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Dynamic Event-Triggered Consensus for Fractional-Order Multi-Agent Systems without Intergroup Balance Condition

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Abstract: This paper deals with the problem of group consensus for a fractional-order multi-agent system (FOMAS) without considering the intergroup balance condition. By adopting a dynamic event-triggered mechanism, the updating frequency of control input is significantly reduced while the consensus performance is maintained. By utilizing the Lyapunov direct method and the properties of a fractional-order derivative, several novel criteria are presented for analyzing the Mittag–Leffler stability of error systems and excluding the Zeno behavior in the triggering mechanism. An example and its simulations are demonstrated to prove the validity of the theoretical results.

Keywords: fractional-order multi-agent systems; leader–follower group consensus; dynamic event-triggered mechanism; Zeno behavior

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1. Introduction

The last decades have already witnessed the rapid growth on the research of distributed cooperation for multi-agent systems (MASs) due mainly to its comprehensive applications in many fields such as physics, engineering, and military (see [1–7]). As one of typical cooperative processes, the consensus control aims to design a strategy (by taking use of the local information from neighbor agents) such that all agents finally converge to a common state. Until now, a great number of papers have emerged to focus on the consensus control of MASs [8–14]. For instance, in [9], the distributed quasi-consensus control has been considered for stochastic nonlinear multi-agent systems with a general network topology. Within the adaptive fuzzy control strategy, the consensus algorithms has been proposed in [13] for MAS with unknown nonlinearities and a directed network topology.

It should be noted that most existing results have investigated the issue that all agents finally approach a common dynamics under the control protocol [15]. In practical applications, agents may need to complete different part of a complex cooperative task, which requires agents in a communication network to achieve different agreements. Hence, it is significant to consider the issue of group consensus control of MASs. Specifically, all agents in a network are usually classified to several different subgroups. The interaction of agents happens not only in the same subgroup but also among different subgroups. By employing a proper consensus protocol, each group of agents tend to their own common dynamics. In other word, the main object of group consensus is to design an appropriate control strategy by which the agents in a same subgroup achieve the consensus performance while the agents in different subgroups cannot achieve consensus as time goes on. Compared to the conventional consensus control, the group consensus shows more flexibility in complex practical applications due mainly to the existence of more than one common dynamics. Recently, several research efforts have been devoted to the group consensus of MASs; please see references [16–19] and papers cited therein.

In most papers mentioned above, the state evolution for each agent has been modeled by a differential equation with integer-order derivative. Actually, fractional-order differential equations show more advantages in reflecting the characteristics of hereditary and the properties of memory for past processes (see [20–25]). Based on this reason, fractional-order dynamical systems have been widely introduced in several practical applications including electrical engineering, control systems, and robotics (see [26,27] and references cited therein). For instance, the model-free adaptive sliding mode control of fractional-order systems has been extensively investigated in [26,27]. Further, more and more research attention has been focused on the consensus of fractional-order multi-agent systems (FOMAS) [28–32]. For example, in [30], the leader–following consensus issue has initially investigated for FOMAS with nonlinear dynamics. It is worth noting that the intergroup balance condition is required for design of consensus protocol. Up to now, there have been few papers published to consider the group consensus of FOMAS without intergroup balance condition. Therefore, it is valuable to further investigate how to design the consensus control protocol with no intergroup balance condition.

Generally speaking, agents often communicate with each other via a shared digital channel, which means the signal is sampled first before it is sent to neighboring agents [33–39]. The traditional periodic triggering mechanism (PTM) is convenient to operate but would inevitably result in the redundant consumption in energy and communication resources. In order to decrease the waste of communication resources, the event-triggering mechanism (ETM) has been proposed in the sampling-data control synthesis. For implementing an ETM, a formula is firstly constructed to be the rule of event generator. Then, the signal will be sampled and transmitted at which the event generator is triggered. The parameters for event generator can be jointly designed with the control gain so as to ensure a desired performance. Compared to the traditional PTM, it has been proven that ETM can save the limited energy and communication resources more effectively [40]. So far, a large number of research articles have been published to deal with the event-triggering consensus control for MASs [41–43]. In specific, the ETM control has been introduced into a class of MASs with fractional-order derivative in [41]. All agents achieve the required consensus through only communicating with local neighbors at those certain instants determined by pre-defined events. The combinational measurement and the iterative event-triggered algorithm have been adopted in [42,43] to avoid continuous monitor to agent states. It should be pointed out that those ETMs adopted in above papers are static. That is to say, the parameters for triggering are given previously. In recent times, several initial researchers have proven that the dynamics event-triggering mechanism (DETM) shows higher efficiency than the static one in decreasing the triggering frequency [44–46]. Therefore, it is more practical to introduce the DETM to the issue of group consensus control of FOMASs. However, as far as we know, little research attempts have been devoted to the problem of group consensus of FOMAS via DETM, and thereby the second motivation of the current research is to abridge such a gap.

Motivated by above discussion, we aims to design a novel control strategy within DETM by which the leader–following group consensus of FOMASs can be achieved without considering intergroup balance condition. To complete this work, we need to cope with two main challenges. The first one is to design an effective DETM by taking use of the state information of FOMASs and the auxiliary internal dynamic variables. The second one is to solve the design issue for the group consensus protocols by which the required performance can be maintained without the intergroup balance condition. The novelties of this paper are outlined to be three aspects.

(1) By employing the related state information between each agent and its neighbors, a control protocol is developed to guarantee the leader–following group consensus of fractional-order multi-agent systems without intergroup balance condition.
(2) To reduce the frequency of state information updates, the DETM is firstly adopted to the leader–following group consensus of fractional-order multi-agent systems.
(3) Several easy-to-check criteria are derived to assure the required consensus performance and exclude the Zeno behavior.

Section 2 provides some preliminaries and the model description. Section 3 gives several criteria by which the leader–following group consensus of FOMAS is achieved and the Zeno phenomenon is excluded. In Section 4, an example and its simulations are illustrated for the effectiveness of results. Finally, some conclusions are presented in Section 5.

Notations: Let $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ be the sets of n-dimensional vectors and $m \times n$ matrices, respectively. $|\mathcal{G}|$ denotes the node number in graph $\mathcal{G}$. The Kronecker product is represented as $\otimes$. $M > 0$ implies M is positive definite. Let $\lambda_{min}(A)$ (or $\lambda_{max}(A)$) represent the minimum (or maximum) eigenvalue of a matrix A. Let $\mathbf{1}_n$ stand for the n-dimensional column vector with all components being 1.

2. Model Description and Preliminaries

2.1. Graph Theory

Assume all agents generate a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ with the adjacent matrix $\mathcal{A} = (a_{ij})_{N\times N}$ satisfying $a_{ij} = a_{ji} \neq 0$ for $(i,j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. In this paper, we suppose that there is no self-edges in $\mathcal{G}$.

By splitting all nodes into S disjoint groups, we obtain $\mathcal{G} = \cup_{s=1}^{S} \mathcal{G}_s$ with $\mathcal{G}_i \cap \mathcal{G}_j = \emptyset$ for $i \neq j$. In a same group, the relationship of interaction among agents is assumed to be cooperative; that is to say, for each s $(1 \leqslant s \leqslant S)$, the weights $a_{ij} \geqslant 0$ when $i, j \in \mathcal{G}_s$. It is naturally noted that $\mathcal{G}_s(1 \leqslant s \leqslant S)$ is the induced subgraph of $\mathcal{G}$ with associated Laplacian matrix $\mathcal{L}_s = (l_{ij})_{|\mathcal{G}_s|\times|\mathcal{G}_s|}$ in which $l_{ii} = \sum_{j\in\mathcal{G}_s, j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. Obviously, $\mathcal{L}_s \geq 0$ takes 0 as an eigenvalue with $\mathbf{1}_{|\mathcal{G}_s|}$ being a eigenvector. We denote a block matrix $\mathcal{L}$ in which S diagonal blocks are chosen to be $\mathcal{L}_1, \dots, \mathcal{L}_S$ and elements in other positions be corresponding ones of $-\mathcal{A}$.

We take $\Theta = \{\theta_1, \theta_2, \dots, \theta_S\}$ as the class of leader agents. $Q_s = \mathrm{diag}\{a_{1\theta_s}, \dots, a_{N\theta_s}\}$ is for the communication between leaders and followers. To describe the overall communication topology, let us introduce a new node 0 and add the new edge between agent i and 0 if $\sum_{s=1}^{S} a_{i\theta_s} > 0$. The overall communication is finally characterized by

$$H = \mathcal{L} + \sum_{s=1}^{S} Q_s = \begin{bmatrix} H_1 & H_{12} & \cdots & H_{1S} \\ H_{21} & H_2 & \cdots & H_{2S} \\ \vdots & \vdots & \ddots & \vdots \\ H_{S1} & H_{S2} & \cdots & H_S \end{bmatrix}, \tag{1}$$

where $H_s \in \mathbb{R}^{|G_s|\times|G_s|}$.

2.2. Fractional-Order System with Caputo-Type Derivative

The Grünwald–Letnikov, Riemann–Liouville, and Caputo are three common definitions of fractional-order derivatives. Here, we consider the Caputo derivative.

Definition 1 ([20])**.** *For a positive scalar $0 < \alpha < 1$ and $x(t) \in C^1([t_0, +\infty), \mathbb{R})$, the derivative of $x(t)$ with order α is*

$${}_{t_0}^{C}D_t^{\alpha} x(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{x'(\tau)}{(t-\tau)^{\alpha}} d\tau,$$

where $t \geqslant t_0$ and $\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt$ is Gamma function with variable z being a complex number and satisfying the real part $Re(z) > 0$. For simplicity, ${}_{t_0}^{C}D_t^{\alpha} x(t) \triangleq D^{\alpha} x(t)$.

Remark 1. *Caputo-type derivative plays a major role in modeling the fractional-order dynamical systems. The main reason is that the initial value for Caputo-type fractional-order system can usually be taken as the same form to systems with integer-order derivative. As such, Caputo-type fractional systems have a broader application range.*

Definition 2. *The Mittag–Leffler function is*

$$E_{\alpha,\beta}(z) = \sum_{j=0}^{+\infty} \frac{z^j}{\Gamma(\alpha j + \beta)}$$

where $\alpha > 0, \beta > 0, z \in \mathbb{C}$, $\Gamma(\cdot)$ is the Gamma function given in Definition 1. Particularly, if $\beta = 1$, then Mittag–Leffler function degenerates to

$$E_\alpha(z) = E_{\alpha,1}(z) = \sum_{j=0}^{+\infty} \frac{z^j}{\Gamma(\alpha j + 1)}.$$

For a fractional-order differential equation (FODE)

$$\begin{cases} D^\alpha p(t) = h(t, p(t)), \\ \quad p(t_0) = p_0, \end{cases} \tag{2}$$

in which $\alpha \in (0,\ 1)$, the vector-value function $h(t, p(t))$ defined on $[t_0, +\infty) \times \mathbb{R}^n$ is Lipschitz continuous. Let $\bar{p} = 0$ be an equilibrium point.

Definition 3 ([25] ML-stable)**.** *System (2) is Mittag–Leffler stable provided that there are positive numbers λ, b and a nonnegative function $m(x) \geqslant 0$ such that*

$$\|p(t)\| \leqslant \left[m(x_0) E_\alpha\left(-\lambda (t - t_0)^\alpha\right)\right]^b,$$

in which $m(x)$ satisfies $|m(x)| \leq m_0 \|x\|$.

2.3. Model Description

Assume the dynamics of leader is governed through the Caputo-type FODE as follows

$$D^\alpha x_{\theta_s}(t) = Af(x_{\theta_s}(t)), \quad s \in \{1, \ldots, S\} \tag{3}$$

and the dynamics of follower is governed by

$$D^\alpha x_i(t) = Af(x_i(t)) + Bu_i(t), \quad i \in \{1, \ldots, N\}, \tag{4}$$

in which $x_{\theta_s}(t)$ and $x_i(t)$ are the n-dimensional states of agents. $u_i(t) \in \mathbb{R}^m$ stands for the control input. A and B are matrices with appropriate dimensions. The overall communication matrix for (3) and (4) is $H = \mathcal{L} + \sum_{s=1}^{S} Q_s$ defined in (1).

Several assumptions are provided for developing the main result about consensus control of FOMAS (3) and (4).

Assumption 1. *There is a $d > 0$ such that $f(x)$ satisfies*

$$\|f(v_1) - f(v_2)\| \leqslant d\ \|v_1 - v_2\|$$

for $v_1, v_2 \in \mathbb{R}^n$.

Assumption 2. *For any agent i, there is a path for reaching the corresponding leader θ_s, and $\sum_{s'=1}^{S} a_{i\theta_{s'}} \geqslant 0$.*

Assumption 3. *For any $i \in \mathcal{G}_s$ and $s' \neq s$, we assume that $a_{i\theta_{s'}} = -\sum_{j \in \mathcal{G}_{s'}} a_{ij}$.*

Remark 2. *In [17], authors have required $a_{i\theta_{s'}} = 0$ for any $i \in \mathcal{G}_s$ and $s' \neq s$, namely, the communication topology satisfies the intergroup balance condition. It is readily observed from Assumption 3 that for $i \in \mathcal{G}_s$, the total information from the other subgroup $\mathcal{G}_{s'}$ is allowed to be arbitrary constants, which includes the intergroup balance condition as its special case.*

Definition 4 ([37])**.** *The FOMAS* (3) *and* (4) *is the leader–following group consensus under the consensus protocol $u_i(t)$, if for any $s \in \{1, 2, \cdots, S\}$, there exist positive numbers β, λ, and μ, such that for $i \in \mathcal{G}_s$ and the corresponding leader agent θ_s*

$$\|x_i(t) - x_{\theta_s}(t)\| \leqslant \beta\big[E_\alpha\big(-\lambda(t-t_0)^\alpha\big)\big]^\mu, \quad t \geqslant t_0.$$

For the simplicity of representation, we introduce the following notations

$$x(t) = \left[x_1^T(t), x_2^T(t), \dots, x_N^T(t)\right]^T \in \mathbb{R}^{nN},$$
$$\bar{x}_{\theta_s}(t) = \left[x_{\theta_s}^T(t), x_{\theta_s}^T(t), \dots, x_{\theta_s}^T(t)\right]^T \in \mathbb{R}^{n|\mathcal{G}_s|},$$
$$\bar{x}_{\theta}(t) = \left[\bar{x}_{\theta_1}^T(t), \bar{x}_{\theta_2}^T(t), \dots, \bar{x}_{\theta_S}^T(t)\right]^T \in \mathbb{R}^{nN}.$$

We obtain that

$$\begin{cases} D^\alpha \bar{x}_\theta(t) = (I_N \otimes A) f(\bar{x}_\theta(t)), \\ D^\alpha x(t) = (I_N \otimes A) f(x(t)) + (I_N \otimes B) u(t), \end{cases} \tag{5}$$

in which $f(x) = \left[f(x_1^T), f(x_2^T), \dots, f(x_N^T)\right]^T$, $u = \left[u_1^T, u_2^T, \dots, u_N^T\right]^T$

Denoting by $\xi(t) = x(t) - \bar{x}_\theta(t)$ the consensus error, we derive that

$$D^\alpha \xi(t) = (I_N \otimes A)\tilde{f}(\xi(t)) + (I_N \otimes B)u(t), \tag{6}$$

with $\tilde{f}(\xi(t)) = f(x(t)) - f(\bar{x}_\theta(t))$.

Remark 3. *According to Definitions 3 and 4, it is clearly seen that the leader–following group consensus of FOMAS* (5) *can sufficiently be ensured by proving the global Miattag–Leffler stability of consensus error system* (6)*.*

3. Main Results

In this section, by taking use of the related state between agents, we propose a distributed consensus protocol such that the group consensus is finally achieved for FOMAS (5). By introducing an internal dynamical variable generated from an auxiliary fractional-order differential equation, the DETM is designed to dramatically reduce the updating frequency of control signals. In addition, the Zeno phenomenon is fully excluded in the triggering mechanism and the control gain matrix is finally derived via solving LMIs.

Let $\{t_k^i \mid k \in \mathbb{Z}^+\}$ represent the event-triggering instants, which is determined later. We suppose that the follower agent i can communicate with its neighbors in a distributed manner and can also receive the information from all leader agents θ_s at triggering instant t_k^i. As such, a control strategy $u_i(t)$ is designed to be

$$\begin{cases} u_i(t) = cKq_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \\ q_i(t_k^i) = \sum\limits_{j \in G_s \cup \theta_s} a_{ij}\left(x_j(t_k^i) - x_i(t_k^i)\right) + \sum\limits_{s' \neq s} \sum\limits_{j \in G_{s'}} a_{ij}\left(x_j(t_k^i) - x_{\theta_{s'}}(t_k^i)\right), \end{cases} \tag{7}$$

in which c is the coupling strength and K is the gain matrix to be designed in current work.

Remark 4. *Owing to the coupling structure of MAS, the control protocol used in our paper depends on the related state error between agent and its neighbors, which is different from the control of isolated fractional order dynamic systems involved in [25,26]. In addition, the event-triggering mechanism makes the control signal update in a aperiodic-sampling manner, which is distinguished from the continuous-time adaptive control used in [25,26]. In this paper, the coupling strength c is considered as the inherent link attribute between agents. Hence, we only need to design the gain matrix K so as to guarantee the required consensus performance.*

For $t \in [t_k^i, t_{k+1}^i)$, let $e_i(t) = x_i(t) - x_i(t_k^i) - \left[x_{\theta_s}(t) - x_{\theta_s}(t_k^i)\right]$ be the measured error between the follower agent i and its corresponding leader agent θ_s. The instants sequence $\{t_k^i |\, k \in \mathbb{Z}^+\}$ is decided by the rule

$$t_{k+1}^i = \inf\Big\{t > t_k^i \mid \Delta_i(t) \geqslant \eta_i(t)\Big\}. \tag{8}$$

where $t_0 = 0$ and $\Delta_i(t) = \sigma_i \|e_i(t)\|^2 - \gamma_i \|q_i(t_k^i)\|^2$ with constants $\gamma_i > 0$ and $\sigma_i > 0$ are triggering parameters. $\eta_i(t)$ is the dynamic variable generated by an auxiliary FODE

$$D^\alpha \eta_i(t) = -\pi_i \eta_i(t) - \rho_i \Delta_i(t), \tag{9}$$

in which $\eta_i(0) > 0$, $\pi_i > 0$, and $\rho_i > 0$.

Remark 5. *The internal dynamic variable $\eta_i(t)$ is generated by using fractional order differential equation with the same order to MAS* (5)*, which can provide a more natural and more accurate reflection to the change of states.*

For the aim of clarity, the block diagram of control loop is shown in Figure 1.

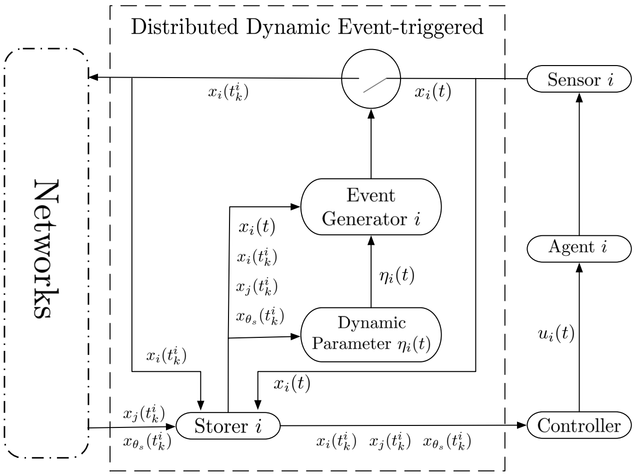

Figure 1. Dynamic event-triggered control framework for FOMAS.

By letting $e(t) = \left[e_1^T(t), e_2^T(t), \ldots, e_N^T(t)\right]^T$, $x(t_k) = \left[x_1^T(t_k^1), x_2^T(t_k^2), \ldots, x_N^T(t_k^N)\right]^T$, one obtains

$$e(t) = x(t) - \bar{x}_\theta(t) - x(t_k) + \bar{x}_\theta(t_k).$$

Recalling the definition of $q_i(t_k^i)$ and denoting $q(t_k) = [q_1^T(t_k^1), q_2^T(t_k^2), \cdots, q_N^T(t_k^N)]^T$, we deduce that

$$q(t_k) = (H \otimes I_n)(\bar{x}_\theta(t_k) - x(t_k)),$$

which, together with $\xi(t) = x(t) - \bar{x}_\theta(t)$, yields that $e(t) - \xi(t) = \bar{x}_\theta(t_k) - x(t_k)$ and

$$q(t_k) = (H \otimes I_n)(e(t) - \xi(t)). \tag{10}$$

By combining with (7), it is noted that

$$u(t) = c(I_N \otimes K)(H \otimes I_n)(e(t) - \xi(t)). \tag{11}$$

By substituting (11) into (6), it is concluded the compact vector form for the consensus error dynamics

$$D^\alpha \xi(t) = (I_N \otimes A)\tilde{f}(\xi(t)) + c(H \otimes BK)(e(t) - \xi(t)). \tag{12}$$

The following lemma is proposed to illustrate the internal dynamic variable $\eta_i(t)$ remains positive for all $t > 0$ provided that the initial value $\eta_i(0)$ is positive.

Lemma 1. *If the prescribed scalars $\eta_i(0) > 0, \rho_i > 0$ and $\pi_i > 0$, then for all $t \geqslant 0$, the internal dynamic variable $\eta_i(t)$ satisfies*

$$\eta_i(t) > 0. \tag{13}$$

Proof. It is not difficult to see $\cup_{k=0}^{\infty}\left[t_k^i, t_{k+1}^i\right) = [0, \infty)$. As such, for any $t \geqslant 0$, there must exist a $k^* \in \mathbb{Z}^+$ such that $t \in \left[t_{k^*}^i, t_{k^*+1}^i\right)$. Considering the mechanism of event-triggering strategy, it is noted that no event occurs between two consecutive instants $t_{k^*}^i$ and $t_{k^*+1}^i$. Based on this reason, for $t \in [t_{k^*}^i, t_{k^*+1}^i)$, we conclude that

$$\Delta_i(t) < \eta_i(t),$$

which further leads to

$$\begin{aligned} D^{\alpha}\eta_i(t) &= -\pi_i\eta_i(t) - \rho_i\Delta_i(t) \\ &\geqslant -\pi_i\eta_i(t) - \rho_i\eta_i(t) \\ &\geqslant -(\pi_i + \rho_i)\eta_i(t). \end{aligned}$$

By letting $p_i = -(\pi_i + \rho_i)$, it is readily deduced that

$$\eta_i(t) \geqslant \eta_i(t_k^i)E_{\alpha}\left[p_i\left(t - t_k^i\right)^{\alpha}\right], \quad t \in [t_k^i, t_{k+1}^i),$$

namely, for any $t \in [t_k^i, t_{k+1}^i)$, we derive that

$$\eta_i(t) \geqslant \eta_i(0)\left(\prod_{j=1}^{k} E_{\alpha}\left[p_i(t_j - t_{j-1})^{\alpha}\right]\right)E_{\alpha}[p_i(t - t_k^i)^{\alpha}].$$

By recalling the definition and properties of Mittag–Leffler function, one deduces $\prod_{j=1}^{k} E_{\alpha}\left[p_i(t_j^i - t_{j-1}^i)^{\alpha}\right]$ is a positive constant, which together with $\eta_i(0) > 0$, further implies

$$\eta_i(t) > 0,$$

for any $t \in [t_k^i, t_{k+1}^i)$. This completes the proof. □

Remark 6. *Obviously, when $\eta_i(t) = 0$, the triggering mechanism (8) reduces to the static one, which is more conservative. To be specific, for any given instant t_k^i, the next t_{k+1}^i determined by the DETM (8) always larger than the one determined by the static ETM. In the proposed method, the constant parameters are jointly designed with control gain K by using information of the consensus error, which is similar to those static ETM adopted in [10,43,47]. However, the internal dynamical variable $\eta_i(t)$ is generated by a preset fractional-order differential equation, which makes our method different.*

Theorem 1. *Let Assumptions 1–3 hold and c be provided. The system (6) is globally ML-stable via the control protocol (7), if there exist two matrices $P > 0$ and K satisfying*

$$\begin{aligned} \Psi = {} & 2d(I_N \otimes PA) - 2c(H \otimes PBK) \\ & + 2c\left(HH^T \otimes PBK(P^TP)^{-1}K^TB^TP^T\right) + \left(HH^T \otimes I_n\right) < 0, \end{aligned} \tag{14}$$

and the triggering parameters γ_i, ρ_i and σ_i satisfy

$$\gamma_i < \min\{\frac{1}{\rho_i}, \frac{\sigma_i}{m_1}\}, \tag{15}$$

with $m_1 = \lambda_{max}\{c[(I_N \otimes P^T P) + (HH^T \otimes I_n)]\}$.

Proof. To address the ML-stability for the error (12), we take both the dynamical evolution governed by (12) and the auxiliary dynamical variable generated by (9) into consideration and thereby construct a function as follows

$$W(t) = V(\xi(t)) + \sum_{i=1}^{N} \eta_i(t), \tag{16}$$

with $V(\xi(t)) = \xi^T(t)(I_N \otimes P)\xi(t)$. It is readily observed $W(t) > V(\xi(t)) > 0$ for all $t \geqslant 0$.

By employing Assumption 1, we calculate the derivative of $V(\xi(t))$ according to (12)

$$\begin{aligned} D^\alpha V(\xi(t)) \leqslant & 2\xi^T(t)(I_N \otimes P)D^\alpha \xi(t) \\ \leqslant & 2\xi^T(t)[d(I_N \otimes PA) - c(H \otimes PBK)]\xi(t) + 2c\,\xi^T(t)(H \otimes PBK)e(t). \end{aligned} \tag{17}$$

Noting that $2x^T y \leqslant x^T Q x + y^T Q^{-1} y$ for vectors x, y and $Q > 0$, it is seen that

$$2\xi^T(t)(H \otimes PBK)e(t) \leqslant \xi^T(t)(HH^T \otimes PBK(P^T P)^{-1} K^T B^T P^T)\xi(t) + e^T(t)\Big(I_N \otimes P^T P\Big)e(t). \tag{18}$$

By substituting (18) into (17) and noting that constants $c > 0$, $d > 0$, we obtain that

$$\begin{aligned} D^\alpha V(\xi(t)) \leqslant\; & 2\xi^T(t)\Big[d(I_N \otimes PA) - c(H \otimes PBK) + c\Big(HH^T \otimes PBK(P^T P)^{-1} K^T B^T P^T\Big)\Big]\xi(t) \\ & + ce^T(t)\Big(I_N \otimes P^T P\Big)e(t) \\ \leqslant\; & 2\xi^T(t)\Big[d(I_N \otimes PA) - c(H \otimes PBK) + c\Big(HH^T \otimes PBK(P^T P)^{-1} K^T B^T P^T\Big)\Big]\xi(t) \\ & + ce^T(t)\Big(I_N \otimes P^T P\Big)e(t) - q^T(t_k)q(t_k) + q^T(t_k)q(t_k). \end{aligned} \tag{19}$$

It follows from $q(t_k) = (H \otimes I_n)(e(t) - \xi(t))$ that

$$q^T(t_k)q(t_k) \leqslant \xi^T(t)\Big(HH^T \otimes I_n\Big)\xi(t) + e^T(t)\Big(HH^T \otimes I_n\Big)e(t). \tag{20}$$

Combining (19) with (20), we have

$$\begin{aligned} D^\alpha V(\xi(t)) \leqslant & \xi^T(t)\Big[2d(I_N \otimes PA) - 2c(H \otimes PBK) + 2c\Big(HH^T \otimes PBK(P^T P)^{-1} K^T B^T P^T\Big) \\ & + \Big(HH^T \otimes I_n\Big)\Big]\xi(t) + ce^T(t)\Big[\Big(I_N \otimes P^T P\Big) + \Big(HH^T \otimes I_n\Big)\Big]e(t) - q^T(t_k)q(t_k) \\ \leqslant & -\delta\xi^T(t)\xi(t) + m_1 e^T(t)e(t) - q^T(t_k)q(t_k). \end{aligned} \tag{21}$$

where $-\delta = \lambda_{max}(\Psi)$ and $m_1 = \lambda_{max}\{c[(I_N \otimes P^T P) + (HH^T \otimes I_n)]\}$. Taking the dynamic event-triggering mechanism (9) into consideration, we derive that

$$\begin{aligned} D^\alpha W(t) &= D^\alpha V(\xi(t)) + \sum_{i=1}^{N} D^\alpha \eta_i(t) \\ &\leqslant -\delta\xi^T(t)\xi(t) - \sum_{i=1}^{N} \pi_i \eta_i(t) + \sum_{i=1}^{N} (m_1 - \rho_i \sigma_i)\|e_i(t)\|^2 + \sum_{i=1}^{N} (\rho_i \gamma_i - 1)\|q_i(t_k)\|^2. \end{aligned} \tag{22}$$

According to inequality (15), one selects a scalar $\iota_i \in (0,\ \pi_i)$ satisfying $\frac{m_1-\rho_i\sigma_i}{\sigma_i} < \iota_i < \frac{1-\rho_i\gamma_i}{\gamma_i}$, which together with (22), further implies that

$$\begin{aligned} D^{\alpha}W(t) &\leqslant -\delta\xi^T(t)\xi(t) - \sum_{i=1}^{N}\pi_i\eta_i(t) + \sum_{i=1}^{N}\iota_i\sigma_i\|e_i(t)\|^2 - \sum_{i=1}^{N}\iota_i\gamma_i\|q_i(t_k)\|^2 \\ &\leqslant -\delta\xi^T(t)\xi(t) - \sum_{i=1}^{N}\pi_i\eta_i(t) + \sum_{i=1}^{N}\iota_i\big(\sigma_i\|e_i(t)\|^2 - \gamma_i\|q_i(t_k)\|^2\big). \end{aligned} \tag{23}$$

Noting that the event-triggering mechanism (8) indicates $\sigma_i\|e_i(t)\|^2 - \gamma_i\|q_i(t_k)\|^2 \leqslant \eta_i(t)$ for all i, one obtains from (23) that

$$\begin{aligned} D^{\alpha}W(t) &\leqslant -\delta\xi^T(t)\xi(t) + \sum_{i=1}^{N}\iota_i\eta_i(t) - \sum_{i=1}^{N}\pi_i\eta_i(t) \\ &\leqslant -\delta\xi^T(t)\xi(t) + \sum_{i=1}^{N}(\iota_i - \pi_i)\eta_i(t). \end{aligned} \tag{24}$$

Let $-\delta_1 = \max\{\iota_i - \pi_i, (-\delta/\rho_{\max}(P))\}$ and we have $-\delta_1 < 0$. Thus, we obtain that

$$\begin{aligned} D^{\alpha}W(t) &\leqslant -\delta_1\left[\xi^T(t)(I_N \otimes P)\xi(t) + \sum_{i=1}^{N}\eta_i(t)\right] \\ &\leqslant -\delta_1 W(t), \end{aligned} \tag{25}$$

which further indicates

$$W(t) \leqslant W(t_0)E_{\alpha}\left[-\delta_1(t-t_0)^{\alpha}\right], \quad t \geqslant t_0.$$

Noting that

$$\lambda_{max}(P)\xi^T(t)\xi(t) \leqslant V(\xi(t)) \leqslant W(t), \tag{26}$$

it is readily observed that

$$\|\xi(t)\|^2 \leqslant \frac{W(t_0)}{\lambda_{max}(P)}E_{\alpha}\left[-\delta_1(t-t_0)^{\alpha}\right],$$

which further implies that

$$\|\xi(t)\| \leqslant \beta\big(E_{\alpha}\left[-\delta_1(t-t_0)^{\alpha}\right]\big)^{\frac{1}{2}}, \tag{27}$$

with $\beta = \left(\frac{W(t_0)}{\lambda_{max}(P)}\right)^{\frac{1}{2}}$. Therefore, the leader–following group consensus is guaranteed for FOMASs (5) under the control protocol (7). Thus, the proof is competed. □

Theorem 2. *Let Assumptions 1–3 hold and c be given. The consensus error system (6) achieves the global ML-stability by (7) if there are two matrices $\bar{P} > 0$ and Y satisfying*

$$\Omega = \begin{bmatrix} W & H \otimes BY & H \otimes \bar{P} \\ * & -2c(I_N \otimes I_n) & 0 \\ * & * & -(I_N \otimes I_n) \end{bmatrix} < 0. \tag{28}$$

The triggering parameters γ_i, ρ_i and σ_i fulfill

$$\gamma_i < \min\{\frac{1}{\rho_i}, \frac{\sigma_i}{m_1}\}, \tag{29}$$

with $W = 2(I_N \otimes dA\bar{P}) - 2c(H \otimes BY)$ *and* $m_1 = \lambda_{max}\{c[(I_N \otimes P^T P) + (HH^T \otimes I_n)]\}$. *Further, K can be designed as* $K = Y\bar{P}^{-1}$.

Proof. According to Schur Complement lemma, $\Psi < 0$ is equivalent to

$$\begin{bmatrix} \Psi_{11} & H \otimes PBKP^{-1} & H \otimes I_n \\ * & -2c(I_N \otimes I_n) & 0 \\ * & * & -(I_N \otimes I_n) \end{bmatrix} < 0, \tag{30}$$

where $\Psi_{11} = 2d(I_N \otimes PA) - 2c(H \otimes PBK)$.

Multiplying (30) by $diag\{I_N \otimes P^{-1}, I_N \otimes I_n, I_N \otimes I_n\}$ from the left and right sides, we have

$$\begin{bmatrix} \tilde{\Omega}_{11} & H \otimes BKP^{-1} & H \otimes P^{-1} \\ * & -2c(I_N \otimes I_n) & 0 \\ * & * & -(I_N \otimes I_n) \end{bmatrix} < 0, \tag{31}$$

with $\tilde{\Omega}_{11} = 2d(I_N \otimes AP^{-1}) - 2c(H \otimes BKP^{-1})$. Obviously, the term KP^{-1} in (31) is nonlinear with respect to matrix variables, which indicates (31) cannot be solved by MATLAB LMI toolbox directly. Based on this reason, we denote $\bar{P} = P^{-1}$ and $Y = K\bar{P}$ and thereby obtain inequality (27). In other words, the LMI (27) is equivalent to inequality (14) in Theorem 1. Thus, we infer that

$$\|\xi(t)\| \leqslant \beta\left(E_\alpha\left[-\delta_3(t-t_0)^\alpha\right]\right)^{\frac{1}{2}}, \tag{32}$$

with $\beta = \left(\frac{W(t_0)}{\lambda_{max}(P)}\right)^{\frac{1}{2}}$. Accordingly, we complete the proof. □

Remark 7. *It is observed from Theorem 2 that the control method shows some distribution characteristics due mainly to the related state information between each agent and its neighbors. In addition, the DETM provides more flexibilities to the control process with resources limitation. The control gain matrix K can be designed by using linear matrix inequalities toolbox. Furthermore, there are only two matrix variables* $\bar{P}$ *and* Y *in LMI (28), which implies a lower computational burden of our method.*

Theorem 3. *Let Assumptions 1–3 hold. For ETM (8) and (9), there exist constants*

$$\zeta_i = e^{\frac{1}{\alpha}\ln\left(\frac{\Gamma(\alpha+1)\sqrt{\gamma_i}}{b\sqrt{\sigma_i}-a\sqrt{\gamma_i}}\right)} > 0 \tag{33}$$

such that $t^i_{k+1} - t^i_k \geqslant \zeta_i$, *which means the Zeno phenomenon is excluded.*

Proof. According to (8), (9) and Lemma 1, we calculate the α-order derivative for $\|e_i(t)\|$ and yields

$$\begin{aligned} D^\alpha\|e_i(t)\| &\leqslant D^\alpha\|x_i(t) - x_{\theta_s}(t)\| \\ &\leqslant \|Af(x_i(t)) - Af(x_{\theta_s}(t)) + cBKq_i(t^i_k)\| \\ &\leqslant \|A\tilde{f}(e_i(t)) + cBKq_i(t^i_k)\| \\ &\leqslant \|dA\|\,\|e_i(t)\| + \|cBK\|\,\|q_i(t^i_k)\| \\ &\leqslant a\|e_i(t)\| + b\|q_i(t^i_k)\|, \end{aligned} \tag{34}$$

where $a = \|dA\|$ and $b = \|cBK\|$.

It follows from $e_i(t_k^i) = 0$ that

$$e_i(t) \leqslant \frac{1}{\Gamma(\alpha)} \int_{t_k^i}^{t} \Big(a\|e_i(\tau)\| + b\|q_i(t_k^i)\| \Big)(t-\tau)^{\alpha-1} d\tau$$
$$\leqslant \frac{b\|q_i(t_k^i)\|}{\Gamma(\alpha+1)}(t-t_k^i)^{\alpha} + \frac{1}{\Gamma(\alpha)} \int_{t_k^i}^{t} a\|e_i(\tau)\|(t-\tau)^{\alpha-1} d\tau. \tag{35}$$

By recalling ETM, one induces $\|e_i(t_k^i)\| = 0$ and $\|e_i(t)\| \leqslant \|e_i(t^i_{(k+1)^-})\| = \lim_{t \to t^i_{(k+1)^-}} \|e_i(t)\|$ for $t \in \left[t_k^i, t_{k+1}^i\right)$. Thus, we have

$$e_i(t) \leqslant \frac{b\|q_i(t_k^i)\| + a\|e_i(t^i_{(k+1)^-})\|}{\Gamma(\alpha+1)}(t-t_k^i)^{\alpha}, \tag{36}$$

Let t approach $t^i_{(k+1)^-}$ in (36). We obtain

$$e_i(t^i_{(k+1)^-}) \leqslant \frac{b\|q_i(t_k^i)\| + a\|e_i(t^i_{(k+1)^-})\|}{\Gamma(\alpha+1)}(t_{k+1}^i - t_k^i)^{\alpha}. \tag{37}$$

Hence, one has

$$\|e_i(t^i_{(k+1)^-})\| \leqslant \frac{b\|q_i(t_k^i)\|(t_{k+1}^i - t_k^i)^{\alpha}}{\Gamma(\alpha+1) + a(t_{k+1}^i - t_k^i)^{\alpha}}$$
$$\leqslant \frac{b\|q_i(t_k^i)\|(t_{k+1}^i - t_k^i)^{\alpha}}{\Gamma(\alpha+1)}, \tag{38}$$

Noting that the auxiliary variable $\eta_i(t) > 0$, we conclude that (8) implies

$$\sigma_i\|e_i(t^i_{(k+1)^-})\|^2 - \gamma_i\|q_i(t_k^i)\|^2 \geqslant \eta_i(t^i_{(k+1)^-}) > 0, \tag{39}$$

which further yields

$$\sqrt{\frac{\gamma_i}{\sigma_i}}\|q_i(t_k^i)\| \leqslant \|e_i(t^i_{(k+1)^-})\|. \tag{40}$$

By taking both (38) and (40) into consideration, we deduce that

$$\sqrt{\frac{\gamma_i}{\sigma_i}}\|q_i(t_k^i)\| \leqslant \frac{b\|q_i(t_k^i)\|(t^i_{(k+1)^-} - t_k^i)^{\alpha}}{\Gamma(\alpha+1) + a(t^i_{(k+1)^-} - t_k^i)^{\alpha}}, \tag{41}$$

namely,

$$(t_{k+1}^{i-} - t_k^i)^{\alpha} \geqslant \frac{\Gamma(\alpha+1)\sqrt{\frac{\gamma_i}{\sigma_i}}}{b} \geqslant \frac{\Gamma(\alpha+1)\sqrt{\gamma_i}}{b\sqrt{\sigma_i}} > 0. \tag{42}$$

By denoting

$$\zeta_i = e^{\frac{1}{\alpha}\ln\left(\frac{\Gamma(\alpha+1)\sqrt{\gamma_i}}{b\sqrt{\sigma_i}}\right)}, \tag{43}$$

we derive that for any $k \in \mathbb{Z}^+$

$$t_{k+1}^i - t_k^i \geqslant \zeta_i > 0. \tag{44}$$

In other words, there is no Zeno phenomenon for the ETM. The proof is complete. □

4. An Example and Illustrations

In this section, the validity and flexibility of the proposed theoretical results is checked by an instance. For convenience of analysis and simulation, we consider a FOMAS consisting of eight agents (two leaders and six followers), namely, leader agents θ_1, θ_2 and follower agents $i = 1, 2, \cdots, 6$. The undirected graph is provided as follows.

The model is taken to be (3) and (4) with parameters

$$A = \begin{bmatrix} -1.9 & 1.2 \\ 0.9 & -3.1 \end{bmatrix}, \quad B = \begin{bmatrix} -1.5 \\ -3.6 \end{bmatrix}.$$

In this example, we choose $\alpha = 0.618$ to be the order of Caputo-type fractional derivative. Let vector function $f(x(t)) = 0.065 \sin(x(t))$ be for the nonlinear dynamics of individual agent, which obviously implies Assumption 1 holds with $d = 0.073$.

It is observed from Figure 2 that all follower agents are classified to two groups such as $\mathcal{V}_1 = \{1, 2, 3, 4\}$, $\mathcal{V}_2 = \{5, 6\}$ with leader agents θ_1, θ_2, respectively. Denote by $\mathcal{G}$ the graph generated by the communication topology of six follower agents. It must be emphasized that $\mathcal{G}$ is also divided into two subgraphs $\mathcal{G}_1$, $\mathcal{G}_2$. By taking the adjacency weights and topology of subgraphs into consideration, we deduce that Assumptions 2 and 3 are satisfied.

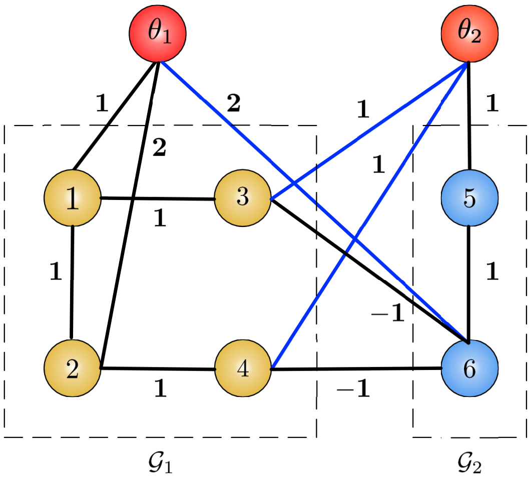

Figure 2. The graph of communication topology for FOMAS.

Let us firstly consider the case of open-loop system, namely, all control input $u_i(t) = 0$. The initial values are taken to be $x_1(0) = [-7, 1]^T$, $x_2(0) = [1, -7]^T$, $x_3(0) = [6, -6]^T$, $x_4(0) = [-7, 2]^T$, $x_5(0) = [6, -1]^T$, $x_6(0) = [-1, 7]^T$ and $x_{\theta_1}(0) = [9, -9]^T$, $x_{\theta_2}(0) = [-1, 1]^T$, respectively. Under such a case, the state evolution for all agents of FOMASs are drawn in Figure 3 that explicitly displays that the follower agents in the same group cannot follow the corresponding leader agent. Hence, the required group consensus is not achieved ultimately.

By employing the weight parameters for every edge in graph $\mathcal{G}$ shown in Figure 2, it is not difficult to obtain the weighted adjacency matrix $\mathcal{A}$ is described as

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix},$$

To facilitate the design of the consensus control protocol for each follower agents, we calculate the block matrix $\mathcal{L}$ for the topology graph $\mathcal{G}$ to be

$$\mathcal{L} = \mathcal{D} - \mathcal{A} = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{bmatrix},$$

At the same time, we note that the communication between leader agents θ_1, θ_2 and follower agents are characterized by the following matrices

$$Q_1 = diag\{1, 2, 0, 0, 0, 2\},$$
$$Q_2 = diag\{0, 0, 1, 1, 1, 0\},$$

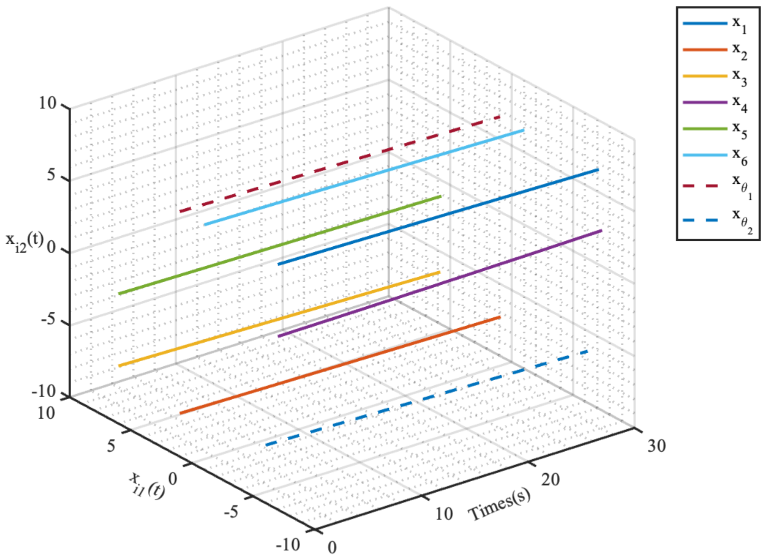

Figure 3. The evolution of states of leader x_{θ_s} and follower x_i with $u_i(t) = 0$.

By introducing $\bar{\mathcal{G}}$ to describe the total graph generated by all leader and follower agents, we deduce that the communication topology of Figure 2 is characterized by the weighted matrix as follows

$$H = \mathcal{L} + Q_1 + Q_2 = \begin{bmatrix} 3 & -1 & -1 & 0 & 0 & 0 \\ -1 & 4 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{bmatrix}.$$

For the aim of pushing all follower agents towards to their corresponding leaders, we consider the distributed consensus protocol taking the form of (7). In specific, we take the parameter $c = 3$ as the coupling strength for control input. According to Theorem 2, a sort of solutions for (28) is derived as follows:

$$\bar{P} = \begin{bmatrix} 0.1973 & 0.0385 \\ 0.0385 & 0.2635 \end{bmatrix}, \quad Y = \begin{bmatrix} -0.1352 & -0.0483 \end{bmatrix}.$$

As such, we obtain the control gain matrix to be

$$K = \begin{bmatrix} -0.6683 & -0.0853 \end{bmatrix}.$$

For the dynamic event-triggered mechanism (8), we choose a set of weight parameters as $\sigma_i = 0.95$, $\gamma_i = 0.23$, $\pi_i = 0.45$, and $\rho_i = 0.52$ for all $i = 1, 2, \cdots, 6$. It is undoubtedly shown that the condition (29) holds. According to Theorem 2, the FOMAS will finally achieve the leader–following group consensus using the distributed dynamic event-triggering consensus control strategy (7). The simulations are presented in Figures 4–9. Specifically, the dynamical evolution for agent states are shown in Figures 4–6, which show that all 6 follower agents are divided into two groups and each follower agent in the same group converges to its corresponding leader agent ultimately. The triggering sequences are provided in Figure 7, which shows that the sampling frequency of the control signal is considerably diminished for each follower agent while the desired group consensus performance is still maintained. In particular, the dynamical evolution of consensus error vector $\xi(t)$ is presented in Figures 8 and 9. It is clearly seen that the consensus error system (6) is a global Mittag–Leffler stability, which further implies the leader–following group consensus of FOMAS. It should be pointed out that our method is also applicable to FOMAS with more agents, which would be our future research topic.

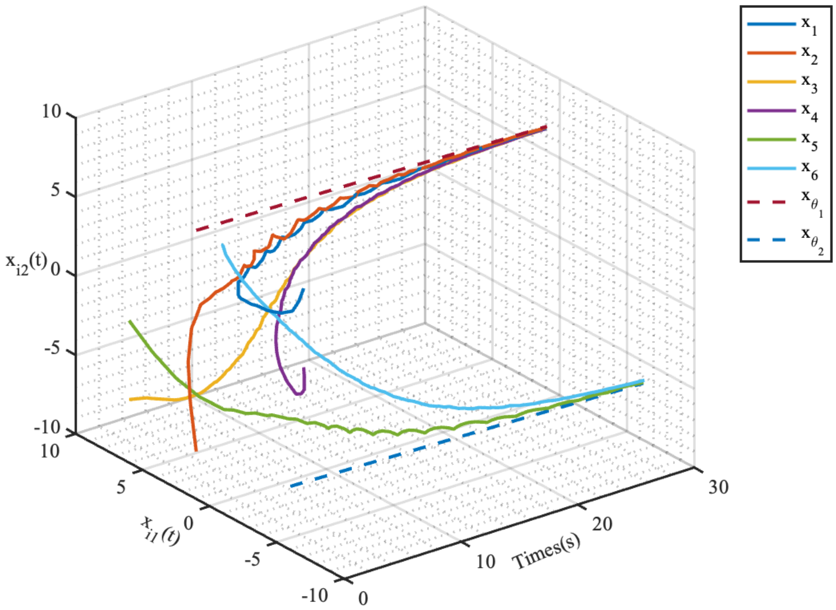

Figure 4. The evolution of states of fractional-order multi-agents systems with times.

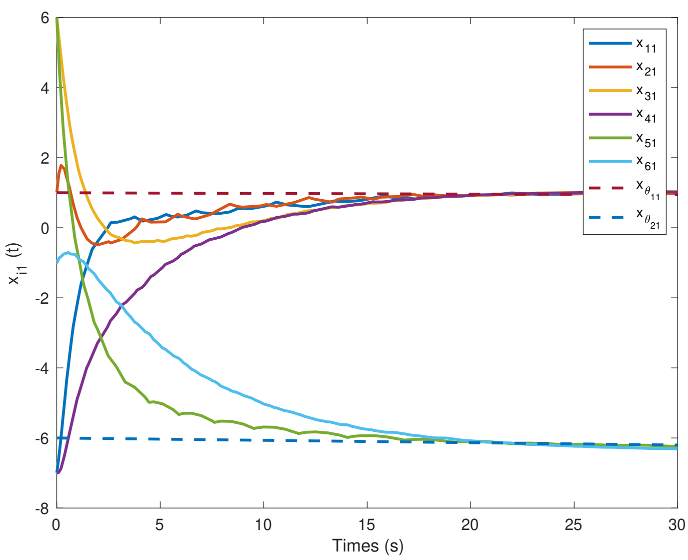

Figure 5. The evolution of states of leader $x_{\theta_s 1}$ and follower x_{i1}.

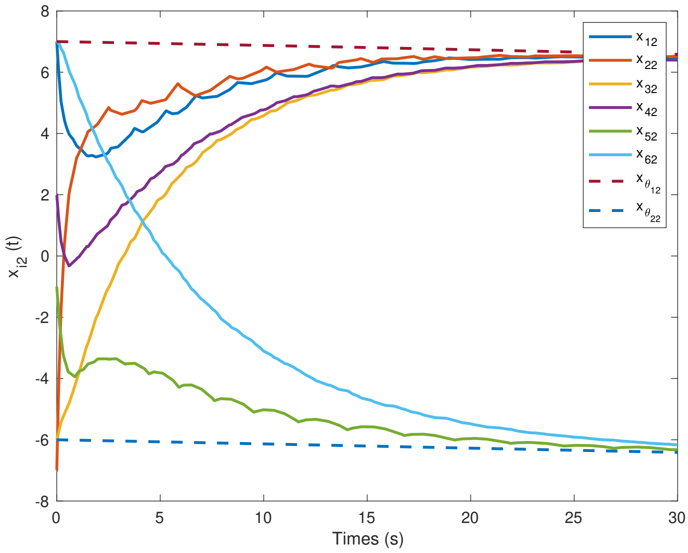

Figure 6. The evolution of states of leader $x_{\theta_s 2}$ and follower x_{i2}.

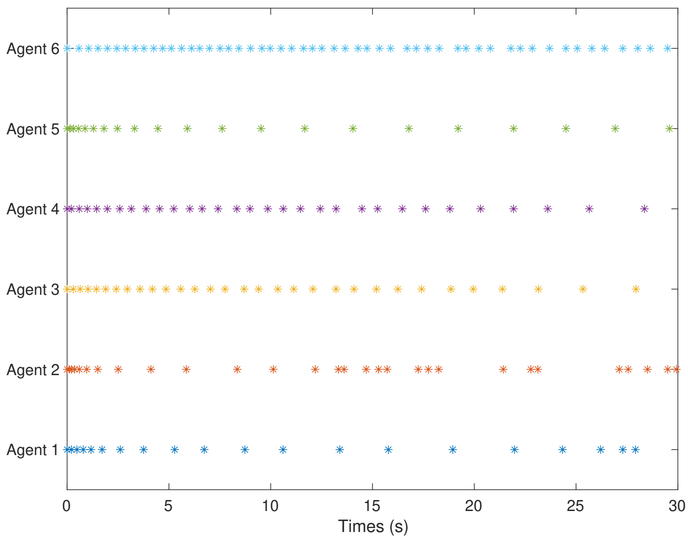

Figure 7. The instants t_k^i of DETM.

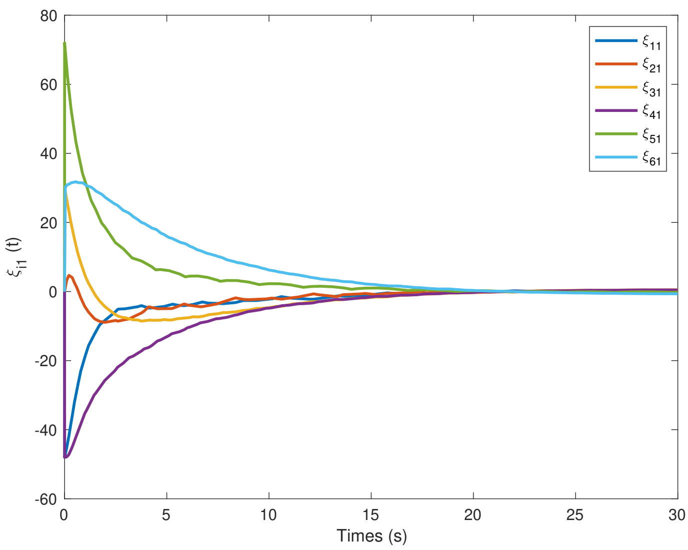

Figure 8. The evolution of the states of consensus error ξ_{i1}.

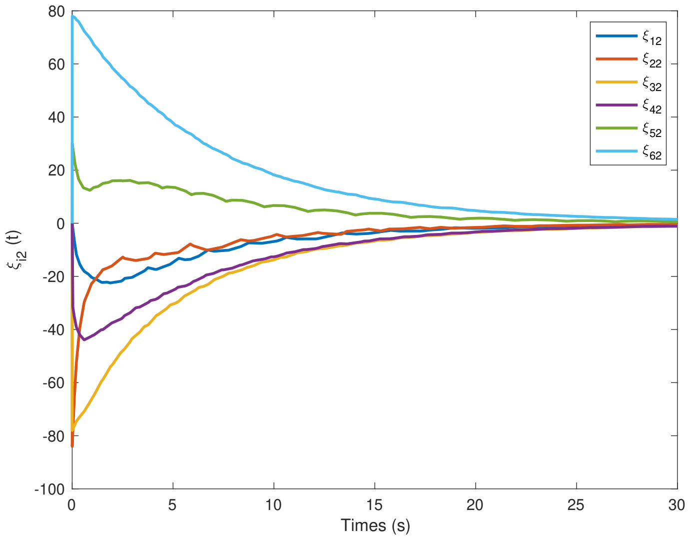

Figure 9. The evolution of the states of consensus error ξ_{i2}.

5. Conclusions

This paper focused on the group consensus for a class of FOMASs under DETM. In the network, each follower agent has been assumed to belong to a certain subgroup with a corresponding leader agent. The leaders may interact with followers in different subgroups. By taking use of the relate state information between follower agents and its neighbors, a general distributed control protocol has been proposed to force each follower agent to approach its corresponding leader. By constructing an auxiliary FODE, the DETM has been introduced for the aim of significantly diminishing the control transmission without loss of the desired target. By employing the properties of fractional-order calculus and Lyapunov direct method, some sufficient criteria have been established to analyze the stability of error system and exclude the Zeno phenomenon in event-triggering mechanism. The gain matrix and the triggering parameters have been co-designed for the control protocol by solving LMIs. A numerical example and its simulations have been presented for supporting our results. The theoretical finding in this paper can enrich theory of consensus control for FOMASs. Moreover, the methodology proposed in this paper can also be used in practical applications such as group control and formation regulation of intelligent vehicles. It should be pointed out that the models considered in this paper are relatively simple and still differ from some real systems with unknown nonlinearity and parameter uncertainty [48,49]. In the near future, we would like to study the group consensus tracking for FOMASs with general unknown nonlinearity and parameter uncertainty.

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References

1. Kwa, H.; Kit, J.; Bouffanais, R. Optimal swarm strategy for dynamic target search and tracking. In Proceedings of the 19th International Conference on Autonomous Agents and MultiAgent Systems, Auckland, New Zealand, 9–13 May 2020; pp. 672–680.
2. Hu, Z.; Ma, L.; Wang, B.; Zou, L.; Bo, Y. Finite-time consensus control for heterogeneous mixed-order nonlinear stochastic multi-agent systems. *Syst. Sci. Control Eng.* **2021**, *9*, 405–416. [CrossRef]
3. He, W.; Xu, B.; Han, Q.L.; Qian, F. Adaptive consensus control of linear multi-agent systems with dynamic event-triggered strategies. *IEEE Trans. Cybern.* **2020**, *50*, 2996–3008. [CrossRef] [PubMed]
4. Du, S.L.; Liu, T.; Ho, D.W. Dynamic event-triggered control for leader-following consensus of multi-agent systems. *IEEE Trans. Syst. Man Cybern. Syst.* **2018**, *50*, 3243–3251. [CrossRef]
5. Ge, X.; Han, Q.L.; Ding, L.; Wang, Y.L.; Zhang, X.M. Dynamic event-triggered distributed coordination control and its applications: A survey of trends and techniques. *IEEE Trans. Syst. Man Cybern. Syst.* **2020**, *50*, 3112–3125. [CrossRef]
6. Yi, X.; Liu, K.; Dimarogonas, D.V.; Johansson, K.H. Distributed dynamic event-triggered control for multi-agent systems. In Proceedings of the 2017 IEEE 56th Annual Conference on Decision and Control (CDC), Melbourne, VIC, Australia, 12–15 December 2017; IEEE: Piscataway, NJ, USA, 2017; pp. 6683–6698.
7. Su, Y.; Cai, H.; Huang, J. The Cooperative Output Regulation by the Distributed Observer Approach. *Int. J. Netw. Dyn. Intell.* **2022**, *1*, 20–35. [CrossRef]
8. Bao, G.; Ma, L.; Yi, X. Recent advances on cooperative control of heterogeneous multi-agent systems subject to constraints: A survey. *Syst. Sci. Control Eng.* **2022**, *10*, 539–551. [CrossRef]
9. Li, B.; Wang, Z.; Han, Q.L.; Liu, H. Distributed Quasiconsensus Control for Stochastic Multiagent Systems Under Round-Robin Protocol and Uniform Quantization. *IEEE Trans. Cybern.* **2022**, *52*, 6721–6732. [CrossRef]
10. Xing, M.; Lu, J.; Liu, Y.; Chen, X. Event-Based Bipartite Consensus of Multi-Agent Systems Subject to DoS Attacks. *IEEE Trans. Netw. Sci. Eng.* **2023**, *10*, 68–80. [CrossRef]
11. Du, H.; Wen, G.; Chen, G.; Cao, J.; Alsaadi, F.E. A distributed finite-time consensus algorithm for higher-order leaderless and leader-following multiagent systems. *IEEE Trans. Syst. Man Cybern. Syst.* **2017**, *47*, 1625–1634. [CrossRef]
12. Min, X.; Baldi, S.; Yu, W. Finite-Time Distributed Control of Nonlinear Multiagent Systems via Funnel Technique. *IEEE Trans. Syst. Man Cybern. Syst.* **2023**, *53*, 1256–1267. [CrossRef]
13. Liu, Z.; Huang, H.; Park, J.H.; Huang, J.; Wang, X.; Lv, M. Adaptive Fuzzy Control for Unknown Nonlinear Multi-Agent Systems with Switching Directed Communication Topologies. *IEEE Trans. Fuzzy Syst.* **2023**, 1–8. [CrossRef]
14. Niu, K.; Wardi, Y.; Abdallah, C.T.; Hayajneh, M. Consensus Controller for Heterogeneous Multi-Agent Systems Using Output Prediction. *IEEE Control Syst. Lett.* **2023**, *7*, 673–678. [CrossRef]
15. Wang, Q.; Dong, X.; Wang, B.; Hua, Y.; Ren, Z. Finite-time Observer-based H_∞ Fault-tolerant Output Formation Tracking Control for Heterogeneous Nonlinear Multi-agent Systems. *IEEE Trans. Netw. Sci. Eng.* **2023**, 1–13.. [CrossRef]
16. Shang, Y.; Ye, Y. Leader-follower fixed-time group consensus control of multiagent systems under directed topology. *Complexity* **2017**, *2017*, 3465076. [CrossRef]
17. Zhang, Y.; Wu, H.; Cao, J. Group Consensus in Finite Time for Fractional Multiagent Systems With Discontinuous Inherent Dynamics Subject to Hölder Growth. *IEEE Trans. Cybern.* **2022**, *52*, 4161–4172. [CrossRef]
18. Hu, A.; Cao, J.; Hu, M.; Guo, L. Event-triggered group consensus for multi-agent systems subject to input saturation. *J. Frankl. Inst.* **2018**, *355*, 7384–7400. [CrossRef]
19. Wen, G.; Huang, J.; Wang, C.; Chen, Z.; Peng, Z. Group consensus control for heterogeneous multi-agent systems with fixed and switching topologies. *Int. J. Control* **2016**, *89*, 259–269. [CrossRef]
20. Podlubny, I. *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications*; Academic Press: Cambridge, MA, USA, 1998.
21. Diethelm, K. *The Analysis of Fractional Differential Equations: An Application-Oriented Exposition Using Differential Operators of Caputo Type*; Springer Science & Business Media: Berlin/Heidelberg, Germany, 2010.
22. Ren, G.; Yu, Y.; Zhang, S. Leader-following consensus of fractional nonlinear multi-agent systems. *Math. Probl. Eng.* **2015**, *2015*, 919757. [CrossRef]
23. Li, Y.; Chen, Y.; Podlubny, I. Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag–Leffler stability. *Comput. Math. Appl.* **2010**, *59*, 1810–1821. [CrossRef]
24. Ma, C.Q.; Liu, T.Y.; Kang, Y.; Zhao, Y.B. Leader-Following Cluster Consensus of Multiagent Systems With Measurement Noise and Weighted Cooperative–Competitive Networks. *IEEE Trans. Syst. Man Cybern. Syst.* **2023**, *53*, 1150–1159. [CrossRef]
25. Zhang, S.; Yu, Y.; Wang, H. Mittag-Leffler stability of fractional-order Hopfield neural networks. *Nonlinear Anal. Hybrid Syst.* **2015**, *16*, 104–121. [CrossRef]
26. Chen, Y.; Tang, C.; Roohi, M. Design of a model-free adaptive sliding mode control to synchronize chaotic fractional-order systems with input saturation: An application in secure communications. *J. Frankl. Inst.* **2021**, *358*, 8109–8137. [CrossRef]
27. Roohi, M.; Zhang, C.; Chen, Y. Adaptive model-free synchronization of different fractional-order neural networks with an application in cryptography. *Nonlinear Dyn.* **2020**, *100*, 3979–4001. [CrossRef]
28. Chen, Y.; Song, Q.; Zhao, Z.; Liu, Y.; Alsaadi, F.E. Global Mittag-Leffler stability for fractional-order quaternion-valued neural networks with piecewise constant arguments and impulses. *Int. J. Syst. Sci.* **2022**, *53*, 1756–1768. [CrossRef]

29. Wang, F.; Yang, Y. Leader-following consensus of nonlinear fractional-order multi-agent systems via event-triggered control. *Int. J. Syst. Sci.* **2017**, *48*, 571–577. [CrossRef]
30. Ye, Y.; Su, H. Leader-following consensus of nonlinear fractional-order multi-agent systems over directed networks. *Nonlinear Dyn.* **2019**, *96*, 1391–1403. [CrossRef]
31. Pan, L.; Shao, H.; Li, Y.; Li, D.; Xi, Y. Event-Triggered Consensus of Matrix-Weighted Networks Subject to Actuator Saturation. *IEEE Trans. Netw. Sci. Eng.* **2023**, *10*, 463–476. [CrossRef]
32. Wang, M.; Wang, R. Group consensus of fractional order multi-agent systems with fixed topology. In Proceedings of the 2017 Chinese Automation Congress (CAC), Jinan, China, 20–22 October 2017; IEEE: Piscataway, NJ, USA, 2017; pp. 331–336.
33. Dai, S.; Wu, Z.; Zhang, P.; Tan, M.; Yu, J. Distributed Formation Control for a Multi-Robotic Fish System with Model-Based Event-Triggered Communication Mechanism. *IEEE Trans. Ind. Electron.* **2023**, 1–10. [CrossRef]
34. Ren, Y.; Xian, C.; Liu, Y.; Zhao, Y. Distributed Adaptive Event-Triggered TVGF for Generic Linear Multi-Agent Systems under Directed Graph. *IEEE Trans. Circuits Syst. II Express Briefs* **2023**. [CrossRef]
35. Wang, Z.; Shi, S.; He, W.; Xiao, M.; Cao, J.; Gorbachev, S. Observer-Based Asynchronous Event-Triggered Bipartite Consensus of Multi-Agent Systems Under False Data Injection Attacks. *IEEE Trans. Control Netw. Syst.* **2023**, 1–12. [CrossRef]
36. Li, Z.; Hu, J.; Li, J. Distributed filtering for delayed nonlinear system with random sensor saturation: A dynamic event-triggered approach. *Syst. Sci. Control Eng.* **2021**, *9*, 440–454. [CrossRef]
37. Zhang, Y.; Wu, H.; Cao, J. Global Mittag-Leffler consensus for fractional singularly perturbed multi-agent systems with discontinuous inherent dynamics via event-triggered control strategy. *J. Frankl. Inst.* **2021**, *358*, 2086–2114. [CrossRef]
38. Xu, B.; Li, B. Event-Triggered State Estimation for Fractional-Order Neural Networks. *Mathematics* **2022**, *10*, 325. [CrossRef]
39. Wang, X.; Sun, Y.; Ding, D. Adaptive Dynamic Programming for Networked Control Systems Under Communication Constraints: A Survey of Trends and Techniques. *Int. J. Netw. Dyn. Intell.* **2022**, 85–98. [CrossRef]
40. Sun, Y.; Tian, X.; Wei, G. Finite-time distributed resilient state estimation subject to hybrid cyber-attacks: A new dynamic event-triggered case. *Int. J. Syst. Sci.* **2022**, *53*, 2832–2844. [CrossRef]
41. Chen, B.; Hu, J.; Zhao, Y.; Ghosh, B.K. Leader-following Consensus of Linear Fractional-order Multi-agent Systems via Event-triggered Control Strategy. *IFAC-PapersOnLine* **2020**, *53*, 2909–2914. [CrossRef]
42. Fan, Y.; Feng, G.; Wang, Y.; Song, C. Distributed event-triggered control of multi-agent systems with combinational measurements. *Automatica* **2013**, *49*, 671–675. [CrossRef]
43. Hu, W.; Liu, L.; Feng, G. Consensus of linear multi-agent systems by distributed event-triggered strategy. *IEEE Trans. Cybern.* **2015**, *46*, 148–157. [CrossRef]
44. Qian, Y.Y.; Liu, L.; Feng, G. Distributed dynamic event-triggered control for cooperative output regulation of linear multi-agent systems. *IEEE Trans. Cybern.* **2019**, *50*, 3023–3032. [CrossRef]
45. Hu, W.; Yang, C.; Huang, T.; Gui, W. A distributed dynamic event-triggered control approach to consensus of linear multi-agent systems with directed networks. *IEEE Trans. Cybern.* **2018**, *50*, 869–874. [CrossRef]
46. Cai, J.; Wang, J.; Feng, J.; Chen, G.; Zhao, Y. Observer-Based Consensus for Multi-Agent Systems with Semi-Markovian Jumping Via Adaptive Event-Triggered SMC. *IEEE Trans. Netw. Sci. Eng.* **2023**, 1–14. [CrossRef]
47. Tripathy, N.S.; Kar, I.N.; Chamanbaz, M.; Bouffanais, R. Robust stabilization of a class of nonlinear systems via aperiodic sensing and actuation. *IEEE Access* **2020**, *8*, 157403–157417. [CrossRef]
48. Roohi, M.; Aghababa, M.P.; Haghighi, A.R. Switching adaptive controllers to control fractional-order complex systems with unknown structure and input nonlinearities. *Complexity* **2015**, *21*, 211–223. [CrossRef]
49. Aghababa, M.P.; Haghighi, A.R.; Roohi, M. Stabilisation of unknown fractional-order chaotic systems: An adaptive switching control strategy with application to power systems. *IET Gener. Transm. Distrib.* **2015**, *9*, 1883–1893. [CrossRef]