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Applying a Mathematical Model for Calculating the Ideal Nutrition for Sheep

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Abstract: The principal economic sector devoted to the breeding, raising, and production of farm animals is known as the livestock industry. There are precise standards for making high-quality feed in animal husbandry. Precision livestock feeding is a crucial component, with the potential to significantly impact the profitability of livestock; it permits the provision of diets to animals that are precisely tailored to their specific daily nutritional needs. Through simulation modeling, a single model can be created for automated systems to determine daily rations for farm animals. For the purposes of this document, precision livestock feeding refers to the practice of tailoring feed to individual animals or groups of animals, taking into account their changing nutritional needs over time and individual differences in terms of nutritional requirements. The practice aims to optimize animal health and performance while reducing feed waste. This paper presents a formal model for determining the quantities of components needed to achieve a minimum cost mixture that satisfies compositional and quantitative criteria. The present research calculates the amount of hay and silage required to feed an animal per day at the most economical cost by applying an optimization approach that involves defining and solving an optimization problem. The problem is solved using a well-known software package, which is necessary for the practical application of the resulting model. Real data from livestock production in Bulgaria are used to numerically test the model.

Keywords: mathematical model; ration; animal feed; feed quality; animal production

MSC: 91B02; 93A30; 90B30

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1. Introduction

There are several strategies for calculating optimal animal nutrition, each with its own advantages and disadvantages. The traditional Ration Formulation approach involves formulating rations manually based on the nutrient requirements of animals, typically using tables of nutrient values for various feed ingredients. Its advantage is that it makes it possible to refine rations based on specific nutritional needs and available feed resources. Potential disadvantages of this approach are that it can be time-consuming and requires knowledge of animal nutrition and the composition of feed ingredients. Also, it may not account for variability in the nutrient content of feed ingredients. As technology advances and enters the agricultural sector, the concept of Precision Feeding (PF) is gaining relevance. Several authors include PF among Precision Livestock Farming (PLF) practices, as the two are strictly connected in terms of their basic principles, goals, and benefits for animals and the environment [1–4]. Employing PLF can assist farmers with managing tasks, including monitoring animal health and performance and optimizing feeding strategies [5,6]. Precision feeding involves the use of advanced technologies or precision farming techniques to monitor and adjust feed rations in real time based on the nutritional needs of individual animals. This approach ensures that each animal receives the appropriate amount of nutrients, which can lead to improved health and productivity. It enables the implementation of personalized feeding strategies that are tailored to the specific needs of each animal, which can enhance efficiency and decrease feed waste [7–9]. However, implementation can be costly, requiring investment in technology and infrastructure. The potential benefits for both farmers and animals motivate us to focus on precision feeding itself and the implementation of innovative technologies in general in animal feeding.

In practice, farmers may use a combination of different approaches, depending on their resources, expertise, and production goals. The ideal strategy for calculating nutrition for animals will depend on factors such as available resources and the level of precision required to meet production goals while optimizing cost-effectiveness and animal welfare.

Progress in precision livestock feeding requires the application of new feeding concepts and mathematical models which are capable of estimating the nutritional needs of specific animals in real-time. This research aims to develop a quantitative model for estimating the necessary components in the feed mix to obtain benefits from animal production. The relationships between content and dietary composition are formalized in an optimization problem, the solution of which provides optimal values for the components of the animal feed. This research presents the solution to this optimization problem with a well-known software suite, which is a prerequisite for the practical application of the derived model. The added value of this research concerns the development of a formal model for the definition of animal feed; its inclusion is a specific optimization problem. The solution to the problem is illustrated with the Excel suite, which has wide popularity and usage.

2. Materials and Methods

2.1. The Basic Concepts of Precision Livestock Feeding

Precision livestock feeding is a crucial component of PLF, with the potential to significantly impact the profitability of livestock. It permits the provision of diets to animals that are precisely tailored to their specific daily nutritional requirements.

Precision feeding has the advantage of facilitating the individual feeding of farmed animals [10,11]. Depending on their health status, age, and other factors [12], animals may require diverse feeding strategies such as different diet compositions, amounts, or daily regimes.

For this document, "precision feeding of livestock" refers to the practice of providing tailored feed to individual or groups of animals, considering their changing nutritional needs over time and individual differences in terms of nutritional requirements. The practice seeks to optimize animal health and performance while reducing feed waste and environmental impact. It is defined as the precise assessment of the nutrients present within feed and feed ingredients, the precise creation of diets, and the evaluation of each animal's or each group's nutritional needs [13,14].

The implementation of precision animal feeding on farms necessitates the amalgamation of three crucial activities: (1) automated data collection, (2) data processing, and (3) actions related to the control and management of the system on the farm [15–17]. For precision animal feeding to be implemented at a personalized level, measurements, data processing, and control actions must be applied to individual animals [18].

2.1.1. Automated Collection of the Data

Digital technology and data collection can enhance precision animal nutrition even further by utilizing real-time information to adjust and optimize multiple factors that ultimately improve performance [19].

Accurate livestock nutrition necessitates measuring animal, feed, and environmental indicators directly and, when feasible, continuously. These indicators include feed intake (e.g., amount consumed and behavior), physical condition (e.g., body weight and composition), and behavioral and health indicators (e.g., physical activity and animal interactions). Employing these measures makes it possible to obtain precise data. The rapid development and availability of modern devices and sensor technologies, such as biosensors, Internet of Things (IoT), Wearable Internet of Things (W-IoT), and smart terminals, are of tremendous importance for animal monitoring in precision livestock feeding. Connecting a large number of standalone devices (sensors and controllers) to the Internet, a local network, or a cloud server is the aim of the Internet of Things. This results in the creation of an automated intelligent system that uses device data to achieve unified and effective administration. The Internet of Things (IoT) is evolving into the so-called portable Internet of Things (W-IoT) with the help of portable sensors. This offers numerous advantages, such as easy decision-making, timely data analysis, convenient maintenance, modification, and update, and real-time data tracking [20–22]. Portable sensors (yellow dots in Figure 1) attached to animals in the form of, e.g., collars, ear tags, or visual trackers, enable individual monitoring of animal status, including health status, nutrition, food intake quality, milk yield, and estrus (Figure 1 [23,24]). The collected data are analyzed and provided to farmers through Internet platforms, allowing for timely adjustments to be made to certain parameters and for informed decision-making [25,26].

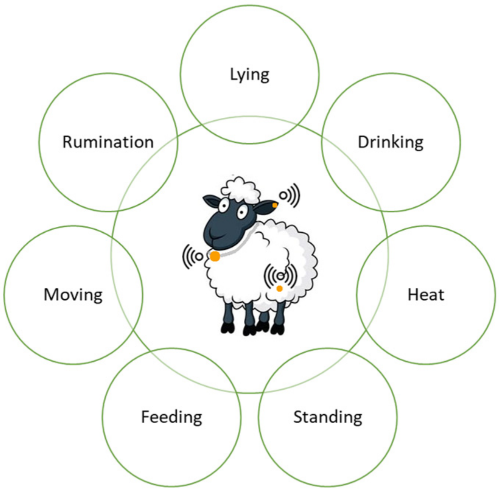

Figure 1. Sensors for monitoring animal behavior.

2.1.2. Data Processing

To implement control activities in precision livestock feeding, it is necessary to process the data collected from the continuous monitoring of animals. Mathematical modeling can serve as a basis for data processing in PLF control (management) systems. While a model is a simplification of the system it depicts, it should still reflect the key factors related to animal responses that need to be controlled in precision livestock feeding. Using systematic measurements, mathematical models for cattle precision feeding have to be built to function in real-time. Because of this, these models differ structurally from conventional feeding models, which typically operate retrospectively in order to mimic and improve well-known production scenarios. Since not all models are appropriate for precision feeding, it is necessary to revisit the fundamentals of model creation. The information that is now accessible and the intended management system architecture should inform the model's structure.

Depending on their objectives and structure, mathematical models can take various forms. They can be based on real-time or future models, mechanical or empirical, deterministic or stochastic, static or dynamic. For additional details about how mathematical models are developed in animal science, see [27].

Empirical or mechanical: There are various methods that are available to predict animal growth [28]. The empirical approach uses mathematical equations to describe animal growth; it employs a "black box" methodology. The authors of [22] indicated that these models were developed to describe the reactions of a system without explaining the system itself and without being restricted by biological principles. Mechanistic models are favored over empirical models because of their flexibility and ability to accurately predict outcomes across a range of situations [29,30]. Additionally, these models offer insight into the biological phenomena that underlie the responses of the system [27]. Consequently, the difficulty of determining the appropriate reference population for the calibration of empirical and mechanistic models limits their usefulness in precision cattle feeding. Furthermore, compared to a reference population, real populations and individual animals may exhibit distinct patterns of feed intake and growth [31].

Deterministic or stochastic: Deterministic models present a given set of initial conditions uniformly. These models offer a distinctive forecast for each specific input variable set. Subjective evaluations are absent. No associated probability (likelihood distribution) exists in deterministic models. Probabilistic models, by comparison, contain randomness. They calculate the likelihood of future system behavior, based on previous conduct. Stochastic models incorporate chance elements into the model so that they predict not only the expected value of a performance attribute (metric, characteristic), but also its variance [27]. Variability is a significant and inherent property of living systems, with animal variability making a considerable contribution to nutrient utilization efficiency [32].

Static or dynamic: Static models do not consider temporal factors, unlike dynamic models, which illustrate the changes within the system from one state to another. Static models primarily focus on the relationship between one variable and the value or status of other variables. Dynamic models, on the other hand, are typically represented by differential equations. In livestock production, particularly in animal feeding, most models are dynamic in nature, as animal responses and nutritional requirements change over time.

Real-time or prospective: These procedures generate models that continuously modify their responses to inputs and outputs by estimating system parameters in real time. In PLF or precision livestock feeding applications, examples of these models are included in some research [33–36].

2.1.3. Control and Management of the System

The main goal of precision livestock feeding is to automatically and continuously monitor, manage, and regulate animal feeding. While data processing aids in system monitoring (e.g., disease detection) and the evaluation of optimal production strategies, data collection and monitoring tools give farmers comprehensive information on the actual state and performance of the animal, as well as the use of farm resources. A decision controller that makes decisions automatically might then use this information. When discussing precision livestock feeding, this usually refers to the amount and type of feed given to a single animal or group of animals. The programming of the controller may reflect the production goals, such as maximizing growth rate, minimizing feed costs, reducing nutrient excretion, or achieving other objectives.

The subsections below highlight practical applications of precision animal nutrition. They explain the need for a quantitative assessments of feed composition, emphasizing how feed is the primary and costliest factor in animal production. As a result, it is crucial to calibrate feed carefully and meet the energy requirements of animals throughout their lifetime, as this can prevent overfeeding and the loss of precious nutrients into the environment.

2.2. Requirements for the Quantitative Assessment of the Composition of Animal Feed Stuff

Feeding ruminants can be described as a skill that requires a delicate balance between the quantity and quality of the ration. Failure to achieve this balance can lead to broken synchronicity. Some farmers fail to pay adequate attention to quality and combine different feeds ineffectively, resulting in imbalanced rations [37].

For optimal results, a balanced diet is essential. This entails ensuring that the diet is proportionate in terms of essential nutrients, namely, crude protein, energy, crude fiber, crude fat, digestible protein, micro- and macro elements, and vitamins. Unfortunately, a significant portion of these nutrients often go overlooked, resulting in diminished productivity and poor animal health. It is important to acknowledge that each nutrient has a distinct function in supporting animal growth, performance, and metabolism [38].

Energy enables the body to perform various functions, including growth, lactation, reproduction, digestion, and movement. It is required in large amounts.

Protein, composed of amino acids, is the fundamental building block of the body. The quality of protein in the diet is critical for supporting growth, milk production, and reproduction.

Minerals play a crucial role in the growth, bone development, and reproduction of animals; their quantities are influenced by the type and quality of the feed. Often, feed alone fails to provide the minerals required by animals, necessitating supplementation in animals' rations.

Additionally, essential vitamins, such as A, D, and E, are vital for the growth and reproductive processes of animals, as emphasized in [39].

Some farmers collaborate with nutritionists to determine the optimal rations for their livestock, while others create their own rations; a further group feeds their animals whatever is available on the farm. The paper will examine several common errors, including inappropriate feed combinations, the use of more than two protein sources in rations, and disregarding feeding norms.

The process of creating a comprehensive ration involves combining various feeds, minerals, and vitamins, ensuring they are well mixed to discourage any selective eating by animals. Roughages, including silages and hay, account for up to 70% of the preparation; the remaining 30% is made up of cereals, protein forages, minerals, and vitamins. The objective of rationing is to maximize production and attain favorable economic outcomes [40].

The rising costs of electricity, heat, and fuel, coupled with heightened technological demands and a need for reduced production expenses, necessitate the creation of information systems to optimize parameters in livestock farming. A crucial parameter in this regard is animal feed, which plays a significant role in husbandry and general well-being. Over a third of the final consumer price comprises expenses related to feed.

Optimization has thus far been employed to address issues in livestock production and restore coherence with agricultural data regarding livestock distribution and populations. Such issues have been tackled using alternative models, including linear, quadratic, or mixed integer methods [41].

This article emphasizes the crucial role of feed utilization in promoting competitiveness and explores its enhancement through better genetics, feeding practices, and breeding techniques [42].

The primary goal of the present research was to create a mathematical model that would allow one to calculate the necessary amounts of each component to create a minimum cost mixture while still meeting the compositional and quantitative requirements of the mixture [43].

2.3. Livestock Food Resources

Proper and balanced nutrition necessitates providing livestock with sufficient, but not excessive, amounts of food. Subjectivity should be avoided. It is imperative to ensure that sheep receive a menu rich in protein, vitamins, and minerals, as this not only minimizes the risk of complications, but also prevents reproductive difficulties in ewe lambs, thus increasing birth rates.

Due to inadequate nutrition, sheep gain weight at a slower rate, require additional care, and are prone to different ailments. To ensure that they receive enough energy in their diet, adding carbohydrates to their cereal feeds and silages is essential. Carbohydrate deficiency may cause various issues, including reduced fertility and milk yield, stunted growth, poor fertilization rates, weight reduction, and other complications [19].

Farms undertake marketing campaigns to maximize profits and secure a larger market share while also striving to mitigate the risk of failure, as highlighted in [44].

One of the primary objectives of food laws, as stated in Regulation (EC) No 178/2002 of the European Parliament and of the Council of 28 January 2002, laying down the general principles and requirements of food law, creating the European Food Safety Authority, and establishing procedures in matters of food safety [45], is to attain a high degree of health protection for both humans and animals.

Using both large, premium compound feeds and well-selected vitamin—mineral premixes is crucial to ensure the best possible health outcomes for animals and to avert potential problems. Any seasoned farmer will recognize the significance of this. The gestation period and age of sheep, and whether it's winter or summer must be taken into account when formulating meals (see Figures 2 and 3). The feed must be optimally balanced to cater to the ewe's specific needs of protein, energy, vitamins, and minerals. Such a diet will allow them to perform at their best with a regular metabolic rate, ultimately promoting their overall well-being.

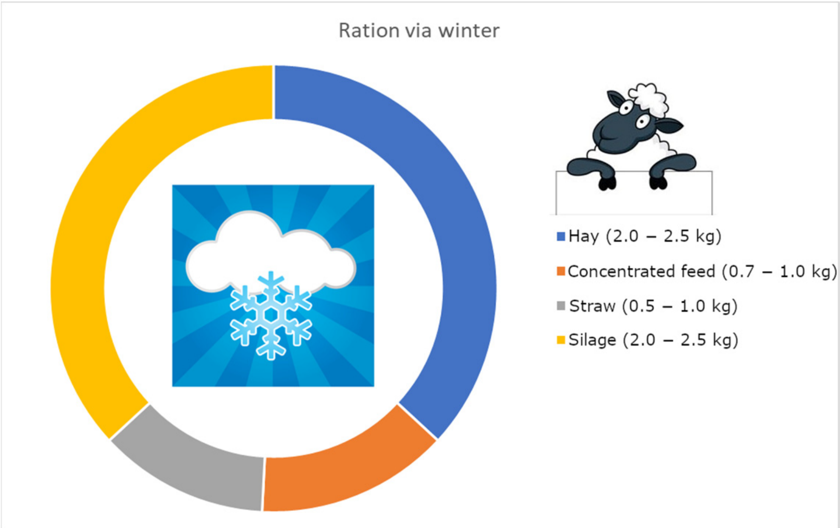

Figure 2. Sheep rations in winter.

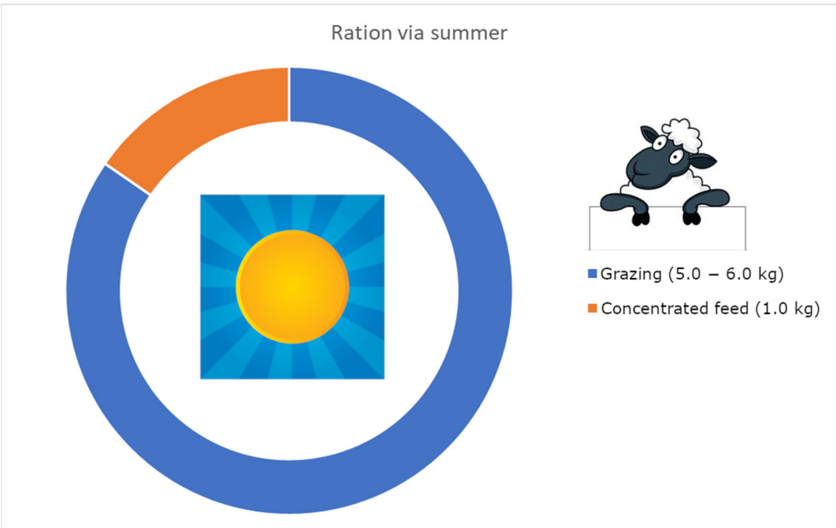

Figure 3. Sheep rations in summer.

Sheep need a diverse diet; this could consist of grazing on pastures containing grass, consuming legumes such as soya, and eating high-nutrition fodder, depending on the farm. Depending on the time of day, geography, and season, sheep exhibit different dietary preferences [46]. To promote voluntary milking [47], farms that permit free-ranging animals need to take these preferences into account.

An optimal means of lowering costs and achieving maximum profitability in livestock production may be developed through the use of specialized programs. These programs utilize non-linear forecasting methodologies which factor in the influences of all relevant aspects [48].

The Solver function in Microsoft Excel 2016 (Version 16) can optimize certain agricultural economic issues presented in tables. The tables provide a rough, initial solution to the problem, although they may not be optimal. The objective function of a linear problem is determined by the quantity to be optimized, which may involve maximizing feed values or minimizing feed cost per day [49]. The conditions and dependencies of the problem are portrayed as linear relationships in the table. The Solver function menu permits the inclusion of further limitations. The principal parameters of the table are subject to optimization.

3. Results

3.1. Mathematical Model of the Problem

The problem must be formalized as an optimization problem. The goal of the optimization is chosen to be the cost of a mixture for animal feed. The constraints consider the required levels of nutrients, based on an appropriate quantity of the components of s = feed. The research applies linear relations to simplify the optimization evaluations.

The quantity of the j-th ingredient in the mixture is known as z_j, while the specific costs of the ingredients are known as c_j (BGN/kg). a_{ij} (gram/kg for the ingredient) is the amount of the i-th substance in a unit of the j-th component.

Model objective performance:

$$\min \mathrm{Y} = \sum_{j=1}^{s} c_j z_j, \quad (1)$$

where z_j is the quantity of the components of the feed. The costs are denoted with coefficients c.

Constraints of the model:

$$\sum_{j=1}^{s} a_{ij} z_j \geq b_j, \quad (2)$$

b_j—minimum quantity of ingredients.

$$Q_{min} \leq \sum_{j=1}^{s} z_j \leq Q_{max}, \quad (3)$$

Q_{min}—Minimum total quantity of forage;
Q_{max}—Maximum total quantity of forage;
$Z_1 < 7$—Maximum quantity of hay;
$Z_2 < 3$—Maximum quantity of silage;
z_1 = kg of hay in the daily mix;
z_2 = kg of silage in the daily mix.

The total quantity of nutrition eaten on a daily, monthly, or annual basis varies depending on the product, the weight of the animal, and several other factors. Usually, the estimate is predicated on dry concentrates, which must form the cornerstone of the diet. The aim is to develop a ration that is economical and satisfies specific nutritional requirements. Table 1 lists the recommended daily intake of nutrients, together with the quantity of each food item that makes up one kilogram of it. Since it is recommended that sheep receive no more than 10 kg of feed per day, numbers Z_1 and Z_2, representing the maximum amounts of hay and silage, are entered into the Solver function as limits for the best possible solution to the problem.

Table 1. Parameters of the optimization problem (According to the National Farm Advice Service) [50].

Nutritional Composition, g/kg	Hay	Silage
Fodder units	0.24	0.53
Calcium	1	13
Phosphorus	2	0.68
Potassium	1.2	3.25
Carotene	0.7	0.4
Protein Digestible In The Gut	53	23
Raw Protein	45	27
Price/BGN	0.3	0.16

The constraints of the problem are defined as linear inequalities, which formalize the relative content of the nutrition component in the total feed. Table 1 shows a set of nutrition components which have to be present in the animal feed. Their relative contents in two important types of feed, hay, and silage are also presented as g/kg.

The amount of silage and hay selected depends on the needs of the animal. According to the National Farm Advice Service, Table 2 indicates how many feed dinners sheep must receive each day.

Table 2. Necessary feed dinners (According to the National Farm Advice Service) [50].

Nutritional Composition, g/kg	Minimum Required Quantity
Fodder units	3.25
Calcium	40
Phosphorus	8
Potassium	6.5
Carotene	6.8
Protein digestible in the gut	84
Raw Protein	130

3.2. Formal Definition of the Optimization Problem

The objective of the task is to determine the daily blend at the lowest possible cost.

Kg of silage (z_2) and kg of hay (z_1) in the daily mix are the decision variables of the model.

Minimizing the overall daily cost (in BGN) of the forage mix is the goal.

$$\min \mathrm{Y} = 0.3 \times z_1 + 0.16 \times z_2. \tag{4}$$

The following is a mathematical representation of the limitations:

- Total amount of food consumed (8 kg);

  $z_1 + z_2 \geq 8,$
  $z_1 + z_2 \leq 10.$

- CEM composition of the combination ($\geq$3.25 g/day):

  $0.24 \times z_1 + 0.53 \times z_2 \geq 3.27.$

- Calcium composition of the combination ($\geq$40 g/day):

  $1 \times z_1 + 13 \times z_2 \geq 46.$

- Phosphorus composition of the combination ($\geq$4 g/day):

  $2 \times z_1 + 0.68 \times z_2 \geq 16.04.$

- Regarding the potassium composition of the combination ($\geq$6.5 g/day):

  $1.2 \times z_1 + 3.25 \times z_2 \geq 18.15.$

- Regarding the carotene composition of the combination (≥6 g/day):
  $0.7 \times z_1 + 0.4 \times z_2 \geq 6.1.$
- Regarding the intestinal digestible protein composition of the combination (≥770 g/day):
  $53 \times z_1 + 23 \times z_2 \geq 440.$
- For Crude Protein composition of the combination (≥1580 g/day):
  $45 \times z_1 + 27 \times z_2 \geq 396.$

3.3. Using a "Solver" to Solve the Problem

The answer to the problem is represented by the Solver function, as shown in Figure 4.

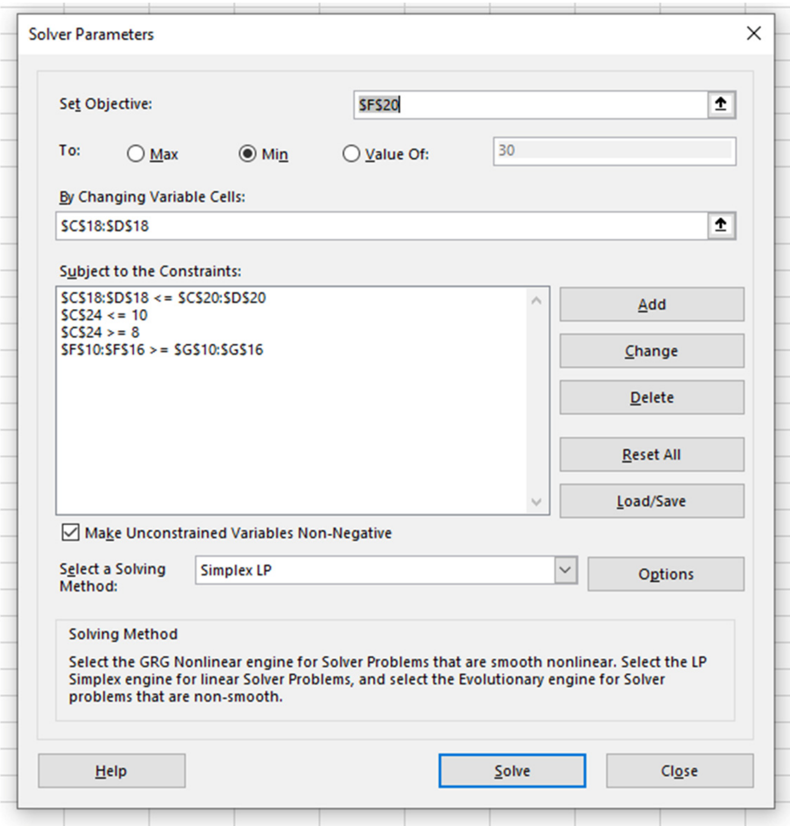

Figure 4. The Solver problem model.

The Set Objective field is utilized to input the minimal expense of life or the aim function [48].

The cells pertaining to hay and silage quantities (C18:D18) that require modification are inputted into the Changing Variable Cells field. The method for determining the optimal feeding cost is called "simplex".

3.4. The Excel Solution to the Problem

This subsection of the paper presents the resolution of the aforementioned issues using the Solver function; see Figure 5.

To solve the problem using Excel, objective function Y of the model is calculated in cell C22 using the formula = SUMPRODUCT(C18:D18,C17:D17). We then use this formula to determine the concentration of each nutrient in the mixture in cells F10 through F16. Next, we determine the daily total of feed in cell C26 by applying formula = SUMPRODUCT(C20:D20).

Nutritional Composition, g / kg	Hay	Silage		Calculation	Constraints
Fodder units	0.24	0.53		3.27	3.25
Calcium	1	13		46.00	40
Phosphorus	2	0.68		16.04	8
Potassium	1.2	3.25		18.15	6.5
Carotene	0.7	0.4		6.10	6.8
Protein Digestible In The Gut	53	23		440.00	84
Raw Protein	45	27		396.00	130
Price / BGN	0.3	0.16			
z	7.0	3.0			
kg per day	10.0	10.0			
y	2.58				
Total amount of feed	10.0				

Figure 5. The task as it appears in a worksheet.

These diets provide sufficient levels of CF, CEM, and PFA, together with the required amounts of calcium and phosphorus. Consequently, there is no need to add extra minerals to the meals. For precisely this reason, rather than using more costly, intricate combinations to balance these meals, hay and silage will be employed.

The task of locating silage in the necessary quantity was also completed. The solution makes it evident that, despite the enormous (10 kg) volume of silage, it is insufficient to provide the required amount of nutrients. The cost and quantity of feed are therefore doubled.

Every day, each sheep needs 8.5 kg of fodder, which includes the necessary amounts of nutrients. The minimum cost of this feed combination per sheep/per day is $y = 0.3 \times 7 + 0.16 \times 3 = 2.58$, subject to limitations on quantity, which must not exceed ten.

These meals provide sufficient dry matter, feed, and digestible protein, together with the appropriate ratios of calcium and phosphorus. Consequently, there is no need to add extra minerals to the meals. For precisely this reason, rather than using more costly, intricate combinations to balance these meals, hay and silage will be employed.

When evaluating the cost of fodder, the animals' current diet is maintained if they are given only hay and silage. The results regearding the levels of nutrients in each choice and related costs are shown in Figure 6. The figure displays:

- The nutrient requirements for sheep,
- The values obtained after solving the problem, are a combination of hay and silage,
- The values obtained after solving the problem on hay only,
- The values obtained after solving the problem of silage only.

Nutrition Composition, g/kg	Desired value	Optimal (Silage and Hay)	Only Silage	Only Hay
Fodder units	3.25	6.73	5.3	2.13
Calcium	40	93	130	8.87
Phosphorus	8	15.36	6.8	17.47
Potassium	6.5	25.15	32.5	10.64
Carotene	6	6.4	4	6.21
Protein Digestible in The Gut	84	417	230	470
Raw Protein	130	369	270	399
Quantity/kg	8 – 10	9	10	8.9
Price/BGN		1.72	1.6	2.66

Figure 6. Results of various mathematical model decisions.

The grey color indicates a nutrient value that is less than the desired or required value. The orange color indicates a higher value of the amount of hay and silage and its price. The results show that feeding only hay and silage does not achieve the nutrient requirements. The amount of silage is 1 kg or more, which gives rise to greater storage space requirements and efforts to maintain silage quality. The results show that the use of hay alone is not recommended, as the cost of feeding a sheep per day is twice as high. From the obtained results, it can be seen that the best and cheapest feed for sheep is a combination of hay and silage. Solving the problem shows the exact amount of hay and silage that should be combined to achieve the desired results at the lowest cost.

The pie chart in Figure 7 shows the feed cost per day/per sheep. The cost of using silage only is lower than the others, but this would not meet the nutritional needs of the sheep on two indicators.

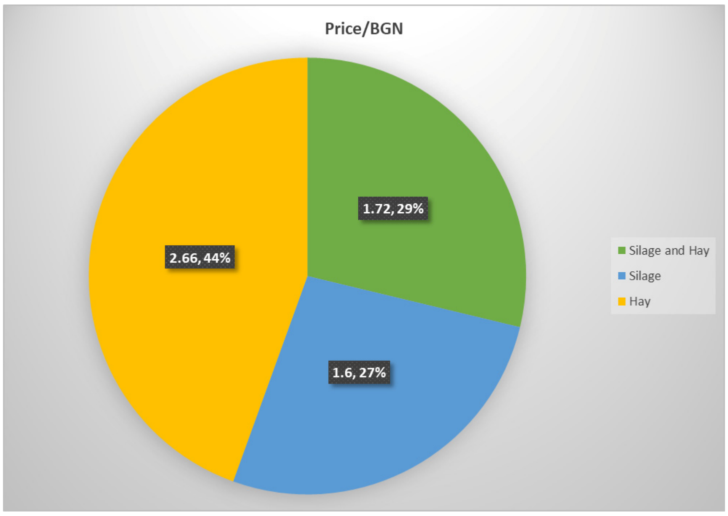

Figure 7. Feed cost per day.

4. Discussion

Effective farm management is a critical aspect of agricultural activities. It establishes a framework for organizing daily life on a farm, allocating resources, and directing activities. A variety of strategies and methods have been implemented to promote productivity, sustainability, and profitability, all of which are paramount to a farm's success.

Utilizing Precision Livestock Farming can help farmers with their management responsibilities, like keeping an eye on the health and performance of their animals and maximizing the efficiency of feeding plans. PLF permits the constant monitoring, modeling, and management of animal biological responses.

The precision feeding of livestock is a component of PLF which can greatly affect animal profitability. It enables the provision of diets that meet the daily nutritional requirements of livestock, leading to economic gains, a reduction in harmful substances released into the environment, and improved resource efficiency.

It is crucial to develop feeding concepts and models that are specifically designed for the precision feeding of livestock. Also, integrating a PLF system to detect changes, in a timely manner, in animal health status based on a reduction in feed intake would be greatly beneficial.

A formal model for determining the optimal quantity of feed for sheep is presented in this paper. The model employs mathematical dependence with performance criteria in the objective function while conforming to set constraints for the problem at hand. The nutritional values of the feed determine its distribution, which is critical for the appropriate rearing of sheep. Using an approach that involves defining and solving an optimization problem, the quantity of hay and silage required to feed an animal per day at the lowest cost is calculated. Suggestions have been made for estimating the required quantity of forage (hay and silage), based on the findings obtained. Contemporary methods are employed by sheep farms to enhance financial productivity. The optimal utilization of feed is imperative on account of its high cost, and efficient farm management significantly contributes to overall earnings.

Future work in this area includes optimizing animal nutrition through the use of specific nutrients, feed additives, or supplements. The aim of this research is to achieve maximum nutrient utilization without any loss in productivity. Additionally, research into feeding strategies, such as modifying feed ingredients or additives, could help reduce the environmental impact of livestock production while maintaining or improving animal performance. For example, protein intake can be reduced by about 25%, and nitrogen release to the environment can be reduced by about 40%. At the same time, farm profitability can be increased by about 10%.

5. Conclusions

One of the most crucial aspects of agricultural activities is farm management. It influences the structure of farm life, the distribution of resources, and the ways in which tasks are completed. It covers a range of tactics and approaches to raise and sustain farm productivity and sustainability. To achieve sustainability and productivity in farms, precision livestock farming must incorporate precision feeding. By controlling various parameters, this significantly mitigates the risks and problems faced by farmers. The aim of such initiatives is to create a feed that satisfies specific nutritional standards and is economically efficient. The results obtained numerically in our research by using a mathematical model and defining and solving an appropriate optimization problem provide useful and practically applicable results. The quantity of hay and silage required to feed an animal per day at the most economical cost has been calculated.

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