mathematics

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Neural Network-Based Distributed Consensus Tracking Control for Nonlinear Multi-Agent Systems with Mismatched and Matched Disturbances

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Abstract: In practice, disturbances, including model uncertainties and unknown external disturbances, are always widely present and have a significant impact on the cooperative control performance of a networked multi-agent system. In this work, the distributed consensus tracking control problem for a class of multi-agent systems subject to matched and mismatched uncertainties is addressed. In particular, the dynamics of the leader agent are modeled with uncertain terms, i.e., the leader's higher-order information, such as velocity and acceleration, is unknown to all followers. To solve this problem, a robust consensus tracking control scheme that combines a neural network-based distributed observer, a barrier function-based disturbance observer, and a tracking controller based on the back-stepping method was developed in this study. Firstly, a neural network-based distributed observer is designed, which is able to achieve effective estimation of leader information by all followers. Secondly, a tracking controller was designed utilizing the back-stepping technique, and the boundedness of the closed-loop error system was proved using the Lyapunov-like theorem, which enables the followers to effectively track the leader's trajectory. Meanwhile, a barrier function-based disturbance observer is proposed, which achieves the effective estimation of matched and mismatched uncertainties of followers. Finally, the effectiveness of the robust consensus tracking control method designed in this study was verified through numerical simulations.

Keywords: consensus tracking; neural network; distributed observer; barrier function; multi-agent systems

MSC: 93C10; 93D50

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Citation: Xu, L.; Qin, K. Neural Network-Based Distributed Consensus Tracking Control for Nonlinear Multi-Agent Systems with Mismatched and Matched Disturbances. *Mathematics* **2024**, *12*, 1319. https://doi.org/10.3390/math12091319

Academic Editors: Zhi Li and Sihai Guan

Received: 7 March 2024
Revised: 14 April 2024
Accepted: 23 April 2024
Published: 26 April 2024

1. Introduction

In recent years, there has been a significant increase in research attention toward the cooperative management of multi-agent systems, owing to its extensive applications in engineering, particularly in scenarios such as multi-UAV flights [1], multi-robot exploration [2], smart grids [3], and multi-sensor networks [4]. In summary, there are three typical architectures for the cooperative control of existing works in multi-agent systems, i.e., centralized, decentralized, and distributed [5,6]. Among them, distributed cooperative control architectures have been widely studied due to their advantages such as scalability and robustness. Existing studies on distributed cooperative control have concentrated on the following behaviors, i.e., consensus [7], formation [8], containment [9], and flocking [10] and distributed estimation [11]. Among them, consensus is the fundamental one, which refers to the eventual convergence of the states of all the agents in a multi-agent system. On the basis of the presence or absence of a leader agent, the issue of consensus for the multi-agent system can be further divided into two distinct categories: leaderless consensus and consensus tracking.

Consensus tracking refers to the capability of all followers in a multi-agent system to track the leader agent's trajectory. Recently, researchers have conducted thorough analyses on the constraints associated with multi-agent systems at the informational and physical layers and designed consensus tracking control schemes such as event-triggered control [12,13], finite-time control [14,15], bipartite consensus [16–18], security control [19], and fault-tolerant control schemes [20]. The issue of consensus tracking control for first-order multi-agent systems under a directed network topology was studied in [21]. On this basis, the authors in [22] focused on analyzing the problem of distributed consensus tracking for multi-agent systems that exhibit Lipschitz node dynamic models. They devised a consensus tracking protocol relying solely on the neighboring agents' relative states. They also demonstrated that with an appropriate selection of control parameters, achieving consensus tracking under a switching-directed topology is feasible. In addition, considering the bound of the control input, the authors in [23] put forward restrictions on the control input, and studies the tracking consensus under linear systems. Further, the authors in [24] used output feedback to design a distributed adaptive control input to implement output asymptotic tracking consensus. The adaptive protocol proposed was not reliant on system parameters and solely utilizes the relative outputs of adjacent agents. In addressing the convergence speed of consensus tracking control for multi-agent systems, the authors in [14] investigated the finite-time consensus tracking control issue formulated in the form of non-strict feedback. In utilizing the Lyapunov stability theory and the back-stepping method, an adaptive control input was developed to guarantee that the tracking error converges to a small neighborhood of zero in finite time. Furthermore, for high-order systems, a novel adaptive fixed-time consensus tracking control input was formulated through the utilization of fuzzy adaptive methods and fixed-time control theory in [25]. Considering the behavior expansion of consensus tracking, the authors in [26] studied the distributed bipartite tracking consensus problem of linear multi-agent systems under a single leader with a signed graph, in which the control input of the leader agent is permitted to be non-zero, while each follower's control input remains unknown. In addition, the authors in [27] considered a switched network topology and studied the distributed bipartite tracking consensus control problem under discrete systems. To address limited network communication resources, the sliding mode control approach with a dynamic event-triggered mechanism was employed in [28] to tackle the consensus tracking challenge in discrete-time multi-agent systems; meanwhile, the authors integrated a dynamic event-triggered mechanism into the sliding mode control system to reduce unnecessary data transmission. Furthermore, in [29], a novel approach was presented, involving the introduction of a fixed-time distributed observer with an event-triggered mechanism. Additionally, to effectively stabilize the tracking error system, an event-based fixed-time controller with an adaptive dynamic surface was developed.

It is noteworthy that the above literature focuses more on nominal multi-agent systems, i.e., there are no disturbances or uncertainties. However, due to the complexity of the environment and the inaccuracy of modeling, agents are inevitably subject to external disturbances and model uncertainties. Hence, there is a need to develop more robust control schemes for multi-agent consensus tracking. Overall, there are two main research ideas available for designing robust consensus tracking control schemes, i.e., feedback control [30–32] and feedforward control [33–36]. Feedback control mainly refers to further improving the performance of the system by suppressing disturbances or uncertainties. Typical control methods mainly include robust adaptive control [30], sliding model variable structure control [31], and H_∞ control [32]. In employing the fractional Lyapunov direct method, the robust consensus tracking problem in uncertain fractional-order multi-agent systems was investigated in [30]. An algorithm based on neural networks was designed to achieve distributed robust adaptation, ensuring exponential convergence of the consensus tracking error with fixed topology. Additionally, to tackle and alleviate the detrimental chattering effects associated with discontinuous controllers, a continuously distributed robust adaptive control scheme based on neural networks was introduced. In [31], the finite-time consensus tracking issue for multi-agent systems with disturbances was investigated through the application of integral sliding mode control (ISMC). Further, the adaptive mechanism and ISMC were integrated into the system to achieve disturbance suppression. Furthermore, in [32], the authors studied H_∞ consensus tracking control problem for linear multi-agent systems with directed and switching graphs while accounting for unknown disturbances. In this paper, the design criteria for consensus protocols are expressed in the form of linear matrix inequalities, leveraging the topologically dependent multiple Lyapunov function method and algebraic graph theory. It is demonstrated that ensuring the solvability of the consensus tracking issue for multi-agent systems under dynamic, directed topologies hinges on meeting specific switching conditions dictated by the average dwell time of the topology.

Different from feedback control, feedforward control mainly refers to estimating the disturbances or uncertainties through detection or a feedback channel, as well as further generating the feedforward term to achieve the compensation of disturbances and uncertainties through the control protocol. The key challenge of this method is to design an effective and easy estimation scheme to generate usable feedforward control terms. Typical robust consensus tracking control schemes based on disturbance and uncertainty estimation and compensation have been proposed in the literature, including active disturbance rejection control (ADRC) [33], an uncertainty and disturbance estimator UDE [34], a high-gain observer HGO [35], a disturbance observer (DOB) [36], and so on. The authors in [33] addressed the consensus tracking issue of multi-agent systems with second-order dynamics and unknown disturbances, employing the ADRC method. The tracking consensus protocol with random disturbance estimation was proposed to ensure system convergence. At the same time, real-time compensation was implemented for the random disturbance affecting each agent. By employing the UDE-based control method, ref. [34] delved into the robust consensus tracking control issue of switched multi-Lagrangian systems. Moreover, the UDE-based method was used to accurately adjust and asymptotically estimate the model uncertainty and disturbance. It is worth noting that several unbounded specified external disturbances can be handled with the help of applying diverse filters. Furthermore, the authors in [35] accomplished the design of a control protocol for multi-agent robust global consensus tracking. In employing a predetermined high-gain design technique, a control input based on the state feedback method was introduced to attain global consensus tracking and disturbance suppression within these systems, considering the dynamics of the agents and network topology. A distributed disturbance observer was developed in [36] to estimate the disturbances affecting followers. Subsequently, in leveraging this disturbance observer, a novel distributed control method was presented to address the consensus tracking issue with disturbance suppression within a fixed directed network topology, and its effectiveness was proved. It is worth noting that the above literature has not yet taken into account the uncertainty that the leader agent may have, and mostly focuses only on the robust control of the follower agents.

According to the above discussions, it is evident that few existing studies related to consensus tracking control have simultaneously considered the following constraints, i.e., external disturbances, mismatched uncertainties, and the leader agent being subject to uncertain dynamics. Therefore, this study focuses on the problem of robust multi-agent consensus tracking control with the above constraints. It should be noted that due to the unknown uncertainty of the leader's dynamics, only some of the followers are able to obtain the leader agent's position, while other higher-order information, such as velocity, acceleration, etc., is not available to all the followers. The main contributions of this paper are summarized as follows:

(1) A robust consensus tracking control scheme is proposed, which consists of three components: a neural network-based distributed observer, a barrier function-based disturbance observer, and a back-stepping-based tracking controller. Each of the three components plays a different role, and in complementing each other's functions, it enables the follower to track the leader agent's trajectory. Moreover, the proposed control scheme is

effective in achieving the convergence of the consensus tracking error and the uncertainty estimation error.

(2) A distributed observer based on neural networks was designed, and an adaptive update law of the parameters is provided, which can effectively realize the online estimation of leader information. The proposed distributed observer can estimate the leader's dynamic states (e.g., velocity, acceleration) for all follower agents despite the absence of direct higher-order information, enhancing the system's adaptability and responsiveness in uncertain environments.

(3) A barrier function-based disturbance observer was designed for a follower agent to estimate the unknown matched/mismatched uncertainties. In turn, the effective compensation of disturbances and uncertainties can be achieved using a simple feedforward control component design. Furthermore, the boundedness of the closed-loop error system was rigorously proved, while extra assumptions on the derivatives of the uncertainty terms were avoided.

The subsequent sections of this paper are structured as follows. Some theories that need to be used in this paper are introduced in Section 2, including graph theory, barrier function, etc. Section 3 then firstly establishes the dynamic model of the followers and leaders and describes the problem studied. The proposed robust coherent tracking control scheme is presented in Section 4. Firstly, a distributed observer based on neural networks is introduced, followed by the design of a perturbation observer based on the barrier function, and then the back-stepping method is introduced for the design of the robust tracking controller. Section 5 verifies the effectiveness of the control method designed through some simulation examples. Lastly, Section 6 summarizes the full work and discusses some of the future research directions.

2. Preliminaries

The relevant mathematical theories related to graphs (for network connections) and barrier functions will be covered in this part. Next, we shall provide the following definitions for several types of widely used notations.

Notation $\mathbf{col}_i^n[\delta_i] \triangleq [\delta_1^T, \delta_2^T, \ldots, \delta_n^T]^T$ or $\mathbf{col}^n[\delta] \triangleq [\delta^T, \delta^T, \ldots, \delta^T]_n^T$ generates a vector in the form of a column. The vector $\mathbf{1}_n$ is thus represented as $\mathbf{1}_n \triangleq \mathbf{col}^n[1]$. $\|\cdot\|$ represents the Euclidean norm. For a matrix $\mathcal{M} \in \mathbb{R}^{n\times n}$ with all the eigenvalues being real, $\mathcal{M}$'s maximum and minimum eigenvalues are represented as $\lambda_{\min}(\mathcal{M})$ and $\lambda_{\min}(\mathcal{M})$. Moreover, $\exp(\cdot)$ denotes an exponential function.

2.1. Graph Theories

A connected undirected graph with n agents is denoted as $G = (V, E)$, in which a node is represented as an agent and $V = \{1, 2, \ldots, n\}$ is the node set of the multi-agent system. The set of edges $E \subseteq V \times V$ defines the communication topology relationship between the agents, where the presence of an edge $(i, j) \in E$ signifies the existence of information exchange between agent i and agent j. The adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ represents the connectivity relationship of the multi-agent system. $L = [l_{ij}] \in R^{n\times n}$ is a Laplacian matrix, which is defined as the difference between the degree matrix D and the adjacency matrix A. Specifically, the degree matrix D is represented by $D = \mathrm{diag}[d_1, \ldots, d_n]$ with $d_i = \sum_{j=1, i\neq j}^{n} a_{ij}$.

This paper explores leader-tracking issues, with the leader being treated as an external entity in a multi-agent system. Specifically, the node set related to $\mathcal{G}$ excludes the leader agent node. To express the relationship between the leader and a follower, we define $B = \mathbf{diag}_i^n[b_i]$, where the ith agent can receive state information from the leader agent; we denote this using $b_i > 0$ to represent information weight, or else $b_i = 0$. Hence, it can be derived that $(L + B)$ forms a positive definite matrix.

2.2. Barrier Function

Definition 1 ([37])**.** *A barrier function is defined for some $\lambda > 0$ as follows:*

$$F_p(x) = \frac{|x|}{\lambda - |x|}. \tag{1}$$

It is a continuous even function with the following three properties:

- $F_p : x \in (-\lambda, \lambda) \to F_p(x) \in [0, \infty)$ *is strictly increasing in the interval* $[0, \lambda)$.
- $\lim_{|x| \to \lambda} F_p(x) = +\infty$.
- *The function* $F_p(x)$ *has a unique minimum as* $F_p(0) = 0$.

Lemma 1 ([37])**.** *Take into account the following system:*

$$\dot{\delta}(t) = u(t) + d(t), \tag{2}$$

in which $0 \le |d(t)| \le d_{\max}$, *and* $d_{\max}$ *is a positive constant that is unknown. Assume that the function* $u(t)$ *is represented as*

$$u(t) = -F(t, \delta(t))\operatorname{sgn}(\delta(t)), \tag{3}$$

and

$$F(t, \delta(t)) = \begin{cases} F_\delta(t), \dot{F}_\delta(t) = \bar{F}|\delta(t)|, \ if\ 0 < t \le \bar{t}, \\ F_p(\delta(t)), \ if\ t > \bar{t}, \end{cases} \tag{4}$$

where $F_\delta(0)$ *and* $\bar{F}$ *hold the following conditions* $F_\delta(0) > 0$ *and* $\bar{F} > 0$, *respectively.* $\operatorname{sgn}(\cdot)$ *denotes a signum function. Then, we can obtain that the variable* $\delta(t)$ *can converge to the domain* $|\delta(t)| \le \delta_1$ *within a finite amount of time* T_δ, *where* $\delta_1 = \lambda\left(\frac{d_{\max}}{d_{\max}+1}\right)$.

3. Problem Statement

In this paper, we consider the following second-order nonlinear multi-agent system with both unmatched and matched uncertainties:

$$\begin{cases} \dot{x}_{i1}(t) = x_{i2}(t) + d_{i1}(t), \\ \dot{x}_{i2}(t) = f_i(x_{i1}(t), x_{i2}(t)) + g_i(x_{i1}(t), x_{i2}(t))u_i(t) + d_{i2}(t), \\ y_i(t) = x_{i1}(t), \end{cases} \tag{5}$$

where $x_{i1} \in \mathcal{R}$ and $x_{i2} \in \mathcal{R}$ are the system states, $y_i \in \mathcal{R}$ is the system output, $f_i(x_{i1}, x_{i2}) \in \mathcal{R}$ is the known nonlinear function, $g_i(x_{i1}, x_{i2}) \in \mathcal{R}\backslash\{0\}$ is the non-zero control gain, and $d_{i1} \in \mathcal{R}$ and $d_{i2} \in \mathcal{R}$ represent the aggregated uncertainties across various channels, encompassing model and parameter uncertainties as well as external disturbances. The agents are indexed by $i \in \{1, 2, \ldots, n\} \triangleq \mathcal{I}$. The system described by Equation (5) is taken from [38], which describes the motion of objects in general, such as angular motion.

Assumption 1. *The mismatched uncertainties* d_{i1} *and external disturbances* d_{i2} *are bounded, i.e.,* $|d_{i1}| \le D_1$ *and* $|d_{i2}| \le D_2$ *hold, where* D_1 *and* D_2 *are unknown positive constants.*

Furthermore, the target agent with an index of 0 can be modeled as a differential equation:

$$\begin{cases} \dot{x}_0(t) = f_0(x_0, t), \\ y_0(t) = x_0(t), \end{cases} \tag{6}$$

where the smooth function $f_0(x_0, t)$ is unknown to all other agents. $x_0(t)$ represents the state variable. The agent designated as the leader is referred to as the target agent, while the other ith agents ($i \in \mathcal{I}$) are referred to as followers. The output $y_0(t)$ corresponds to the information that is accessible to at least one follower.

Definition 2 (Bounded Consensus Tracking Control)**.** *The control protocol $u_i(t)$ in (5) is said to be a bounded consensus tracking control if the state variables $x_i(t)$ of all the followers end up boundedly tracking the leader's, i.e., for all $i \in \mathcal{I}$*

$$|x_{i1}(t) - x_0(t)| \leq b, \forall t \geq t_b, \tag{7}$$

where $b \geq 0$ is the ultimate bound of the tracking error, and $t_b \geq 0$ is the corresponding settling time.

In order to realize the bounded consensus tracking control problem proposed by the above definition, we need to design the corresponding control strategy for the followers.

4. Main Results

In this section, a two-module robust consensus tracking control scheme is proposed for the *i*th agent shown in Figure 1 to tackle the issue at hand. The scheme combines a neural network-based distributed observer and a back-stepping-based tracking controller. Firstly, the neural network-based distributed observer is able to efficiently estimate the information of the leader through all the followers. Then, the back-stepping-based tracking controller was designed to enable the followers to track the estimates of the trajectory of the leader, which is generated by a distributed observer. Eventually, the tracking of the leader's trajectory can be achieved.

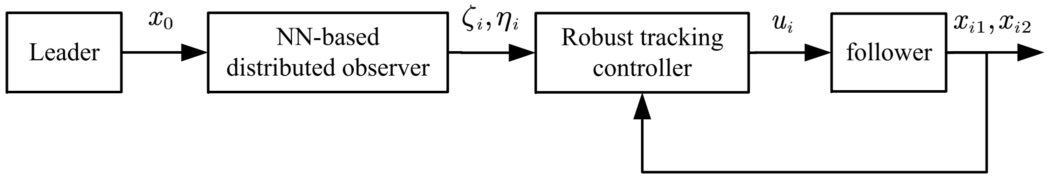


Figure 1. The proposed control scheme.

As discussed in the preceding section, there are two issues that need to be addressed in this process: (1) the leader dynamics involve an uncertainty term, thereby causing each follower agent to lack precise velocity and acceleration information about the leader agent; and (2) each follower agent is subject to matched and mismatched uncertainties. Thus, followers need not only a valid estimate of the leader's information, but also compensation for the unknown uncertainty terms.

4.1. Neural Network-Based Distributed Observer Design

The initial step in addressing the robust consensus tracking issues involves mitigating uncertainties within the leader agent model. In order to achieve this goal, we need to use neural networks to fit unknown function values. Therefore, we need the following assumption:

Assumption 2. *$\gamma(x,t) \triangleq f_0(x,t)$ can be expressed on a prescribed compact set $\Omega_{\gamma i} \subset \mathbb{R}^2$ using linearly parameterized neural networks as follows:*

$$\gamma(x,t) = \phi_\gamma^T(x,t)\theta_\gamma + e_\gamma, \tag{8}$$

where $\phi_\gamma^T(x,t) = \boldsymbol{col}_k^{h_{\gamma i}}[\phi_{\gamma i,k}(x,t)] \in \mathbb{R}^{h_{\gamma i}}$; the parameter $\theta_\gamma = \boldsymbol{col}_k^{h_{\gamma i}}[\theta_{\gamma i,k}] \in \mathbb{R}^{h_{\gamma i}}$ is an unknown constant vector; and e_γ is the NN approximation error.

Remark 1. *In traditional adaptive control theories, extensive research has focused on linearly parameterized models of unknown nonlinear dynamics [39,40]. Assumption 2 will be satisfied once the fundamental function ϕ_γ is suitably chosen, and the receptive fields cover the respective value ranges of the smooth functions $\gamma(x,t)$.*

Throughout the paper, $\hat{\alpha}$ is used to represent the estimate of a quantity α, and $\tilde{\alpha} \triangleq \hat{\alpha} - \alpha$, to denote its estimation error. Then, the estimate of the ith agent for $\gamma(x,t)$ is $\hat{\gamma}_i(x,t) \triangleq \phi_\gamma^T(x,t)\hat{\theta}_{\gamma i}$, and the corresponding estimate error is designated by $\tilde{\gamma}_i(x,t)$. The following formula shows that

$$\underline{\tilde{\gamma}_i}(x,t) \triangleq \phi_\gamma^T(x,t)\tilde{\theta}_{\gamma i} = \phi_\gamma^T(x,t)[\hat{\theta}_{\gamma i} - \theta_\gamma], \tag{9}$$

and the estimation errors can also be represented as

$$\tilde{\gamma}_i(x,t) = \underline{\tilde{\gamma}_i}(x,t) - e_\gamma. \tag{10}$$

Based on the NN's approximation theorems, we can make the following assumption.

Assumption 3. *The approximation error e_γ is bounded by unknown constants δ_γ in the corresponding compact set Ω_γ. That is, $|e_{\gamma i}| \le \delta_{\gamma i}$.*

In order to obtain the estimated value of the leader's state, we can construct the following neural network-based distributed observer:

$$\begin{aligned}
\dot{\zeta}_i =& \eta_i, \\
\dot{\eta}_i =& -k_\epsilon \epsilon_{1i} - k_p p_{2i} + \phi_\gamma^T(\zeta_i,t)\dot{\hat{\theta}}_{\gamma i} + \frac{\partial}{\partial \zeta}\phi_\gamma^T(\zeta_i,t)\hat{\theta}_{\gamma i}\eta_i + \frac{\partial}{\partial t}\phi_\gamma^T(\zeta_i,t)\hat{\theta}_{\gamma i} \\
& - \mathrm{rec}(p_{2i})(|k_\epsilon \epsilon_{1i}| + |k_\epsilon \epsilon_{1i}|\hat{\delta}_{\gamma i}), \\
\epsilon_{1i} =& \sum_{j=1}^{n} l_{ij}(\zeta_i - \zeta_j) + b_i(\zeta_i - x_0),
\end{aligned} \tag{11}$$

where ζ_i and η_i are distributed observer states, and k_ϵ and k_p are distributed observer gains. Moreover, $\mathrm{rec}(\alpha)$ is the safe reciprocal function such that $\mathrm{rec}(\alpha) \cdot \alpha$ equals unity for non-zero α, or equals zero otherwise.

Through formulating the subsequent coordinate transformation,

$$\begin{aligned}
p_{1i} &= \zeta_i - x_0, \\
p_{2i} &= \eta_i - \hat{\gamma}_i(\zeta_i,t),
\end{aligned} \tag{12}$$

one obtains

$$\begin{aligned}
\dot{p}_{2i} =& \dot{\eta}_i - \phi_\gamma^T(\zeta_i,t)\dot{\hat{\theta}}_{\gamma i} - \frac{\partial}{\partial \zeta}\phi_\gamma^T(\zeta_i,t)\hat{\theta}_{\gamma i}\dot{\zeta}_i - \frac{\partial}{\partial t}\phi_\gamma^T(\zeta_i,t)\hat{\theta}_{\gamma i} \\
=& \dot{\eta}_i - \phi_\gamma^T(\zeta_i,t)\dot{\hat{\theta}}_{\gamma i} - \frac{\partial}{\partial \zeta}\phi_\gamma^T(\zeta_i,t)\hat{\theta}_{\gamma i}\eta_i - \frac{\partial}{\partial t}\phi_\gamma^T(\zeta_i,t)\hat{\theta}_{\gamma i}.
\end{aligned} \tag{13}$$

Accordingly,

$$\dot{p}_{2i} = -k_\epsilon \epsilon_{1i} - k_p p_{2i} - \mathrm{rec}(p_{2i})(|k_\epsilon \epsilon 1i| + |k_\epsilon \epsilon 1i|\hat{\delta}_{\gamma i}). \tag{14}$$

Let $X \triangleq \mathbf{col}_i^n[\zeta_i]$, $P_1 \triangleq \mathbf{col}_i^n[p_{1i}]$, $P_2 \triangleq \mathbf{col}_i^n[p_{2i}]$, $X_0 \triangleq x_0\mathbf{1}_n$, $\Delta = \mathbf{col}_i^n[\mathrm{rec}(p_{2i})(|k_\epsilon \epsilon_{1i} + |k_\epsilon \epsilon_{1i}|\hat{\delta}_{\gamma i})]$, $E_1 \triangleq \mathbf{col}_i^n[\epsilon_{1i}]$, $\hat{R} \triangleq \mathbf{col}_i^n[\hat{\gamma}_i(\zeta_i,t)]$, $\underline{\tilde{R}} \triangleq \mathbf{col}_i^n[\underline{\tilde{\gamma}}(\zeta_i,t)]$, and $E_\gamma = \mathbf{col}_i^n[e_{\gamma i}]$. Apparently, $E_1 = (L+B)P_1$.

The matrix forms of state variables p_{1i} and p_{2i} are as follows:

$$\begin{aligned}
P_1 &= X - X_0, \\
P_2 &= \dot{X} - \hat{R} = \dot{X} - R - \underline{\tilde{R}} + E_\gamma,
\end{aligned} \tag{15}$$

with the following compact models:

$$\begin{aligned} \dot{P}_1 &= \dot{X} - \dot{X}_0, \\ \dot{P}_2 &= -k_\epsilon (L+B) P_1 - k_p P_2 - \Delta. \end{aligned} \tag{16}$$

Their adaptive rules are

$$\begin{aligned} \dot{\hat{\theta}}_{\gamma i} &= -k_f k_\epsilon \epsilon_{1i} \phi_\gamma(\zeta_i, t), \\ \dot{\hat{\delta}}_{\gamma i} &= k_\delta |k_\epsilon \epsilon_{1i}|. \end{aligned} \tag{17}$$

Remark 2. *As shown in (17), the adaptive law of $\hat{\delta}_{\gamma i}$ contains an absolute value function $|k_\epsilon \epsilon_{1i}|$. This implies that $\hat{\delta}_{\gamma i}$ is an increasing function until the consensus tracking error ϵ_{1i} is 0. When this parameter $\hat{\delta}_{\gamma i}$ remains constant, it also indicates that the consensus tracking control objective of this paper is achieved. It should be noted that this absolute value function $|k_\epsilon \epsilon_{1i}|$ does not produce chattering. In fact, this estimation value $\hat{\delta}_{\gamma i}$ compensates for the neural network fitting error $e_{\gamma i}$. In the proof, how to ensure the convergence of the closed-loop error system will be further described later on.*

Theorem 1. *The distributed observer given by (11) with the adaptive algorithm established in (17) achieves the estimation of the leader's states, i.e.,* $\lim_{t\to\infty}(\zeta_i - x_0) = 0$ *for all* $i \in \mathcal{I}$*, under Assumptions 2 and 3, if*

$$\frac{\partial}{\partial x}\gamma(x,t) < 0. \tag{18}$$

Proof. The positive definite Lyapunov function is

$$V_1 = \frac{k_\epsilon}{2} P_1^T (L+B) P_1 + \frac{1}{2} P_2^T P_2 + \frac{1}{2k_f} \sum_{i=1}^{n} \tilde{\theta}_{\gamma i}^T \tilde{\theta}_{\gamma i} + \frac{1}{2k_\delta} \sum_{i=1}^{n} \tilde{\delta}_{\gamma i}^T \tilde{\delta}_{\gamma i}, \tag{19}$$

and its derivative is

$$\begin{aligned} \dot{V}_1 =& k_\epsilon P_1^T (L+B)(\dot{X} - \dot{X}_0) - k_\epsilon (\dot{X} - R - \underline{\tilde{R}} + E_\gamma)^T (L+B) P_1 - P_2^T \Delta \\ &+ \frac{1}{k_f} \sum_{i=1}^{n} \tilde{\theta}_{\gamma i}^T \dot{\hat{\theta}}_{\gamma i} + \frac{1}{k_\delta} \sum_{i=1}^{n} \tilde{\delta}_{\gamma i}^T \dot{\hat{\delta}}_{\gamma i} - k_p P_2^T P_2 \\ =& k_\epsilon (R - \dot{X}_0)^T (L+B) P_1 + k_\epsilon \underline{\tilde{R}}^T E_1 + \frac{1}{k_f} \sum_{i=1}^{n} \tilde{\theta}_{\gamma i}^T \dot{\hat{\theta}}_{\gamma i} \\ &- k_p P_2^T P_2 - k_\epsilon E_\gamma^T E_1 - P_2^T \Delta + \frac{1}{k_\delta} \sum_{i=1}^{n} \tilde{\delta}_{\gamma i}^T \dot{\hat{\delta}}_{\gamma i}. \end{aligned} \tag{20}$$

According to (17), we can obtain that

$$k_\epsilon \tilde{R}^T E_1 = \frac{1}{k_f} \sum_{i=1}^{n} \tilde{\theta}_{\gamma i}^T \dot{\hat{\theta}}_{\gamma i} \tag{21}$$

and

$$\begin{aligned} -k_\epsilon E_\gamma^T E_1 - P_2^T \Delta + \frac{1}{k_\delta} \sum_{i=1}^{n} \tilde{\delta}_{\gamma i}^T \dot{\hat{\delta}}_{\gamma i} &= \sum_{i=1}^{n} \Big[-k_\epsilon \epsilon_{1i} e_{\gamma i} - |k_\epsilon \epsilon_{1i}| (\hat{\delta}_{\gamma i} - \tilde{\delta}_{\gamma i}) \Big] \\ &\le \sum_{i=1}^{n} \Big(|k_\epsilon \epsilon_{1i}| \cdot |e_{\gamma i}| - |k_\epsilon \epsilon_{1i}| \cdot \delta_{\gamma i} \Big) \\ &\le 0, \end{aligned} \tag{22}$$

Moreover, the composition rule reveals that $\dot{x}_0 = \gamma(x_0, t)$; then,

$$\begin{aligned} R - \dot{X}_0 &= \mathbf{col}_i^n[\gamma(\zeta_i, t) - \dot{x}_0] \\ &= \mathbf{col}_i^n[\gamma(\zeta_i, t) - \gamma(x_0, t)] \\ &= \mathbf{col}_i^n[\frac{\partial}{\partial x}\{\gamma(\xi_i, t)\}(\zeta_i - x_0)] \\ &= -\mathbf{diag}_i^n[-\frac{\partial}{\partial x}\gamma(\xi_i, t)]P_1 \\ &\triangleq -\Xi P_1. \end{aligned} \tag{23}$$

where $\xi_i \in [\min(\zeta_i, x_0), \max(\zeta_i, x_0)]$. Ξ is positive definite; consequently,

$$\dot{V}_1 \le -k_\epsilon P_1^T \Xi (L + B) P_1 - k_p P_2^T P_2 \le 0. \tag{24}$$

Therefore, V_1 keeps decreasing until $P_1 \equiv P_2 \equiv 0$, which implies that $P_1(\infty) \to 0$, i.e., $\lim_{t\to\infty}(\zeta_i - x_0) = 0$ for all $i \in \mathcal{I}$. The theorem is proven. □

4.2. Robust Tracking Controller Design

In the following, the design for a robust tracking controller will be shown for each follower to track the corresponding estimates generated through a neural network-driven distributed observer (11). Ultimately, consensus tracking from the followers to the leader is achieved.

Based on the idea of the back-stepping method, the controller design process of the second-order system (5) can be divided into two steps.

4.2.1. Step 1

Introduce a virtual control input v_i and define the tracking errors as follows:

$$e_{i1} = x_{i1} - \zeta_i, \tag{25}$$

$$e_{i2} = x_{i2} - v_i. \tag{26}$$

When considering (5), (25), and (26), the dynamics of e_{i1} are

$$\dot{e}_{i1} = e_{i2} + v_i + d_{i1} - \eta_i. \tag{27}$$

Design the virtual control input v_i as follows:

$$v_i = -k_{1i}e_{i1} - \hat{d}_{i1} + \eta_i, \tag{28}$$

where k_{1i} is a positive constant, while $\hat{d}_{i1}$ stands for the estimate of d_{i1} produced by a disturbance observer, which is shown later.

Consider the following Lyapunov candidate function:

$$V_{B1} = \frac{1}{2}{e_{i1}}^2. \tag{29}$$

Taking the derivative of V_{B1} leads to

$$\begin{aligned} \dot{V}_{B1} &= -k_{1i}{e_{i1}}^2 - e_{i1}\left(\tilde{d}_{i1} - e_{i2}\right) \\ &\le -k_{1i}|e_{i1}|^2 + |e_{i1}|\left(|\tilde{d}_{i1}| + |e_{i2}|\right) \\ &= -k_{1i}(1-\sigma)|e_{i1}|^2 - k_{1i}\sigma|e_{i1}|^2 + |e_{i1}|\left(|\tilde{d}_{i1}| + |e_{i2}|\right) \\ &= -k_{1i}(1-\sigma)|e_{i1}|^2 - |e_{i1}|\left(k_{1i}\sigma|e_{i1}| + |\tilde{d}_{i1}| + |e_{i2}|\right), \end{aligned} \tag{30}$$

where $0 < \sigma < 1$, and $\tilde{d}_{i1} \triangleq \hat{d}_{i1} - d_{i1}$ is the estimation error of d_{i1}.

From the above equation, it can be derived that

$$\dot{V}_{B1} \leq -k_{1i}(1-\sigma)|e_{i1}|^2, \forall |e_{i1}| \geq b_{B1}, \tag{31}$$

where $b_{B1} = \frac{|\tilde{d}_{i1}|+|e_{i2}|}{k_{1i}\sigma}$.

According to Theorem 4.18 in [41], there exists $t_{B1i} \geq 0$ such that

$$|e_{i1}(t)| \leq b_{B1i}, \forall t \geq t_{B1i}. \tag{32}$$

Thus, the tracking error e_{i1} is bounded if e_{i2} and $\tilde{d}_{i1}$ are bounded.

In the following, the disturbance observer is shown for estimating d_{i1} based on the barrier function to ensure that $\tilde{d}_{i1}$ is bounded.

An auxiliary system is formulated as follows:

$$s_{i1} = \varphi_{i1} - x_{i1}, \tag{33}$$

where the dynamic equation of φ_{i1} is

$$\dot{\varphi}_{i1} = x_{i2} + \hat{d}_{i1}. \tag{34}$$

Combining (5), (33), and (34) yields

$$\dot{s}_{i1} = \hat{d}_{i1} - d_{i1}. \tag{35}$$

Based on the barrier function, $\hat{d}_{i1}$ is given by

$$\hat{d}_{i1} = -K_1(s_{i1}(t))\mathrm{sgn}(s_{i1}), \tag{36}$$

and

$$K_1(s_{i1}(t)) = \begin{cases} K_{a1}(t), \dot{K}_{a1}(t) = k_{a1}|s_{i1}|, 0 < t \leq t_1, \\ K_{p_1}(s_{i1}) = \frac{|s_{i1}|}{\lambda_1 - |s_{i1}|}, t > t_1, \end{cases} \tag{37}$$

where k_{a1} and λ_1 are positive constants, and t_1 is the time when $|s_{i1}(t)| \leq \lambda_1$ is satisfied for the first time.

Theorem 2. *When considering the system (35) under Assumption 1, if we adopt the disturbance observer as detailed in (36) and (37) based on the barrier function specified in Definition 1, it can be established that s_{i1} achieves convergence to the domain $|s_{i1}(t)| \leq \lambda_1\left(\frac{d_{\max}}{d_{\max}+1}\right)$ within finite time. Additionally, the estimation error for d_{i1} exhibits bounded convergence.*

Proof. From (35), it is obvious that

$$\dot{s}_{i1} = \tilde{d}_{i1}. \tag{38}$$

By Definition 1, when $s_{i1} \in [-\lambda_1, \lambda_1]$, $K_{p1}(s_{i1}) \in [0, \infty]$. According to Lemma 1, it can be inferred that when $t > t_1$, $|s_{i1}| < \lambda_1$, and thus, $|\hat{d}_{i1}|$ is bounded. Meanwhile, it follows from Lemma 1 that s_{i1} converges to $|s_{i1}(t)| \leq \lambda_1\left(\frac{d_{\max}}{d_{\max}+1}\right)$ in finite time. In combination with with Assumption 1, both $\hat{d}_{i1}$ and d_{i1} are bounded such that $\tilde{d}_{i1}$ is bounded, and the boundedness of the integral $|s_{i1}|$ further ensures that the fluctuations of $\tilde{d}_{i1}$ are limited. The proof is complete. □

Remark 3. *Barrier function-based disturbance observers (36) and (37) have only the requirement that the disturbance be integrable and bounded, i.e., that it is capable of estimating bounded non-smooth nonlinear disturbances.*

4.2.2. Step 2

Next, we will show the design of the control input u_i to ensure that e_{i2} is bounded.

Combining (26) with the second equation of (5) yields the dynamic equation of e_{i2}:

$$\dot{e}_{i2} = f_i(x_{i1}, x_{i2}) + g_i(x_{i1}, x_{i2})u_i + d_{i2} + k_{1i}(x_{i2} + d_{i1} - \eta_i) + \dot{\hat{d}}_{i1} - \dot{\eta}_i. \tag{39}$$

We designed u_i as follows:

$$u_i = \frac{1}{g_i(x_{i1}, x_{i2})}\left(-k_{2i}e_{i2} - f_i(x_{i1}, x_{i2}) - \hat{d}_{i2} - k_{1i}\left(x_{i2} + \hat{d}_{i1} - \eta_i\right) + \dot{\eta}_i\right), \tag{40}$$

where $k_{2i} > 0$, and $\hat{d}_{i2}$ is the estimate of d_{i2}.

Consider the following Lyapunov candidate function:

$$V_{B2} = \frac{1}{2}{e_{i2}}^2. \tag{41}$$

The derivative of V_{B2} is

$$\begin{aligned} \dot{V}_{B2} &= -k_{2i}{e_{i2}}^2 - e_{i2}\left(\tilde{d}_{i2} + k_{1i}\tilde{d}_{i1} - \dot{\hat{d}}_{i1}\right) \\ &\leq -k_{2i}|e_{i2}|^2 + |e_{i2}|\left(|\tilde{d}_{i2}| + k_{1i}|\tilde{d}_{i1}| + |\dot{\hat{d}}_{i1}|\right) \\ &= -k_{2i}(1-\sigma)|e_{2i}|^2 - |e_{i2}|\left(k_{2i}\sigma|e_{i2}| + |\tilde{d}_{i2}| + k_{1i}|\tilde{d}_{i1}| + |\dot{\hat{d}}_{i1}|\right) \end{aligned} \tag{42}$$

where $\tilde{d}_{i2} \triangleq \hat{d}_{i2} - d_{i2}$ is the estimation error of d_{i2}.

From (42), it can be derived that

$$\dot{V}_{B2} \leq -k_{2i}(1-\sigma)|e_{i2}|^2, \forall |e_{i2}| \geq b_{B2}, \tag{43}$$

where $b_{B2} = \frac{|\tilde{d}_{i2}| + k_{1i}|\tilde{d}_{i1}| + |\dot{\hat{d}}_{i1}|}{k_{2i}\sigma}$.

According to Theorem 4.18 in [41], there exists $t_{B2i} \geq 0$ such that

$$|e_{i2}(t)| \leq b_{B2i}, \forall t \geq t_{B2i}. \tag{44}$$

Theorem 2 and Equations (36) and (37) guarantee that $\tilde{d}_{i1}$ and $\dot{\hat{d}}_{i1}$ are bounded. Consequently, the tracking error e_{i2} is bounded as long as the disturbance estimation error $\tilde{d}_{i2}$ is bounded.

Similar to the previous step, the disturbance observer for estimating d_{i2} was designed based on the barrier function.

Likewise, the subsequent auxiliary system is formulated as

$$s_{i2} = \varphi_{i2} - x_{i2}, \tag{45}$$

where the dynamics of φ_{i2} are given by

$$\dot{\varphi}_{i2} = f(x_{i1}, x_{i2}) + g(x_{i1}, x_{i2})u_i + \hat{d}_{i2}. \tag{46}$$

In combining the second equations of (5), (45), and (46), it can be obtained that

$$\dot{s}_{i2} = \hat{d}_{i2} - d_{i2}. \tag{47}$$

Then, $\hat{d}_{i2}$ is given by

$$\hat{d}_{i2} = -K_2(s_{i2}(t))\mathrm{sgn}(s_{i2}) \tag{48}$$

and

$$K_2(s_{i2}(t)) = \begin{cases} K_{a2}(t), \dot{K}_{a2}(t) = \bar{k}_{a2}|s_{i2}|, 0 < t \leq t_2, \\ K_{p_2}(s_{i2}) = \frac{|s_{i2}|}{\lambda_2 - |s_{i2}|}, t > t_2, \end{cases} \tag{49}$$

where k_{a2} and λ_2 are positive constants, and t_2 equals the time when $|s_{i2}(t)| \leq \frac{\lambda_2}{2}$ is satisfied for the first time.

It is obvious that Equations (47)–(49) have the same structure as Equations (35)–(37). Therefore, it follows from Theorem 2 that $\tilde{d}_{i2}$ is bounded. As a consequence, e_{i2} and e_{i1} are bounded.

Up to now, with the design idea of the back-stepping technique and based on the barrier function disturbance observers (36), (37), (48) and (49), we obtained the robust tracking controller composed by Equations (28) and (40).

Theorem 3. *Under Assumptions 1–3, considering multi-agent systems (5) and (6) subject to both matched and mismatched disturbances in combination with neural network-based distributed observer (11), the control strategy u_i achieves the bounded consensus tracking from the follower state $x_{i1}(i \in \mathcal{I})$ to the leader state x_0.*

Proof. For all $i \in \mathcal{I}$, define the tracking error of the ith follower with respect to the leader as

$$e_i = x_{i1} - x_0, \tag{50}$$

which can be transformed into

$$e_i = x_{i1} - \zeta_i + \zeta_i - x_0 = e_{i1} + \zeta_i - x_0. \tag{51}$$

From the design process of the robust tracking controller, it is evident that for $b_c = \max\{b_{B1i}(i \in \mathcal{I})\}$, there is a corresponding settling time t_b such that for all $i \in \mathcal{I}$,

$$|e_{i1}(t)| \leq b_c, \forall t \geq t_b. \tag{52}$$

Disturbance observers (36), (37), (48) and (49) guarantee that b_c is bounded.

According to Theorem 1, it is obtained that

$$\lim_{t \to \infty} (\zeta_i - x_0) = 0. \tag{53}$$

Thus, for all $i \in \mathcal{I}$ and for t_b, there exists a boundary $b_d \geq 0$ such that $|\zeta_i - x_0| \leq b_d$.

Hence, it can be obtained that for all $i \in \mathcal{I}$,

$$e_i(t) \leq b, \forall t \geq t_b, \tag{54}$$

where $b = b_c + b_d$ is the ultimate bound of the tracking error.

Subsequently, according to Definition 2, it can be deduced that the control protocol u_i is a bounded consensus tracking control. The proof is complete. □

The robust tracking control scheme was shown in this section. Initially, a distributed observer based on neural networks was introduced to accurately estimate leader information for all follower nodes. Subsequently, a robust tracking controller based on back-stepping technique was developed, demonstrating the boundedness of the closed-loop error system through the application of the Lyapunov-like theorem. Meanwhile, a barrier function-based disturbance observer was designed to accurately estimate matched and mismatched uncertainties among followers. Ultimately, bounded consensus tracking from the followers to the leader trajectory is achieved.

Remark 4. *It can be observed from (36), (37), (48) and (49) that for $j = 1, 2$, the solution s_{ij} reaches $\frac{\lambda_j}{2}$ in finite time t_j. At this point, the adaptive gain K_j switches from K_{aj} to K_{pj} and remains as K_{pj} thereafter. It is evident that as $s_{ij} \to 0$, $K_{pj} \to 0$. Consequently, K_{pj} exhibits the same behavior as $\frac{|s_{ij}|}{\lambda_i}$ in the vicinity of zero, that is, $\frac{s_{ij}}{\lambda_j} \ll 1 \Rightarrow K_{pj}(s_{ij}) = \frac{|s_{ij}|}{\lambda_j - |s_{ij}|} \approx \frac{|s_{ij}|}{\lambda_j}$. This implies that if d_{ij} and s_{ij} monotonically approach zero, $K_p(s_{ij})$ will tend to zero as well. Hence, the discontinuity of* *the signal $\hat{d}_{ij}$ only occurs once at time t_j [37]. It is worth noting that $\hat{d}_{ij}$ becomes continuous from time t_j, and there is no chattering generated by (36), (37), (48) or (49).*

Remark 5. *In the simulation, to mitigate the chattering effect resulting from discontinuous control signals, the emulation of the signum function* $\mathrm{sgn}(x)$ *is carried out utilizing function* $\tanh(ax) = \frac{e^{ax}-e^{-ax}}{e^{ax}+e^{-ax}}$*. Here, parameter a may be set as $a = 10$, allowing for a smoother control signal while closely approximating the characteristics of the signum function.*

5. Simulation Results

Several simulation examples are presented to validate the efficacy of the proposed distributed consensus tracking control scheme in this section. Here, we considered that there are four followers and one leader, and Figure 2 depicts the communication topology among the agents. Then, L and B were derived as follows:

$$L = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & -1 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1.4 & 0 & 0 & 0 \\ 0 & 1.2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

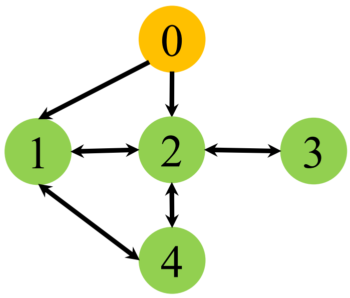


Figure 2. The communication topology among agents.

In addition, we consider that each follower agent's dynamics are characterized by the longitudinal height channel model of a four-rotor UAV system. According to the previous work in [42], the transnational dynamic model of the height channel of the four-rotor UAV system is

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} + d_{i1}, \\ \dot{x}_{i2} &= \frac{\cos\phi_i \cos\theta_i u_i}{m} - g + d_{i2}, \end{aligned} \tag{55}$$

in which ϕ_i represents the rolling angle, and θ_i represents the pitch angle of a follower agent. The quadrotor's mass is denoted by m, while U_1 represents the total thrust force. The acceleration of gravity is expressed as g. Additionally, d_{i_1} and d_{i_2} correspond to the mismatched uncertainty and matched uncertainty, respectively.

The model parameters for the four-rotor UAV were chosen as $m = 3$ kg, $\phi_i = 0°$, $\theta_i = 0°$, and, in the simulation verification scenario of this paper, $g = 9.81$ m/s^2. Furthermore, d_{i1} and d_{i2} were regarded as follows: $d_{i1} = 0.2x_{i1} + \sin(0.2\pi t)$ and $d_{i2} = 0.2\mathrm{sgn}(x_{i1}x_{i2})$.

Moreover, the dynamics of the leading agent were taken into account as follows:

$$\dot{x}_0 = -0.4x_0 + \cos(t^{0.8}) + 0.02t + 1. \tag{56}$$

A radial basis function (RBF) neural network was employed for the estimation of $\gamma(x,t)$ utilizing Gaussian basis functions:

$$\phi_{\gamma}^{T}(x,t) = e^{-\frac{(x-\mu_{\gamma ix,k})^2+(t-\mu_{\gamma it,k})^2}{\eta_{\gamma i,k}^2}}, k \in \{1,2,\ldots,h_{\gamma i}\}, \tag{57}$$

The widths of the Gaussian basis functions were $\eta_{\gamma i,k} = 6$, and the number of nodes was $h_{\gamma i} = 17 \times 17$. The centers $(\mu_{\gamma ix,k}, \mu_{\gamma it,k})$ were uniformly distributed within the range $[-25, 25] \times [0, 50]$. The initial conditions for the system were $x_0(0) = 3.5$, $\hat{\theta}_{\gamma i}(0) = 0$. The observer gain and adaptive law parameters were $k_f = 10$, $k_\epsilon = 5$, $k_p = 5$, and $k_\delta = 5$. Let $K_\theta(0) = 5$ and $K_p(0) = 10$. The step of sampling was chosen to be 0.01 s. Additionally, the control tracking parameters are presented in Table 1.

Table 1. The control parameters of each follower agent.

Parameter	Value	Parameter	Value	Parameter	Value
k_1	5	k_{a1}	100	λ_1	0.1
k_2	10	k_{a2}	100	λ_2	0.02

In [43], the authors addressed the consensus tracking control problem of second-order uncertain multi-agent systems with mismatched and matched disturbances. They designed a neural network-based consensus tracking control scheme and estimated the compound uncertainties utilizing a neural network approximator. In the simulation, we compared the differences between the controllers designed in this study and the controllers designed in the literature [43]. The simulation and comparison results of the numerical examples are illustrated in Figures 3–7. It is evident from Figure 3 that all follower agents successfully tracked the leader's trajectory, signifying the attainment of tracking control within the multi-agent system. Figure 4 demonstrates the trajectory of the consensus tracking error under the two control schemes. It can be seen that the control error judder is more obvious using the control scheme from the literature [43], while the error trajectory is relatively smooth using the control scheme designed in this study. In Figures 5 and 6, the disturbance estimation errors are shown under the two control schemes. It can be seen that the barrier function-based disturbance estimator designed in this study can effectively achieve compensation for unknown disturbances compared to the neural network-based estimation method in the literature [43]. Finally, the control input of each follower agent is shown in Figure 7.

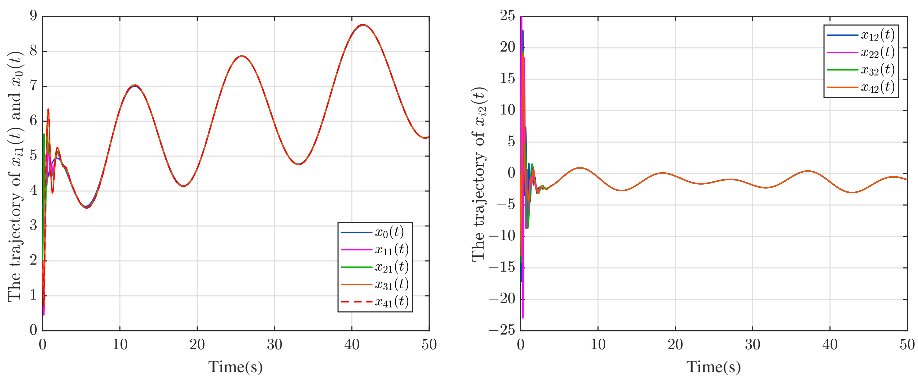
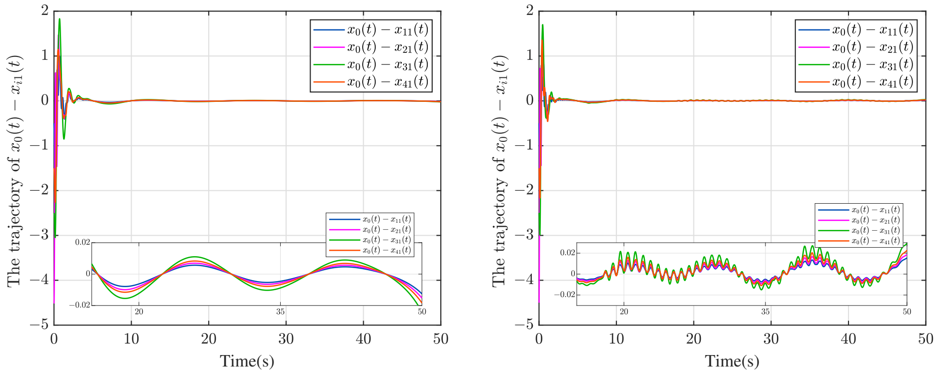
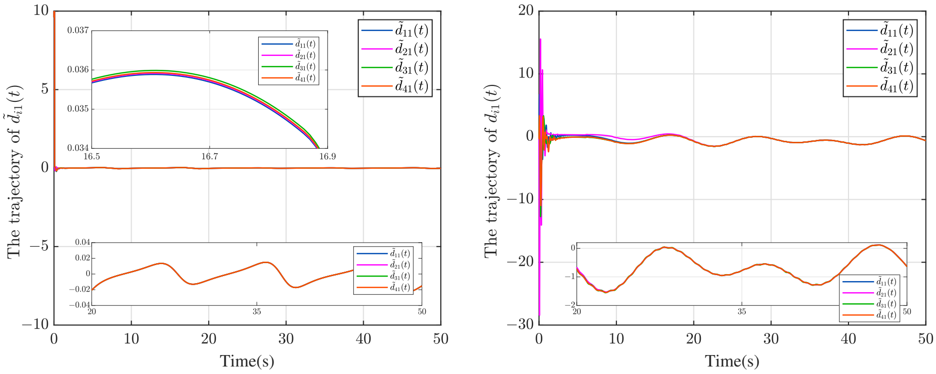


Figure 3. The position and velocity trajectories of agents, respectively.



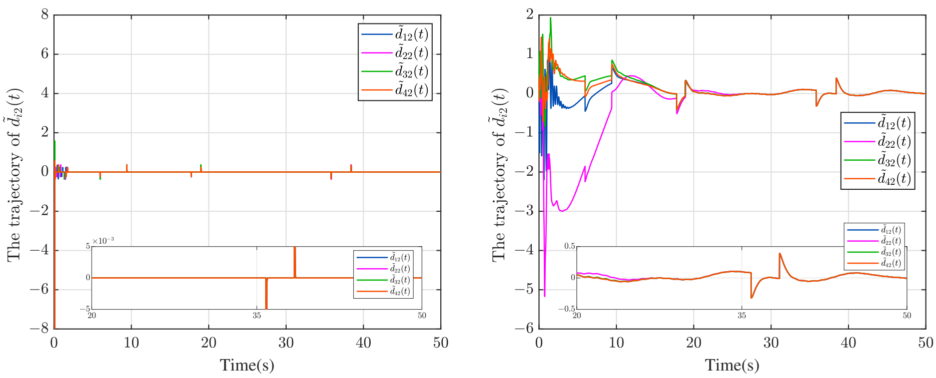
(**a**) (**b**)

Figure 4. The consensus tracking error trajectories of follower agents. (**a**) The proposed controller. (**b**) The NN-based controller in [43].



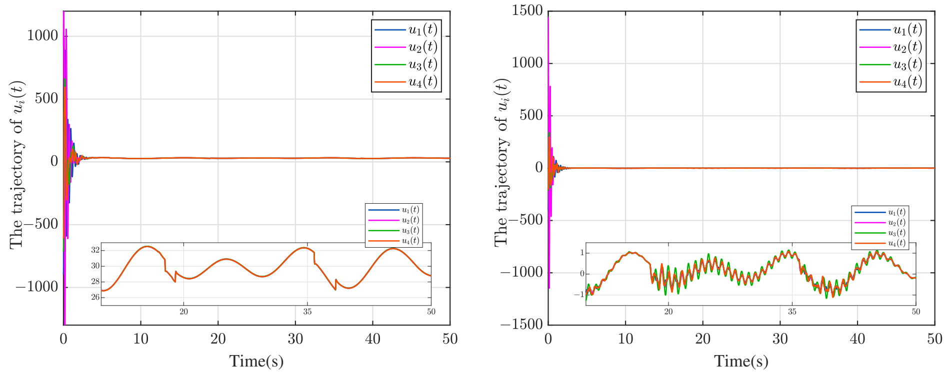
(**a**) (**b**)

Figure 5. The estimation error $\tilde{d}_{i1}$ trajectories of follower agents. (**a**) The proposed controller. (**b**) The NN-based controller in [43].



(**a**) (**b**)

Figure 6. The estimation error $\tilde{d}_{i2}$ trajectories of follower agents. (**a**) The proposed controller. (**b**) The NN-based controller in [43].



(**a**) (**b**)

Figure 7. The control input trajectories of follower agents. (**a**) The proposed controller. (**b**) The NN-based controller in [43].

6. Conclusions

This research delved into the intricacies of addressing the consensus tracking control challenge in second-order multi-agent systems with both mismatched and matched uncertainties. By integrating a neural network-based distributed observer, a barrier function-based disturbance observer, and a back-stepping-based tracking controller, a robust tracking control method was developed. This scheme enables the distributed estimation of leader information, compensation for disturbances, and the effective tracking of the leader's trajectory by followers, despite the presence of uncertainties. Then, the stability of the error system was demonstrated and established using the Lyapunov theory. Additionally, simulation results are provided to validate the efficacy of the distributed consensus tracking scheme. It is important to point out that in this paper, we consider that followers can communicate with each other in both directions. In the future, there will be further exploration of the tracking control problem for uncertain multi-agent systems under a directed topology.

Author Contributions: L.X.: conceptualization, methodology, and writing—original draft. K.Q.: validation and writing—review and editing. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Natural Science Foundation of Sichuan Province (2022NSFSC0037), Sichuan Science and Technology Programs (2022JDR0107, 2021YFG0130, MZGC20230069), Fundamental Research Funds for the Central Universities (ZYGX2020J020), and the Wuhu Science and Technology Plan Project (2022yf23).

Data Availability Statement: The data presented in this study are available on request from the corresponding author.

Conflicts of Interest: No conflict of interest exists in the submission of this manuscript, and all authors approve the manuscript for publication. I would like to declare on behalf of my co-author that the work described was original research that has not been published previously and is not under consideration for publication elsewhere, in whole or in part. All the authors listed have approved the manuscript that is enclosed. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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