts began to narrow in on a recognizable rate. Third, it was only after Monk Yixing's 一行 (673–727) *Dayan li* 大衍曆 of 728 that the "annual difference" gained universal acceptance.

How does one perceive such a thing? In his preface to the *Chunqiu changli* 春秋長曆, we see Du Yu 杜預 (Wei–Jin, 222–285) struggling in vain to reconcile the dates of eclipses recorded in the *Spring and Autumn Annals* (covering 722–481 B.C.). In his words, such things "can be largely known, within limits" 有大量可得而限, but as time goes on, "differences (*cha*) of a hair's breadth" 毫末之差 accumulate and render the task impossible:

> Thus it is that in the *Annals* there are solar eclipses on consecutive months and that there are long years without any at all: there is no single principle, and holding to constant values in computation is why there is no astronomy (*li*) that does not slip ahead or behind. 故春秋日有頻月而蝕者，有曠年不蝕者，理不得一，而算守恒數，故曆無不有先後也.[3]

Du's astronomical work was part of his commentarial project, the *Chunqiu jingzhuan jijie* 春秋經傳集解, and further analysis can be found quoted therefrom in Kong Yingda's *Chunqiu zhengyi* 春秋正義 (K. Chen 2015, pp. 214–20; Wang 2023, pp. 123–49). On the subject of eclipses, Sima Qian 司馬遷 (*ca*. 145–*ca*. 86 B.C.) mentions studying "historical records" (*shiji* 史記) going back a hundred years.[4] The "Tianwen" 天文 (Heavenly Patterns) monographs of his and subsequent histories would further go on to provide the most professional, expansive, and unbroken record of eclipses in human history. But nothing could

outweigh the 37 dated eclipse records in the *Annals* and its traditions; it was a Classic (*jing* 經), supposedly edited by the hand of Confucius (551–479 B.C.), and the eclipses therein go back five centuries before the dawn of imperial record keeping. According to Ban Gu 班固 (32–92), Liu Xin 劉歆 (d. A.D. 23) likewise "created the *Santong li* 三統曆 and chronology (*Shijing* 世經) to explain the *Annals*" 作三統曆及譜以說春秋,[5] and the *Annals* have provided astronomers with critical data for the study of eclipses up to the modern day.[6]

Precession is different. At a human scale, its effects are generational and, thus, invisible. This is not to mention that the sun's position among the stars cannot be directly observed, since the light of the one drowns out the other. The "annual difference" is thus not *seen* but *sought*.

The earliest indirect method of measuring the position of the sun was to use the "star" (lodge) that was "centered" (*zhong* 中, i.e., culminated) on the meridian at dusk and dawn, when the sun is roughly a quarter of the sky away (Figure 2). This is inexact, and it was Jiang Ji 姜岌 of Tianshui 天水 (L. Qin, fl. 384) who first thought to extrapolate the position of the sun from the position of the moon at lunar eclipse, when the two are opposite one another. As per the first point, no one had bothered to systematically conduct and publish such measurements in anticipation of this use, so the historical record was mostly blank, and experts had to reconfigure their observation programs and content themselves with a limited set of recent data. It is also important to note that, in his exhaustive study of the earlier tradition, Guo Shoujing 郭守敬 (1231–1316) concludes that only six astronomical procedure texts (*li*) to his day had based their values on new observations, the rest relying on texts, number crunching, and splitting differences (Qu 2008, pp. 181–83).

The best that ancient scripture had to offer concerning the precession of the equinoxes were the legendary instructions given by King Yao 堯 to the brothers Xi and He 羲和 in the "Yaodian" 堯典 chapter of the *Classic of Documents*:

> When the day is [of] medium [length] the star is Bird, and this is used to determine mid-spring. . . . When the day is long[est] the star is Fire, and this is used to fix mid-summer. . . . When the night is [of] medium [length] the star is Barrens, and this is used to determine mid-autumn. . . . When the day is short[est] the star is Mane, and this is used to fix mid-winter. 日中星鳥，以殷仲春……日永星火，以正仲夏……宵中星虛，以殷仲秋……日短星昴，以正仲冬.[7].

There is a clear parallel between these instructions and the stellar orientations provided for each month in the later "Yueling" 月令 chapter of the *Record of Rites*. For example:

> In the late autumn month, the sun is in Chamber; Barrens is centered at dusk, and Willow is centered at dawn. 季秋之月，日在房，昏虛中，旦柳中.[8]

Neither of these passages is as useful as the *Annals*' eclipse records; they are undated, months and lodges are imprecise, and the *Documents* neglects to mention the hour. The *Documents* and *Rites* would also seem to contradict one another as to the month that a star like Barrens is centered (mid-autumn vs. late autumn). Nevertheless, these were some of the best data available on the position of the sun at the solstices and equinoxes prior to A.D. 384 for reasons of observational reach and sacred authority.

Lastly, we should ask why any of this matters. The facile answer is that astronomy was important, and that these are the sort of things that literally keep astronomers up at night.[9] More specifically, it is worth emphasizing that, while an "annual difference" model held significant explanatory potential, its predictive applications were limited: it enabled the expert to compensate for errors in calculated position so minute as to be undetectable at the scale of a single human lifetime, with little to retrodict in the past and no hope of verifying one's predictions in the future. We can thus say that curiosity was a factor, but it is difficult to label this "science for science's sake". As shown by the example of eclipses, such things were as much matters of "Confucian studies" as they were "astronomy", and most of the aforementioned "astronomers" were polymaths whose accomplishments in other fields made them natural vectors by which knowledge circulated from one to the other. These men had, for that reason, a habit of bringing together all the elements necessary

to reveal an irreconcilable conflict: that, in this case, between scripture and experience, between philology and experiment, and between Sage and man. They were also the ones obliged to close the matter, be it by concession or by feat of mental gymnastics.

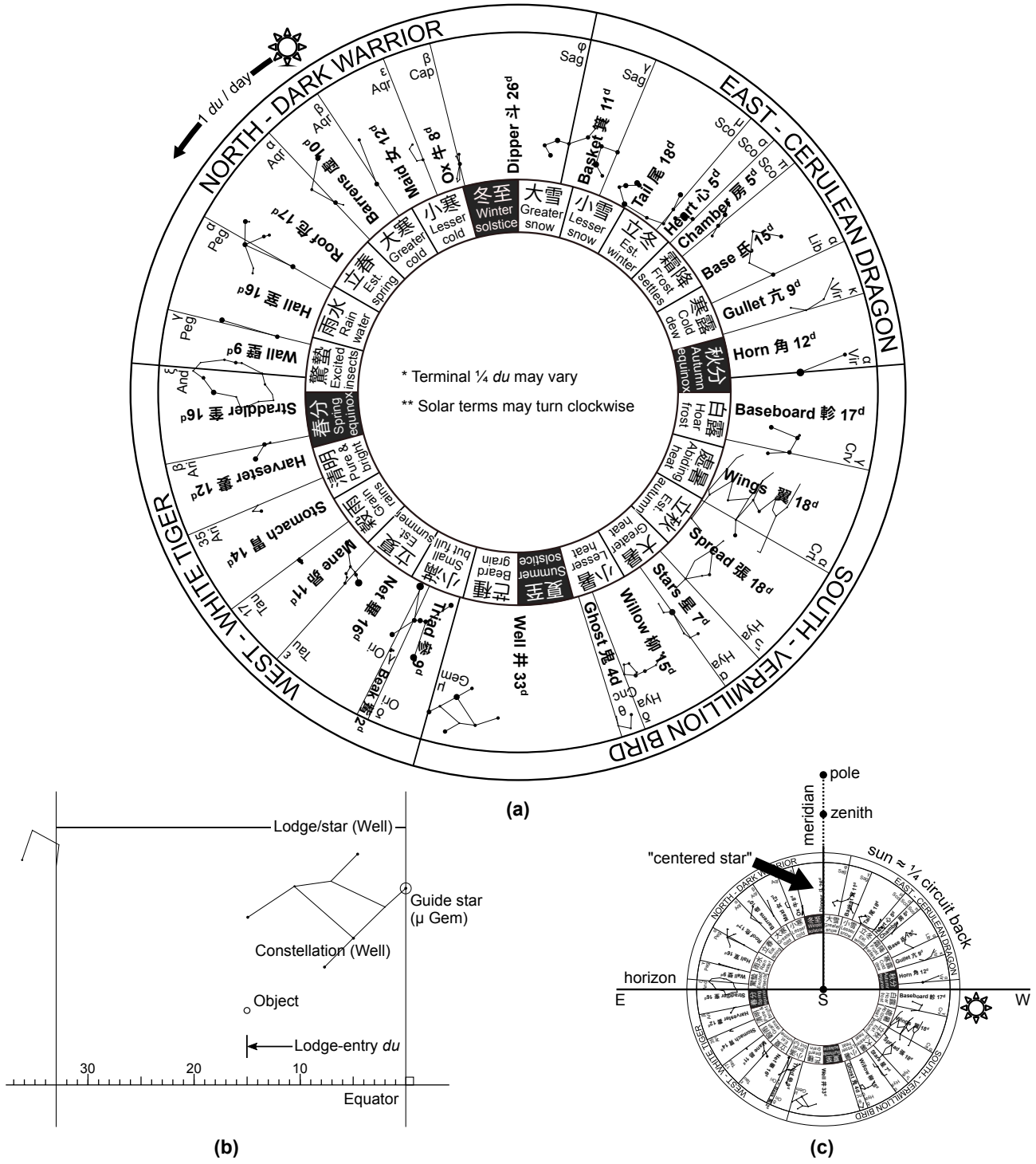

Figure 2. (**a**) The 28 lodges and 24 solar terms. (**b**) "Lodge entry-*du*" (*ruxiu du* 入宿度). (**c**) A "centered star" (*zhongxing* 中星). The *du* 度 is a linear measure of the circumference of an astronomical great circle defined as the daily displacement of the mean sun, meaning that there are as many *du* in a circle as there are days in the sidereal year and, roughly speaking, $365\frac{1}{4}\,du = 360°$, and $1\,du = 0.99°$. The specific lodge widths and solar term association are as per He Chengtian 何承天 (370–447).

3. Men of the Southern Sky

The story of precession generally begins with Yu Xi 虞喜 of Kuaiji 會稽 (Jin, fl. 307–345; for place names, see the map in Figure 3), whom Yixing credits as "awakening to it, making Heaven Heaven, and the year the year, then establishing a [precise value for their] difference to seek their transformations, making [the solstices and equinoxes] retreat 1 *du* every 50 years" 虞喜覺之，使天為天，歲為歲，乃立差以追其變，使五十年退一度.[10] For that reason, it seems wise to preface Yu Xi with the dream from which he was stirred.

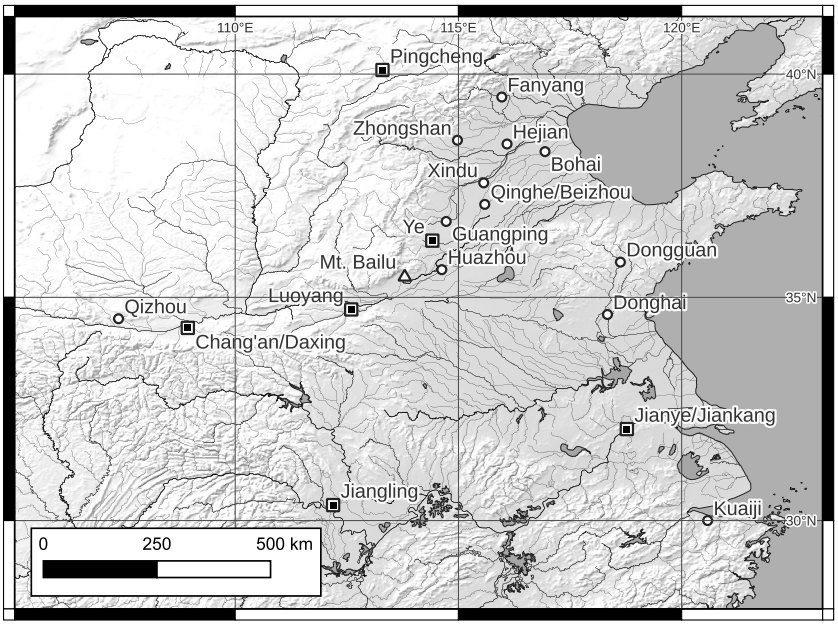

Figure 3. Map. Note that peoples' places of origin are given throughout at the level of the commandery (*jun* 郡) or prefecture (*zhou* 州).

In his commentary to the aforementioned passage from the *Documents*, pseudo-Kong Anguo 孔安國 (W. Han) glosses "Bird" as "the seven lodges of the Vermilion Bird of the South" 南方朱鳥七宿, and he interprets the following lodges as synecdoche by merit of their central position in the other quadrants:

> Barrens is the cent[ral] star of the Dark Warrior, which is also to say that [its] seven stars (lodges) are all visible on the date of the autumn equinox. 虛，玄武之中星，亦言七星皆以秋分日見.

In their respective commentaries, Ma Rong 馬融 (79–166) and his disciple Zheng Xuan 鄭玄 (127–200) disagree, insisting that "these all raise the [specific] stars (lodges) that are directly cent[ered on the meridian], not the fact that the quadrant [to which they belong] is complete[ly visible]" 皆舉正中之星，不為一方盡. Agreeing, the contrarian Wang Su 王肅 (195–256) insists that, *actually*, the parallel structure of the text implies a progression through the three months of each season such that each constellation refers to the "centered star" of *the subsequent month*.[11] The reason for this, presumably, is to align the "Yaodian" with the "Yueling" (above) and, more or less, with the observational reality of his own day. In sum, Ma Rong and Zheng Xuan broke from Kong Anguo to insist that the passage in question was, in fact, a list of modern-style scientific data, and Wang Su proposed a radical rereading to reconcile the two Classics with both one another and experience. Even Wang Su could trust Zheng Xuan, apparently, because Zheng Xuan was equally trained in astronomy and mathematics.[12]

Returning to Yu Xi, it is only ten centuries later, in the *Songshi* 宋史 (1346), that we get the first and only clue as to his line of reasoning:

> Yu Xi said, "On winter solstice in the time of Yao, 'The day is at its shortest, and the star is Mane'. Today, more than 2700 years [later], Eastern Wall is centered,

> so we know that this was brought about by a gradual difference every year". 虞喜云：「堯時冬至『日短星昴』，今二千七百餘年，乃東壁中，則知每歲漸差之所至」.[13]

In short, Yu Xi accepted the aforementioned premises, but instead of adapting his reading, he adapted the science. This required attributing an immaculately conceived, rainbow-eyebrowed sage king at the dawn of human civilization with a precise historical date. As studied in Wang (2022, pp. 236–41), Yu Xi had his choice of at least six chronologies placing the ascension of Yao between 2413 and 2145 B.C., and he apparently went with that used for the contemporary stele at "Yao's tomb" in Chengyang 城陽 District. All we know about this chronology is that the stele states that "from Yao's ascension to Yongjia year 3 (A.D. 309) were 2721 years" 堯即位至永嘉三年，二千七百二十有一載.[14]

As for the modern "centered star", Wang (2022, p. 241) argues that Yu Xi used the ancient "Yueling"—"In the mid-winter month, the sun is in Dipper, and the Eastern Wall is centered at dusk" 仲冬之月，日在斗，昏東壁中.[15] Alternatively, he may have used the latest solar table—from A.D. 174 (Cullen 2007)—and chose the solar term prior, Greater Snow, at the *beginning* of the mid-winter month and centered at dusk at Wall $\frac{7}{12}$ *du*. Yu Xi then correctly calculated from his chosen parameters the rate necessary for the "centered star" to retreat 54 *du* from mid-Mane to mid-Wall over 2700 years: $2700 \div -54 = 50$ years per -1 *du* (error -20.6 years). It is worth noting, of course, that this does not account for how far the centered stars would have moved from the date of the "Yueling" (or Han solar table) to Yu Xi's day in the fourth century.

We do not know where Yu Xi wrote this other than, socially speaking, in reclusion. It could have been in his six-volume monograph on scientific cosmology, the *Antian lun* 安天論, later dismissed as unserious by He Chengtian 何承天 (370–447) and Li Chunfeng (Morgan 2019, pp. 146–58), but it could have just as easily been in one of his several commentaries to the Classics. Another outside possibility is his thirty-volume collection *Zhilin xinshu* 志林新書, which later bibliographers classified under "Rujia" 儒家 (Confucianism), but whose contents range from history to local cults and early Daoist movements around Kuaiji.[16] Xi's younger brother Yu 預 led a successful career in the Palace Library and in historiography, and while Xi saw frequent solicitation after the Jin court's exodus south to Jiankang in 317, he was a man outside of time. Xi was the great-grandson of Fan 翻 (164–233), the celebrated scholar of the *Changes* and the outspoken courtier who saw himself banished from the Sun-Wu court to Jiaozhou 交州, in modern Vietnam. Of Fan's four prominent sons, Si 汜 (218–*ca*. 271) was purged in a succession struggle, returning to Jiaozhou as its regional inspector; Zhong 忠 (d. 280) died before surrendering as a prisoner of war to the Jin; Bing 昺, conversely, resigned his command of the northern army to join them; and Xi's grandfather Song 聳 kept his head down, transitioning from a governorship under the one to under the other. In the sciences, Fan was a friend of Lu Ji 陸績 (188–219) and a mutual acquaintance of Kan Ze 闞澤 (d. 243) through Ding Gu 丁固 (198–273), while Si served alongside Wang Fan 王蕃 (228–266) as a court attendant. The Yus were thus part of the scientific heyday of the Sun-Wu court, and one of the brothers—Song or Bing—wrote his own monograph on scientific cosmology (*Qiongtian lun* 穹天論). But that was another time: the south was defeated, its savants were dead or scattered, Yu Xi was an orphan, and the next century and a half saw no court activity in astronomy outside the Gansu Corridor and occupied Chang'an (Morgan 2021). Not only did Yu Xi choose to refuse office, live alone, and die childless, he was, in astronomy, a lost soul speaking into the void.[17]

This brings us to our second orphan. A generation later, in 365–371, Xu Guang 徐廣 of Dongguan 東莞 (Jin–Song, 351–425) began what would become a forty-year personal observation program pursued over his own career as an archivist and historian in Jiankang. His greatest contribution to the history of precession lies however in raising his fatherless nephew-in-law He Chengtian of Donghai 東海. His adopted son would take over for him, continuing forty more years, and he would orient his observations towards gnomonics and errors in the *timing* of the solstices and equinoxes. In 443, finally, He Chengtian would

submit his *Yuanjia li* 元嘉曆 to Emperor Wen of the Liu-Song (r. 424–453), sparking the first court discussion of "calendar reform" in Chinese-held territory since 237.

In his memorial, He focuses on the contemporary winter solstice, emphasizing how his family observation program helped detect errors of 4 *du* in position and 3+ days in timing and, of course, how the *Yuanjia li* stood to correct that. He does not use the word "annual difference", but his argument and sources are familiar:

> The "Yaodian" states that "The day is at its longest, and the star is Fire;—you may thus exactly determine the mid-summer". Today, Fire (i.e., Heart) is centered in the late summer month. [It] further [states] that "The night is of medium length, and the star is Barrens;—you may thus exactly determine the mid-autumn". Today, Barrens is centered in the late autumn month. Investigating this through centered stars more than 2700 years hence, [one finds that] the difference (*cha*) is 27 to 28 *du*, so on the winter solstice decreed by Yao, the sun was around 10 *du* in[to] Maid. 堯典云「日永星火，以正仲夏」。今季夏則火中。又「宵中星虛，以殷仲秋」。今季秋則虛中。爾來二千七百餘年，以中星檢之，所差二十七八度。則堯令冬至，日在須女十度左右也.[18]

The major difference is that He Chengtian uses *two* of the *Documents'* four relevant lines, ignoring that used by Yu Xi (mid-winter: Mane), and the resulting rate is half that of his predecessor. As to the rest, to summarize from Zheng (2007) and Wang (2022), He apparently switches to the chronology of Liu Xin's *Shijing* or Huangfu Mi's 皇甫謐 (214–282) *Diwang shiji* 帝王世紀, which place Yao's ascension around 2300 B.C.,[19] so as to keep Yu Xi's figure of "2700 years hence" a century later. The centered stars of "today" are likewise not those found in the *Yuanjia li*'s new solar table but in the ancient "Yueling".[20] He's argument is that the months stand for *qi*, the two-*qi* (= $\frac{1}{12}$ year = 30.44 day) discrepancy between the "Yaodian" and "Yueling" amounts, at 1 *du*/day, to a discrepancy of about 27 *du*, and $2700 \div -27 = 100$ years per -1 *du* (error $+29.4$ years). The position given for the ancient winter solstice—"10 *du* in[to] Maid"—is either calculated 27 *du* back from Jiang Ji's value (Dipper 17 *du*, in 384) or, if 十 is a mistake for 七, from He Chengtian's (Dipper $14\frac{1}{12}$ *du*). Either way, this argument is a pastiche, and the numbers do not add up.

Three things are odd about this memorial. First, this pastiche presents a stark contrast with the rigorous, functional mathematics of the extant procedure text of the *Yuanjia li* that it prefaces in the *Songshu* 宋書 (492/493). Second, the *Yuanjia li* does not, tellingly, use an "annual difference" at all.[21] Third, He is famous in Buddhist sources for his exchange with a monk over "the astronomy of Buddhist countries", and while several features of his *Yuanjia li* smack of Indian influence (Zheng and Jiang 2007; Niu and Jiang 1997), he does not seem to have borrowed anything concerning precession. He Chengtian was otherwise a rather standard Confucian scholar-official, moving from director of the watches for the Heir Apparent to become an editor and academician, and he was known, outside of astronomy, for compiling an encyclopedia (*Huanglan* 皇覽, 123 vols), a ritual compendium (*Lilun* 禮論, 300 vols), and a commentary to the *Spring and Autumn Annals* (*Chunqiu qianzhuan* 春秋前傳, 10 vols). Indeed, the other thing for which He Chengtian was remembered in Buddhist sources was his staunch ethnocentric criticism of its doctrines (Martin and Chaussende 2020, pp. 201–2). One might say that He Chengtian was the first serious astronomer to work on precession, but his solution was no better, he was as much of an exegete as were Yu Xi and Zheng Xuan, and he kept whatever he knew about precession out of his actual astronomy.

After some debate and revisions, Emperor Wen ordered the state adoption of the *Yuanjia li* in 445. A mere seventeen years later, under Wen's son Xiaowu (r. 453–465), the scholar-official Zu Chongzhi 祖沖之 of Fanyang 范陽 (429–500) would submit a new astronomy, with the aim of replacing the *Yuanjia li*, that finally incorporated an "annual difference". The proposal met a point-by-point rebuttal by the emperor's former tutor and current favorite, Dai Faxing 戴法興 (414–465), which stymied discussion until Xiaowu's death in 465. Under the Southern Qi (479–502), Prince Zhangmao 長懋 (458–493) brought the mat-

ter back up to Emperor Wu (r. 482–493) before their own untimely deaths, and it was only another dynasty later, in 504, that Chongzhi's son Geng 暅 (fl. 504–525) managed to convince the great Buddhist monarch, Emperor Wu of Liang (r. 502–549), to see his father's work into state policy. The idea seems to have gained traction, as when Emperor Wu commissioned a new astronomy in 544, Yu Kuo 虞劇 et al. likewise submitted one with an "annual difference".[22]

Neither of their rates were an improvement; Zu Chongzhi arrived at 45.9 years per −1 *du* (error −24.7 years), and Yu Kuo et al. at 183 years per −1 *du* (error +112.4 years). All we know about the latter is that Yu Kuo reports comparing Jiang Ji, He Chengtian, and Zu Chongzhi's winter solstice positions with contemporary "centered star" observations, i.e., a purely empirical approach from recent expert data.[23] Zu Chongzhi's, on the other hand, was part of a larger project. As detailed in his extant memorial, "there are three truths (i.e., a priori principles) to the establishment of [his] methods" 設法之情有三: (1) that Barrens 1 *du*, at the position $zi._{\text{B01}}$ 子 in the middle of the northern quadrant (Figure 2), is the natural start point of the winter solstice at high origin; (2) that $jiazi._{01}$ 甲子 is the natural start point in the sexagenary cycle for both the year and day at high origin; and (3) that, as per tradition, all astronomical cycles must start at zero at high origin.[24] In addition to the symbolic perfection of these choices, Zu had also developed a method for interpolating the exact moment of the solstice from multiple shadow measures, which led to a massive improvement in contemporary data. It is worth emphasizing, however, that the alignment of the burgeoning number of astronomical cycles to a single high origin required compromise, and as the most invisible and experimental of them all, his value for the "annual difference" was likely stretched to fit to his choice of origin (Qu 2008, pp. 175–77, 182).

Nothing further is known about Yu Kuo. As concerns Zu Chongzhi and his son Geng, their biographies and bibliographies present similar profiles to that of He Chengtian: scholar-officials equally accomplished in Classical exegesis and the sciences. With that said, Chongzhi was foremost an inventor and mathematician, attributed with the later Classic *Zhuishu* 綴述 and a commentary to the *Jiuzhang suanshu* 九章算術, and he is also attributed with having "written *yi* 義 ("meaning" commentaries) to the *Changes*, *Lao*[*zi*], and *Zhuang*[*zi*]" 著易老莊義.[25] The birth of theoretical mathematics in Liu Hui's 劉徽 commentary to the *Jiuzhang suanshu* (263) was intertwined with "dark learning" (*xuanxue* 玄學) philosophy (Guo 1984), so it is only natural that Zu—the second great Chinese mathematician—might have had an equal exegetical interest in its core texts.

In summary, the history of the precession of the equinoxes in China to A.D. 544 is one of a problem in Confucian exegesis that was consciously developed into one of mathematical astronomy and, gradually, saw a shift in epistemic weight away from ancient scripture and towards contemporary observations and mathematics. This was not because experts were improving or even narrowing in on a single answer, nor because the idea bore pragmatic fruit, but rather, it would seem, because they were curious. There were a variety of characters involved in this history, but they would appear to share three things in common. First, those at the frontier between mathematical astronomy and Classical exegesis tended, unsurprisingly, to be polymaths established in both fields. Second, everyone in this history from Yu Xi to Yu Kuang was active in and around Jiankang (Morgan 2021, pp. 32–34). Third, to pursue the question using the *Documents* meant rejecting both Kong Anguo's authority and his perfectly sensible position that one should be careful about overreading the "Yaodian".

4. Isolation, Conquest, and the New Underdogs

Following a parallel transmission from the Gansu Corridor in 439, the Northern Wei (386–534) capital at Pingcheng 平城 saw an even greater and more sustained explosion of activity in astronomy than did Jiankang in the same years. This too involved a variety of characters. Most were scholar-officials (e.g., Li Yexing 李業興 [484–549]). Authors also included a well-connected hermit (Li Mi 李謐 [484–515]), a *śramaṇa* (Tong Daorong 統道融 [fl. 518]), and two high-level Daoists: Zhang Bin 張賓 (fl. 568–584) and Kou Qianzhi's

寇謙之 (365–448) patron Cui Hao 崔浩 (d. 450). Through the court's move to Luoyang 洛陽 (493) and schism between the Eastern (534–550) and Western Wei (535–557), what *these men* shared in common was that none of them ever spoke of the "annual difference". This is despite two known transmissions from south to north. One was from Zu Geng to Xindu Fang 信都芳 (d. 543/550), after Xindu helped secure Zu's release as a prisoner of war *circa* 520. Another was the Western Wei's capture and integration of the human and material resources in astronomy at the Liang's (502–557) new capital of Jiangling 江陵 in 555, which saw the southern experts Yu Jicai 庾季才 (516–603) and Ming Kerang 明克讓 (525–594) commissioned to produce a north–south synthesis to inaugurate the Northern Zhou (557–581).

Silence remained the mainstream position on the "annual difference" at Ye 鄴 through the Eastern Wei and Northern Qi (550–577), as it did in Chang'an 長安 through the Western Wei and Northern Zhou into the Sui (581–618). Like Yu Jicai and Ming Kerang before him, the Daoist master Zhang Bin led a joint team of experts from Chang'an and Ye to author a new astronomical synthesis to inaugurate the Sui—the *Kaihuang li* 開皇曆 (584)—for which "[the northerners] Bin and company used [the southerner] He Chengtian's model with minor modifications" 賓等依何承天法，微加增損.[26] Like He Chengtian's now long-surpassed *Yuanjia li*, the extant parameters of the *Kaihuang li* show no distinction of the tropical and sidereal years.[27] No relevant Confucian commentary survives from this period, but to get a sense of the situation there, we can consult Zhen Luan 甄鸞 of Zhongshan's 中山 (N. Zhou, fl. 535–570) *Wujing suanshu* 五經算術, a two-volume study of math problems in the Confucian Classics and their commentaries. From the *Documents*, the *Wujing suanshu* offers three problems of astronomy, and one of numeration. It cites the Kong Anguo and Zheng Xuan commentaries as authorities, and, like Kong and Zheng, it does not single out the passage on "centered stars" as one requiring mathematical treatment.[28] Neither do the parameters of Zhen Luan's *Tianhe li* 天和曆 distinguish between the tropical and sidereal year.[29]

The *Tianhe li* and *Kaihuang li* share a number of other technical features in common with all northern works from the fifth century on: the "obscuration" (*bu* 蔀), large denominators, and the continued use of an eclipse year (*hui* 會) where the south had shifted to the nodical month (*yinyang li* 陰陽曆).[30] This common regional tradition in mathematical astronomy, however, is the only thing that Zhang Bin and Zhen Luan shared. Likened to Kou Qianzhi, Zhang Bin was a Daoist master (*daoshi* 道士) and the author of Emperor Wu of Zhou's (r. 560–578) conversion and pogrom against the Buddhist *samgha*.[31] Called in for a second opinion, Zhen Luan, on the other hand, authored what is perhaps the most thorough, witty, and devastating criticism of religious Daoism in Chinese history: the *Xiaodao lun* 笑道論 (Kohn 2008). Zhen Luan disappears from the historical record thereafter, while Zhang Bin would go on to become the chief propagandist behind the rise of the Sui.

To reiterate, northern experts at the courts in Luoyang, Chang'an, and Ye did not use or even speak of the "annual difference" for the first two and a half centuries since its discovery. This is despite the transmission of southern works (520), the integration of southern experts (555), and the nominal interest in synthesizing the two traditions (557–584). As to why, Morgan (2021) argues that this is because northern astronomers were first unaware, then unconvinced, having formed their own tradition over a century of isolation from their southern analogues. Whatever the case, an explicit conflict arose from strange quarters aimed at Zhang Bin's coalition.

To further summarize from Liang Li (2019) and Morgan (2021), as soon as Zhang Bin's *Kaihuang li* was enacted, three no-name former Northern Qi subjects from the North China Plain began to demand an immediate rollback of the monumental reform in favor of their own respective astronomies: Liu Xiaosun 劉孝孫 of Guangping 廣平 (d. 594/597), Liu Zhuo 劉焯 of Xindu 信都 (543–610), and Zhang Zhouxuan 張冑玄 of Bohai 渤海 (d. 605/617). Liu Xiaosun was an activist—a thorn in the side both the throne and the Directorate—who outlived Zhang Bin but died stymied by his own obstinacy. Zhang Zhouxuan was a no-name Directorate technician, who briefly served alongside Liu Xiaosun. Liu Zhuo, lastly, was a promising scholar-official who had fallen from grace for his involvement with his child-

hood friend Liu Xuan 劉炫 of Hejian's 河間 (*c*.546–*c*.613) scheme to pass off forgeries in the Imperial Library's book drive. Zhang Zhouxuan and Liu Zhuo were allies and protégés of Liu Xiaosun, Liu Xiaosun was a student of Zhang Zixin 張子信 of Qinghe 清河 (d. 577), and Zhang Zixin was not just their intellectual forefather but something of a revolutionary martyr figure. Zhang Zixin had died in the fall of the Northern Qi, and while he had briefly served the court as a doctor, he had mostly refused office to live in the woods on Mount Bailu 白鹿山, a sanctuary and site of solitary cultivation for Buddhists and Daoists alike.[32] In a similar vein, Zhang Zixin also fled the Ge Rong 葛榮 rebellion (526–528) by secluding himself on a sea isle, where he would devote himself for more than thirty years to observation and cultivating mathematical astronomy. The result was so revolutionary that it would mark a new age for Chinese astronomy and raise questions about foreign influence (Jiang 1991, pp. 276–83). But it was untested, and the live trial secured by his students Liu Xiaosun and Zhang Mengbin 張孟賓 of Guangping in 576 was cut short by the fall of Ye and the death of their master.

After Liu Xiaosun's death, Zhang Zhouxuan and Liu Zhuo had a falling out, each authored and petitioned for his own astronomy, and both, in different ways, won the day. In 597, Zhang was granted a live trial, he both destroyed and ousted the competition, his *Daye li* 大業曆 was instated, and he and Yuan Chong 袁充 (544–618), his newly arrived ally from the former Chen (557–589) court at Jiankang, were appointed to replace the Chang'an faction at the Directorate. Liu Zhuo's *Huangji li* 皇極曆 had the support of the heir apparent, but nothing came of it when he took the throne in 604. He and Liu Xuan were students of the influential Xu Zunming 徐遵明 (475–529)–Xiong Ansheng 熊安生 (d. 578) lineage of Rites scholarship in the North China Plain. Given their repeated failures at court, they thus wedded this with mathematics and the Zhang Zixin school of astronomy to make their living running a sort of private Confucian polytechnic and, as part of the curriculum, composing extensive subcommentaries to the Classics. At this they were successful. Among their students were Kong Yingda of Xindu and Zhang Shiheng 張士衡 of Hejian (d. 645), teacher to Jia Gongyan 賈公彥 of Guangping (fl. 637–650). And in the *Wujing zhengyi* 五經正義 (compiled 638–653), Kong Yingda specifies that he is simply reworking his teachers' subcommentaries to the *Documents*, *Odes*, and *Zuo Tradition* (McMullen 1988, pp. 74–75).

The "annual difference" would also appear to have emerged the victor from this rivalry. Their principal innovations aside, both Zhang Zhouxuan and Liu Zhuan's procedure texts adopt the "annual difference", and they manage to significantly improve on historical rates to that point; the *Daye li* uses 84.8 years per −1 *du* (error +14.2 years), and the *Huangji li* 76.5 years per −1 *du* (error +6.0 years). According to Li Chunfeng, "Zhouxuan both studied Zu Chongzhi and transmitted his master method" 冑玄學祖冲之，兼傳其師法.[33] If Shen Kuo 沈括 (1031–1095) is to be believed, however, this could also trace back to the North China Plain, because "it was only in the Northern Qi that Zhang Zixin first knew the annual difference through watching (observation)" 至北齊張子信方侯知歲差.[34] The northern and southern traditions had now truly been synthesized, if not replaced, and the new rivalry was between two offshoots of the Zhang Zixin school, both of which used the "annual difference".

A generation later, when Emperor Gaozu of the Tang (r. 618–635) took the throne, the Astronomical Directorate was run by the southerner Yu Jicai's grandson Jian 儉 and, in second command, Fu Yi 傅弈 of Ye (555–639). They recommended Fu Renjun 傅仁均 of Huazhou 滑州 to author the inaugural astronomical procedure text, and he came back with a finished procedure text the same year, which he sold to the emperor in seven points aimed at harmonizing the values of science, scripture, and symbolism. Two dealt with the long-debated idea of "fixing" (*ding* 定) the civil calendar to the true, astronomical new moon. Another two concerned the Zu Chongzhi-like choice of an appropriately meaningful high origin, appealing both to *Changes* numerology and the sexagenary year and day of the Tang founding. The other three affirm its accord with astronomical records in the "Yaodian", the *Odes*, and the *Spring and Autumn Annals* weft *Mingli xu* 命曆序. Fu's *Wuyin yuan li* 戊寅元曆 was rushed into service the following year without trial or discussion, but little was

new: "Renjun's astronomical methods followed Zhouxuan['s] as their ancestor, with minor reference to Liu Xiaosun's old appraisal" 仁均曆法祖述冑玄，稍以劉孝孫舊議參之.[35]

In sum, disciples of the hermit Zhang Zixin's North China Plain school forced a confrontation at the Sui court concerning, among other things, the "annual difference". They split, and one of the two new factions managed to forge an alliance with a newly arrived expert from Jiankang to chase out the old guard. The names had changed, but the southern–North China Plain alliance still controlled the Directorate in 618, and the Tang's (618–907) new astronomy was only naturally an embellishment and minor improvement on the previous historical winner's. The "annual difference", *it would seem*, was finally on the right track and here to stay.

5. New Stakes, New Positions

Backed by the Astronomical Directorate, recent history, and the founding emperor, rather than settling the expert opinion on the "annual difference", Fu Renjun's *Wuyin yuan li* was an inflection point around which the discussion spontaneously realigned into a new pair of opposing parties. The *Wuyin yuan li* went slightly backwards, using a rate of 55.5 years per −1 *du* (error −15.1 years), but even the author of the *Xin Tangshu* 新唐書 "Li zhi" 曆志, in all his hindsight, is ambivalent about whether Fu's work was actually worse than its eventual replacement:

> It was mostly greatly looser than [Li] Chunfeng (below). Still, the two were at odds (as concerns predictions), and Chunfeng was unable to exceed the hits that it got. 其大最疎於淳風。然更相出入，其有所中，淳風亦不能逾之.[36]

Either way, in its second year of service, the *Wuyin yuan li* disastrously predicted eclipses on 23 February (lunar), 10 March (solar), and 2 September 620 (solar) that, while they did actually occur in and around Tang territory, were reported as successive failures.[37] Zu Xiaosun 祖孝孫 of Fanyang and the Directorate's astronomy erudite Wang Xiaotong 王孝通 launched an investigation upon imperial order. Reporting back in 623, their first, principal (and wholly unrelated) criticism concerned the "annual difference"; they pointed to contradictions within the "Yaodian" militating against a neat date from a strict reading, and they affirmed the inherent relation between the current solstitial lodges (Dipper and Well) and the seasonal extremes of cold and heat. In doing so, their language echos that of the Kong Anguo commentary:

> "When the day is short[est] the star is Mane, and this is used to fix mid-winter". The seven lodges are all visible, but only the central lodge is evoked. If the central lodge is evoked, then the remaining stars can be known.「日短星昴，以正仲冬。」七宿畢見，舉中宿言耳。舉中宿，則餘星可知.[38]

Fu Renjun responded with a learned explanation grounded in both astronomy and the Classics, but the throne nonetheless ordered Wang and Cui Shanwei 崔善為 of Beizhou 貝州 (d. 627) to amend several dozen items.[39]

Once Zu and Wang had succeeded in drawing blood, Li Chunfeng of Qizhou 岐州 took over. At the time, Li was a 24-year-old auxiliary at the Directorate with his career ahead of him and, it would seem, a lot to prove. Li petitioned the throne three separate times for further amendments. The first, in 627, concerned eighteen unspecified matters, of which seven were adopted after review by Cui Shanwei. The second concerned a 3 *ke* 刻 (≈45-min) gap in the official calendar between what would be a momentous conjunction of the new moon and winter solstice on day $jiazi_{.01}$ on 19 December 640. Li had developed "new procedures" 新術 that happened to nudge the new moon past midnight, onto the more auspicious day, and they won the unconditional support of both Directorate staff and Kong Yingda, then chancellor of the National University and an influential voice on the Rites and imperial ceremonial. The very next year, Li was appointed as an erudite at the Ministry of Rites and, shortly after, saw himself promoted to assistant director of the Astronomical Directorate. The third petition, in 644, involved the symbolic "unfixing" of the civil calendar to the true, astronomical moon to avoid four "big" 30-day months in a

row. As it happened, by the time that the throne grew tired of amending the *Wuyin yuan li* in 664, Li was the director of the Astronomical Directorate, he was ready in waiting with his own *Linde li* 麟德曆, and this too was immediately adopted without trial or debate.[40]

The biggest sign of how the times had changed is that the *Linde li* was "referentially operated" 參行 with the *Jingwei li* 經緯曆 of the new Indian-origin director Gautama Luo 瞿曇羅. Indeed, appended to the version of the *Linde li* preserved in the *Jiu Tangshu* 舊唐書 (945) is a discussion of how eclipse prediction was handled in "the Indian methods of Kāśyapa Xiaowei and company" 迦葉孝威等天竺法.[41] If this essay was Li Chunfeng's, then it had no apparent impact on his work. Understandably, his two major influences were tied to his more immediate engagements. The first, as argued in Liang Li (2019), is that when Li Chunfeng was charged with writing the "Lüli zhi" 律曆志 for the court's new histories of the Jin (265–420) and the Five Dynasties (502–618) in 641–648, he solidified his knowledge of mathematical astronomy through study of Liu Zhuo's *Huangji li*, canonizing both the work and its author, in defiance of the conventions of state history, as a celebration of the underdog. The ultimate structural similarities between the *Linde li* and *Huangji li* several decades later were thus, according to Li (2019), a way for Li Chunfeng to secure by proxy the political and intellectual place that he felt his intellectual idol was due. In so doing, however, the *Linde li* abandoned the "annual difference", bringing us to Li Chunfeng's second source of inspiration.

Somewhere along the way, as it happens, Li also adopted the position of the first piece of criticism to garner immediate imperial support for challenging the scientific establishment. According to the rebuttal in Yixing's *Liyi* 曆議 a century later, "Chunfeng held that the winter solstice had always been at Dipper 13 *du*" 淳風以冬至常在斗十三度. His argument is threefold. First, "Chunfeng held that ancient procedures were loose and confused, [to the point that] they were unaware of discrepancies [as much as] 15 *du* with heaven at quarter and full moon, dusk and dawn" 淳風以為古術踈舛，雖弦望昏明，差天十五度而猶不知. In other words, astronomy had come a long way, and we should question the reliability of earlier sources given what we know of their inaccuracy on other fronts.[42] Second, he emphasizes that "today, at the medial *qi* of the first month of spring (Rain Water), the sun is in House, and the centered stars at dusk and dawn are no different than in the 'Yueling'", 今孟春中氣，日在營室，昏明中星，與月令不殊. Indeed, the precession of the equinoxes was sufficiently slow, and the "Yueling" sufficiently recent and vague, that a good half of the latter's solar and centered-star positions still held.[43] Third, as illustrated in Figure 4, Li Chunfeng hammers the contradiction inherent in reading the key passage from the "Yaodian" as a list of expert, modern-style "centered stars":

> If Mane was centered at the winter solstice, then neither the Stellar Fire nor the Stellar Barrens would yet be due west at the summer solstice and autumn equinox. And if Fire was centered at the summer solstice, and Barrens centered at the autumn equinox, then Mane would already be due east at the winter solstice. There is [some] expansion and contraction between these, and [thus] this is insufficient proof of the annual difference. 若冬至昴中，則夏至秋分星火、星虛，皆在未正之西。若以夏至火中，秋分虛中，則冬至昴在巳正之東。互有盈縮，不足以為歲差證.[44]

This is why Yu Xi conveniently ignored the summer solstice and autumn equinox, why He Chengtian ignored the winter solstice, and why the result was a rate of precession half that of Yu Xi's.

The stakes had clearly shifted, and so too did the divisions. This was no longer about regional cliques. The North China Plain school had won, almost everyone involved was now from the neighborhood, and the only clear distinction between regional bases was that between the Chinese and Indian-origin staff at the Directorate. Neither was this purely about the finer distinctions between the *Daye li* and *Huangji li*, because their consensus on the "annual difference" was now up for question by serious experts. Here too, lastly, the debate involved major figures in the "Three Teachings" without any obvious relation to

their affiliations. Fu Renjun was a Daoist master.[45] So too once was his Directorate sponsor Fu Yi, who famously militated for the abolition of Buddhism as a morally depraved foreign scourge in 621–626.[46] Kong Yingda, by contrast, was one of the leading voices of Confucian scholarship, but Li Chunfeng, the leader of the opposition, was also the son of a Daoist master and was at least posthumously attributed with a considerable output in religious Daoist writings (Goodman 2019).

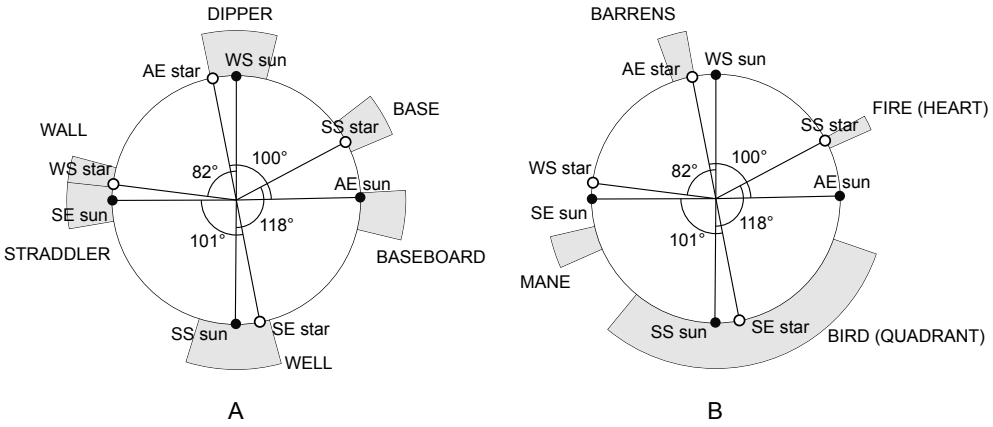

Figure 4. Distribution of solar positions and centered stars according to He Chengtian's solar table for 443 (A) compared to the "Yaodian", as per Liu Chunfeng's analysis (B). As such, either the centered stars for the autumn equinox (AE) and summer solstice (SS) work, and Mane is too far east, or the winter solstice (WS) works and Barrens and Fire are too far west. Diagram based on Qu (2008, p. 188, Figure 3-14).

The one neat parallel with the previous generation is that Li Chunfeng was an underdog on a mission to redeem his own scientific master/martyr figure. This worked out for him. Not only did he succeed in canonizing Liu Zhuo in state history and astronomical policy, but Li would also be charged in 656 with fixing and commentating a canon of Computational Classics: the so-called *Suanjing shishu* 算經十書, the equivalent in mathematics of Kong Yingda's *Wujing zhengyi*. This also worked out for Wang Xiaotong, one notes, whose *Jigu suanshu* 緝古算術 Li Chunfeng elevated to the status of a Classic and included in his canon.[47] It is tempting to interpret Li Chunfeng's position on the "annual difference" as part of a cynical strategy to distinguish himself so as to advance these goals. Indeed, it does not help that Li had, in the same years, affirmed the sage-time origin of the armillary sphere in asking for money to build one, only to reverse his position when it came time to write a proper history.[48] But Li was not the only one of Liu Zhuo's disciples to reject the "annual difference" in Liu's name.

In his preface to the *Shangshu zhengyi* 尚書正義, Kong Yingda describes himself as synthesizing the subcommentaries of Liu Zhuo and Liu Xuan to find an appropriate middle ground. His specific complaint about the former is "the way, for example, that [Liu Zhuo's] words necessarily fall back on numbers to which the text of the Classic is [invoked] only by way of complement" 若其言必託數，經悉對文.[49] Despite Kong's own frequent tangents into astronomy and mathematics in the *Wujing zhengyi*,[50] the key passage in the "Yaodian" is indeed stripped of any mathematical analysis that was presumably once there in Liu Zhuo's original. Instead, Kong Yingda lays out the positions of the early commentators, concluding that:

> Mr. Wang's explanation (that the "centered stars" are those of the following month) violates the flow of the text. The way that Mr. Kong [Anguo] directly gets "they (the lodges of the appropriate quadrant) are all visible" is slightly tortuous, [but] it is superior to Wang [Su] and Ma [Rong and Zheng Xuan] in terms of its reasoning (i.e., that these are modern-style "centered stars"). 王氏之說非文勢也。孔氏直取「畢見」，稍為迂闊，比諸王、馬，於理最優.[51]

Kong Yingda then spells out how this looked for three of the four relevant lines, omitting the mid-winter month:

> It is calculated (*ji* 計) that in mid-spring the sun is in Straddler–Harvester, and it sets at *you*.B10 (18:00, due west) on the horizon. At the time that dusk begins, Well–Ghost is at *wu*.B07 (12:00, the meridian); Willow–Stars–Spread is at *si*.B06 (10:00, two hours from culmination); and Axletree–Wings is at *chen*.B05 (08:00, four hours from culmination). This is to say that the seven lodges of the Vermillion Bird can all be seen… 計仲春日在奎、婁而入於酉地，則初昏之時井、鬼在午，柳、星、張在巳，軫、翼在辰，是朱鳥七宿皆得見也……
>
> It is calculated that in mid-summer the sun is in Eastern Well, and it sets at *you*.B10 on the horizon. At the time that dusk begins, Horn–Gullet is at *wu*.B07; Base–Chamber–Heart is at *si*.B06; and Tail–Basket is at *chen*.B05. This is to say that the seven lodges of the east are all visible… 計仲夏日在東井而入於酉地，即初昏之時角、亢在午，氐、房、心在巳，尾、箕在辰，是東方七宿皆得見也……
>
> It is calculated that in mid-autumn the sun is in Horn–Gullet, and it sets at *you*.B10 on the horizon. At the time that dusk begins, Dipper–Ox is at *wu*.B07; Maid–Barrens–Rooftop is at *si*.B06; Hall–Wall is at *chen*.B05. Speaking of this by raising the cent[ral] star Barrens is also to say that all seven stars (lodges) are visible together at the hour of dusk on the day of the autumn equinox. 計仲秋日在角、亢而入於酉地，初昏之時斗、牛在午，女、虛、危在巳，室、壁在辰，舉虛中星言之，亦言七星皆以秋分之日昏時並見.[52]

This suffices to illustrate Kong Anguo's point: that the lodges of the relevant quadrants "are all visible". However, Kong Yingda does not account for the difference in the time and direction of sunset at the solstices, which makes this nonsensical if read as an attempt at a coherent mathematical solution (compare Figure 5 to Figure 4). Kong Yingda would have known that, given his background, so the only sensible conclusion is that he consciously refused to treat this as a problem of mathematical astronomy and, thus, as proof of the "annual difference".

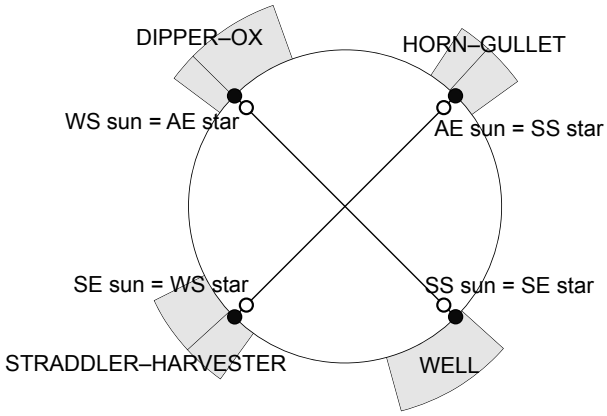

Figure 5. Illustration of Kong Yingda's description of the position of the sun and "centered stars" in the "Yaodian".

The discussion of the "annual difference" had seen a hard turn back to Classical evidence in the rhetoric of both Fu Renjun and his critics in the 620s. Whatever we may say about Wang Xiaotong, Li Chunfeng and Kong Yingda knew the subject, and their stated reasons for rejecting this scientific construct both involve considerations of antiquarian authority, philological rigor, and the sensibility of Kong Anguo's reading.

Fu Yi, Fu Renjun, and Li Chunfeng were all religious Daoists, but the Directorate was a subsidiary of the Ministry of Rites, and the idiom of astronomy was that of the Confucian Classics, so it was only a matter of time until the Confucian commentarial tradition reasserted itself on the matter. Indeed, the north had gotten along just fine without the "an-

nual difference" until Zhang Zhouxuan had had his way in 597, and it is entirely possible that the silent resistance to it there was rooted in similar considerations.

6. Conclusions

The story of the "annual difference" fades out for another generation or two before reversing course one last time in the eighth century. Now it was Buddhists fighting among themselves, with the monk Yixing, backed by Vice Director Nangong Yue 南宮說, opposed by Gautama Luo's son, Director Gautama Siddhārtha 瞿曇悉達.[53]

After his untimely death in 727, Yixing would posthumously emerge the victor and close the debate with Nangong Yue's editorial and political aid. Yixing's *Dayan li* 大衍曆 would both replace the *Linde li* in state policy and best Gautama's translated work, the *Navagraha li* (*Jiuzhi li* 九執曆), in live trial.[54] And as concerns the "annual difference", the detailed analysis in Yixing's *Liyi* would square the circle by harmonizing the growing number of reliable historical observations with a clever rereading of the "Yaodian" (Qu 2008, pp. 187–95). Li Chunfeng and his contemporaries' skepticism was thereupon relegated to the dustbin of history. However, the monk's astronomy would have no impact on the place of Kong Yingda's *Wujing zhengyi* in Classical education and the civil service examinations. Centuries later, this led to a division among exegetes. Some, like Huang Zhencheng 黃鎮成 (1287–1362), expanded upon Kong Yingda's reasoning.[55] Others, like Chen Dayou 陳大猷 (1198–1250), would conclude that "it is probably that no one between the two Kongs (Anguo and Yingda) and Wang Su knew that astronomers had a model for the annual difference" 蓋二孔、王肅皆不知曆家有歲差之法.[56] Indeed, how could a "Confucian" (*Rujia*) like Kong Yingda have known anything about the work of an "astronomer" (*lijia* 曆家) like Liu Zhuo?

The history of the "annual difference" as a scientific construct is not linear, nor is it one that is neatly contained within the bounds of such simple, retrospective labels. It was equally a problem of the radically different *epistemes* of astronomy and exegesis, it passed through the hands of some of the most important figures in the history of Buddhism, Daoism, and Confucianism, and it was alternately ignored and rejected for four centuries in astronomy. One of the things that helps make sense of this history is thus placing the voices of the exegetes back in dialogue with those of the astronomers, as do K. Chen (2015) and Wang (2022). In this case, it reminds us that the entire scientific debate was, essentially, between Kong Anguo and Zheng Xuan's respective exegetical positions, and that the former's skepticism was no less legitimate, authoritative, or sensible so many centuries later. Equally important is situating the discussion in regard to geographical divisions and regional networks. After all, it is probably misleading to treat this as a "discussion" when the idea was exclusive to Jiankang until adopted by a peripheral Northern Qi school as part of their gambit to overturn the scientific establishment of their conquerors in Chang'an (Morgan 2021).

Lastly, while modern and premodern scholars alike tend to think of the history of astronomy as unfolding according to some teleology of progress or "accumulation" (*ji* 積) (Henderson 2006; Morgan 2017, pp. 177–12), Li Chunfeng and Kong Yingda's sudden reversal from their master Liu Zhuo's position in their influential canons reminds us that things are not always so simple. It is possible to interpret this reversal at face value as a return to Kong Anguo, to responsible philology, and to the skepticism befitting both an astronomer and an exegete faced with a theory about intangible historical forces. However, given the very tangible forces of personal, regional, and political affiliations informing the history of the "annual difference" throughout, it is tempting to read this reversal through the lens of Collins (1998). Namely, it would appear that everything is fluid, and that each generation in an intellectual lineage like theirs stakes out a new, sometimes diametrically opposed set of positions in a bid to divide (or consolidate) the limited attention space of its contemporaries in favor of its distinction and perpetuation. Be it the one, the other, or both, what we are reading here in the timeless, state-sponsored "correct meaning" (*zhengyi* 正義) of Kong Yingda's Confucian canon is an informed, context-specific dissenting opinion

on a scientific matter in accordance with that personally advanced by him at court—one tacitly opposite the position of his master and presumed source text, and which would be definitively disproved two generations later by a Buddhist monk.

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Abbreviations

The following abbreviations are used in this manuscript:

SKQS	*Siku quanshu* 四庫全書. Rpt. of *Wenyuange Siku quanshu* 文淵閣四庫全書. 1500 vols. Taibei: Taiwan Shangwu yinshuguan, 1983.
SSJ	*Chongkan Song ben Shisanjing zhushu fu jiaokan ji* 重刊宋本十三經注疏附校勘記. Ruan Yuan 阮元, ed. Taibei: Yiwen yinshuguan, 1965.
T	*Taishō shinshū daizō kyō* 大正新脩大藏經. 100 vols. Takakusu Junjirō 高楠順次郎 and Watanabe Kaikyoku 渡邊海旭 et al., eds. Tōkyō: Taishō issaikyō kankō-kai, 1924–1934.

Notes

1 For example, J. Chen (2000), Goodman (2005), Cullen (2009), Lien (2011), Zuo (2018), and Morgan (2022).

2 I thank Reader 1 and Guo Jinsong 郭津嵩 for both bringing Wang Yutong's 王雨桐 work to my attention during the review process.

3 Cited in *Jinshu* 晉書 (Zhonghua shuju 中華書局 edn.), *j*. 18, pp. 563–64. On the perceived "limits" of empirical knowledge, see Sivin (1986).

4 *Shiji* 史記 (Zhonghua shuju edn.), *j*. 27, p. 1349.

5 *Hanshu* 漢書 (Zhonghua shuju edn.), *j*. 21A, p. 979. On Liu Xin's *Shijing*, see Cullen (2001) and Wang (2023).

6 See for example their use in the evaluation of the *Shoushi li* 授時曆 of 1280 (Sivin 2009, pp. 311–66) and in the study of the earth's rotation (Stephenson 1997, pp. 221–27).

7 *Shangshu zhushu* 尚書注疏 (*SSJ*)，*j*. 2, pp. 21a–b

8 *Liji zhushu* 禮記注疏 (*SSJ*), *j*. 17, p. 337a.

9 On the motivations for practicing astronomy, see Jiang (1991) and Morgan (2017).

10 *Xin Tangshu*, *j*. 27A, p. 600.

11 These four commentaries are quoted and summarized in *Shangshu zhushu*, *j*. 2, pp. 21b, 24b.

12 See Cullen (2009), Morgan (2022), Chemla and Zhu (2022), and the many earlier articles by Zhu Yiwen cited in the latter two.

13 *Songshi* 宋史 (Zhonghua shuju edn.), *j*. 74, p. 1689.

14 *Xushu zhengji* 續述征記, cited in *Taiping yulan* 太平御覽 (Sibu congkan edn), *j*. 80, pp. 8a–b.

15 *Liji zhushu*, *j*. 17, p. 344b.

16 For its classification, see *Suishu* 隋書 (Zhonghua shuju edn.), *j*. 34, p. 999. Fragments of the lost text are collected in *Quan shanggu sandai Qin Han Sanguo Liuchao wen* 全上古三代秦漢三國六朝文 (Hebei jiaoyu chubanshe, 1997), vol. 5, *j*. 82, pp. 853–54. I thank Johan Rols for directing me to his dissertation (Rols 2021) and helping me better understand the nature and contents of this work in a personal communication 15–17 June 2023.

17 See the Yus' biographies in *Jinshu*, *j*. 82, pp. 2143–2147; *j*. 91, pp. 2348–49, *Sanguo zhi* 三國志 (Zhonghua shuju edn.), *j*. 57, pp. 1317–27, and Martin and Chaussende (2020), pp. 601–2. On their scientific work, see M. Chen (2003, pp. 237–38).

18 *Songshu* 宋書 (Zhonghua shuju edn.), *j*. 12, p. 261.

19 On the *Shijing*, see Note 5.On the *Diwang shiji* 帝王世紀, see Nagel-Angermann (1999).

20 He Chengtian's solar table of 443 has the centered star at Greater Heat (: late summer) as "Tail $8\frac{8}{12}$" 尾八太弱 and that at Frost Settles (: late autumn) at "Maid $11\frac{5}{12}$" 女十一半弱 (*Songshu*, *j*. 13, p. 280). Compare this to "In the late summer month, the sun is in Willow, and Fire is centered at dusk" 季夏之月，日在柳，昏火中 (*Liji zhushu*, *j*. 16, p. 318b) and "In the late autumn month, the sun is in Chamber, and Barrens is centered at dusk" (above).

21 For a translation and study of the *Yuanjia li*, see Liu (2003, pp. 241–84).

22 *Songshu*, *j*. 13, pp. 304–17, *Nanshi* 南史 (Zhonghua shuju edn.), *j*. 72, p. 1774, and *Suishu*, *j*. 17, pp. 416–17.

23 *Xin Tangshu*, *j*. 27A, pp. 616–17; cf. Qu (2008, pp. 193–94).

24 *Songshu*, *j*. 13, pp. 289–90.

25 *Nanshi*, *j*. 72, p. 1774.

26 *Suishu*, *j*. 17, pp. 420–21. From Ye, Zhang Bin's team included the academician and failed astronomical reformer Zheng Yuanwei 鄭元偉 (fl. 576–581). The famous Northern Qi scholar Li Delin 李德林 (532–592) is also attributed with a *Kaihuang li* (*Jiu Tangshu* 舊唐書 [Zhonghua shuju edn.], *j*. 47, p. 2038), suggesting that he too was part of the picture. The southerner Yu Jicai remained the acting director of the Astronomical Directorate at this time, and while he and his clique did not participate in the authorship of the *Kaihuang li*, they were later convicted for failing to "investigate its accuracy" 審疎密 and "vainly operating this astronomy" 虛行此曆 [upon imperial order] despite their expertise and position (*Suishu*, *j*. 17, pp. 434–35).

27 *Suishu*, *j*. 17, pp. 421–22, and *Kaiyuan zhanjing* 開元占經 (*SKQS*), *j*. 105, pp. 12a–14a.

28 Qian (1998), pp. 334–37. For a recent study of the *Wujing suanshu* and math problems in Confucian commentary, see Chemla and Zhu (2022).

29 *Kaiyuan zhanjing*, *j*. 105, pp. 11a–b.

30 The details of these elements are not particularly important for the current article, and I am in the process of expanding upon these structural distinctions in other writings. Nevertheless, the curious reader can learn more about them in Liu (2003), Qu (2008), and Cullen (2017).

31 *Fozu tongji* 佛祖統紀 (*T* no. 2035), *j*. 42, p. 386b, and *Fozu lidai tongzai* 佛祖歷代通載 (*T* no. 2036), *j*. 9, p. 554c.

32 See for example *Xu Gaoseng zhuan* 續高僧傳 (*T* no. 2060), *j*. 16, p. 559, *j*. 19, p. 584, *j*. 20, p. 590, *j*. 23, p. 626, *j*. 28, p. 686, and *Sandong qunxian lu* 三洞群仙錄 (*Zhengtong Daozang Hanfenlou* 正統道藏　涵芬樓 *edn.*), *j*. 13, p.8a.

33 *Suishu*, *j*. 17, pp. 435.

34 *Mengxi bitan jiaozheng* 夢溪筆談校證 (Shanghai chuban gongsi, 1956), *j*. 7, p. 294.

35 *Xin Tangshu*, *j*. 25, p. 536.

36 *Xin Tangshu*, *j*. 25, p. 536.

37 These are lunar eclipse 06316 and solar eclipses 06240 and 06241 in Espenak and Meeus (2007) and Espenak and Meeus (2009).

38 *Xin Tangshu*, *j*. 25, p. 534. Compare Zu and Wang's language to the earlier quotation from Kong Anguo above and "Fire is the central asterism (*zhongxing*) of the Cerulean Dragon, and we know by evoking its center that its seven stars (lodges) are visible" 火，蒼龍之中星，舉中則七星見可知 (*Shangshu zhushu*, *j*. 2, p. 21b; cf. Wang (2022, p. 249).

39 *Xin Tangshu*, *j*. 25, pp. 534–36.

40 *Xin Tangshu*, *j*. 25, pp. 534–36; *j*. 26, p. 559; cf. Goodman (2019).

41 *Jiu Tangshu*, *j*. 33, p. 1205. On this passage and the Indian-origin astronomers at the Tang court, see Jiang (1992).

42 This point ties into the position on human progress developed in Li Chunfeng's historical writings, studied in Morgan (2017), pp. 177–12.

43 I have yet to reconstruct the calculations necessary to give an exact number of matches between the "Yueling" and Li Chunfeng's *Linde li*. However, a comparison of its 36 monthly data points with those for the medial *qi* in He Chengtian's solar table of 443 shows 16 matches, 17 mismatches, and 3 additional points that one can consider matches despite the fact that they use the defunct lodges Establishment Star (Jianxing 建星, in Dipper) and Bow (Hu 弧, in Well). On solar tables and the procedures by which they were replaced, see Zhang et al. (2008, pp. 27–49).

44 Cited in *Xin Tangshu*, *j*. 27A, pp. 600–18; cf. Qu (2008, p. 188).

45 *Jiu Tangshu*, *j*. 25, p. 534.

46 *Jiu Tangshu*, *j*. 79, pp. 2714–17; cf. *Fozu lidai tongzai* (*T* no. 2036), *j*. 11, pp. 564a–68b.

47 On the mathematical canon and its role in later education, see (Keller and Volkov 2014, esp. pp. 59–64).

48 Morgan (2019, p. 151; 2017, pp. 188–96). Note that Wang (2022, p. 257n3) disagrees with this reading, affirming that Li Chunfeng was consistent on the armillary sphere's sage-time origins.

49 *Shangshu zhushu*, preface, p. 4a.

50 See the works cited in Note 56.

[51] *Shangshu zhushu*, *j*. 2, p. 24b.

[52] *Shangshu zhushu*, *j*. 2, p. 24a–25a.

[53] Gautama Siddhārtha was named after the Buddha, he translated an astronomical work from Sanskrit, we know that his ancestors Prajñāruci and Dharmajñāna learned, taught, and translated Buddhist texts, and Wang Pu 王朴 (d. 959) erroneously identifies Gautama Siddhārtha as "a foreign monk from India" 天竺番僧 (Sen 1995). It thus seems safe to assume that their contemporaries would have associated the Gautamas with Buddhism in some sense.

[54] On Gautama's translation, see Yabuuti (1979).

[55] *Shangshu tongkao* 尚書通考 (*SKQS*), *j*. 1, pp. 44a–54b.

[56] *Shu jizhuan huowen* 書集傳或問 (*SKQS*), *j*. 1, p. 12b. A similar statement is to be found in Lin Zhiqi's 林之奇 (1112–1176) commentary in *Shangshu quanjie* 尚書全解 (Zhonghua shuju edn.), *j*. 1, pp. 23a–b.

References

Chemla, Karine, and Zhu Yiwen 朱一文. 2022. Contrasting Commentaries and Contrasting Subcommentaries on Mathematical and Confucian Canons: Intentions and Mathematical Practices. In *Mathematical Commentaries in the Ancient World: A Global Perspective*. Edited by Karine Chemla and Glenn W. Most. Cambridge, MA: Cambridge University Press, pp. 278–33.

Chen, Jinhua. 2000. The Birth of a Polymath: The Genealogical Background of the Tang Monk-scientist Yixing (673–727). *T'ang Studies* 18–19: 1–39.

Chen, Kanli. 陳侃理. 2015. *Ruxue, shushu yu zhengzhi: Zaiyi de zhengzhi wenhua shi* 儒學、數術與政治—災異的政治文化史. Beijing: Beijing daxue chubanshe.

Chen Meidong 陳美東. 2003. *Zhongguo Kexue Jishu Shi: Tianwenxue Juan* 中國科學技術史：天文學卷. Beijing: Kexue Chubanshe.

Collins, Randall. 1998. *The Sociology of Philosophies: A Global Theory of Intellectual Change*. Cambridge, MA: The Belknap Press of Harvard University Press.

Cullen, Christopher. 2001. The Birthday of the Old Man of Jiang County and Other Puzzles: Work in Progress on Liu Xin's *Canon of the Ages*. *Asia Major* 14: 27–60.

Cullen, Christopher. 2007. Huo Rong's Observation Programme of AD 102 and the *Han li* Solar Table. *Journal for the History of Astronomy* 38: 75–98. [CrossRef]

Cullen, Christopher. 2009. People and Numbers in Early Imperial China: Locating "Mathematics" and "Mathematicians" in Chinese Space. In *Oxford Handbook of the History of Mathematics*. Edited by Eleanor Robson and Jacqueline A. Stedall. Oxford: Oxford University Press, pp. 591–18.

Cullen, Christopher. 2017. *The Foundations of Celestial Reckoning: Three Ancient Chinese Astronomical Systems*. New York: Routledge.

Espenak, Fred, and Jean Meeus. 2007. *Five Millennium Canon of Solar Cclipses: −1999 to +3000 (2000 BCE to 3000 CE)*. Washington, DC: National Aeronautics and Space Flight Administration.

Espenak, Fred, and Jean Meeus. 2009. *Five Millennium Canon of Lunar Eclipses: −1999 to +3000*. Washington, DC: National Aeronautics and Space Flight Administration.

Goodman, Howard. 2005. Chinese Polymaths, 100–300 AD: The Tung-kuan, Taoist Dissent, and Technical Skills. *Asia Major* 18: 101–174.

Goodman, Howard. 2010. *Xun Xu and the Politics of Precision in Third-Century AD China*. Leiden: Brill.

Goodman, Howard. 2019. The Life and Intellectual World of Li Chunfeng (602–670). In *Monographs in Tang Official History: Perspectives from the Technical Treatises of the History of Sui (Sui shu)*. Edited by Daniel Patrick Morgan, Damien Chaussende and Karine Chemla. Cham: Springer Nature Switzerland AG, pp. 29–49.

Guo, Shuchun. 郭書春. 1984. Liu Hui Sixiang Tanyuan 劉徽思想探源. *Zhongguo Zhexueshi Yanjiu* 中國哲學史研究 2: 57–62.

He, Miaofu. 何妙福. 1978. Suicha Zai Zhongguo de Faxian Ji Qi Fenxi 歲差在中國的發現及其分析. *Keji Shi Wenji* 科技史文集 1: 22–30.

Henderson, John. 2006. Premodern Chinese Notions of Astronomical History and Calendrical Time. In *Notions of Time in Chinese Historical Thinking*. Edited by Huang Chun-chieh and John Henderson. Hong Kong: Chinese University Press, pp. 97–113.

Jiang Xiaoyuan 江曉原. 1991. *Tianxue Zhenyuan* 天學真原 (rpt. 2007 ed.). Shenyang: Liaoning Jiaoyu Chubanshe.

Jiang, Xiaoyuan. 江曉原. 1992. Liuchao Sui Tang Chuanru Zhongtu Zhi Yindu Tianxue 六朝隋唐傳入中土之印度天學. *Hanxue Yanjiu* 漢學研究 10: 253–277.

Keller, Agathe, and Alexei Volkov. 2014. Mathematics Education in Oriental Antiquity and Middle Ages. In *Handbook on the History of Mathematics Education*. Edited by A. Karp and G. Schubring. New York: Springer, pp. 55–83.

Kohn, Livia. 2008. *Laughing at the Dao: Debates Among Buddhists and Daoists in Medieval China*. Magdalena: Three Pines Press.

Latour, Bruno. 1993. *We Have Never Been Modern*. Cambridge, MA: Harvard University Press.

Li, Liang. 李亮. 2019. The Compilation of the Calendrical Portion of the *Book of Sui* "Monograph on Tono-metrology and Mathematical Astronomy" and its Impacts. In *Monographs in Tang Official History: Perspectives from the Technical Treatises of the History of Sui (Sui shu)*. Edited by Daniel Patrick Morgan, Damien Chaussende and Karine Chemla. Cham: Springer Nature Switzerland AG, pp. 117–41.

Lien, Yeong-Chung. 2011. *Zhang Heng, Eastern Han Polymath, His Life and Works*. Ph.D. thesis, University of Washington, Seattle, WA, USA.

Liu, Hongtao. 劉洪濤. 2003. *Gudai Lifa Jisuanfa* 古代曆法計算法. Tianjin: Nankai Daxue Chubanshe.

Martin, François, and Damien Chaussende, eds. 2020. *Dictionnaire biographique du Haut Moyen Âge chinois: Culture, politique et religion de la fin des Han à la veille des Tang (III^{e}-VI^{e} siècles)*. Paris: Les Belles Lettres.
McMullen, David. 1988. *State and Scholars in T'ang China*. Cambridge: Cambridge University Press.
Morgan, Daniel Patrick. 2017. *Astral Sciences in Early Imperial China: Observation, Sagehood and the Individual*. Cambridge: Cambridge University Press.
Morgan, Daniel Patrick. 2019. Heavenly Patterns. In Monographs in Tang Official History: Perspectives from the Technical Treatises of the History of Sui (Sui shu). Edited by Daniel Patrick Morgan, Damien Chaussende, and Karine Chemla. Cham: Springer Nature Switzerland AG, pp. 143–179.
Morgan, Daniel Patrick. 2021. Regional Networks in Chinese Mathematics and Astronomy, 317–618 CE. *East Asian Science, Technology, and Medicine* 53: 1–53. [CrossRef]
Morgan, Daniel Patrick. 2022. Calling out Zheng Xuan (127–200 CE) at the Crossroads of Ritual, Maths, Sport and Classical Commentary. In *Mathematical Commentaries in the Ancient World: A Global Perspective*. Edited by K. Chemla and G. W. Most. Cambridge: Cambridge University Press, pp. 141–96. With Collaboration of Karine Chemla.
Nagel-Angermann, Monique. 1999. *Das Diwang Shiji des Huangfu Mi (215–282)*. Ph.D. thesis, Westfälischen Wilhelms-Universität, Münster, Germany.
Niu, Weixing 鈕衛星, and Jiang Xiaoyuan 江曉原. 1997. He Chengtian gaili yu Yindu tianwenxue 何承天改曆與印度天文學. *Ziran Bianzhengfa Tongxun* 自然辯證法通訊 1: 39–44. 80.
Pingree, David Edwin. 1972. Precession and Trepidation in Indian Astronomy Before A.D. 1200. *Journal for the History of Astronomy* 3: 27–35. [CrossRef]
Qian, Baocong. 錢寶琮. 1998. In *Suanjing shishu* 算經十書. Volume 4 of *Li Yan, Qian Baocong kexueshi quanji* 李儼、錢寶琮科學史全集. Edited by Shiran Du 杜石然 and Shuchun Guo 郭書春 and Dun Liu 劉鈍. Shenyang: Liaoning Jiaoyu Chubanshe.
Qu, Anjing. 曲安京. 2008. *Zhongguo Shuli Tianwenxue* 中國數理天文學. Beijing: Kexue Chubanshe.
Rols, Johan. 2021. *Les interdits de destruction de la faune et de la flore de l'Antiquité tardive au haut Moyen-Âge chinois*. Ph.D. thesis, EPHE-PSL Research University, Paris, France.
Sen, Tansen. 1995. Gautama Zhuan: An Indian astronomer at the Tang court. *China Report* 31: 197–208. [CrossRef]
Sivin, Nathan. 1978. On the Word "Taoist" as a Source of Perplexity: With Special Reference to the Relations of Science and Religion in Traditional China. *History of Religions* 17: 303–330. [CrossRef]
Sivin, Nathan. 1986. On the Limits of Empirical Knowledge in the Traditional Chinese Sciences. In *Time, Science, and Society in China and the West*. Edited by J. T. Fraser, N. M. Lawrence and F. C. Haber. Amherst: University of Massachusetts Press, pp. 151–69.
Sivin, Nathan. 2009. *Granting the Seasons: The Chinese Astronomical Reform of 1280, with a Study of its Many Dimensions and a Translation of its Records*. New York: Springer.
Stephenson, F. Richard. 1997. *Historical Eclipses and Earth's Rotation*. Cambridge, MA: Cambridge University Press.
Swerdlow, Noel. 2010. Tycho, Longomontanus, and Kepler on Ptolemy's Solar Observations and Theory, Precession of the Equinoxes, and Obliquity of the Ecliptic. In *Ptolemy in Perspective: Use and Criticism of His Work from Antiquity to the Nineteenth Century*. Number 23 in Archimedes: New Studies in the History and Philosophy of Science and Technology. Edited by Alexander Jones. Dordrecht: Springer, pp. 151–202.
Wang, Yutong. 王雨桐. 2022. 'Yaodian' Zhongxing Yu Ridu Suicha: Han Tang Zhijian de Jingdian Quanshi He Lisuan Shijian 《堯典》中星與日度歲差——漢唐之間的經典詮釋和曆算實踐. *Guoxue yanjiu* 國學研究 47: 231–258.
Wang, Yutong. 王雨桐. 2023. *Han Tang Jian de Jingxue, Lishu Yu Gushi Niandai Jiangou* 漢唐間的經學、曆數與古史年代建構. Ph.D. thesis, Peking University, Beijing, China.
Yabuuti, Kiyosi. 藪內清. 1979. Researches on the *Chiu-chih li*: Indian Astronomy under the T'ang Dynasty. *Acta Asiatica* 36: 7–48.
Zhang, Peiyu 張培瑜, Chen Meidong 陳美東, Bo Shuren 薄樹人, and Hu Tiezhu 胡鐵珠. 2008. *Zhongguo Gudai Lifa* 中國古代曆法. Beijing: Zhongguo Kexue Jishu Chubanshe.
Zheng, Cheng. 鄭誠. 2007. He Chengtian Suicha Kao 何承天歲差考. *Shanghai Jiaotong Daxue Xuebao (Zhexue Shehui Kexue Ban)* 上海交通大學學報（哲學社會科學版） 15: 50–57.
Zheng, Cheng 鄭誠, and Jiang Xiaoyuan 江曉原. 2007. *He Chengtian Wen Foguo Lishu Gushi de Yuanliu Ji Yingxiang* 何承天問佛國曆術故事的源流及影響. *Zhongguo Wenhua* 中國文化 25–26: 61–71.
Zuo, Ya. 2018. *Shen Gua's Empiricism*. Cambridge: Harvard University Asia Center.