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Editorial

Advances in Fractional-Order Neural Networks, Volume II

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Fractional-order neural network models have become an active research subject and have attracted increasing attention in many fields. For instance, fractional-order neural networks are recognized as effective tools for the modeling, validation and guaranteed learning of dynamical processes in biology, biochemistry, neurocomputing, engineering, physics, economics, etc. [1–5]. Advances in fractional calculus lead to the development of new fractional-order neural network models. From the other side, challenges and knowledge from research in science and engineering motivate new advancements in the area of fractional-order neural networks [6–9].

After the successful production of Volume I of this Special Issue, we invited investigators to contribute original research articles as well as review articles focused on the latest achievements in the modeling, control and applications of fractional-order neural networks.

Volume II of the Special Issue was successful; twelve research papers were published that addressed advances in fractional-order neural networks.

Four papers addressed interesting topics related to the synchronization of fractional-order neural networks, which is one of the significant research directions in such systems.

For example, Wang et al. studied the synchronization in finite time of fractional-order complex-valued gene networks with time delays. They established several synchronization in finite time criteria based on feedback controllers and adaptive controllers. The setting time of the response is estimated using the theory of fractional calculus.

He, Li and Liu investigated the asymptotic synchronization of fractional-order complex dynamical networks with different structures and parameter uncertainties. The proposed controller is more adaptable and effective and, as such, the derived results extend some of those found in the existing literature. The validity and feasibility of the theoretical outcomes are confirmed via two simulation instances.

Song, Cao and Abdel-Aty considered the synchronization problem for a more general kind of master–slave memristor-based neural networks with fractional derivatives. By applying a continuous-frequency-distributed equivalent model tool, some new outcomes and sufficient conditions on the robust synchronization of the master–slave neural networks with uncertainty are proposed via linear matrix inequality.

Chen et al. studied the finite-time synchronization problem of fractional-order stochastic memristive bidirectional associative memory neural networks with discontinuous jumps. They proposed a novel criterion for finite-time synchronization by utilizing the properties of quadratic fractional-order Gronwall inequality with time delay and the comparison principle. This criterion provides a new approach to analyzing the finite-time synchronization problem of neural networks with stochasticity.

Variable-order fractionals are a relatively recent development in the field of fractional neural networks. This extremely interesting topic has been investigated in two papers.

Karoun et al. introduced a discrete-time Hopfield neural network with non-commensurate fractional variable orders for three neurons. Its chaotic behavior was studied via phase-plot portraits, bifurcation and Lyapunov exponent diagrams.

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Alsaade et al. proposed a model-free and finite-time super-twisting control technique for a variable-order fractional Hopfield-like neural network. The controller proposed in their research is able to regulate the system even when its complex variable-order fractional dynamic is completely unknown.

One of the most important concepts in neural network models in a periodic environment is that of periodicity. However, in real-world problems, the exact periodicity of the states is usually too strong and has limited applicability.

Feckan and Danca considered some aspects related to the non-periodicity of a class of complex maps defined in the sense of Caputo-like fractional differences and related to the asymptotical stability of fixed points. The presented results are exemplified in the case of the Mandelbrot set of fractional order.

Stamov et al. studied the almost periodic behavior of factional-order impulsive delayed reaction–diffusion gene regulatory networks with Caputo-type fractional-order derivatives and impulsive disturbances at unfixed instants of time. In addition, using Lyapunov-like impulsive functions, perfect Mittag–Leffler stability criteria were proposed.

Very interesting results related to fractional factorial split-plot designs, with replicated settings of the whole plot factors from the viewpoint of clear effects, are proposed by Zhao.

Xu and Li studied the problem of a group consensus for a fractional-order multi-agent system without considering the intergroup balance condition. By adopting a dynamic event-triggered mechanism, the updating frequency of the control input is significantly reduced while the consensus performance is maintained.

The study by Wei et al. is devoted to one of the main challenges in using a fractional-order neural network modeling approach, namely the fact that a long memory property is necessary, whereas infinite memory is undesirable. Their study puts particular emphasis on the topic, developing some remarkable properties such as the equivalence relation, the nabla Taylor formula and the nabla Laplace transform of such nabla-tempered fractional calculus.

Finally, the paper authored by Wang, Wang and Chu investigated Hopfield-type neural networks with fractional derivatives of incommensurate fractional orders. Numerous fundamental and qualitative properties were studied, including dissipative properties and stabilization.

The editors of this Special Issue "Advances in Fractional-Order Neural Networks, Volume II" would like to express their sincere gratitude to all the authors who contributed their valuable works, and also to the reviewers for their exceptional efforts in reviewing the manuscripts. We believe that the selected papers will enrich the readers' knowledge and will stimulate the continuing efforts to develop the theory and applications of fractional-order neural networks, which will continue to be one of the dominant themes in mathematics and mathematics applications due to its theoretical and practical significance.

Conflicts of Interest: The authors declare no conflict of interest.

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