



Article Finite-Time Adaptive Event-Triggered Control for Full States Constrained FONSs with Uncertain Parameters and Disturbances

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Abstract: This article focuses the event-triggered adaptive finite-time control scheme for the states constrained fractional-order nonlinear systems (FONSs) under uncertain parameters and external disturbances. The backstepping scheme is employed to construct the finite-time controller via a series of barrier Lyapunov function (BLF) to solve that all the state constraints are not violated. Different from the trigger condition with fixed value, the event-triggered strategy is applied to overcome the communication burden of controller caused by the limited communication resources. By utilizing fractional-order Lyapunov analysis, all variables in the resulted system are proven to be bounded, and the tracking error converges to the small neighborhood around origin in finite time and without the Zeno behavior. Finally, the effectiveness of the proposed control scheme is verified by the simulation analysis of a bus power system.

Keywords: fractional-order systems; state constraints; barrier Lyapunov functions; event-triggered; finite-time

1. Introduction

Fractional-order nonlinear systems (FONSs) established by fractional calculus can present the physical systems accurately for lots of different areas such as infectious diseases [1,2], image processing [3,4], secret communication [5,6], circuit [7,8], vehicle engineering [9,10], etc. Furthermore, many controller design problem for FONSs have been researched, in which the adaptive technique as a very powerful tool have been investigated by combining with backstepping mechanism to deal with the uncertainties in FONSs [11–25].

All above mentioned achievements ignore the state constraints. In plentiful physical systems, it is generally widespread that the system states can change within a specified range, and the control system may become unstable when the system states violate these constraints. Therefore, it is practical significance to control FONSs with state constraints. To tackle the state constraints issue for FONSs, the barrier Lyapunov function (BLF) as the predominant approach by applying error constraint is used to limit the state [26–29]. The authors in [28] design the adaptive controller for the FONSs under tate constraints, and the states remain in the bounds by introducing the BLF. In [29], an adaptive dynamic surface control scheme by using asymmetric BLF is proposed for FONSs with states constraints and input nonlinearity to achieve the tracking performance.

In fact, the above literature can theoretically guarantee the infinite-time stability of the control system, namely, the control system obtain the desired performance only when the time tends to infinity. In order to complete faster transient performance and better disturbance rejection ability by using the finite-time control, many contributions in this field have been reported for both integer-order nonlinear systems [30–32] and FONSs [22,33–38]. In [22], a finite-time adaptive neural controller is developed for a class of the FONSs, and the errors converge to zero in finite time. The author in [33] develops fractional order finite-time output feedback controllers for nonlinear interconnected systems to ensure the finite time stability. A sliding mode controller is developed for FONSs to ensure all



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Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). variables arrive at a domain within the fixed time in [34]. The author in [35] presentes the fractional-order fuzzy controller with finite-time performance for tumor systems with finite-time stability. In [36], an adaptive neural finite-time controller for the FONSs under unknown dynamics is proposed to obtain finite-time convergence. The authors in [37] develop the finite-time adaptive controller for uncertain FONSs by using the command filter. For a class of FONSs with faults, the authors in [38] present a fuzzy adaptive dynamic sliding mode controller. However, the communication with time-trigger method may lead to the waste of resource coming from the huge data transmission.

Event-triggered control (ETC) can reduce the limited computation resources and communication, which is only triggered when the condition is met. In [39], the ETC strategy for the FONSs is designED to ensure the stability and reduce computational burden. In [40], the distributed ETC for multiagent fractional-order systems is designed to sSolve the problem of limited communication resources. For the FONS with unmodeled dynamic in [41], the neural adaptive ETC is presented to reduce computational burden. For the integrator FONS under disturbances and unknown dynamic in [42], an adaptive ETC method is proposed to reduce the amount of computation required for transmission. In [43], an adaptive fuzzy ETC scheme is developed for FONS under uncertainty to reduce the transmission of the control signal. In [44], an adaptive fuzzy hybrid ETC for the uncertain time-delay FONS under actuator fault is proposed to improve the efficiency of computing resource. To our best knowledge, there is a lack of research on the finite-time adaptive event-triggered controller of the FONSs with full-state constraints and uncertain parameters bringing forward the challenge.

Based on above discussion, this article will design a finite-time event-triggered adaptive controller for full-states constrained FONSs with uncertain parameters and external disturbances by combining backstepping technique and event-triggered scheme. The significance and contributions are as follows:

- (1) Compared with full-state constraints results in [28,29] without finite time performance, the finite-time event-triggered adaptive controller is exploited by combing BLFs and backstepping technology, and the finite-time convergence of the close-loop signals can be guaranteed. Compared with the finite-time controller in [36–38], it is further resolved that the state constraints are not violated.
- (2) Different from the conventional periodic controllers in [28,29,36–38], an event triggered adaptive controller is proposed and the stability is proved by using finite-time fractional-order Lyapunov criterion, in which the control signals are updated only via the event-triggering mechanism largely reducing the consumption of network resources and communication burden.

2. Problem Descriptions

Consider the following strict-feedback FONS with uncertain parameters and external disturbances:

$$\begin{cases} D^{\alpha}x_{1} = f_{1}(x_{1}) + g_{1}(x_{1})x_{2} + \theta_{1}^{1}\varphi_{1}(x_{1}) + d_{1}(t) \\ D^{\alpha}x_{2} = f_{2}(\underline{x}_{2}) + g_{2}(\underline{x}_{2})x_{3} + \theta_{2}^{T}\varphi_{2}(\underline{x}_{2}) + d_{2}(t) \\ \vdots \\ D^{\alpha}x_{n-1} = f_{n-1}(\underline{x}_{n-1}) + g_{n-1}(\underline{x}_{n-1})x_{n} + \theta_{n-1}^{T}\varphi_{n-1}(\underline{x}_{n-1}) + d_{n-1}(t) \\ D^{\alpha}x_{n} = f_{n}(x) + g_{n}(x)u + \theta_{n}^{T}\varphi_{n}(x) + d_{n}(t) \\ y = x_{1} \end{cases}$$
(1)

where $\underline{x}_j = (x_1 \ x_2 \ \dots \ x_j)^T \in \mathbb{R}^j (j = 1, 2, \dots, n-1)$ and $x = (x_1 \ x_2 \ \dots \ x_n)^T \in \mathbb{R}^n$ denote the state vectors, $y \in \mathbb{R}$ denotes the output, *u* denoted controller input, $d_i(t)$ is the

external disturbance, and $f_i(\cdot)$ and $g_i(\cdot)$ denote known smooth functions. α is the system fractional order. the Caputo fractional derivative of z(t) is denoted as [45,46]:

$$D_{t_0}^{\alpha} z(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t (t-\tau)^{n-\alpha-1} z^{(n)}(\tau) d\tau$$
(2)

where $\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} dt$, $n-1 < \alpha < n, n \in Z^+$. $D_{t_0}^\alpha$ is defined as D^α , when $t_0 = 0$.

Control objective: (1) output *y* can track the signal $y_r(t)$ in finite time; (2) all the states are within the bounds of the constraints; and (3) Zeno behavior is avoided to occur.

Assumption 1. For $\forall k_{c_1} > 0$, there are positive A_0 , A_1 , and A_2 , s.t. $|y_r(t)| \leq A_0 < k_{c_1}$, $|D^{\alpha}y_r(t)| \leq A_1$, and $|D^{2\alpha}y_r(t)| \leq A_2$.

The desired signal $y_r(t)$ is the known bounded signal in real application. Then, according to the maximum value of $|y_r(t)|$, $|D^{\alpha}y_r(t)|$ and $|D^{2\alpha}y_r(t)|$, the bounds A_0 , A_1 and A_2 can be obtained.

Assumption 2. There are unknown constants $g_{i\min}$ and $g_{i\max}$, s.t. $0 < g_{i\min} \le |g_i(\underline{x}_i)| \le g_{i\max}$. Without loss of generality, $0 < g_{i\min} \le g_i(\underline{x}_i) \le g_{i\max}$.

Assumption 3. $d_i(t)$ is bounded, and $|d_i(t)| \leq \bar{d}_i$ with $\bar{d}_i > 0$.

Lemma 1. In [47,48]. Let $z(t) \in \mathbb{R}^n$, $Q = Q^T > 0$, then $D^{\alpha}(z^T(t)Qz(t)) \le 2z^T(t)QD^{\alpha}(z(t))$ holds for $\forall t \ge t_0$.

Lemma 2. In [49]. Let $Z_1(\cdot), Z_2(\cdot) \in \mathbb{R}$. Assume that $Z_1(Z_2)$ is convex (i.e., $\partial^2 Z_1(Z_2) / \partial Z_2^2 \ge 0$), then, $D^{\alpha}Z_1(Z_2) \le \partial Z_1(Z_2) / \partial Z_2 \cdot D^{\alpha}Z_2$ holds for $\forall t \ge 0$.

Lemma 3. *In* [50]: For $\forall q_1, q_2$ and $\forall s_1, s_2, s_3 > 0$, *it is hold that:*

$$|q_1|^{s_1}|q_2|^{s_2} \le \frac{s_1}{s_1+s_2}s_3|q_1|^{s_1+s_2} + \frac{s_2}{s_1+s_2}s_3^{-\frac{s_1}{s_2}}|q_2|^{s_1+s_2}$$
(3)

Lemma 4. In [51,52]. When $\bar{\varsigma}(t)$ satisfy $|\bar{\varsigma}(t)| \leq k_{b_0}$ for $\forall k_{b_0} > 0$, we obtain

$$\ln \frac{k_{b_0}^2}{k_{b_0}^2 - \bar{\varsigma}^2(t)} \le \frac{\bar{\varsigma}^2(t)}{k_{b_0}^2 - \bar{\varsigma}^2(t)} \tag{4}$$

Lemma 5. In [53]: Suppose $h(t) \in C^1((0, +\infty], \mathbb{R}), c \in (0, 1]$, and $d \in [1, \infty)$, then

$$D^{c}h^{d}(t) = \frac{\Gamma(1+d)\Gamma(2-c)}{\Gamma(1+d-c)}h^{d-1}(t)D^{c}h(t)$$
(5)

3. Finite-Time Adaptive Controller with State Constraints

In this section, a finite-time ETC adaptive method will be developed for the systems (1) combined with the backstepping technology. The event-triggered actual controller, virtual controller and adaptive parameter estimation will be structured by the choice of BLF, for which the detailed design process is presented as follows.

Step 1: Define $\chi_1 = y - y_r(t)$, from system (1) the Caputo fractional derivative of χ_1 is

$$D^{\alpha}\chi_{1} = f_{1}(x_{1}) + g_{1}(x_{1})(\chi_{2} + \vartheta_{1}) + \theta_{1}^{\mathrm{T}}\varphi_{1}(x_{1}) + d_{1}(t) - D^{\alpha}y_{r}(t)$$
(6)

Denote $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, where $\hat{\theta}_i$ is the estimate parameter of θ_i . Consider the BLF candidate function as

$$V_1 = \frac{1}{2} \ln \frac{k_{b_1}^2}{k_{b_1}^2 - \chi_1^2} + \frac{1}{2\gamma_1} \tilde{\theta}_1^2$$
(7)

where $k_{b_1} = k_{c_1} - A_0$ and $\gamma_1 > 0$ is a design parameter. $k_{b_1} > 0$ and its definition will be given later.

According to Lemmas 1 and 2, the derivative of V_1 is

$$D^{\alpha}V_{1} = \frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}} D^{\alpha}\chi_{1} - \frac{1}{\gamma_{1}}\tilde{\theta}_{1}^{\mathrm{T}}D^{\alpha}\hat{\theta}_{1}$$

$$= \frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}}\hat{\theta}_{1}^{\mathrm{T}}\varphi_{1}(x_{1}) - \frac{1}{\gamma_{1}}\tilde{\theta}_{1}^{\mathrm{T}}\left(D^{\alpha}\hat{\theta}_{1} - \gamma_{1}\frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}}\varphi_{1}(x_{1})\right)$$

$$+ \frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}}(f_{1}(x_{1}) + g_{1}(x_{1})(\chi_{2} + \vartheta_{1}) + d_{1}(t) - D^{\alpha}y_{r}(t))$$
(8)

The Young's inequality can be applied to obtain

$$\frac{\chi_1}{k_{b_1}^2 - \chi_1^2} d_1(t) \le \frac{\chi_1^2}{2a_1^2 \left(k_{b_1}^2 - \chi_1^2\right)^2} + \frac{1}{2}a_1^2 \vec{d}_1^2 \tag{9}$$

where $a_1 > 0$.

Substituting (9) into (8), yields

$$D^{\alpha}V \leq \frac{g_{1}(x_{1})\chi_{1}\chi_{2}}{k_{b_{1}}^{2} - \chi_{1}^{2}} + \frac{1}{2}a_{1}^{2}\vec{d}_{1}^{2} - \frac{1}{\gamma_{1}}\tilde{\theta}_{1}^{T}\left(D^{\alpha}\hat{\theta}_{1} - \gamma_{1}\frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}}\varphi_{1}(x_{1})\right) \\ + \frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}}\left(\hat{\theta}_{1}^{T}\varphi_{1}(x_{1}) + f_{1}(x_{1}) + g_{1}(x_{1})\vartheta_{1} - D^{\alpha}y_{r}(t) + \frac{\chi_{1}}{2a_{1}^{2}\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)}\right)$$
(10)

The virtual control signal ϑ_1 and parameter estimation $D^{\alpha}\hat{\theta}_1$ are designed as follows

$$\vartheta_1 = \frac{1}{g_1(x_1)} \left(-\frac{b_1 \chi_1^{2\bar{\rho}-1}}{\left(k_{b_1}^2 - \chi_1^2\right)^{\bar{\rho}-1}} - f_1(x_1) - \hat{\theta}_1^{\mathrm{T}} \varphi_1(x_1) - \frac{\chi_1}{2a_1^2 \left(k_{b_1}^2 - \chi_1^2\right)} + D^{\alpha} y_r(t) \right) \quad (11)$$

$$D^{\alpha}\hat{\theta}_{1} = \gamma_{1}\frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}}\varphi_{1}(x_{1}) - \zeta_{1}\hat{\theta}_{1}$$
(12)

where $\bar{\rho} \in (0, 1)$, $b_1 > 0$ and $\zeta_1 > 0$.

Substituting (11) and (12) into (10), $D^{\alpha}V_1$ is presented as

$$D^{\alpha}V_{1} \leq \frac{g_{1}(x_{1})\chi_{1}\chi_{2}}{k_{b_{1}}^{2} - \chi_{1}^{2}} - \frac{b_{1}\chi_{1}^{2\bar{\rho}}}{\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)^{\bar{\rho}}} + \frac{1}{2}a_{1}^{2}\vec{d}_{1}^{2} + \frac{\zeta_{1}}{\gamma_{1}}\tilde{\theta}_{1}^{T}\hat{\theta}_{1}$$
(13)

Step 2: The Caputo fractional derivative of $\chi_2 = x_2 - \vartheta_1$ yields

$$D^{\alpha}\chi_{2} = f_{2}(\underline{x}_{2}) + g_{2}(\underline{x}_{2})(\chi_{3} + \vartheta_{2}) + \theta_{2}^{\mathrm{T}}\varphi_{2}(\underline{x}_{2}) + d_{2}(t) - D^{\alpha}Y_{1}$$
(14)

The Lyapunov candidate function is selected as

$$V_{2} = V_{1} + \frac{1}{2} \ln \frac{k_{b_{2}}^{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}} + \frac{1}{2\gamma_{2}} \tilde{\theta}_{2}^{\mathrm{T}} \tilde{\theta}_{2}$$
(15)

where $k_{b_2} > 0$ and $\gamma_2 > 0$. From (14) and (15), the derivative of V_2 is

$$D^{\alpha}V_{2} = D^{\alpha}V_{1} - \frac{1}{\gamma_{2}}\tilde{\theta}_{2}^{\mathrm{T}}D^{\alpha}\hat{\theta}_{2} + \frac{\chi_{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}}\tilde{\theta}_{2}^{\mathrm{T}}\varphi_{2}(\underline{x}_{2}) + \frac{\chi_{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}} \left(\hat{\theta}_{2}^{\mathrm{T}}\varphi_{2}(x_{2}) + f_{2}(\underline{x}_{2}) + g_{2}(\underline{x}_{2})(\chi_{3} + \vartheta_{2}) + d_{2}(t) - D^{\alpha}Y_{2,l}\right)$$
(16)

Using the Young's inequality, yields

$$\frac{\chi_2}{k_{b_2}^2 - \chi_2^2} d_2(t) \le \frac{\chi_2^2}{2a_2^2 \left(k_{b_2}^2 - \chi_2^2\right)^2} + \frac{1}{2}a_2^2 d_2^2 \tag{17}$$

where $a_2 > 0$ is a design parameter.

Substituting (17) into (16), yields

$$D^{\alpha}V_{2} \leq D^{\alpha}V_{1} + \frac{1}{2}a_{2}^{2}d_{2}^{2} - \frac{1}{\gamma_{2}}\tilde{\theta}_{2}^{\mathrm{T}}D^{\alpha}\hat{\theta}_{2} + \frac{\chi_{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}}\tilde{\theta}_{2}^{\mathrm{T}}\varphi_{2}(\underline{x}_{2}) + \frac{\chi_{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}} \left(\hat{\theta}_{2}^{\mathrm{T}}\varphi_{2}(\underline{x}_{2}) + f_{2}(\underline{x}_{2}) + g_{2}(\underline{x}_{2})(\chi_{3} + \vartheta_{2}) - D^{\alpha}\vartheta_{1} + \frac{\chi_{2}}{2a_{2}^{2}(k_{b_{2}}^{2} - \chi_{2}^{2})}\right)$$
(18)

The virtual controller ϑ_2 and the parameter estimation $D^{\alpha}\hat{\theta}_2$ are designed as

$$\vartheta_{2} = \frac{1}{g_{2}(\underline{x}_{2})} \left(-\frac{b_{2}\chi_{2}^{2\bar{\rho}-1}}{\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)^{\bar{\rho}-1}} - f_{2}(\underline{x}_{2}) - \hat{\theta}_{2}^{\mathrm{T}}\varphi_{2}(\underline{x}_{2}) - \frac{\chi_{2}}{2a_{2}^{2}\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)} + D^{\alpha}\vartheta_{1} - \frac{g_{1}(\underline{x}_{1})\chi_{1}\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)}{\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)} \right)$$
(19)
$$D^{\alpha}\vartheta_{1} = \varphi_{1} - \frac{\chi_{2}}{2a_{2}^{2}\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)} + D^{\alpha}\vartheta_{1} - \frac{g_{1}(\underline{x}_{1})\chi_{1}\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)}{\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)} \right)$$
(20)

$$D^{\alpha}\hat{\theta}_{2} = \gamma_{2}\frac{\chi_{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}}\varphi_{2}(\underline{x}_{2}) - \varsigma_{2}\hat{\theta}_{2}$$

$$\tag{20}$$

where $b_2 > 0$ and $\varsigma_2 > 0$ are the design parameters. According to (19) and (20), (18) can be rewritten as

$$D^{\alpha}V_{2} \leq D^{\alpha}V_{1} + \frac{\chi_{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}}g_{2}(\underline{x}_{2})\chi_{3} - \frac{b_{2}\chi_{2}^{2\bar{\rho}}}{\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)^{\bar{\rho}}} - \frac{g_{1}(\underline{x}_{1})\chi_{1}\chi_{2}}{\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)} + \frac{1}{2}a_{2}^{2}d_{2}^{2} + \frac{\varsigma_{2}}{\gamma_{2}}\bar{\theta}_{1}^{T}\hat{\theta}_{2}$$

$$\leq -\frac{b_{1}\chi_{1}^{2\bar{\rho}}}{\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)^{\bar{\rho}}} - \frac{b_{2}\chi_{2}^{2\bar{\rho}}}{\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)^{\bar{\rho}}} + \frac{g_{2}(\underline{x}_{2})\chi_{2}\chi_{3}}{k_{b_{2}}^{2} - \chi_{2}^{2}} + \frac{1}{2}a_{1}^{2}d_{1}^{2} + \frac{\varsigma_{1}}{\gamma_{1}}\tilde{\theta}_{1}^{T}\hat{\theta}_{1} + \frac{1}{2}a_{2}^{2}d_{2}^{2} + \frac{\varsigma_{2}}{\gamma_{2}}\tilde{\theta}_{2}^{T}\hat{\theta}_{2}$$

$$(21)$$

Step i(i = 3, 4, ..., n - 1): Define $\chi_i = x_i - \vartheta_{i-1}$, and we can obtain

$$D^{\alpha}\chi_{i} = f_{i}(\underline{x}_{i}) + g_{i}(\underline{x}_{i})(\chi_{i+1} + \vartheta_{i}) + \theta_{i}^{\mathrm{T}}\varphi_{i}(\underline{x}_{i}) + d_{i}(t) - D^{\alpha}\vartheta_{i-1}$$
(22)

Consider the Lyapunov candidate function as

$$V_{i} = V_{i-1} + \frac{1}{2} \ln \frac{k_{b_{i}}^{2}}{k_{b_{i}}^{2} - \chi_{i}^{2}} + \frac{1}{2\gamma_{i}} \tilde{\theta}_{i}^{\mathrm{T}} \tilde{\theta}_{i}$$
(23)

where $k_{b_i} > 0$, $\gamma_i > 0$, and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. According to (22) and (23), the derivative of V_i is

$$D^{\alpha}V_{i} = D^{\alpha}V_{i-1} + \frac{g_{i}(\underline{x}_{i})\chi_{i}\chi_{i+1}}{k_{b_{i}}^{2} - \chi_{i}^{2}} - \frac{1}{\gamma_{i}}\tilde{\theta}_{i}^{\mathrm{T}}D^{\alpha}\hat{\theta}_{i} + \tilde{\theta}_{i}^{\mathrm{T}}\varphi_{i}(\underline{x}_{i})\frac{\chi_{i}}{k_{b_{i}}^{2} - \chi_{i}^{2}} + \frac{\chi_{i}}{k_{b_{i}}^{2} - \chi_{i}^{2}}\left(f_{i}(\underline{x}_{i}) + g_{i}(\underline{x}_{i})\vartheta_{i} + \hat{\theta}_{i}^{\mathrm{T}}\varphi_{i}(\underline{x}_{i}) + d_{i}(t) - D^{\alpha}\vartheta_{i-1}\right)$$

$$(24)$$

Using the Young's inequality, yields

$$\frac{\chi_i}{k_{b_i}^2 - \chi_i^2} d_i(t) \le \frac{\chi_i^2}{2a_i^2 \left(k_{b_i}^2 - \chi_i^2\right)^2} + \frac{1}{2}a_i^2 \vec{d}_i^2$$
⁽²⁵⁾

where $a_i > 0$.

Substituting (25) into (24), yields

$$D^{\alpha}V_{i} \leq D^{\alpha}V_{i-1} + \frac{g_{i}(\underline{x}_{i})\chi_{i}\chi_{i+1}}{k_{b_{i}}^{2} - \chi_{i}^{2}} + \frac{\chi_{i}^{2}}{2a_{i}^{2}\left(k_{b_{i}}^{2} - \chi_{i}^{2}\right)^{2}} + \frac{1}{2}a_{i}^{2}d_{i}^{2}$$

$$+ \frac{\chi_{i}}{k_{b_{i}}^{2} - \chi_{i}^{2}}\left(f_{i}(\underline{x}_{i}) + g_{i}(\underline{x}_{i})\vartheta_{i} + \hat{\theta}_{i}^{T}\varphi_{i}(\underline{x}_{i}) - D^{\alpha}\vartheta_{i-1}\right)$$

$$- \frac{1}{\gamma_{i}}\tilde{\theta}_{i}^{T}D^{\alpha}\hat{\theta}_{i} + \tilde{\theta}_{i}^{T}\varphi_{i}(\underline{x}_{i})\frac{\chi_{i}}{k_{b_{i}}^{2} - \chi_{i}^{2}}$$

$$(26)$$

Construct the virtual control ϑ_i and the parameter estimation $D^{\alpha}\hat{\theta}_i$ as

$$\vartheta_{i} = \frac{1}{g_{i}(\underline{x}_{i})} \left(-\frac{b_{i}\chi_{i}^{2\bar{\rho}-1}}{\left(k_{b_{i}}^{2} - \chi_{i}^{2}\right)^{\bar{\rho}-1}} - f_{i}(\underline{x}_{i}) - \hat{\theta}_{i}^{\mathrm{T}}\varphi_{i}(\underline{x}_{i}) - \frac{\chi_{i}}{2a_{i}^{2}\left(k_{b_{i}}^{2} - \chi_{i}^{2}\right)} + D^{\alpha}\vartheta_{i-1} - \frac{g_{i-1}(\underline{x}_{i-1})\chi_{i-1}\left(k_{b_{i}}^{2} - \chi_{i}^{2}\right)}{k_{b_{i-1}}^{2} - \chi_{i-1}^{2}} \right)$$
(27)

$$D^{\alpha}\hat{\theta}_{i} = \gamma_{i} \frac{\chi_{i}}{k_{b_{i}}^{2} - \chi_{i}^{2}} \varphi_{i}(\underline{x}_{i}) - \varsigma_{i}\hat{\theta}_{i}$$
⁽²⁸⁾

where $b_i > 0$ and $\zeta_i > 0$.

Then, the $D^{\alpha}V_i$ can be rewritten as

$$D^{\alpha}V_{i} \leq D^{\alpha}V_{i-1} - \frac{b_{i}\chi_{i}^{2\bar{\rho}}}{\left(k_{b_{i}}^{2} - \chi_{i}^{2}\right)^{\bar{\rho}}} + \frac{g_{i}(\underline{x}_{i})\chi_{i}\chi_{i+1}}{k_{b_{i}}^{2} - \chi_{i}^{2}} - \frac{g_{i-1}(\underline{x}_{i-1})\chi_{i-1}\chi_{i}}{k_{b_{i-1}}^{2} - \chi_{i-1}^{2}} + \frac{1}{2}a_{i}^{2}d_{i}^{2} + \frac{\varsigma_{i}}{\gamma_{i}}\tilde{\theta}_{i}^{T}\hat{\theta}_{i} \leq -\sum_{j=1}^{i}\frac{b_{j}\chi_{j}^{2\bar{\rho}}}{\left(k_{b_{j}}^{2} - \chi_{j}^{2}\right)^{\bar{\rho}}} + \sum_{j=1}^{i}\frac{\varsigma_{j}}{\gamma_{j}}\tilde{\theta}_{i}^{T}\hat{\theta}_{j} + \sum_{j=1}^{i}\frac{1}{2}a_{j}^{2}d_{j}^{2} + \frac{g_{i}(\underline{x}_{i})\chi_{i}\chi_{i+1}}{k_{b_{i}}^{2} - \chi_{i}^{2}}$$
(29)

Step *n*: The Caputo fractional derivative of $\chi_n = x_n - \vartheta_{n-1}$ is given as

$$D^{\alpha}\chi_n = g_n(x)u + f_n(x) + \theta_n^{\mathrm{T}}\varphi_n(x) + d_n(t) - D^{\alpha}\vartheta_{n-1}$$
(30)

Consider the Lyapunov candidate function as

$$V_n = V_{n-1} + \frac{1}{2} \ln \frac{k_{b_n}^2}{k_{b_n}^2 - \chi_n^2} + \frac{1}{2\gamma_n} \tilde{\theta}_n^{\rm T} \tilde{\theta}_n$$
(31)

where $\gamma_n > 0$, $k_{b_n} > 0$, and $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$. The derivative of V_n is

$$D^{\alpha}V_{n} = D^{\alpha}V_{n-1} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}\tilde{\theta}_{n}^{\mathrm{T}}\varphi_{n}(x) - \frac{1}{\gamma_{n}}\tilde{\theta}_{n}^{\mathrm{T}}D^{\alpha}\hat{\theta}_{n} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}\left(f_{n}(x) + g_{n}(x)u + \hat{\theta}_{n}^{\mathrm{T}}\varphi_{n}(x) + d_{n}(t) - D^{\alpha}\vartheta_{n-1}\right)$$
(32)

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From the Young's inequality it is true that

$$\frac{\chi_n d_n(t)}{k_{b_n}^2 - \chi_n^2} \le \frac{\chi_n^2}{2a_n^2 \left(k_{b_n}^2 - \chi_n^2\right)^2} + \frac{1}{2}a_n^2 \bar{d}_n^2 \tag{33}$$

where $a_n > 0$.

The controller *u* with event-triggered mechanism and the parameter estimation $D^{\alpha}\hat{\theta}_n$ are constructed as follows

$$u(t) = \Theta(t_k), \forall t \in [t_k, t_{k+1})$$
(34)

$$t_{k+1} = \inf\{t \in \mathbb{R} ||\mathbf{Y}(t)| \ge \lambda_1^* |u(t)| + \lambda_2^*\}$$
(35)

$$\Theta(t) = -(1+\lambda_1^*) \left(\vartheta_n \tanh\left(\frac{\vartheta_n \chi_n}{\kappa^* \left(k_{b_n}^2 - \chi_n^2\right)}\right) + \bar{\lambda}_2^* \tanh\left(\frac{\bar{\lambda}_2^* \chi_n}{\kappa^* \left(k_{b_n}^2 - \chi_n^2\right)}\right) \right)$$
(36)

$$D^{\alpha}\hat{\theta}_{n} = \gamma_{n} \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \varphi_{n}(x) - \varsigma_{n}\hat{\theta}_{n}$$
(37)

where

$$\vartheta_{n} = \frac{1}{g_{n}(x)} \left(-\frac{b_{n}\chi_{n}^{2\bar{\rho}-1}}{\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)^{\bar{\rho}-1}} - f_{n}(x) - \hat{\theta}_{n}^{\mathrm{T}}\varphi_{n}(x) - \frac{\chi_{n}}{2a_{n}^{2}\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)} + D^{\alpha}\vartheta_{n-1} - \frac{g_{n-1}(\underline{x}_{n-1})\chi_{n-1}\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)}{k_{b_{n-1}}^{2} - \chi_{n-1}^{2}} \right)$$
(38)

and $c_n > 0$, $\varsigma_n > 0$ and $\kappa^* > 0$. $t_k (k \in \mathbb{Z}^+)$ is the update time. $Y(t) = \Theta(t) - u(t)$ is the sampling error, $\lambda_1^* \in (0, 1)$, $\bar{\lambda}_2^* > \lambda_2^*/1 - \lambda_1^*$ and $\lambda_2^* > 0$ are known parameters.

From $Y(t) = \Theta(t) - u(t)$ and (35), we have $\Theta(t) = (1 + \lambda_1^* \lambda_1(t))u(t) + \lambda_2^* \lambda_2(t)$, where $|\lambda_1(t)| \le 1$ and $|\lambda_2(t)| \le 1$. Then

$$u(t) = \frac{\Theta(t) - \lambda_2^* \lambda_2(t)}{1 + \lambda_1^* \lambda_1(t)}$$
(39)

Substitute (33)–(37) and (39) into (32), yields

$$D^{\alpha}V_{n} = D^{\alpha}V_{n-1} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}\tilde{\theta}_{n}^{\mathrm{T}}\varphi_{n}(x) - \frac{1}{\gamma_{n}}\tilde{\theta}_{n}^{\mathrm{T}}D^{\alpha}\hat{\theta}_{n} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(f_{n}(x) + g_{n}(x)u + \hat{\theta}_{n}^{\mathrm{T}}\varphi_{n}(x) + d_{n}(t) - D^{\alpha}\vartheta_{n-1}\right) = D^{\alpha}V_{n-1} + \frac{\chi_{n}g_{n}(x)}{k_{b_{n}}^{2} - \chi_{n}^{2}}\frac{\Theta(t)}{1 + \lambda_{1}^{*}\lambda_{1}(t)} - \frac{\chi_{n}g_{n}(x)}{k_{b_{n}}^{2} - \chi_{n}^{2}}\frac{\lambda_{2}^{*}\lambda_{2}(t)}{1 + \lambda_{1}^{*}\lambda_{1}(t)} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(f_{n}(x) + \hat{\theta}_{n}^{\mathrm{T}}\varphi_{n}(x) - D^{\alpha}\vartheta_{n-1} + \frac{\chi_{n}}{2a_{n}^{2}}\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)\right) + \frac{1}{2}a_{n}^{2}d_{n}^{2} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}\tilde{\theta}_{n}^{\mathrm{T}}\varphi_{n}(x) - \frac{1}{\gamma_{n}}\tilde{\theta}_{n}^{\mathrm{T}}D^{\alpha}\hat{\theta}_{n}$$

$$(40)$$

Theorem 1. Considering the FONSs (1) under Assumptions 1–3, suppose that initial constraint $x_i(0) \in \Omega_{x_i} = \{x_i | |x_i(0)| < k_{c_i}\}$ is satisfied, by designing the event-triggered actual controller (34), with virtual controllers (11), (19), (27), the parameter estimation (12), (20), (28) and (37), it confirms that: (1) all the signals are bounded and Zeno behavior is avoided; (2) the tracking signal can be well

tracked and the state variables x(t) keep within the set Ω_{x_i} in finite-time T; and (3) for $\forall t \geq T$, error χ_i satisfies $\chi_i \in \Omega_{\chi_i}$, i = 1, 2, ..., n, where

Proof. Due to $|\lambda_1(t)| \le 1$, $|\lambda_2(t)| \le 1$, we obtain

$$\frac{\Theta(t)\chi_n}{1+\lambda_1^*\lambda_1(t)} \le \frac{\Theta(t)\chi_n}{1+\lambda_1^*}$$

$$\frac{\lambda_2^*\lambda_2(t)}{1+\lambda_1^*\lambda_1(t)} \le \left|\frac{\lambda_2^*}{1-\lambda_1^*}\right|$$
(42)

Using Assumption 2 and substituting (38)–(34) and (42) into (40), one can get

$$D^{\alpha}V_{n} \leq D^{\alpha}V_{n-1} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(-\frac{b_{n}\chi_{n}^{2\bar{\rho}-1}}{\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)^{\bar{\rho}-1}} - \frac{g_{n-1}(\underline{x}_{n-1})\chi_{n-1}\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)}{k_{b_{n-1}}^{2} - \chi_{n-1}^{2}} \right) + g_{n}(x) \left(-\frac{\vartheta_{n}\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \tanh\left(\frac{\vartheta_{n}\chi_{n}}{\kappa^{*}\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)}\right) - \frac{\bar{\lambda}_{2}^{*}\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \tanh\left(\frac{\bar{\lambda}_{2}^{*}\chi_{n}}{\kappa^{*}\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)}\right) \right) - \frac{\chi_{n}g_{n}(x)}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(\frac{\lambda_{2}^{*}\chi_{n}}{1 + \lambda_{1}^{*}\lambda_{1}(t)} - g_{n}(x)\vartheta_{n}\frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(\frac{h_{n}\chi_{n}}{1 + \lambda_{1}^{*}\lambda_{1}(t)} - g_{n}(x)\vartheta_{n}\frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(\frac{h_{n}\chi_{n}}{1 + \lambda_{1}^{*}\lambda_{1}(t)} - g_{n}(x)\vartheta_{n}\frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(\frac{h_{n}\chi_{n}}{1 + \lambda_{1}^{*}\lambda_{1}(t)} - g_{n}(x)\vartheta_{n}\frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \right) + \frac{1}{2}a_{n}^{2}d_{n}^{2} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \left(-\frac{h_{n}\chi_{n}^{2\bar{\rho}-1}}{(k_{b_{n}}^{2} - \chi_{n}^{2})} - \frac{g_{n-1}(\underline{x}_{n-1})\chi_{n-1}\left(k_{b_{n}}^{2} - \chi_{n}^{2}\right)}{k_{b_{n-1}}^{2} - \chi_{n-1}^{2}}}\right) + 0.557\kappa^{*}g_{n}(x) + \frac{1}{2}a_{n}^{2}d_{n}^{2} + \frac{\chi_{n}}{k_{b_{n}}^{2} - \chi_{n}^{2}} \tilde{\theta}_{n}^{T}\varphi_{n}(x) - \frac{1}{\gamma_{n}}\tilde{\theta}_{n}^{T}\varphi_{n}(x) - \frac{1}{\gamma_{n}}\tilde{\theta}_{n}^{T}D^{\alpha}\hat{\theta}_{n}}$$

$$\leq -\sum_{i=1}^{n} \frac{b_{i}\chi_{i}^{2\bar{\rho}}}{\left(k_{b_{i}}^{2} - \chi_{i}^{2}\right)^{\bar{\rho}}} + \sum_{i=1}^{n} \frac{\xi_{i}}{\gamma_{i}}\tilde{\theta}_{i}^{T}\hat{\theta}_{i} + \sum_{i=1}^{n} \frac{1}{2}a_{i}^{2}d_{i}^{2} + 0.557\kappa^{*}g_{n}\max}$$

According to parameter estimation error $ilde{ heta}_i = heta_i - \hat{ heta}_i$, it holds

$$\tilde{\theta}_i^{\mathrm{T}} \hat{\theta}_i \le \frac{1}{2} \theta_i^{\mathrm{T}} \theta_i - \frac{1}{2} \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i \tag{44}$$

Then, one can obtain

$$\sum_{i=1}^{n} \frac{\zeta_{i}}{\gamma_{i}} \tilde{\theta}_{i}^{\mathrm{T}} \hat{\theta}_{i} \leq \sum_{i=1}^{n} \frac{\zeta_{i}}{2\gamma_{i}} \theta_{i}^{\mathrm{T}} \theta_{i} - \sum_{i=1}^{n} \frac{\zeta_{i}}{2\gamma_{i}} \tilde{\theta}_{i}^{\mathrm{T}} \tilde{\theta}_{i}$$

$$\tag{45}$$

From (45) and (43) can be rewritten as

$$D^{\alpha}V_{n} \leq -\sum_{i=1}^{n} \frac{b_{i}\chi_{i}^{2\bar{\rho}}}{\left(k_{b_{i}}^{2}-\chi_{i}^{2}\right)^{\bar{\rho}}} - \sum_{i=1}^{n} \frac{\zeta_{i}}{2\gamma_{i}}\tilde{\theta}_{i}^{\mathrm{T}}\tilde{\theta}_{i} + \sum_{i=1}^{n-1} \frac{q_{i+1}}{2}$$
$$+\sum_{i=1}^{n} \frac{\zeta_{i}}{2\gamma_{i}}\theta_{i}^{\mathrm{T}}\theta_{i} + \sum_{i=1}^{n} \frac{1}{2}a_{i}^{2}d_{i}^{2} + 0.557\kappa^{*}g_{n\max}$$
$$\leq -\bar{\sigma}\left(\sum_{i=1}^{n} \left(\frac{\chi_{i}^{2}}{2\left(k_{b_{i}}^{2}-\chi_{i}^{2}\right)}\right)^{\bar{\rho}} + \sum_{i=1}^{n} \frac{1}{2\gamma_{i}}\tilde{\theta}_{i}^{\mathrm{T}}\tilde{\theta}_{i}\right) + \Delta$$
(46)

where

$$\bar{\sigma} = \min\{2^{\bar{\rho}}b_i, \varsigma_i, i = 1, 2, \dots, n\}$$

$$\Delta = \sum_{i=1}^{n-1} \frac{q_{i+1}}{2} + \sum_{i=1}^n \frac{\varsigma_i}{2\gamma_i} \theta_i^{\mathrm{T}} \theta_i + \sum_{i=1}^n \frac{1}{2} a_i^2 \bar{d}_i^2 + 0.557 \kappa^* g_{n\max}$$
(47)

According to Lemma 3, choose $q_1 = 1, q_2 = \sum_{i=1}^{n} \frac{1}{2\gamma_i} \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i, s_1 = 1 - \bar{\rho}, s_2 = \bar{\rho}$ and $s_3 = \bar{\rho}^{\frac{\bar{\rho}}{1-\bar{\rho}}}$, then one can obtain

$$\left(\sum_{i=1}^{n} \frac{1}{2\gamma_i} \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i\right)^{\rho} \le (1-\bar{\rho})s_3 + \sum_{i=1}^{n} \frac{1}{2\gamma_i} \tilde{\theta}_i^{\mathrm{T}} \tilde{\theta}_i$$
(48)

Based on (48) and (46) can be described as

$$D^{\alpha}V_{n} \leq -\bar{\sigma}\sum_{i=1}^{n} \left(\frac{\chi_{i}^{2}}{2\left(k_{b_{i}}^{2}-\chi_{i}^{2}\right)}\right)^{\bar{\rho}} - \bar{\sigma}\left(\sum_{i=1}^{n}\frac{1}{2\gamma_{i}}\tilde{\theta}_{i}^{\mathrm{T}}\tilde{\theta}_{i}\right)^{\bar{\rho}} + \mathbf{M}$$

$$\tag{49}$$

where $M = \Delta + \bar{\sigma}(1 - \bar{\rho})s_3$.

According to Lemma 4, one can obtain

$$D^{\alpha}V_{n} \leq -\bar{\sigma}V_{n}^{\rho} + \mathbf{M}$$

$$\tag{50}$$

For $\forall \omega \in (0,1)$, (50) is rewritten as $D^{\alpha}V_n \leq -\bar{\sigma}\omega V_n^{\bar{\rho}} - \bar{\sigma}(1-\omega)V_n^{\bar{\rho}} + M$. If $V_n > (M/(\bar{\sigma}(1-\vartheta)))^{\frac{1}{\bar{\rho}}}$, we obtain

$$D^{\alpha}V_n \le -\bar{\sigma}\varpi V_n^{\rho} \tag{51}$$

Let $V = V_n^{1-\bar{\rho}}$, then $D^{\alpha}V^{\frac{1}{1-\bar{\rho}}} \leq -\bar{\sigma}\omega V^{\frac{\bar{\rho}}{1-\bar{\rho}}}$. From Lemma 5, we obtain

$$D^{\alpha}V^{\frac{1}{1-\bar{\rho}}} = \frac{\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}}\right)\Gamma(2-\alpha)}{\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}}-\alpha\right)}V^{\frac{\bar{\rho}}{1-\bar{\rho}}}(t)D^{\alpha}V$$
(52)

Then

$$D^{\alpha}V \leq -\bar{\sigma}\omega \frac{\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}}-\alpha\right)}{\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}}\right)\Gamma(2-\alpha)}$$
(53)

According to $V = V_n^{1-\bar{\rho}}$ and Caputo fractional integral, we obtain

$$V_n^{1-\bar{\rho}}(t) - V_n^{1-\bar{\rho}}(0) \le -\bar{\sigma}\omega \frac{\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}} - \alpha\right)}{\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}}\right)\Gamma(2-\alpha)} \frac{t^{\alpha}}{\Gamma(\alpha+1)}$$
(54)

From (54) and $V_n > (M/(\bar{\sigma}(1-\vartheta)))^{\frac{1}{p}}$, the finite time *T* can be designed as follow:

$$t < \left(\frac{\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}}\right)\Gamma(2-\alpha)\Gamma(\alpha+1)}{\bar{\sigma}\vartheta\Gamma\left(\frac{2-\bar{\rho}}{1-\bar{\rho}}-\alpha\right)}\left(V_n^{1-\bar{\rho}}(0) - \left(\frac{M}{\bar{\sigma}(1-\vartheta)}\right)^{\frac{1}{\bar{\rho}}}\right)\right)^{\frac{1}{\alpha}}$$
(55)

Then, we obtain $V_n \leq (M/(\bar{\sigma}(1-\omega)))^{\frac{1}{\bar{\rho}}}$. Using the definition of V_n , it holds

$$\frac{1}{2} \ln \frac{k_{b_i}^2}{k_{b_i}^2 - \chi_i^2} \le \left(M / (\bar{\sigma}(1 - \omega)) \right)^{\frac{1}{\bar{\rho}}}$$
(56)

This implies that

$$|\chi_k| \le k_{b_k} \sqrt{1 - e^{-2(\mathbf{M}/(\bar{\sigma}(1-\omega)))^{1/\bar{\rho}}}}, \forall t \ge T$$
(57)

It can be found that tracking error converges to Ω_{χ_i} in finite time *T*.

From $V_n \leq \left(M/(\bar{\sigma}(1-\omega))\right)^{\frac{1}{\bar{\rho}}}$, the boundedness of $\ln \frac{k_{b_k}^2}{k_{b_i}^2 - \chi_i^2}$ as well as $\tilde{\theta}_i$ can be obtained, which means that $|\chi_i|$ satisfies $|\chi_i| < k_{b_i}$. Due to $x_1 = \chi_1 + y_r(t)$ with the boundedness of χ_1 and $y_r(t)$, it holds that state x_1 is bounded. From (11), the virtual controller ϑ_1 is a function of χ_1 and $\hat{\theta}_1$. Then, ϑ_1 is also bounded with the supremum $\bar{\vartheta}_1$ of ϑ_1 . Based on the definition of $\chi_2 = x_2 - \vartheta_1$, it holds that ϑ_1 and x_2 are bounded. In the same way, the boundedness of $x_i(i = 3, ..., n)$, $\vartheta_j(j = 2, ..., n-1)$ and controller u can be obtained.

In order to avoid Zeno behavior, it requires to prove that there is a constant $t^* > 0$ such that for $k \in Z^+$, $t_{k+1} - t_k \ge t^*$. Based on the sampling error $Y(t) = \Theta(t) - u(t)$, it holds $D^{\alpha}|Y(t)| = sign(Y(t))D^{\alpha}Y(t) \le |D^{\alpha}\Theta(t)|$. From the definition of $\Theta(t)$ in (36), it holds that $\exists \zeta > 0$ such that $|D^{\alpha}\Theta(t)| < \zeta$. According to $Y(t_k) = 0$ and $\lim_{t \to t_{k+1}} Y(t) = \lambda_2^*$,

one gets $t_{k+1} - t_k \ge \lambda_2^* / \zeta = t^*$, which means that the Zeno behavior is avoided to occur.

Due to $x_1 = \chi_1 + y_r(t)$ with $|y_r(t)| \le A_0$ from Assumption 1, we get $|x_1| \le |\chi_1| + |y_r(t)| < k_{b_1} + A_0$. Define $k_{b_1} = k_{c_1} - A_0$, one can obtain $|x_1| < k_{c_1}$. From $x_2 = \chi_2 + \vartheta_1$, it holds $|x_2| \le |\chi_2| + |\vartheta_1| < k_{b_2} + \overline{\vartheta}_1$. Define $k_{b_2} = k_{c_2} - \overline{\vartheta}_1$, one can have $|x_2| < k_{c_2}$. Similarly, one can get $|x_i| < k_{c_i}$, i = 3, ..., n. Thus, all x(t) do not transgress the set Ω_{x_i} in finite-time *T*. \Box

4. Simulation

In this section, the validity of the devised finite time constraint ETC strategy will be illustrated by a single-machine-infinite bus power system presented as [54]:

$$D^{\alpha} x_{1} = 0.1 x_{1} + x_{2} + 0.03 \sin(t)$$

$$D^{\alpha} x_{2} = -0.02 x_{2} + 0.1 \cos(x_{1}) + 0.2$$

$$-\sin(x_{1}) + u + 0.2593 \sin(t)$$

$$y = x_{1}$$
(58)

where x_1 and x_2 are the system states, y is the output of systems, u is the controller input, $\alpha = 0.85$, $f_1(x_1) = 0$, $f_2(x) = -0.02x_2+0.2 - \sin(x_1)$, $\varphi_1(x_1) = x_1$, $\varphi_2(x) = \cos(x_1)$, $\theta_1 = \theta_2 = 0.1$, and $g_1(x_1) = g_2(x) = 1$. $d_1(t) = 0.03 \sin(t)$ and $d_2(t) = 0.2593 \sin(t)$ are the external disturbances. The virtual control laws are constructed as

$$\vartheta_{1} = -\frac{b_{1}\chi_{1}^{2\rho-1}}{\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)^{\rho-1}} - f_{1}(x_{1}) - \hat{\theta}_{1}^{\mathsf{T}}\varphi_{1}(x_{1}) - \frac{k_{b_{1}}^{2} - \chi_{1}^{2}}{2a_{1}^{2}\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)} + D^{\alpha}y_{r}(t) \vartheta_{2} = -\frac{b_{2}\chi_{2}^{2\rho-1}}{\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)^{\rho-1}} - f_{2}(x_{2}) - \hat{\theta}_{2}^{\mathsf{T}}\varphi_{2}(x_{2}) - \frac{\chi_{2}}{2a_{2}^{2}\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)} + D^{\alpha}\vartheta_{1} - \frac{g_{1}(x_{1})\chi_{1}\left(k_{b_{2}}^{2} - \chi_{2}^{2}\right)}{\left(k_{b_{1}}^{2} - \chi_{1}^{2}\right)}$$
(59)

The adaptive laws are given as

$$D^{\alpha}\hat{\theta}_{1} = \gamma_{1}\frac{\chi_{1}}{k_{b_{1}}^{2} - \chi_{1}^{2}}\varphi_{1}(x_{1}) - \zeta_{1}\hat{\theta}_{1}$$

$$D^{\alpha}\hat{\theta}_{2} = \gamma_{2}\frac{\chi_{2}}{k_{b_{2}}^{2} - \chi_{2}^{2}}\varphi_{2}(x_{2}) - \zeta_{2}\hat{\theta}_{2}$$
(60)

Design the following event-triggered strategy

$$\Theta(t) = -(1+\lambda_1^*) \left(\vartheta_2 \tanh\left(\frac{\vartheta_2 \chi_2}{\kappa^* \left(k_{b_2}^2 - \chi_2^2\right)}\right) + \bar{\lambda}_2^* \tanh\left(\frac{\bar{\lambda}_2^* \chi_2}{\kappa^* \left(k_{b_2}^2 - \chi_2^2\right)}\right) \right)$$

$$u(t) = \Theta(t_k), \forall t \in [t_k, t_{k+1})$$

$$t_{k+1} = \inf\left\{ t \in \mathbb{R} ||\mathbf{Y}(t)| \ge \lambda_1^* |u(t)| + \lambda_2^* \right\}$$
(61)

The design parameters in (59)–(61) are chosen as $a_1 = 0.065$, $a_2 = 0.1$, $b_1 = 1.1$, $b_2 = 1.6$, $\kappa^* = 2.1$, $\bar{\rho} = 0.5$, $\gamma_1 = 0.5$, $\gamma_2 = 1$, $\zeta_1 = 0.001$, $\zeta_2 = 0.08$, $\lambda_1^* = 0.0001$, $\lambda_2^* = 0.5$ and $\bar{\lambda}_2^* = 0.6001$. The initial conditions are selected as $x_1(0) = x_2(0) = 0$ and $\hat{\theta}_1(0) = \hat{\theta}_2(0) = 0$. The desired signal is $y_r(t) = 0.8 \sin(t)$, and the constraints are presented as $k_{c_1} = 1$ and $k_{c_2} = 1.3$.

Figures 1 and 2 show the trajectories of the reference signal, system output and system state. It is obvious that all system states maintain the given constraints by using the proposed finite-time ETC scheme. The event-triggered actuator control input *u* is described in Figure 3. Figure 4 presents the trajectories of parameters estimation. It can be found that the parameter estimation $\hat{\theta}_2$ is not smooth causing the compensation of nonlinear uncertainty to fluctuate, which is the main reason that the state trajectory of x_2 is not smooth in Figure 2. The sequence of steps of event-triggered sampling and the number of accumulated events are shown in Figure 5 and Figure 6, respectively. One can obtain that the stability and good tracking performance can be both guaranteed with the reduced communication burden.



Figure 1. System output *y* and tracking signal $y_r(t)$.



Figure 2. State trajectory *x*₂.



Figure 3. Control input *u*.



Figure 4. Parameters estimation $\hat{\theta}_1$ and $\hat{\theta}_2$.



Figure 5. The interval of the triggered transmission.



Figure 6. The number of the accumulated events.

5. Conclusions

An finite-time adaptive ETC scheme for FONSs with full-state constraints, uncertain parameters and external disturbances has been presented. By using the backstepping technology and a series of BLFs, the finite-time adaptive controller and parameter estimator are constructed. The dynamic event-triggered strategy is employed to overcome the limited communication resources. On the basis of the fractional-order Lyapunov analysis, all variables are bounded without the Zeno behavior and the tracking error converges to a small neighborhood around zero in finite time. The provided simulation results have further demonstrated the validity of the proposed control strategy. In the future, the controller design and stability analysis for nonlinear switching fractional order system controllers will be researched.

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