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# Robust Synchronization of Fractional-Order Uncertain Chaotic Systems Based on Output Feedback Sliding Mode Control

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**Abstract:** This paper mainly focuses on the robust synchronization issue for drive-response fractional-order chaotic systems (FOCS) when they have unknown parameters and external disturbances. In order to achieve the goal, the sliding mode control scheme only using output information is designed, and at the same time, the structures of a sliding mode surface and a sliding mode controller are also constructed. A sufficient criterion is presented to ensure the robust synchronization of FOCS according to the stability theory of the fractional calculus and sliding mode control technique. In addition, the result can be applied to identical or non-identical chaotic systems with fractional-order. In the end, we build two practical examples to illustrate the feasibility of our theoretical results.

**Keywords:** robust synchronization; fractional-order; output feedback; sliding mode

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## 1. Introduction

In the last decades, dynamic systems have been intensively studied the fields of natural science and engineering technology. Especially the fractional-order dynamic systems described by the fractional-order derivative have received widespread concern because they are more accurate expressions of real systems with memory and inherited features where such characteristics are neglected or difficult to express with integer-order systems [1–8].

Sliding-mode control (SMC) has had much attention paid to it over the past decades by many researchers due to its low sensitivity to the parameters and high robustness to external disturbances [9–13]. Most of the results on SMC were focused on the matched uncertainties, such as [9,10]. Yan et al. studied the robust synchronization issue of master-slave integer-order chaotic systems by designing an adaptive sliding mode controller and the new proportional-integral switching surface in [9]. Pai obtained some sufficient criteria on the robust synchronization for a large class of uncertain master-slaver chaotic systems via an adaptive sliding mode control scheme and Lyapunov stability theory when not all the system states were fully unavailable [10]. Sliding-mode control is also effective when the parameters or uncertainties are mismatched, as in [11].

Some new application areas of SMC were developed in recent years, for example consensus, chaos control, stability, and synchronization of different systems with external disturbances and parameter uncertainty, which are receiving increasing attention [14–29]. Using the active sliding mode controller, Tavazoei and Haeri presented some new results on the synchronization of identical and non-identical drive-response chaotic systems with fractional-order in [21]. Hosseinnia et al. studied an uncertain Duffing–Holmes chaotic system and obtained new synchronization results based on the sliding mode control method in [22]. Wang et al. was concerned with the stabilization for an uncertain economic system with fractional-order by designing a new fraction-order integral switching surface in [23]. Aghababa addressed a new terminal sliding mode controller with fractional-order to achieve chaos control or synchronization for a class of chaotic or hyperchaotic fractional-order systems in a finite time when the systems were affected by uncertainties and external noises in [24]. He also discussed the finite time control problem for a class of uncertain fractional-order nonlinear systems with model uncertainties and external disturbances via the fractional Lyapunov stability theory in [25].

The results of all these research papers were conservative because they supposed that the system states can be fully known. In fact, not all the states are variable, because it may be too expensive or impossible to measure in real systems. In this case, two of the common feasible methods are output feedback and state observers. The measured output information, for which only a portion of the state information can be used, is a more straightforward approach. However, few results have emerged on the output feedback sliding mode control scheme for fractional-order chaotic systems (FOCS). Thus, it is of both theoretical and practical importance to analyze the issue of robust synchronization of drive-response FOCS by only using the output feedback information.

Inspired by the above discussions, in this paper, we mainly discuss robust synchronization for uncertain FOCS by using the output feedback method. The highlight of this paper is that we devise the appropriate sliding mode controller to realize chaos synchronization of uncertain chaotic systems with fractional-order based on the output feedback information and stability theory of the fractional calculus.

The remainder of the paper is outlined as follows. Some necessary preliminaries and lemmas on fractional calculus are given in Section 2. The model of the drive-response systems and the problem formulation are presented in Section 3. The main conclusions are established in Section 4. In Section 5, numerical results are presented to show the effectiveness of the theoretical method. In Section 6, conclusions and possibilities for future work are provided.

## 2. Preliminaries

In this section, some necessary preliminaries on fractional calculus are recalled, which will be used in the following sections.

There exist three definitions for the fractional derivative that are commonly used in the literature such as the Caputo derivative, the Riemann–Liouville derivative, and the Grünwald–Letnikov derivative. In this letter, we adopt the Caputo derivative due to its clear physical meaning. For more details, please refer to [30].

The fractional-order Caputo derivative is defined as follows:

$$
{}_0^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} \left(\frac{d}{d\tau}\right)^m f(\tau) d\tau,
$$

where $\alpha$ represents the order of the derivative and $m-1 < \alpha \leq m$, $f(t)$ is a time-dependent function, and $\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt$. For simplicity, a simple notation $D^{\alpha} f(t)$ is adopted to indicate the fractional-order Caputo derivative ${}_a^C D_t^{\alpha} f(t)$.

To proceed, the following lemmas are provided.

**Lemma 1** ([31,32])**.** *When $0 < \alpha < 1$, $A \in \mathbb{R}^{n\times n}$ is a real matrix. Consider the following the autonomous fractional-order linear system which can be described by:*

$$
D^{\alpha} x(t) = A x(t), x(0) = x_0.
$$

*Then:*

(i) *the system is asymptotical stable if and only if* $|arg(spec(A))| > \frac{\pi}{2}\alpha$;
(ii) *the system is stable if* $|arg(spec(A))| \geq \frac{\pi}{2}\alpha$ *and those critical eigenvalues that satisfy* $|arg(spec(A))| = \frac{\pi}{2}\alpha$ *have a geometric multiplicity of one, where* $spec(A)$ *is the spectrum of all eigenvalues of* $A$.

**Lemma 2** ([15]). *Let* $0 < \alpha < 1$, *and suppose that* $\sigma(t)$ *is a solution of* $D^{\alpha}\sigma = -ksgn(\sigma), k > 0$, *then a nonzero solution* $\sigma(t)$ *satisfies* $\sigma\dot{\sigma} < 0$, *i.e.,* $\sigma(t)$ *converges asymptotically to* $\sigma = 0$.

**Remark 1.** *When* $\alpha = 1$, *this corresponds to the classical case* $\dot{\sigma} = -ksgn(\sigma)$ *and ensures* $\sigma\dot{\sigma} < 0$ *if* $\sigma \neq 0$.

## 3. Problem Formulation

Consider a class of FOCS, which can be represented as follows:

$$\begin{cases} D^{\alpha}x(t) = Ax(t) + BH(t,x) \\ y(t) = Cx(t) \end{cases}, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $y(t) \in \mathbb{R}^l$ are the system state and the system measurement output information, respectively. $H(t,x)$ is a function of $t$ and $x$. $A, B$, and $C$ are known constant matrices with compatible dimensions.

**Remark 2.** *Fractional-order chaotic systems (1) are quite popular and common. Many fractional-order chaotic system, such as the fractional-order Chen system, the fractional-order Lorenz system, the fractional-order Duffing–Holmes system, and so on, can be transformed into this form.*

We refer to chaotic systems (1) with parameter uncertainties and external disturbances as the drive systems, which are described as follows:

$$\begin{cases} D^{\alpha}x(t) = Ax(t) + BH(t,x) + d(t) + g(t,x) \\ y(t) = Cx(t) \end{cases}, \tag{2}$$

where $d(t) \in \mathbb{R}^n$ and $g(t,x) \in \mathbb{R}^n$ represent, respectively, external disturbances and the parameter uncertainties, which are supposed to satisfy the following standard matching condition.

The controlled response systems are characterized by:

$$\begin{cases} D^{\alpha}x_r(t) = Ax_r(t) + Bu(t) \\ y_r(t) = Cx_r(t) \end{cases}, \tag{3}$$

where $x_r(t) \in \mathbb{R}^n, y_r(t) \in \mathbb{R}^l$ are the system state and the system measurement output information, respectively. $u(t) \in \mathbb{R}^m$ is the controller, which is designed later. The constant matrices $A, B, C$ have appropriate dimensions with $m \leq l < n$.

To obtain the main results, some basic assumptions are given in this section.

**Assumption 1.** *Let* $d(t) \in \mathbb{R}^n$ *and* $g(t,x) \in \mathbb{R}^n$ *satisfy the standard matching condition, i.e., there exist functions* $\hat{g}(t,x)$ *and* $\hat{d}(t)$ *such that* $g(t,x) = B\hat{g}(t,x), d(t) = B\hat{d}(t)$.

Suppose $f(t,x) = \hat{g}(t,x) + \hat{d}(t) + H(t,x)$ represent the lumped uncertainty and nonlinear parts, then system (2) can be rewritten into:

$$\begin{cases} D^{\alpha}x(t) = Ax(t) + Bf(t,x) \\ y(t) = Cx(t) \end{cases}. \tag{4}$$

Considering the bounds of the uncertainties and nonlinear part of system (3), we give the following common assumption, which is widely introduced in the literature; e.g., see [9,10,12].

**Assumption 2.** *The lumped uncertainty is uniformly bounded, i.e.,* $\|f(t,x)\| \le \rho$, *for some* $\rho > 0$.

**Remark 3.** *The states of chaotic systems are always bounded. Therefore, Assumption 2 is reasonable and unrestrictive.*

**Assumption 3.** *Rank* $(CB)$ = *rank* $(B) = m$.

**Assumption 4.** *The triple* $(A, B, C)$ *is controllable and observable.*

Let $e(t)$ denote the synchronization error where $e(t) = x(t) - x_r(t)$, then the error of the dynamical system can be expressed by:

$$\begin{cases} D^\alpha e(t) = Ae(t) + Bf(t,x) - Bu(t) \\ y_e(t) = Ce(t) \end{cases}. \tag{5}$$

The robust synchronization issue is equivalent to the problem of stabilization of the error system (5) by designing a suitable controller $u(t)$. That is to say,

$$\lim_{t\to+\infty} \|x(t) - x_r(t)\| = \lim_{t\to+\infty} \|e(t)\| = 0.$$

## 4. Sliding Surface and Sliding Mode Controller Design

In this section, the sliding surface and sliding mode controller are presented to obtain the stabilization of the error system (5).

Firstly, we define the following sliding surface as follows:

$$s = Hy_e(t),$$

where $H$ is a full-rank matrix.

In the next step, to ensure the existence of the sliding mode motion, the following control strategy can be given by:

$$u(t) = u_s(t) + u_{eq}(t),$$

where $u_s(t)$ is the switching control and $u_{eq}(t)$ is the equivalent control.

By using $s'(t) = 0$, that is $D^\alpha s(t) = 0$, we have:

$$D^\alpha s(t) = HD^\alpha y_e(t) = HCD^\alpha e(t) = HC(Ae(t) - Bu(t)).$$

Thus, one has the equivalent control $u_{eq}(t)$

$$u_{eq}(t) = (HCB)^{-1}HCAe(t).$$

To force the system towards the designed sliding mode surface, the switching control $u_s(t)$ can be designed by:

$$u_s(t) = \gamma(HCB)^{-1}sgn(s).$$

In the end, we get the control law as follows:

$$u(t) = (HCB)^{-1}HCAe(t) + \gamma(HCB)^{-1}sgn(s). \tag{6}$$

Now, the main results of this section will be introduced.

**Theorem 1.** *Consider the synchronization error system (5) and suppose the Assumptions 1–4 hold. If the control law (6) is adopted and* $\gamma > \rho \|HCB\|$*, then its trajectories will converge to the sliding surface within a finite time* $t_r \le (\frac{|s(0)|\Gamma(\alpha+1)}{\gamma-\rho\|HCB\|})^{\frac{1}{\alpha}}$.

**Proof.** By the definition of the sliding mode surface $s(t)$, we derive:

$$\begin{aligned} D^{\alpha}s(t) &= HCD^{\alpha}e(t) \\ &= HC(Ae(t) + Bf(t,x) - Bu(t)) \\ &= -\gamma sgns(t) + HCBf(t,x). \end{aligned}$$

Applying Lemma 2 and the condition $\gamma > \rho\|HCB\|$, we see that the system states will converge to $s(t) = 0$ in a finite time.

In what follows, an upper bound of the reaching time $t_r$ is evaluated.

Integrating both sides of equation $D^{\alpha}s(t) = -\gamma sgns(t)$ from 0–$t_r$, one has:

$$\begin{aligned} s(t_r) - s(0) &= J^{\alpha}[-\gamma sgns(t)] \\ &= -\frac{1}{\Gamma(\alpha)} \int_0^{t_r} (t_r - \tau)^{\alpha-1} \gamma sgns(t) d\tau. \end{aligned}$$

According to the fact that $sgns(t) = sgns(0)$ during this interval, we get:

$$s(t_r) - s(0) = \frac{-\gamma sgns(0)}{\Gamma(\alpha)} \int_0^{t_r} (t_r - \tau)^{\alpha-1} d\tau.$$

Noting that $s(t_r) = 0$, we arrive at:

$$s(0) = \frac{\gamma sgns(0)}{\Gamma(\alpha)} \int_0^{t_r} (t_r - \tau)^{\alpha-1} d\tau = \frac{\gamma sgns(0)}{\Gamma(\alpha+1)} t_r^{\alpha}.$$

With straightforward manipulations, one has:

$$t_r = (\frac{|s(0)|\Gamma(\alpha+1)}{\gamma})^{\frac{1}{\alpha}}.$$

Under the presence of bounded uncertainties, it is straightforward to derive the following estimated time:

$$t_r \le (\frac{|s(0)|\Gamma(\alpha+1)}{\gamma - \rho\|HCB\|})^{\frac{1}{\alpha}}.$$

The proof is complete. □

Now, we construct the sliding mode dynamics.

According to Assumption 3, we know that there exists a transformation matrix $T$ such that $TB = \begin{pmatrix} 0 \\ B_1 \end{pmatrix}$, where $B_1$ is nonsingular. Let $T = \begin{pmatrix} W^T \\ B^T \end{pmatrix}$, where the columns of the matrix $W$ are made of basis vectors of the null space of $B^T$.

Define $z = Te$, with $z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, z_1 \in \mathbb{R}^{n-m}, z_2 \in \mathbb{R}^m$, then the synchronization error (5) is transformed to the following form:

$$\begin{cases} D^\alpha z_1(t) = A_{11}z_1(t) + A_{12}z_2(t) \\ D^\alpha z_2(t) = A_{21}z_1(t) + A_{22}z_2(t) + B_1 f(t,x) - B_1 u(t) \\ y_e(t) = C_1 z_1(t) + C_2 z_2(t) \end{cases}, \quad (7)$$

where: $CT^{-1} = \begin{pmatrix} C_1 & C_2 \end{pmatrix}, TAT^{-1} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$.

Assume that $HC_2$ is a nonsingular matrix, it is clear that:

$$z_2(t) = (HC_2)^{-1}(s(t) - HC_1 z_1(t))$$

For $s(t) = 0$, the synchronization error of system (7) is rewritten as:

$$D^\alpha z_1(t) = (A_{11} - A_{12}KC_1)z_1(t), \quad (8)$$

where $K = (HC_2)^{-1}H$.

**Remark 4.** *The controlled system (8) is robust on the unknown parameter uncertainties and external disturbances. Therefore, using the pole assignment method, one can assign the performance of the system (8) by choosing a suitable matrix K.*

**Theorem 2.** *If there exists a gain matrix K satisfying the following stability criterion:*

$$\min_i |\lambda_i(A_{11} - A_{12}KC_1)| > \frac{\alpha\pi}{2},$$

*then system (8) is asymptotically stable or the drive-response system can achieve global synchronization.*

**Proof.** According the Lemma 1, one has $z_1(t) \to 0$, if and only if there exists a gain matrix $K$ such that:

$$|arg(spec(A_{11} - A_{12}KC_1))| > \frac{\pi\alpha}{2}.$$

That is,

$$\min_i |\lambda_i(A_{11} - A_{12}KC_1)| > \frac{\alpha\pi}{2}.$$

The proof is finished. □

**Remark 5.** *Assumption 4 implies the controllability of* $(A_{11}, A_{12})$*; thus, the control gain K exists.*

**Remark 6.** *Compared with the literature [9,10], the new results of this paper are more general for* $0 < \alpha \leq 1$. *When* $\alpha = 1$, *it degenerates into a classical chaotic system.*

## 5. Two Examples

In this section, two illustrative examples are given to show the feasibility and applicability of our theoretical results.

**Example 1.** *Consider the following fractional-order Chua's circuit as the drive system, which can be described by:*

$$\begin{bmatrix} D^{\alpha}x_1(t) \\ D^{\alpha}x_2(t) \\ D^{\alpha}x_3(t) \end{bmatrix} = \begin{bmatrix} -c & c & 0 \\ 1 & -1 & 1 \\ 0 & -d & -e \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [f(x) + \hat{g}(t,x) + \hat{d}(t)],$$

$$y(t) = [1 \quad 1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

*The corresponding response system can be written as:*

$$\begin{bmatrix} D^{\alpha}x_{r1}(t) \\ D^{\alpha}x_{r2}(t) \\ D^{\alpha}x_{r3}(t) \end{bmatrix} = \begin{bmatrix} -c & c & 0 \\ 1 & -1 & 1 \\ 0 & -d & -e \end{bmatrix} \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \\ x_{r3}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t),$$

$$y_r(t) = [1 \quad 1 \quad 0] \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \\ x_{r3}(t) \end{bmatrix},$$

*where:*

$$f(x) = -c(bx_1 + 0.5(a-b)[|x_1+1| - |x_1-1|]), \hat{g}(t,x) = 0.1sin(2t), \ \ \hat{d}(t) = 0.2.$$

$\alpha = 0.985, a = -1.1726, b = -0.7882, c = 10, d = 10.3035, e = 0.268.$

*According to the proposed method, the switching surface matrix H and transformation matrix T can be given, respectively, as follows:*

$$H = 1, T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

*By straightforward computation, one can obtain the sliding mode controller:*

$$u(t) = (1-c)e_1(t) + (c-1)e_2(t) + e_3(t) + \gamma sgn(s),$$

*and there exists:*

$$K = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

*such that the eigenvalues of $A_{11} - A_{12}KC_1$ are $-1.134 + 3.09088i, -1.134 - 3.09088i$. Therefore, according to Theorem 2, the drive-response system can realize global synchronization.*

**Example 2.** *Let the following Lorenz system with fractional-order as the drive system be represented by:*

$$\begin{bmatrix} D^{\alpha}x_1(t) \\ D^{\alpha}x_2(t) \\ D^{\alpha}x_3(t) \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} [f(x) + \hat{g}(t,x) + \hat{d}(t)],$$

$$y(t) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}.$$

*The corresponding response system can be written as:*

$$\begin{bmatrix} D^\alpha x_{r1}(t) \\ D^\alpha x_{r2}(t) \\ D^\alpha x_{r3}(t) \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \\ x_{r3}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t),$$

$$y_r(t) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \\ x_{r3}(t) \end{bmatrix},$$

*where:*

$$f(x) = \begin{bmatrix} -x_1 x_3 \\ x_1 x_2 \end{bmatrix}, \hat{g}(t,x) = \begin{bmatrix} 0.5sint \\ 0 \end{bmatrix}, \hat{d}(t) = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$\alpha = 0.995, a = 10, b = \frac{8}{3}, c = 28.$

*According to the proposed method, the switching matrix H and the transformation matrix T are chosen, respectively, as:*

$$H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

*By computing, one can obtain the sliding mode controller:*

$$u(t) = \begin{pmatrix} ce_1(t) - e_2(t) \\ -be_3(t) \end{pmatrix} + \gamma sgn(s),$$

*and there exists:*

$$K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

*such that the eigenvalue of* $A_{11} - A_{12}KC_1$ *is* $-10$. *Therefore, According to Theorem 2, the drive-response system can realize global synchronization.*

## 6. Conclusions

This paper was mainly concerned with robust synchronization for a class of drive-response FOCS. Relying on the stability theory of the fractional calculus and the sliding mode control method, a sufficient criterion was obtained to achieve synchronization only using output feedback information when external disturbances and unknown parameters are present. To show the effectiveness of our proposed SMC strategy, two numerical examples were given. In the next stage, the adaptive control will be considered when the upper bounds of parameters are unknown or there exist mismatched parameter uncertainties.

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