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Protecting the Quantum Coherence of Two Atoms Inside an Optical Cavity by Quantum Feedback Control Combined with Noise-Assisted Preparation

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Abstract: We propose a theoretical scheme to enhance quantum coherence and obtain steady-state coherence by combining quantum feedback control and noise-assisted preparation. We investigate the effects of quantum-jump-based feedback control and noise field on the quantum coherence and excited-state population between two atoms inside an optical cavity where a noise field drives one, and the other is under quantum feedback control. It is found that steady quantum coherence can be achieved by adding an external noise field, and the quantum feedback can prolong the coherence time with partial suppression of the spontaneous emission of atoms. In addition, we study the influence of the joint action of quantum feedback and noise-assisted preparation on quantum coherence and show that the combined action of feedback control and noise-assisted preparation is more effective in enhancing steady coherence. The findings of our research offer some general guidelines for improving the steady-state coherence of coupled qubit systems and have the potential to be applied in the realm of quantum information technology.

Keywords: quantum coherence; quantum feedback control; noise-assisted preparation; two-level atoms

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1. Introduction

Quantum coherence is considered as a hot topic in various branches of physics, including quantum thermodynamics [1,2], quantum algorithms [3], and quantum channel discrimination [4,5]. Although the importance of quantum coherence is proverbially acknowledged, there have been very few well-accepted efficient methods of measuring quantum coherence until very recently. Baumgratz, Cramer, and Plenio [6] proposed a rigorous framework for quantifying the quantum coherence where the l_1-norm coherence and the relative entropy of coherence were proved to be the effective quantifiers for quantifying quantum coherence. In quantum information science, quantum coherence has been considered a valuable physical resource which should be protected in various quantum tasks [6,7]. Several coherence measures have been proposed [4,6,8,9] by the studies of quantum coherence as an important resource [7,10,11] over the past few years, and their properties have also been explored [4,12–15].

However, even if quantum tasks were to be performed flawlessly, fundamental factors such as spontaneous emission of atoms and environmental noises would persist, limiting the lifetime of quantum coherence and demanding efficient schemes to protect coherence. For this reason, various approaches have been put forward to protect the quantum coherence from decoherence [16] caused by environment noise or other dissipative channels, such as quantum feedback control [17,18], decoherence-free subspace [19], dynamical decoupling [20], engineering reservoir [21], quantum Zeno effect [22], and schemes utilizing the memory effects [23,24]. Among these schemes, quantum feedback control is widely studied. It alters the future dynamics of quantum systems, which can be used to improve both stability and robustness of quantum systems [18] based on the feeding back of measurement results. In the feedback strategies, Bayesian feedback controls [25] and quantum-jump-based feedback control [18,26] can be used to generate steady-state entanglement in a cavity [27]. It has also been shown that the quantum-jump-based feedback strategy was superior to Bayesian measurement control [17], as it does not need real-time state estimation. At the same time, the interaction between the system of interest and environmental noises results in a loss of coherence. Therefore, it is essential to prevent, reduce, or utilize the impact of environmental noise in practical quantum information tasks. For this reason, several proposals have been put forward, such as loop control strategies [28] and quantum error correction [29]. Numerous seminal ideas that noise can assist the generation of entanglement have been put forward [30–35] instead of attempting to shield the system from environmental noise.

For studying many-body physics, most experimental setups depend on a common cavity mode to facilitate interactions [36,37] and can only afford short-distance or all-to-all connectivity, respectively. The constraint in question has been successfully addressed in recent studies with superconducting qubits [38] and cold atoms [39], showcasing the ability to adjust the connectedness of interactions. Recent experimental advances [40,41] have realized the decoupling of the trapping mechanism from the cavity and highly controllable mechanical frequencies and particle positions by programming specific tweezers to be resonant with the cavity mode, which opens up opportunities for designing a broad range of connectivity. Inspired by the above point of view, we investigate the role of feedback strategy and noise-assisted preparation on quantum coherence and excited state population in an optical quantum electrodynamics (QED) system consisting of two two-level atoms, one of which is driven by a noise-assisted preparation, and the other is under quantum feedback control, instead of acting on a common cavity model, as previously studied [31,42]. By using the Monte Carlo wave function (MCWF) method [43,44], we find that a steady quantum coherence can be achieved by adding a noise field on the first atom, and the value of this steady coherence depends on the form of the initial state. In addition, it is demonstrated that the decay of quantum coherence between two atoms can be slowed down and the spontaneous emission of atoms is partially suppressed by adding quantum feedback control. We also show that the intensity of spontaneous emission is closely related to the effect of quantum feedback and needs to be controlled within a specific range for applying the quantum feedback most efficiently. We can control the first atom indirectly by employing a feedback control on the second atom. Finally, we investigate the influence of the joint action of quantum feedback control and noise-assisted preparation on the quantum coherence dynamics of two atoms. Compared to the case where only noise is added to the first atom, we show that the steady quantum coherence can be enhanced significantly by the joint action of quantum feedback and noise field, which means that this joint action can protect coherence much better in some situations. The improvement of stable quantum coherence is significant to the quantum information tasks.

This paper is organized as follows. In Section 2, we investigate the evolution of quantum coherence with noise-assisted preparation. In Section 3, we introduce the quantum-jump-based feedback control and give the super-operator under the feedback control. The influence of quantum feedback on quantum coherence and excited-state population is explored. Section 4 investigates the joint action of the quantum feedback control and noise field on the quantum coherence shared between two atoms. The conclusion of this paper is given in Section 5.

2. The Steady Quantum Coherence Generated by Noise-Assisted Preparation

We theoretically consider an optical quantum system that consists of a pair of two-level atoms (labeled as atom 1 and 2) placed in a single-model optical cavity [45]. Two atoms are spatially separated, which means that all nonlinear interaction terms, such as dipole–dipole interaction, can be neglected; we can assume that the first atom is driven by a noise-assisted preparation [31], and the other is under quantum feedback control [18,26,42]. From an experimental point of view, our model may be realized by combining a high-finesse optical microcavity, allowing us to work in the robust regime of cavity QED [41], and an acousto-optic deflector enabling single particle control by generating an array of tweezers that are individually controllable [40,46,47]. Under the action of optical tweezers, the position of atoms in the optical cavity is fixed, achieving the distinguishability of atoms. The optical frequency of the tweezers, as well as the positions of the atoms and the mechanical frequencies of their center-of-mass modes, are controlled by programming the radio-frequency inputs of the acousto-optic deflectors. Therefore, it is possible to introduce a noise field of a specific frequency or any specific interaction to a specific atom in the cavity [48]. The Hamiltonian of this atom–cavity system is given by [45] ($\hbar = 1$)

$$H = \omega a^\dagger a + \frac{\omega_0}{2}\sum_{j=1}^{2}(|e\rangle_{jj}\langle e| - |g\rangle_{jj}\langle g|) + g\sum_{j=1}^{2}(a^\dagger|g\rangle_{jj}\langle e| - a|e\rangle_{jj}\langle g|), \tag{1}$$

where the parameter g is the atom–cavity coupling strength constant, $a^\dagger(a)$ denotes the creation (annihilation) operator of cavity, ω is cavity frequency and ω_0 is atomic transition frequency, and $|g\rangle$ and $|e\rangle$ are the atomic ground and excited state, respectively. Tracing out the degree of freedom of the cavity field, the dynamics of the subsystem, which consist of only two atoms inside an optical cavity, can be described by quantum master equations in general Lindblad form,

$$\frac{d\rho_s}{dt} = -i[\widetilde{H_s}, \rho_s] + \mathcal{L}\rho_s, \tag{2}$$

where $\widetilde{H_s}$ is the subsystem Hamiltonian after tracing out cavity field, ρ_s is the density matrix of subsystem that consists of two atoms, and $\mathcal{L}$ is a super-operator that describes the influence of environment,

$$\mathcal{L}\rho_s = \sum_k (L_k\rho_s L_k^\dagger - \frac{1}{2}\left\{\rho_s, L_k^\dagger L_k\right\}), \tag{3}$$

where different ks indicate the different dissipative channels. Dissipative subsystem channels mainly include the spontaneous emission of atoms and energy level transition caused by noise field. Then, the form of L of subsystem reads

$$L_{j=1,2} = \sqrt{2\gamma}|g\rangle_{jj}\langle e|, L_3 = \sqrt{2n_T\Gamma}|e\rangle_{11}\langle g|, L_4 = \sqrt{2n_T\Gamma}|g\rangle_{11}\langle e|, \tag{4}$$

where γ describes the spontaneous emission rate of atoms; the noise intensity is characterized by an adequate photon number n_T and Γ characterizing the coupling strength between atom 1 and the noise field. Because the analytical solution of master Equation (2) is not easy to find, we adopt MCWF [43,44] to obtain the solution numerically. Finally, a mean quantum trajectory can be obtained to describe the system's evolution after sampling over many quantum trajectories. The mean quantum trajectory for the following simulations in the present paper is calculated by sampling 2000 quantum trajectories.

We first investigate the influence of noise-assisted preparation on the system's evolution, and specifically use the l_1-norm coherence [6],

$$C(\rho) = \sum_{i\neq j}|\rho_{ij}|, \tag{5}$$

which is the sum of the off-diagonal element magnitudes of the density matrix of the system. We assume that the cavity field is initially prepared in the vacuum state $|0\rangle$ and the initial state of two atoms is the maximally coherent state $|\psi(t_0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$. The quantum coherence shared between two atoms with (red line) and without (black line) white noise as a function of time t is displayed in Figure 1a for clarifying the influence of white-noise fields on the subsystem. We set the intensity of the noise field to be $n_T = 0.3$, coupling strength between subsystem and noise $\Gamma = 0.01$, spontaneous rate $\gamma = 5 \times 10^{-4}\omega$, detuning $\Delta = \omega_0 - \omega$, and $g^2/\Delta = 0.1$. It is shown that the white-noise field accelerates decoherence in a short time, and the system can finally obtain a steady quantum coherence that will not decay over time. Over an extended period of evolutionary development (see the inset of Figure 1a), the absence of noise will ultimately lead to the decay of coherence to zero, whereas the presence of noise will contribute to the stabilization of coherence at a small yet non-zero stable value. This means that the quantum decoherence effect is completely suppressed by adding a noise field to the first atom, and the steady quantum coherence that we achieved is of great significance for the progress of quantum tasks. As a further study, for the system initially prepared in the state $|\psi(t_0)\rangle = |e\rangle_1|g\rangle_2$ without feedback control, the dynamics of the quantum coherence between atoms in the situation where we only consider the effect of noise field is shown in Figure 1b. It is demonstrated that the system can achieve a steady quantum coherence.

Furthermore, distinct evolutionary behaviors of quantum coherence are evident with different initial states. There is a simple decay for the maximally coherent state, whereas a state of $|e\rangle_1|g\rangle_2$ shows oscillatory behaviors. Several factors could explain this observation. In the maximally coherent state, two atoms exist in a highly entangled, strongly correlated state. Thus, the decay trajectory is smooth when the system evolves without notable oscillations due to the atomic interconnectedness. This atomic interconnection lends the system resistance to mild perturbations of singular atomic states. On the other hand, with one atom in an excited state, the system experiences significant dynamical inhomogeneities. Some dynamic processes (like decoherence) might be severely influenced by another atom in the ground state, resulting in unconventional system decoherence oscillations. Here, atoms in the excited state frequently toggle between the excited and ground states, generating robust fluctuations in the coherence.

The dynamic behavior of the population of the excited state of two atoms with (red line) and without (black line) white Gaussian noise can be found in Figure 2. The definition of the population for the excited state $|e\rangle$ is $N_{ee} = Tr(\sum_i \sigma_i^{ee}\rho(t))$ with $\sigma^{ee} = |e\rangle|e\rangle$, indicating the mean excitation number of the state $|e\rangle$ for the whole system. We plot N_{ee} as a function of time for the initial state of subsystem (a) $|\psi(t_0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$ in Figure 2a; (b)$|\psi(t_0)\rangle = |e\rangle_1|g\rangle_2$ in Figure 2b with the strength of white-noise field $n_T = 0.3$. Interestingly, the excited-state population of two atoms drops to zero in the absence of noise, but N_{ee} maintains a dynamic relative stability value for a long time region with the addition of noise-assisted preparation. Because the excited-state population is related to the spontaneous emission, the stability of N_{ee} means that the spontaneous emission of atoms is entirely suppressed by the noise adding on the first atom, which may explain why adding white noise forms a stable coherence.

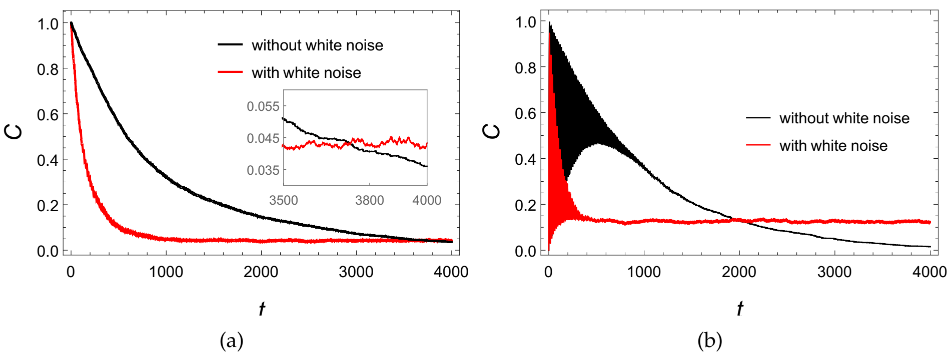


Figure 1. The coherence C shared between two atoms plotted as the function of the time t for different initial states (**a**) $|\psi(t_0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$, (**b**) $|\psi(t_0)\rangle = |e\rangle_1|g\rangle_2$ with and without noise field acting on atom 1. In this figure, other parameters are chosen as $n_T = 0.3, A_x = 0$, $g^2/\Delta = 0.1, \Gamma = 0.01, \gamma = 5 \times 10^{-4}\omega$.

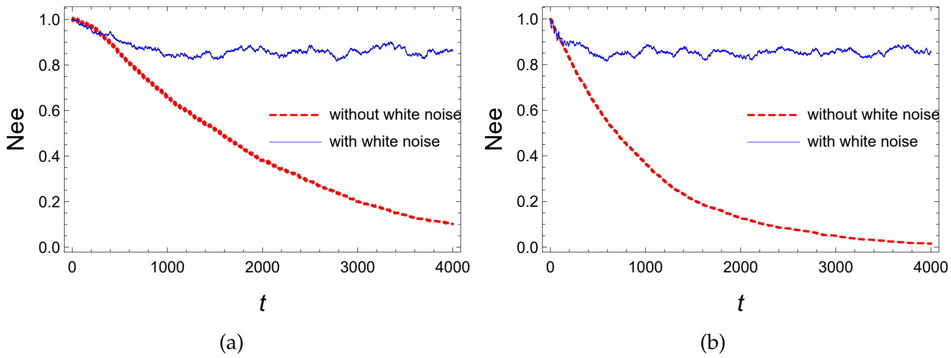


Figure 2. The excited-state population N_{ee} of two atoms plotted as the function of the time t with and without noise field for different initial states (**a**) $|\psi(t_0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$, (**b**) $|\psi(t_0)\rangle = |e\rangle_1|g\rangle_2$ in the absence of feedback control. Other parameters $n_T = 0.3, A_x = 0$, $g^2/\Delta = 0.1, \Gamma = 0.01, \gamma = 5 \times 10^{-4}\omega$.

3. Effect of Feedback Control on Quantum Coherence and Excited-State Population

In order to keep the high quality of quantum information processing for a long time, we adopt the quantum-jump-based feedback control [18,26,42] conditioned on the measurement of quantum jumps, together with an appropriate choice of the control Hamiltonian. In this section, we investigate the effect of quantum feedback on quantum coherence and excited state population for the system, which consists of two atoms inside an optical cavity. First, we outline the procedure of the feedback control performed on atoms. We have a photon detector D that keeps monitoring the output of the cavity, and the quantum feedback control can be applied to atom 2 only after a detection click, i.e., a quantum jump occurs. In particular, we assume that the response time of feedback control is much smaller than the time scale of the system's evolution, so the master equation of this system with feedback control is Markovian. In our article, the new super-operator $\mathcal{L}\rho_s$ with feedback on atom 2 in the master Equation (2) can be written as [18,49]

$$\mathcal{L}\rho_s = \sum_{k=1,3,4} (L_k\rho_s L_k^\dagger - \frac{1}{2}\Big\{\rho_s, L_k^\dagger L_k\Big\}) + U_{FB}L_2\rho_s L_2^\dagger U_{FB}^\dagger - \frac{1}{2}\Big\{\rho_s, L_2^\dagger L_2\Big\}. \tag{6}$$

It is much easier to interpret the feedback control in that the unitary transformation $U_{FB} = e^{iH_f}$ represents the finite amount of evolution imposed by the control Hamiltonian H_f on the atom. The parameter H_f is a Hermitian operator that can be decomposed by Pauli matrices [42] $H_f = A_x\sigma_x + A_y\sigma_y + A_z\sigma_z$; here, A_x, A_y, A_z are real numbers.

We assume that cavity field is initially prepared in the vacuum state $|0\rangle$ and the initial state of two atoms is prepared the maximally coherent state $|\psi(t_0)\rangle = \frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$. The feedback control is only added to atom 2 and does not affect atom 1. Only consider the σ_x control ($A_y = A_z = 0$) in the following for simplicity. Then, the feedback operator of atom 2 reads

$$U_{FB} = I_1 \otimes e^{iA_x\sigma_x} = I_1 \otimes \left(\cos|A_x| + i\frac{\sin|A_x|}{|A_x|}A_x\sigma_x\right), \tag{7}$$

where I_1 is the identity matrix of atom 1. The decoherence process of atom 1 can be represented by the evolution of off-diagonal elements of the reduced density matrix. We can obtain the reduced matrix of the first atom by tracing out atom 2.

$$\rho_1 = Tr_2(\rho_s) = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}, \tag{8}$$

where ρ_{ee} is the excited-state population, ρ_{gg} is the ground-state population, and the coherence of atom 1 can be represented by ρ_{eg} or ρ_{ge}. Because $\rho_{eg} = \rho^*_{ge}, \rho_{ee} + \rho_{gg} = 1$, we study only the coherence $|\rho_{eg}|$ and the excited-state population $|\rho_{ee}|$ for the sake of brevity.

We numerically study the evolution of the reduced density matrix of atom 1 by tracing out atom 2, and the dynamic evolution of excited-state population $|\rho_{ee}|$ and coherence $|\rho_{eg}|$ of atom 1 without (black line) and with (red line) feedback control can be found in Figure 3a,b, where feedback amplitude is chosen as $A_x = \pi/3, A_y = A_z = 0$, and other parameters are $g^2/\Delta = 0.1$, the spontaneous rate $\gamma = 5 \times 10^{-4}\omega$, noise field $n_T = 0$, and coupling strength $\Gamma = 0.01$. It can be seen from Figure 3a that the excited-state population of the first atom decays fast as time goes on due to the atom's spontaneous emission. However, after adding feedback control, the excited-state population decays more slowly compared to the case without the quantum feedback, which means the feedback control added to the second atom can indirectly suppress the spontaneous emission of atom 1. At the same time, we plot $|\rho_{eg}|$, representing the quantum coherence of the first atom, as a function of time t, and we can see from Figure 3b that a more substantial oscillation amplitude and longer coherence time appear in comparison with the case without the quantum feedback. It indicates that the feedback controls acting on the second atom can indirectly affect the dynamics of the first atom, which means that we can control the first atom by adding quantum feedback control on atom 2.

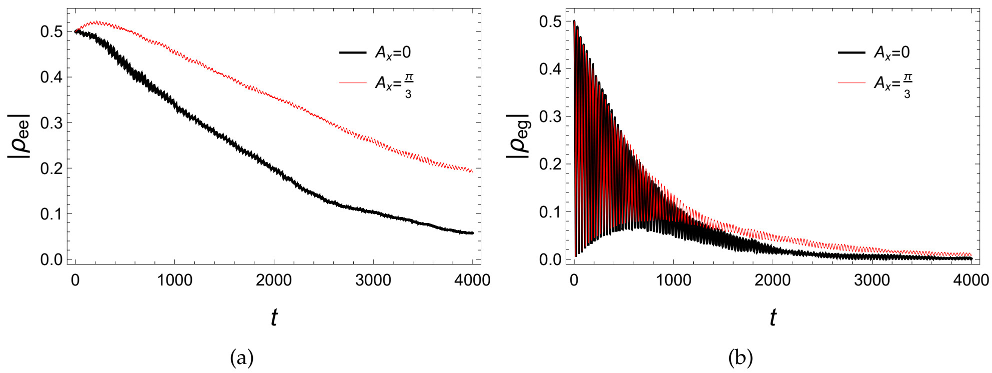


Figure 3. (**a**) In the absence of noise field, the dynamic evolution of the excited-state population $|\rho_{ee}|$ of the first atom with and without feedback control acting on the second atom. Two curves correspond to $A_x = \pi/3, A_y = A_z = 0$ (red line) and $A_x = A_y = A_z = 0$ (black line) for $n_T = 0, g^2/\Delta = 0.1, \Gamma = 0.01, \gamma = 5 \times 10^{-4}\omega$. (**b**) Time evolution of quantum coherence $|\rho_{eg}|$ of the first atom with and without feedback control for the same parameters as (**a**).

For the situation where feedback control is applied to atom 2, we plot the quantum coherence C shared between two atoms versus the time parameter t by choosing different feedback amplitudes $A_x = 0$ (black line), $\pi/4$ (green line), $\pi/3$ (blue dashed line), and $\pi/2$ (red line) in Figure 4, with the spontaneous rate γ to be $5 \times 10^{-4}\omega$, $g^2/\Delta = 0.1$, noise field $n_T = 0$. It is quite clear from Figure 4 that the quantum coherence evolves to zero as time goes on without quantum feedback control. This demonstrates that the system lost coherence due to two atoms' interaction and spontaneous emission. However, the rate of decline can be slowed down when the feedback is applied, which means adding feedback can reduce the loss of quantum coherence for a certain period. The effect of protecting quantum coherence with feedback control $A_x = \pi/4$ is better than the case of controls $A_x = \pi/3, \pi/2$.

To get more insight into the role of the quantum feedback control in affecting the evolution of the system, we plot the excited-state population of two atoms as a function of time t with different feedback amplitudes $A_x = 0$ (black line), $\pi/4$ (green line), $\pi/3$ (blue dashed line), and $\pi/2$ (red line) in Figure 5, and other parameters are the same as the analysis above. Compared with the case without the quantum feedback control, the excited state population increases during the initial period and then decreases in the following period. Because the spontaneous emission is accompanied by a reduction in the population of the excited state, we can see that the spontaneous emission of atoms is partially suppressed by adding feedback control on the second atom, and this may be the main reason for the suppression of decoherence with quantum feedback control. We conclude that the quantum feedback control effectively prolongs the time for atomic spontaneous emission. At the same time, the excitation number in the case of $A_x = \pi/4$ decays more slowly in a large time region than other quantum controls $A_x = \pi/3, \pi/2$, and it is consistent with the results in Figure 4.

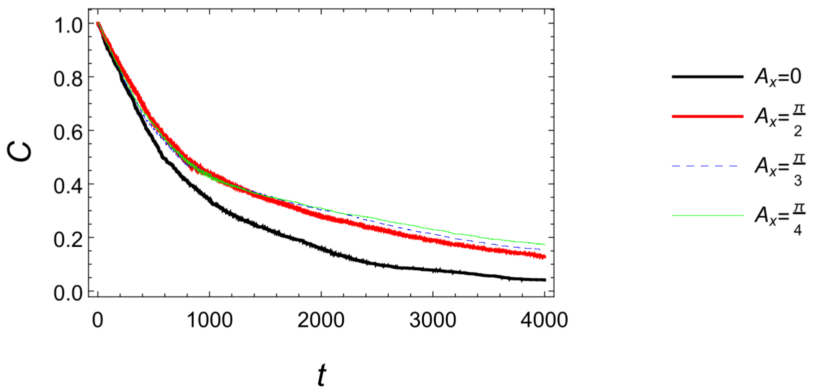


Figure 4. When the feedback control is applied to the second atom, the dynamic of quantum coherence C shared between two atoms is depicted as a function of time t with different control parameters, $A_x = 0, \pi/2, \pi/3, \pi/4$. The parameters are chosen as $n_T = 0, g^2/\Delta = 0.1, \gamma = 5 \times 10^{-4}\omega$.

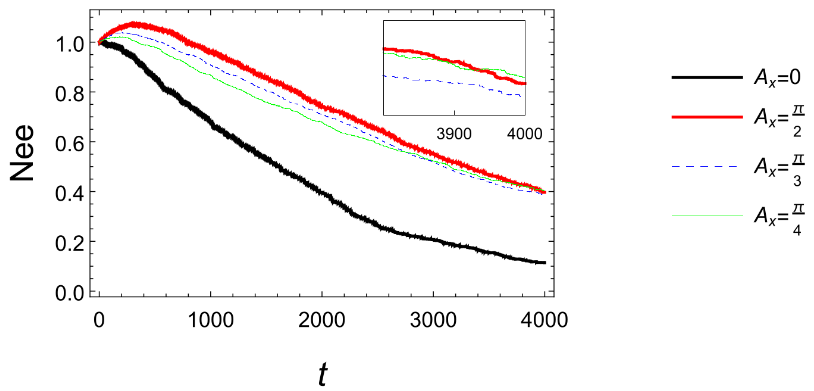
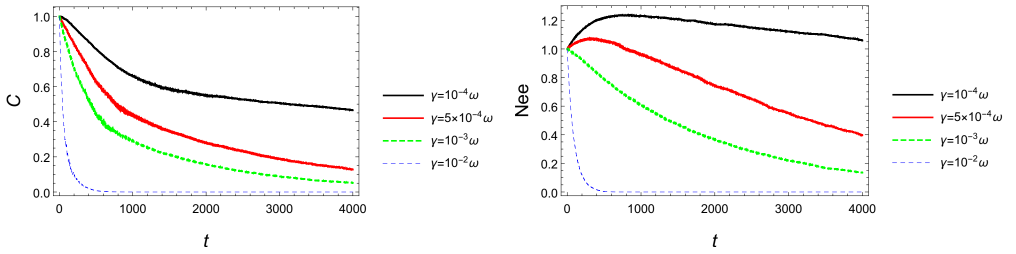


Figure 5. Time evolution of the excited-state population of two atoms for different feedback amplitudes $A_x = 0, \pi/2, \pi/3, \pi/4$ without noise field. The parameters are chosen as $n_T = 0$, $g^2/\Delta = 0.1, \gamma = 5 \times 10^{-4}\omega$.

Because the quantum feedback control can reduce the spontaneous emission, we study the influence of atomic spontaneous emission on the behavior of the quantum coherence and excited-state population of two atoms only under the action of control $A_x = \pi/3$. The intensity of spontaneous emission is chosen as $\gamma = 10^{-4}\omega$ (black line), $5 \times 10^{-4}\omega$ (red line), $10^{-3}\omega$ (green line), and $10^{-2}\omega$ (blue dashed line), and other parameters are chosen as $n_T = 0, \Gamma = 0.01, g^2/\Delta = 0.1$. We can see from Figure 6a that the intensity of spontaneous emission obviously influences the effect of feedback. The quantum feedback control can influence the system more effectively in a small spontaneous emission rate regime as it can slow down the decay of C more effectively. In particular, the decay is almost prohibited for the weak spontaneous emission $\gamma = 10^{-4}\omega$, whereas the coherence drops to zero quickly for the strong spontaneous emission $\gamma = 10^{-2}\omega$. This means that spontaneous emission intensity needs to be controlled within a specific range to apply the quantum feedback most efficiently. It is shown in Figure 6b that the suppression effect of quantum feedback to atomic spontaneous emission disappears when the intensity of spontaneous emission exceeds a specific value. With the decrease in spontaneous emission intensity, the suppressing effect of feedback on the spontaneous emission effect is gradually enhanced.



(a) (b)

Figure 6. (**a**) The quantum coherence C shared between two atoms and (**b**) the excited-state population of two atoms N_{ee} as the function of time t with different intensity of spontaneous emission $\gamma = 10^{-4}\omega, 5 \times 10^{-4}\omega, 10^{-3}\omega, 10^{-2}\omega$ with feedback control acting on atom 2. Other parameters are chosen as $n_T = 0, g^2/\Delta = 0.1, \Gamma = 0.01, A_x = \pi/3$.

4. Effect of the Joint Action of Quantum Feedback and Noise-Assisted Preparation on Quantum Coherence

In Section 2, we showed that steady quantum coherence can be achieved by adding a noise field on the first atom, which indicates significant worth in the quantum information field. In Section 3, we found that the quantum coherence shared between two atoms decays more slowly with quantum feedback control than without quantum feedback. This naturally leads to a question about the above analysis: Does the combination of quantum feedback control and noise-assisted preparation better protect quantum coherence? In this section, we investigate the effect of feedback control on the system's evolution in the presence of a white-noise field.

We investigate how the quantum-feedback strategy can enhance the steady quantum coherence with the effect of a white-noise field. For a clearer view of results, the system is initially prepared in the state $|\psi(t_0)\rangle = |e\rangle_1|g\rangle_2$, and other parameters remain unchanged. There is the same conclusion when the initial state is prepared in the maximally coherent state $\frac{1}{\sqrt{2}}(|e\rangle_1 + |g\rangle_1) \otimes \frac{1}{\sqrt{2}}(|e\rangle_2 + |g\rangle_2)$. The quantum coherence of two atoms as a function of time t with different quantum feedback amplitudes $A_x = 0$ (black line), $\pi/10$ (red line), $3\pi/20$ (blue dashed line), $\pi/4$ (green line), and $\pi/2$ (orange line) with white-noise field $n_T = 0.3$ is displayed in Figure 7. We can see that the white-noise field can induce a steady quantum coherence that does not change over time, and the value of the steady quantum coherence can be increased with the addition of feedback control compared to that in Figure 1. In this sense, the combined action of noise field and quantum feedback control can better protect the quantum coherence in some situations. The enhancement effect of the feedback control on the stability of quantum coherence lies in different choices of the feedback amplitude, and we plot a density diagram of quantum coherence shared between two atoms with $A_x = 0 \sim \pi$ in Figure 8. It should be noted that the feedback amplitude A_x influences the system's evolution with a period of π [42]. This result demonstrates that the quantum coherence of two atoms exhibits an oscillatory behavior in a short time region, and it decays to a constant value for a long time. It is evident from Figure 8 that, for an appropriate feedback amplitude around $A_x \approx 0.8$ and $A_x \approx 2.3$ ($A_y = A_z = 0$), the value of steady quantum coherence can be enhanced.

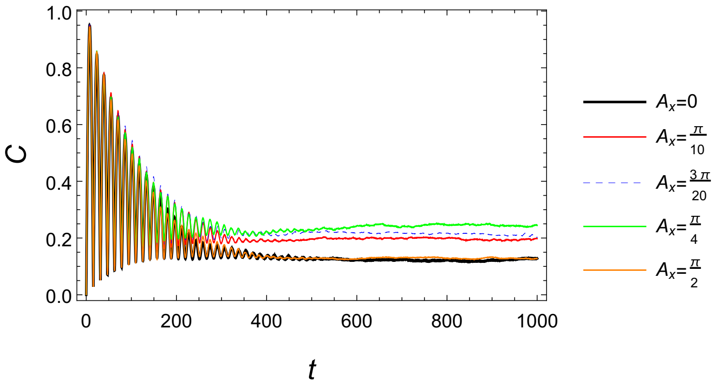


Figure 7. The coherence C shared between two atoms plotted as a function of the time t with noise field acting on the first atom and quantum feedback control acting on the second atom for the initial state $|\psi(t_0)\rangle = |e\rangle_1|g\rangle_2$. The parameters are chosen as $n_T = 0.3, A_x = 0, \pi/10, 3\pi/20, \pi/4, \pi/2,$ $g^2/\Delta = 0.1, \Gamma = 0.01, \gamma = 5 \times 10^{-4}\omega$.

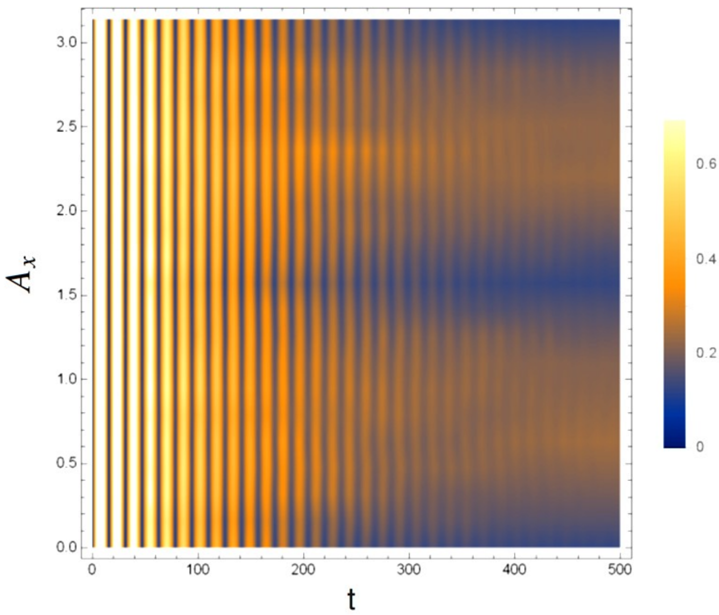


Figure 8. The density diagram of quantum coherence C between two atoms. The coherence is depicted as a function of feedback amplitude A_x and time t for the initial state $|\psi(t_0)\rangle = |e\rangle_1|g\rangle_2$. The parameters are chosen as $n_T = 0.3, A_x = 0 \sim \pi, g^2/\Delta = 0.1, \Gamma = 0.01, \gamma = 5 \times 10^{-4}\omega$.

5. Conclusions and Discussion

In conclusion, we have investigated the effect of quantum-jump-based feedback control on quantum coherence and excited-state population of an optical system consisting of two two-level atoms, where one of them was driven by a noise-assisted preparation, and the other was under quantum feedback control. We found that the quantum coherence shared between two atoms with noise field drops to steady quantum coherence without feedback control. The stability value depends on the initial state of the subsystem using the MCWF numerical simulation. We also found that the spontaneous emission of atoms is entirely suppressed by adding a noise field on the first atom. Furthermore, we demonstrated that the decoherence of the first atom can be suppressed by a local control acting on the second atom, and the decoherence time of the first atom is greatly extended. We also analyzed the dynamic of quantum coherence and excited-state population of two atoms with different quantum feedback control situations without noise field. It was found that the decoherence process is sufficiently slowed down, and the spontaneous emission of atoms is partially suppressed by adding quantum feedback control. Moreover, we observed that the protecting effect of feedback on quantum coherence is affected by the intensity of spontaneous emission. Finally, we explored the influence of the joint action of quantum-jump-based feedback control and white Gaussian noise on the quantum coherence dynamics between two atoms. Notably, the joint action of quantum feedback and noise field can enhance the steady quantum coherence much better than the case with only noise field, which illustrates the necessity of using both quantum feedback and noise-assisted preparation. In addition, the steady quantum coherence value can be effectively increased by adding different feedback controls.

We note that various alternative schemes have been recently proposed to achieve the goal of protecting coherence, including the well-known noise-assistance mechanism acting on the entire cavity field [33], a novel entanglement generation and protection scheme utilizing the spatial indistinguishability [50,51], and a scheme that preserves coherence by preparing a system of resources composed of more atoms [52]. In contrast, the feedback control approach employed in this study is a real-time, resource-efficient, and less demanding experimental condition for quantum information protection strategy, exhibiting potential for practical implementation. Specifically, it is a real-time protection strategy that does not require errors to be detected and corrected until after processing is complete; it does not require redundant quantum bit resources, saving them and reducing the difficulty of implementation; and it works by allowing a certain degree of noise and distortion, and is, therefore, easier to implement in a practical experimental setting [53,54]. On the other hand, as compared to alternative coherence-preserving strategies [31,33,42], our approach yields higher values of nonvanishing steady-state coherence rather than just improving coherence in a short period, which holds significance within the realm of quantum information processing.

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