

## Article

# First-Principles Study on Thermodynamic, Structural, Mechanical, Electronic, and Phonon Properties of tP16 Ru-Based Alloys

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**Abstract:** Transition metal-ruthenium alloys are promising candidates for ultra-high-temperature structural applications. However, the mechanical and electronic characteristics of these alloys are not well understood in the literature. This study uses first-principles density functional theory calculations to explore the structural, electronic, mechanical, and phonon properties of  $X_3\text{Ru}$  ( $X = \text{Sc}, \text{Ti}, \text{V}, \text{Cr}, \text{Mn}, \text{Fe}, \text{Co}, \text{Ni}, \text{Cu}, \text{and Zn}$ ) binary alloys in the tP16 crystallographic phase. We find that  $\text{Mn}_3\text{Ru}$ ,  $\text{Sc}_3\text{Ru}$ ,  $\text{Ti}_3\text{Ru}$ ,  $\text{V}_3\text{Ru}$ , and  $\text{Zn}_3\text{Ru}$  have negative heats of formation and hence are thermodynamically stable. Mechanical analysis ( $C_{ij}$ ) indicates that all tP16- $X_3\text{Ru}$  alloys are mechanically stable except,  $\text{Fe}_3\text{Ru}$  and  $\text{Cr}_3\text{Ru}$ . Moreover, these compounds exhibit ductility and possess high melting temperatures. Furthermore, phonon dispersion curves indicate that  $\text{Cr}_3\text{Ru}$ ,  $\text{Co}_3\text{Ru}$ ,  $\text{Ni}_3\text{Ru}$ , and  $\text{Cu}_3\text{Ru}$  are dynamically stable, while the electronic density of states reveals all the  $X_3\text{Ru}$  alloys are metallic, with a significant overlap between the valence and conduction bands at the Fermi energy. These findings offer insights into the novel properties of the tP16  $X_3\text{Ru}$  intermetallic alloys for the exploration of high-temperature structural applications.

**Keywords:** intermetallics; phase stability; high-temperature structural applications; density functional theory



**Citation:** Mnisi, B.O.; Benecha, M.E.; Tibane, M.M. First-Principles Study on Thermodynamic, Structural, Mechanical, Electronic, and Phonon Properties of tP16 Ru-Based Alloys. *Alloys* **2024**, *3*, 126–139. <https://doi.org/10.3390/alloys3020007>

Academic Editor: Claudio Testani

Received: 4 March 2024

Revised: 9 April 2024

Accepted: 10 April 2024

Published: 15 April 2024



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## 1. Introduction

Intermetallic alloys offer potential as structural materials in extreme environments. Nickel-based superalloys (NBSAs) are the current material of choice in high-temperature structural applications such as in aerospace, marine, nuclear reactor, and chemical industries. This is attributed to the distinctive surface and mechanical properties of NBSAs [1], which arise from the  $\gamma/\gamma'$  microstructures in the  $L1_2$  cubic phase ( $\gamma$ ) and ordered tetragonal phase ( $\gamma'$ ), where the trans-granular and grain boundary weakness are found to play a critical role [2]. For this reason, research on high-temperature structural materials has revolved around NBSAs for a number of years [3–5]. Despite the accomplishment of NBSAs, they are currently limited by their high-temperature capability due to nickel's low melting point [6–8], thus hindering the development of next-generation, high-temperature structural engineering systems.

In this context, several intermetallic structures, including the refractory metals (RMs), platinum group metals (PGMs), as well as their binaries, have been studied in an effort to raise the melting point of NBSAs. However, RMs based on Mo, Nb, Ta, and W suffer intense oxidation in the air above 500 °C (normally referred to as pesting [9]), while the strength of titanium alloys deteriorates with increasing temperature. Although the PGMs (consisting of Pt, Ru, Os, Rh, Pd, and Ir) have comparable chemical properties and similar mineral deposits to NBSAs [10], their use is hampered by weight and cost.

The phase diagrams of the binaries of these metals have also been studied in an effort to find suitable stable alternatives to NBSAs. McAlister and Kahan [11] studied the phase diagram of Pt-Ga, and their findings [8] suggest that  $L_{12}$ -Pt<sub>3</sub>Ga transforms to tetragonal tP16-Pt<sub>3</sub>Ga at excessive temperatures of around 1290 °C [12]. On the other hand, Mishima et al. [13] found a phase transformation in Pt<sub>3</sub>Al from  $L_{12}$  to D0c and  $L_{12}$  to D0c', at around 420 °C and 130 °C, respectively [14,15]. Furthermore, Tibane et al. [16] studied the phase stability of Pt-Cr and Cr-Ru in various phases including A15, B2, tP16, D0c, D0c', and  $L_{12}$  and found tP16 Pt<sub>3</sub>Cr to be the most energetically favorable. In a similar study by Chauke et al. [17], the tP16 phase of Pt<sub>3</sub>Al was found to be the most stable. Other studies have similarly found the tP16 phase to be stable in a number of alloys [12,18].

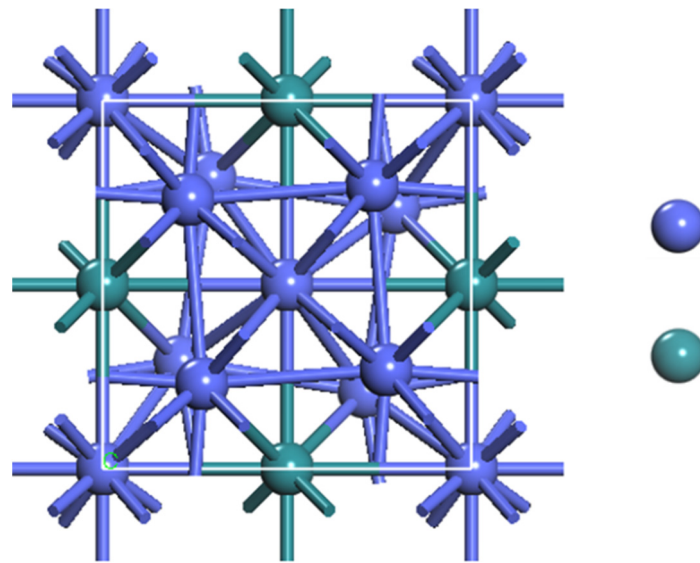
Recently, we have studied a number of [19–21] 3d-transition metal Ru-based alloys due to Ru potential as a high-temperature structural material [16]. Specifically, in A15 Cr<sub>3</sub>Ru and Ru<sub>3</sub>Cr alloys doped with 3d and 4d transition metals (Mn, Mo, Pt, Pd, Fe, Co, Re, and Zr), it was established that doping leads to thermodynamic stability. Furthermore, we investigated A15 X-Ru (X = Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn) [20], and it was found that Mn<sub>3</sub>Ru has a negative heat of formation, which may lead to its experimental realization, among other attractive mechanical properties. In D0c X<sub>3</sub>Ru, we found [19] that the calculated heat of formation for Sc<sub>3</sub>Ru, Ti<sub>3</sub>Ru, V<sub>3</sub>Ru, Mn<sub>3</sub>Ru, and Zn<sub>3</sub>Ru is negative, signifying their thermodynamic stability. The utilization of 3d transition metals is motivated by their capability to form alloys with relatively higher melting points and low densities [22]. Furthermore, the incorporation of transition metal elements is known to improve thermodynamic stability [21] and increase room temperature ductility in various materials [23]. It is therefore of interest to study the properties of Ruthenium-based transition metal alloys in other crystallographic phases in order to understand their full potential for high-temperature structural applications.

In this paper, we investigate the structural, electronic, and mechanical properties of tetragonal tP16 X<sub>3</sub>Ru (X = Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn) alloys using the density functional theory (DFT) technique [24,25]. Specifically, we compute their thermodynamic stability, mechanical stability, and electronic properties to determine their potential for high-temperature structural applications.

## 2. Methods

In this work, the properties of tP16 X<sub>3</sub>Ru (X = Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn) are studied by first-principles density functional theory method using the CASTEP code [25]. The interaction between the core and valence electrons is described by the Vanderbilt ultrasoft pseudopotential [26], with a converged plane wave cut-off of 800 eV, while the electronic exchange correlation is treated by the Perdew–Burke–Ernzerhof Generalized Gradient Approximation (PBE-GGA) functional incorporating the Hubbard parameter (U = 2.5 eV) [27]. The value of U in this study has been chosen to reproduce the experimental lattice constants and magnetic moments of the individual elements X (Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn) and Ru. The properties of the tP16 X<sub>3</sub>Ru (X = Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn) alloys were modeled from their respective unit cells (Figure 1), using a well-converged Monkhorst-Pack [28] k-point sampling of  $15 \times 15 \times 11$ .

For each system, geometry optimization was performed using the BFGS algorithm, with the self-consistent convergence of the total energy and the forces set to  $5.0 \times 10^{-6}$  eV/atom and 0.01 eV/Å. To ensure an accurate representation of the ground state magnetic properties, the smearing width was set to 0.001 eV, allowing an initial random magnetic moment corresponding to a ferromagnetic state. It should be noted that, by symmetry and periodicity of the supercell approach, any calculated magnetic moment is ferromagnetic, since the spin alignment is replicated in all neighboring image supercells. Phonon calculations were conducted using finite displacement method [29–32], utilizing larger supercells size of  $4 \times 4 \times 4$ , with a cut-off radius of 5.0 Å, and a k-grid sampling of  $5 \times 5 \times 3$ .



**Figure 1.** Crystal structure of primitive tetragonal  $X_3Ru$  alloy. The green balls represent Ru atoms, while the purple balls represent X (X = Sc – Zn) atoms.

### 3. Results and Discussion

#### 3.1. Crystal Structure

The  $X_3Ru$  alloy crystallizes in the space group  $P4/mbm$  (Pearson symbol  $tP16$ ) in a primitive tetragonal phase. The crystal structure consists of 16 atoms per unit cell (4Ru and 12X) with Ru on 4f (0, 0.5, 0.258), X1 on 4e (0, 0, 0.251), X2 on 4g (0.231, 0.731, 0), and X3 on 4h (0.29, 0.29, 0.5) sites.

The calculated equilibrium lattice constants, volume, heat of formation, magnetic moments, density, and tetragonality ( $c/a$  ratio) for these alloys are given in Table 1. We note that the proposed  $tP16$   $X_3Ru$  systems are relatively new; hence, their fundamental data are limited in the literature. In order to validate our model, we find that the calculated lattice constant of  $Cr_3Ru$  is in accord with the experiment [33] and literature [21]. Similarly, our calculated lattice constants of the individual metals are in agreement with experimental data [34] (see Supplementary Materials (references [16,35,36] are cited in the Supplementary Materials)).

For a given material system, the heat of formation  $\Delta H_f$  describes the strength of atomic interactions, which indicate the thermodynamic stability of the material, and it is given by Equation (1):

$$\Delta H_f = \left[ \frac{E_{structure} - \sum_i x_i E_i}{x_T} \right] \quad (1)$$

where  $E_{(structure)}$  and  $E_{(i)}$  are the calculated equilibrium total energies of the alloy system and that of the individual elemental species  $i$ , with total atomic concentrations  $x_T$ . The calculated heat of formation for each alloy system is presented in Table 1. Negative heat of formation suggests thermodynamic stability, whereas a positive value indicates thermodynamic instability under equilibrium conditions. The calculated heat of formation is comparable to previous theoretical data in the case of  $Cr_3Ru$ , with a slight discrepancy that can be attributed to the use of different exchange correlation functionals. In this study, the on-site Hubbard  $U$  was incorporated into the generalized gradient approximation (GGA) in order to accurately describe the ground-state magnetic moments, whereas the previous studies employed plain GGA [16].

**Table 1.** Optimized lattice constants (a, c), volume ( $V_0$ ), ratio of c/a, heat of formation ( $\Delta H_f$ ), magnetic moment ( $\mu_B$ ), and density of tP16  $X_3Ru$  ( $X = Sc - Zn$ ) structures.

Alloy	a (Å)	c (Å)	$V_0$ (Å <sup>3</sup> )	$\Delta H_f$ (eV/Atom)	Magnetic Moments ( $\mu_B$ /Atom)	Density (g/cm <sup>3</sup> )	c/a
Sc <sub>3</sub> Ru	6.23	8.97	348.07	−0.18	0.02	4.50	1.44
Ti <sub>3</sub> Ru	5.40	9.28	270.39	−0.41	0.89	6.01	1.72
V <sub>3</sub> Ru	5.73	8.11	266.05	−0.27	2.19	6.34	1.42
Cr <sub>3</sub> Ru	6.04	7.28	265.89	0.11	0.68	6.42	1.21
Cr <sub>3</sub> Ru [16]	5.20			0.24			
Mn <sub>3</sub> Ru	5.75	8.14	269.02	−1.61	3.25	6.56	1.42
Fe <sub>3</sub> Ru	5.07	8.02	206.15	0.07	0.95	8.65	1.58
Co <sub>3</sub> Ru	5.20	7.34	198.30	0.10	1.82	9.31	1.41
Ni <sub>3</sub> Ru	5.15	7.28	192.89	0.92	1.08	9.55	1.41
Cu <sub>3</sub> Ru	5.21	7.37	200.43	0.31	0.00	9.67	1.41
Zn <sub>3</sub> Ru	5.36	7.57	217.32	−0.23	0.00	9.08	1.41
t16-Pt <sub>3</sub> Al [17]	5.4			−0.79			1.45
tP16-Pt <sub>3</sub> Ga [12]	5.5	7.80				6.14	
L1 <sub>2</sub> -Pt <sub>3</sub> Al [37]	3.9						

We find that the heat of formation for Sc<sub>3</sub>Ru, Ti<sub>3</sub>Ru, V<sub>3</sub>Ru, Mn<sub>3</sub>Ru, and Zn<sub>3</sub>Ru is negative. This result indicates that these alloys are thermodynamically stable and hence can be synthesized under equilibrium experimental conditions. Conversely, Ni<sub>3</sub>Ru, Co<sub>3</sub>Ru, Cu<sub>3</sub>Ru, and Fe<sub>3</sub>Ru exhibit positive heat of formation, rendering them unstable, and can only be achieved under non-equilibrium conditions. It is noteworthy that the stable systems of Ti, V, Cr, Mn, Co, Ni, and Cu possess relatively high magnetic moments, which can potentially expand the use of these alloys in spintronic applications.

The density of a material describes its lightness/heaviness and holds an essential responsibility in aerospace applications specifically for spinning components in aircraft wings and turbine blades. It is calculated as follows:

$$\rho^{cal} = \left[ \frac{M_W * N}{Vol * A_0} \right] \quad (2)$$

where  $M_W$  is the molecular weight of the alloy,  $N$  is the number of atoms,  $Vol$  is the volume of the unit cell, and  $A_0$  is Avogadro's number ( $6.022 \times 10^{23}$ ). Table 1 indicates that the densities for  $X_3Ru$  alloys range from 4.5 g/cm<sup>3</sup> to 9.1 g/cm<sup>3</sup>. It is noted that the thermodynamically stable Sc<sub>3</sub>Ru and Ti<sub>3</sub>Ru have the lowest densities compared to the currently used L1<sub>2</sub>-Ni<sub>3</sub>Al density.

We have further determined the tetragonality (c/a) ratio of the  $X_3Ru$  alloys in order to evaluate possible phase changes that might exist in them. The tetragonality ratio is found to range from 1.21 to 1.72, with X = Co, Ni, Cu, and Zn possessing a constant value of 1.41, which indicates the absence of phase transition, while in Ti<sub>3</sub>Ru, c/a = 1.7, indicating a possible phase change.

### 3.2. Mechanical Properties

#### 3.2.1. Elastic Constants

Mechanical properties describe the resistance of crystal structures to external stress or strain applied to them. It was evaluated by applying a minimum strain to the unit cell to

enable the total energy difference to be determined [38,39]. The elastic strain is calculated using Equation (3).

$$U = \frac{\Delta E}{V_0} \frac{1}{2} \sum_i^6 \sum_j^6 C_{ij} e_i e_j \quad (3)$$

where  $\Delta E$  is the total energy change of the deformed unit cell relative to the initial cell,  $V_0$  is the unstrained volume,  $C_{ij}$  ( $i, j = 1$  to 6) are the elastic constants, and  $e_i$  or  $e_j$  corresponds to the strain. In this study, the stress–strain method based on Hook’s law was used to calculate the elasticity and the elastic stiffness tensors, expressed as follows [40]:

$$\{\delta_{ij}\} = [C_{ij}]\{\varepsilon_{ij}\} \quad (4)$$

where  $\{\delta_{ij}\}$  and  $\{\varepsilon_{ij}\}$  are the stress and strain tensors, respectively. For primitive tetragonal  $X_3\text{Ru}$  ( $X = \text{Sc} - \text{Zn}$ ) structures, there are six independent components ( $C_{11}$ ,  $C_{12}$ ,  $C_{13}$ ,  $C_{33}$ ,  $C_{44}$ ,  $C_{66}$ ) calculated at zero pressure. Based on the calculated elastic constants, the Bohr mechanical stability criteria were examined for all the structures as follows:

$$\begin{aligned} C_{11} > 0, C_{33} > 0, C_{44} > 0, C_{66} > 0, C_{11} - C_{12} > 0, \\ (C_{33} + C_{11} - 2C_{13}) > 0, (2C_{11} + 2C_{12} + C_{33} + 4C_{13}) > 0 \end{aligned} \quad (5)$$

Table 2 presents the elastic constants ( $C_{ij}$ ) and melting temperatures for  $X_3\text{Ru}$  compounds. Notably, all structures exhibit  $C_{ij} > 0$ , indicating compliance with the mechanical stability criteria. However, the  $\text{Fe}_3\text{Ru}$  and  $\text{Cr}_3\text{Ru}$  structures are mechanically unstable due to  $C_{66} < 0$  and  $C_{44} < 0$ . A discernible correlation emerges between the elastic constants and the melting temperature of materials [41–44]. Noteworthy is the observation that the melting temperature of the mechanically stable  $\text{Ni}_3\text{Ru}$  surpasses that of the currently used  $\text{L}_{12}\text{Ni}_3\text{Al}$  (Table 2). Additionally,  $\text{Ni}_3\text{Ru}$  has the highest melting temperature, outperforming  $\text{Cr}_3\text{Ru}$ . Consequently,  $\text{Ni}_3\text{Ru}$  proves to be a viable candidate for high-temperature structural applications.

**Table 2.** Elastic constants ( $C_{ij}$ ) and melting temperatures (MT) of  $X_3\text{Ru}$  ( $X = \text{Sc} - \text{Zn}$ ) structures. The data for the currently used alloy  $\text{L}_{12}\text{Ni}_3\text{Al}$  structure are included for comparison.

Phase	$C_{11}$	$C_{12}$	$C_{13}$	$C_{33}$	$C_{44}$	$C_{66}$	MT $\pm 300$ K
$\text{Sc}_3\text{Ru}$	120.46	64.81	62.42	95.48	38.42	8.42	858.60
$\text{Ti}_3\text{Ru}$	172.26	77.43	91.89	162.47	154.62	3.50	1114.49
$\text{V}_3\text{Ru}$	192.82	24.35	73.52	154.00	58.86	47.65	1163.46
$\text{Cr}_3\text{Ru}$	51.53	36.48	53.03	5.39	−3.65	15.73	516.68
$\text{Mn}_3\text{Ru}$	190.79	29.40	83.55	130.89	77.64	25.21	1122.71
$\text{Fe}_3\text{Ru}$	361.31	77.92	141.14	237.96	39.08	−42.41	1794.87
$\text{Co}_3\text{Ru}$	330.98	68.68	170.26	270.15	130.85	36.12	1602.33
$\text{Ni}_3\text{Ru}$	337.15	115.37	166.10	286.49	117.29	63.19	1795.19
$\text{Cu}_3\text{Ru}$	262.26	87.26	153.90	192.07	94.72	28.84	1428.89
$\text{Zn}_3\text{Ru}$	247.59	68.25	122.39	177.61	89.16	53.27	1363.19
$\text{L}_{12}\text{-Ni}_3\text{Al}$ [45]	235	157			128		1691.00
$\text{L}_{12}\text{-Pt}_3\text{Al}$ [37]	317	177			111		

To assess the suitability of the  $X_3\text{Ru}$  ( $X = \text{Sc}, \text{Ti}, \text{V}, \text{Cr}, \text{Mn}, \text{Fe}, \text{Co}, \text{Ni}, \text{Cu}, \text{and Zn}$ ) alloys for high-temperature structural application, the melting temperature ( $T_m$ ) was calculated

using the  $C_{ij}$  elastic parameters. For tetragonal structures, the melting temperature ( $T_m$ ) was calculated from Equation (6):

$$T_m = 354 \text{ K} + \left( \frac{450 \text{ K}}{\text{Mbar}} \right) * \left[ \frac{1}{3} (2C_{11} + C_{33}) \right] \pm 300 \text{ K} \quad (6)$$

where Mbar is a mega bar. There exists a direct proportionality between the elastic constants and the melting temperature of materials, as evidenced in references [41–44]. The melting temperature data of the  $X_3\text{Ru}$  alloys range from 516.68 K to 1795.19 K, with  $\text{Ni}_3\text{Ru}$  having the highest melting temperature. Notably, the thermodynamically and mechanically stable  $\text{Sc}_3\text{Ru}$ ,  $\text{Ti}_3\text{Ru}$ ,  $\text{V}_3\text{Ru}$ ,  $\text{Mn}_3\text{Ru}$ , and  $\text{Zn}_3\text{Ru}$  alloys exhibit melting temperatures of 858.60 K, 1114.49 K, 1163.46 K, 1122.71 K, and 1363.19 K, respectively, which are in the range of the currently utilized  $\text{L}_{12} \text{Ni}_3\text{Al}$ .

### 3.2.2. Bulk, Shear, and Young's Moduli

Based on the  $C_{ij}$ , other elastic parameters can be calculated such as bulk modulus ( $B$ ), shear modulus ( $G$ ), and Young's modulus ( $E$ ), which are obtained from the Voigt, Reuss, and Hill approximations [46,47]. Bulk modulus ( $B$ ) highlights the resistance to compression depending on the crystal structure of a material. High compressibility in materials attributes to large bulk modulus, while low bulk modulus constitutes low compressibility. For tetragonal structures,  $B$  is given as follows:

$$B_V = \frac{1}{9} ((2C_{11} + C_{12}) + C_{33} + 4C_{13}) \quad (7)$$

and the Reuss bounds are given as follows:

$$B_R = \frac{C^2}{M} \quad (8)$$

where

$$M = C_{11} + C_{12} + 2C_{33} - 4C_{13} \quad (9)$$

with  $B_R$ ,  $B = B_H$ , and  $B_V$  being the Bulk modulus for Reuss ( $R$ ), Voigt ( $V$ ), and Hill ( $H$ ) approximations [47]. The shear modulus ( $G$ ) of a material describes its response to shear stress and is a measure of a material's stiffness. For tetragonal structures,  $G$  is expressed as follows:

$$G_V = \frac{1}{30} (M + 3C_{11} - 3C_{12} + 12C_{44} + 6C_{66}) \quad (10)$$

where,  $M = C_{11} + C_{12} + 2C_{33} - 4C_{13}$

$$G_R = 15 \left[ \left( \frac{18B_V}{C^2} \right) + \left( \frac{6}{C_{11} - C_{12}} \right) + \left( \frac{6}{C_{44}} \right) + \left( \frac{3}{C_{66}} \right) \right]^{-1} \quad (11)$$

where  $C^2 = (C_{11} + C_{12})C_{33} - 2C_{13}^2$ .

$G_R$  and  $G_V$  are the Reuss and Voigt bounds [46]. Young's modulus and Poisson's ratio  $\nu$  are independent of the type of crystal structure of a material and are given as follows:

$$E = \left( \frac{9B_X G_X}{3B_X + G_X} \right) \nu = \left( \frac{3B_X - 2G_X}{2(B_X + G_X)} \right) \quad (12)$$

where  $X = V, R$ , and  $H$  approximations.

The Voigt, Reuss, and Hill approximations [46,47] utilize elastic constants to calculate macroscopic quantities, such as bulk ( $B$ ) modulus, shear ( $G$ ) modulus, and Young's ( $E$ ) modulus. For tetragonal structures, the bulk, shear, and Young's modulus were derived from the elastic constants [48,49], as summarized in Table 3. The bulk and shear modulus represent volume and shape changes, respectively, while Young's modulus indicates



material stiffness. Table 3 illustrates that the Ni<sub>3</sub>Ru structure demonstrates the highest bulk, shear, and Young's modulus, whereas Mn<sub>3</sub>Ru exhibits the lowest values. In addition, Cr<sub>3</sub>Ru possesses a negative shear modulus and Young's modulus. The negative shear and Young moduli are due to phase change leading to instability. This instability is also observed in ferro-elastic phase transformation [50]. The negative elastic modulus is due to Landau theory [51] when two local minima form in a strain energy function. Furthermore, solids with negative elastic modulus can be stabilized with sufficient constraint.

**Table 3.** Bulk (GPa), shear (GPa), Young modulus (GPa),  $B_H/G_H$ , Poisson's ratio ( $\nu$ ), and Vickers hardness ( $H_V$ ) of X<sub>3</sub>Ru (X = Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn) structures, where V, R, and H stands for Voigt, Reuss, and Hill approximations. The UND = undefined. Data for L1<sub>2</sub>-Ni<sub>3</sub>Al and L1<sub>2</sub>-Pt<sub>3</sub>Al are included for comparison.

tP16 Phase	B <sub>V</sub>	G <sub>V</sub>	B <sub>R</sub>	G <sub>R</sub>	B <sub>H</sub>	G <sub>H</sub>	E <sub>H</sub>	B <sub>H</sub> /G <sub>H</sub>	$\nu$	H <sub>V</sub>
Sc <sub>3</sub> Ru	79.52	26.84	78.21	19.61	78.86	23.22	63.43	3.40	0.366	0.01
Ti <sub>3</sub> Ru	114.38	78.93	114.35	14.34	114.37	46.64	83.28	2.45	0.321	3.63
V <sub>3</sub> Ru	98.05	57.62	97.94	53.86	97.99	55.74	140.57	1.76	0.261	8.35
Cr <sub>3</sub> Ru	43.73	−0.58	45.44	12.43	44.58	−6.51	−20.53	−6.85	0.577	UND
Mn <sub>3</sub> Ru	100.61	57.16	100.56	42.22	100.58	49.69	127.99	2.02	0.288	5.61
Fe <sub>3</sub> Ru	186.78	47.18	184.48	96.20	185.63	71.69	190.54	2.59	0.329	5.00
Co <sub>3</sub> Ru	194.50	94.42	193.07	67.62	193.79	81.02	213.33	2.39	0.317	6.43
Ni <sub>3</sub> Ru	206.21	93.77	206.21	85.46	206.21	89.61	234.82	2.30	0.310	7.46
Cu <sub>3</sub> Ru	167.41	65.09	167.39	42.36	167.40	53.72	145.59	3.12	0.355	2.44
Zn <sub>3</sub> Ru	144.32	70.30	144.01	58.49	144.17	64.39	168.14	2.24	0.306	5.90
Li <sub>2</sub> -Ni <sub>3</sub> Al [45]					183.00	92.00	237.00	1.98	0.280	
Li <sub>2</sub> -Pt <sub>3</sub> Al [37]								2.43		

Consequently, Ni<sub>3</sub>Ru displays superior resistance to volume and shape changes, emerging as the most compression-resistant material under zero-pressure conditions. At the same time, the G values in X<sub>3</sub>Ru (X = Mn, Fe, Ni, Co, and Zn) alloys are lower than the B values, consistent with observations in reference [52]. This difference suggests that G limits the stability of these alloys, aligning with trends documented in the aforementioned reference.

The assessment of material ductility/brittleness can be effectively performed using the bulk-to-shear modulus ratio (B/G), as introduced by Pugh [53]. A high B/G ratio is associated with ductility, while a low ratio corresponds to brittleness, with the critical value between them being 1.75. The calculated B/G ratios are detailed in Table 3, revealing that all X<sub>3</sub>Ru structures, except Cr<sub>3</sub>Ru, exhibit ductility. Notably, the B/G ratios for all structures surpass those of L1<sub>2</sub>-Ni<sub>3</sub>Al, except for Cr<sub>3</sub>Ru and V<sub>3</sub>Ru. Furthermore, Poisson's ratio ( $\nu$ ) serves as an additional indicator for ductility or brittleness, as outlined by Frantsevich [54]. Materials with  $\nu > 0.3$  are considered ductile, while those with  $\nu < 0.3$  are deemed brittle. The values for Poisson's ratio for all structures exceed 0.3, aligning with the ductile nature predicted by the B/G ratios above, except for the Cr<sub>3</sub>Ru structure.

Additionally, we conducted calculations for Vickers hardness, a metric that gauges a material's resistance to localized plastic deformation caused by either mechanical indentation or abrasion. The hardness of the X<sub>3</sub>Ru alloys is determined using the Vickers hardness equation [55]:

$$H_V = 2(K^2 G H)^{0.585} - 3 \quad (13)$$

where K represents the  $G_H/B_H$  ratio of the shear modulus to the bulk modulus. In mechanical properties, Vickers hardness assumes significant importance, particularly in high-temperature applications, where micro-cracks are prevalent. The calculated Vickers hard-

ness values for the primitive tetragonal  $X_3\text{Ru}$  structures are outlined in Table 3. Notably,  $\text{V}_3\text{Ru}$  exhibits the highest hardness, whereas  $\text{Cu}_3\text{Ru}$  has the lowest value. Consequently,  $\text{V}_3\text{Ru}$  showcases superior resistance to localized plastic deformation induced by abrasion or mechanical indentation.

### 3.2.3. Elastic Anisotropy

Anisotropic behavior refers to a material's directional dependence on a physical property, a phenomenon linked to micro-cracks, phase transformation, precipitation, and dislocation dynamics [40]. This directional dependence is represented by three elastic factors  $A_1$ ,  $A_2$ , and  $A_3$  corresponding to different crystallographic planes, as well as the universal anisotropic index ( $A^U$ ) and the compression percentage ( $A_B$ ) anisotropy, respectively. This holds true particularly for tetragonal structures as given in Equation (14).

$$A_1 = \frac{2C_{66}}{C_{11}-C_{12}}, A_2 = A_3 = \frac{4C_{44}}{C_{11}+C_{33}-2C_{13}} \quad (14)$$

$$A^U = 5\frac{G_V}{G_R} + \frac{B_V}{B_R} - 6 \text{ and } A_B = \frac{B_V-B_R}{B_V+B_R} * 100\%$$

In the context of shear anisotropic factors,  $A_1$ ,  $A_2$ , and  $A_3$  correspond to the (001), (010), and (100) shear planes, respectively. In isotropic structural materials, these factors ( $A_1$ ,  $A_2$ , and  $A_3$ ) should ideally be equal to one. Similarly, the value of the universal anisotropic index ( $A^U$ ) should be zero for isotropy. Deviations from these values, either one or zero, indicate the degree of elastic anisotropy in a crystal. The terms  $B_V$ ,  $B_R$ ,  $G_V$ , and  $G_R$  represent the Voigt and Reuss approximations in bulk modulus (B) and shear modulus (G). The maximum value for the compression percentage ( $A_B$ ) is 100%, and the minimum is zero. A value of zero signifies isotropy, while 100% indicates maximum anisotropy. The calculated Zener anisotropy factor ( $A_1$ ) values indicate that they are both less than and greater than one for  $A_2$  and  $A^U$ , signifying that all the structures exhibit elastic anisotropy under zero-pressure conditions, as shown in Table 4. Additionally, the  $A_B$  value for the bulk modulus percentage is zero for  $\text{Ni}_3\text{Ru}$ ,  $\text{Ti}_3\text{Ru}$ , and  $\text{Cu}_3\text{Ru}$ , suggesting isotropy in their structures, while the rest are characterized by elastic anisotropy.

**Table 4.** Shear anisotropy factors ( $A_1$ ,  $A_2$ ), universal elastic anisotropy index ( $A^U$ ), and the percentage of anisotropy ( $A_B$ ) of  $X_3\text{Ru}$  ( $X = \text{Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn}$ ) structures.

Phase	$A_1$	$A_2$	$A^U$	$A_B$
$\text{Sc}_3\text{Ru}$	0.30	1.69	1.86	0.83
$\text{Ti}_3\text{Ru}$	0.07	4.10	22.53	0.01
$\text{V}_3\text{Ru}$	0.57	1.18	0.35	0.06
$\text{Cr}_3\text{Ru}$	2.09	0.30	−4.80	−1.92
$\text{Mn}_3\text{Ru}$	0.31	2.01	1.77	1.77
$\text{Fe}_3\text{Ru}$	−0.30	0.49	−2.53	3.45
$\text{Co}_3\text{Ru}$	0.28	2.01	1.99	0.37
$\text{Ni}_3\text{Ru}$	0.57	1.61	0.49	0.00
$\text{Cu}_3\text{Ru}$	0.33	2.59	2.68	0.01
$\text{Zn}_3\text{Ru}$	0.59	1.98	1.01	0.11

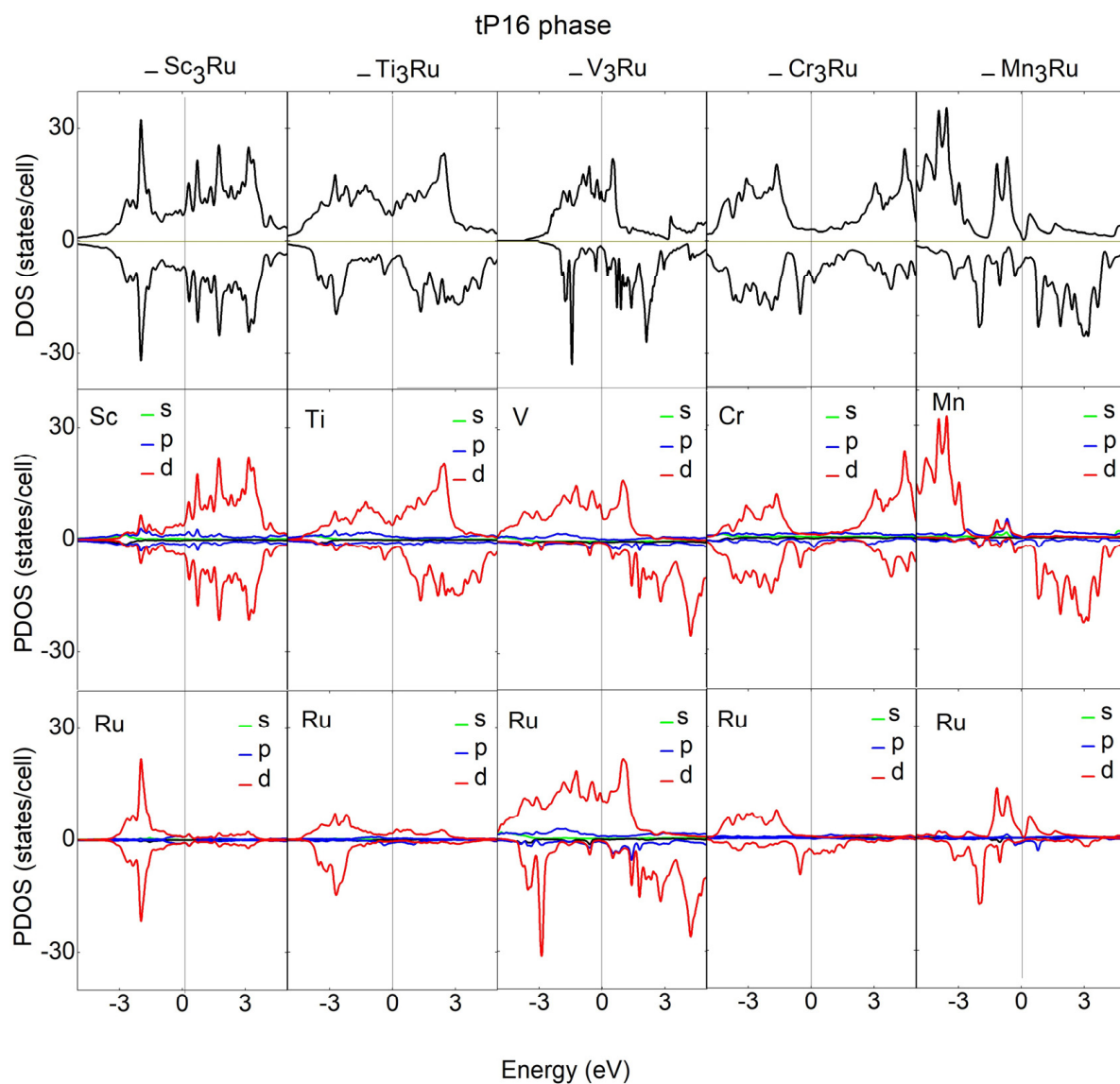
### 3.3. Electronic and Magnetic Properties

To gain insights into the bonding characteristics of tP16  $X_3\text{Ru}$  alloys ( $X = \text{Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn}$ ), their electronic and magnetic properties were investigated. The calculated magnetic moments of these alloys are detailed in Table 1. Notably, the thermodynamically and mechanically stable  $\text{V}_3\text{Ru}$  and  $\text{Mn}_3\text{Ru}$  exhibit the highest magnetic moments of  $2.19 \mu_B$  and  $3.25 \mu_B$ , respectively. Additionally,  $\text{Co}_3\text{Ru}$  and  $\text{Ni}_3\text{Ru}$ , which are mechanically stable, demonstrate magnetic moments of  $1.82 \mu_B$  and  $1.08 \mu_B$ .

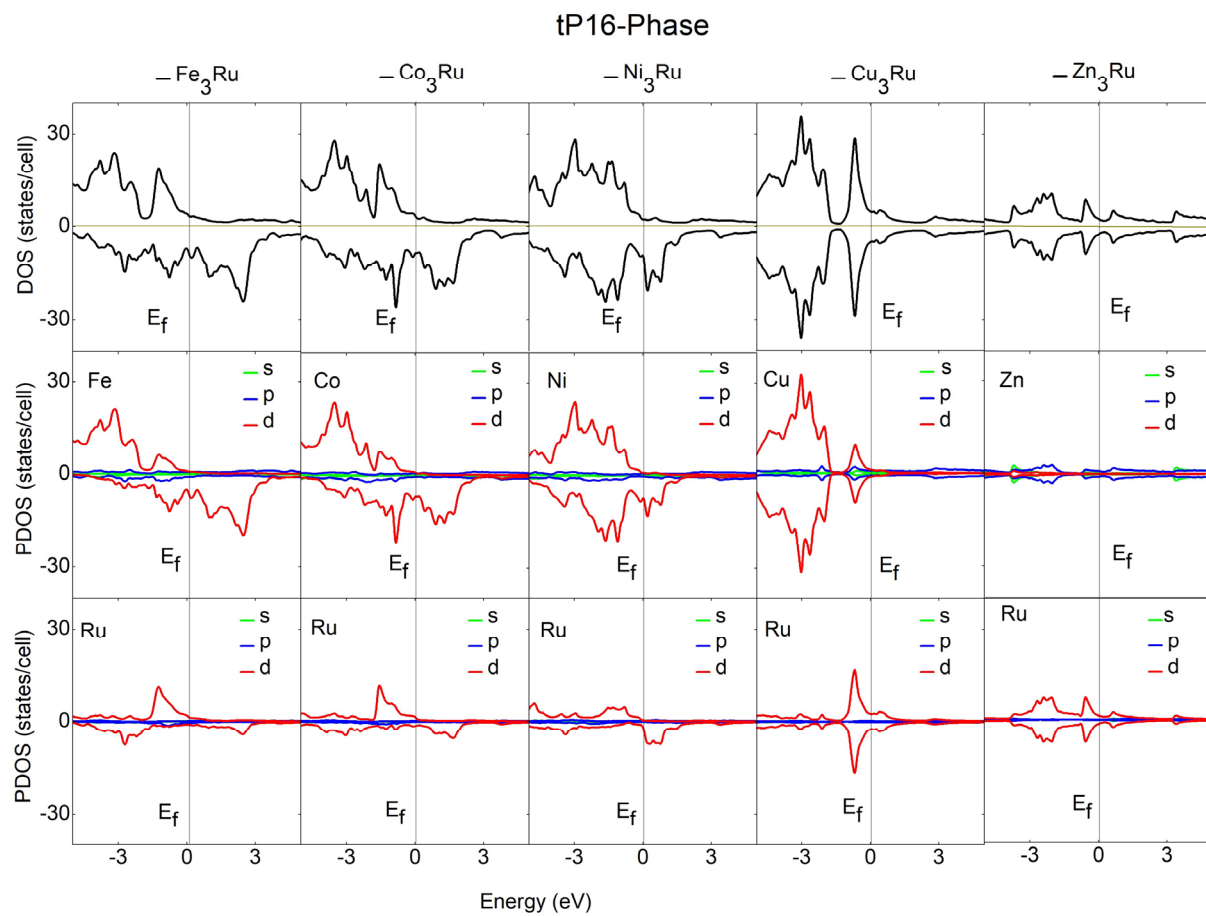


The relationship between a material's structural stability and magnetic moments relies on its electronic properties [56]. Figures 2 and 3 show the total density of states (TDOS) and partial density of states (PDOS) for  $X_3\text{Ru}$  alloys in the tP16 phase. An electronic overlap is observed between the valence and conduction bands around the Fermi energy level of  $X_3\text{Ru}$  alloys, signifying a metallic nature. These findings are consistent with previous theoretical studies [16,18,24]. To evaluate the individual atom contributions to the TDOS, we calculated the PDOS for  $X_3\text{Ru}$  alloys, as depicted in Figures 2 and 3. It is evident that the transition metal and Ru 3d orbitals are the major contributors to the total density of states around the Fermi level, with the s, p orbitals making minimal contribution.

Moreover, the symmetry of the spin-up and spin-down bands is evident in  $\text{Sc}_3\text{Ru}$ ,  $\text{Cu}_3\text{Ru}$ , and  $\text{Zn}_3\text{Ru}$  alloys. This symmetrical pattern signifies non-spin polarization, confirming the absence of magnetic moments (zero magnetic moments) in these alloys. In contrast,  $\text{V}_3\text{Ru}$ ,  $\text{Cr}_3\text{Ru}$ ,  $\text{Mn}_3\text{Ru}$ ,  $\text{Fe}_3\text{Ru}$ ,  $\text{Co}_3\text{Ru}$ ,  $\text{Ni}_3\text{Ru}$ , and  $\text{Zn}_3\text{Ru}$  alloys exhibit asymmetric spin-up and spin-down channels. This asymmetry indicates electron spin polarization at the Fermi energy, leading to a spin polarization effect. This observation aligns with the presence of non-zero magnetic moments, as indicated in Table 1.



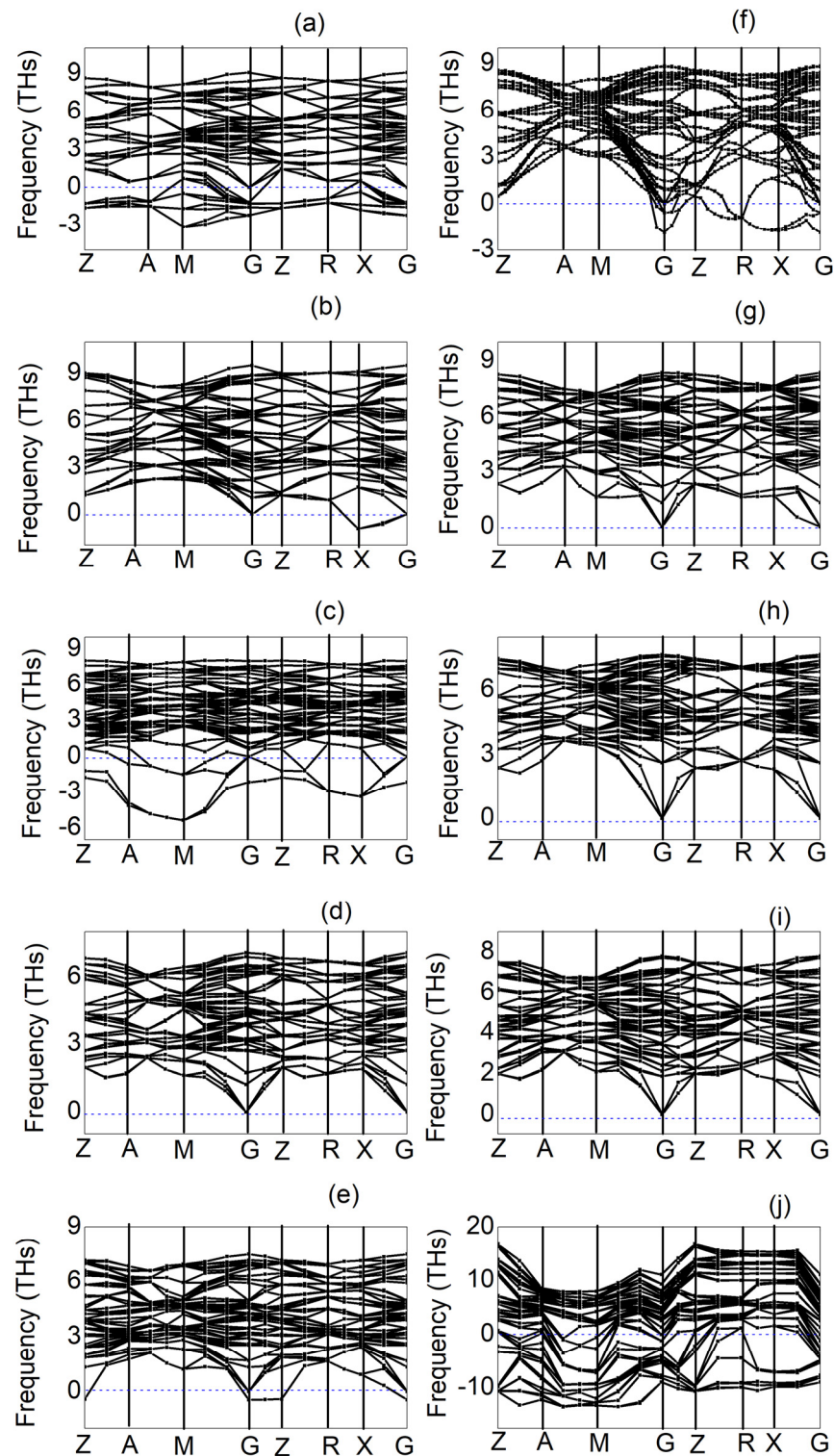
**Figure 2.** Partial density of states of tP16  $X_3\text{Ru}$  ( $X = \text{Sc} - \text{Mn}$ ) structures.



**Figure 3.** Partial density of states of tP16  $X_3Ru$  ( $X = Fe - Zn$ ) structures, where the dotted line is the Fermi energy ( $E_f = 0$ ). The arrows represent the spin-up and -down DOS for all the structures.

### 3.4. Phonon Dispersion Curves

Phonons are crucial for understanding the dynamic behavior and thermal conductivities, which are key areas in the advancement of novel materials research. Phonon dispersion curves offer valuable information about a material's dynamic stability or instability by revealing both positive and negative phonon frequencies. Positive frequencies indicate dynamic stability, whereas negative frequencies signify dynamic instability within a compound [57,58]. Figure 4 shows the dispersion curves of tP16  $X_3Ru$  (where  $X = Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu$ , and  $Zn$ ) alloys, plotted across the highest symmetry k-points of the Brillouin zone. Among these alloys, the phonon dispersion curves of  $Cr_3Ru$ ,  $Co_3Ru$ ,  $Ni_3Ru$ , and  $Cu_3Ru$  demonstrate dynamic stability, as there are no imaginary frequencies present. In contrast, the phonon dispersion plots of  $Sc_3Ru$ ,  $Ti_3Ru$ ,  $Fe_3Ru$ ,  $V_3Ru$ ,  $Zn_3Ru$ , and  $Mn_3Ru$  display negative phonon modes, indicating dynamic instability and, therefore, which may be a limiting factor in applications where dynamic stability is vital.



**Figure 4.** Dispersion curves for (a)  $\text{Sc}_3\text{Ru}$ , (b)  $\text{Ti}_3\text{Ru}$ , (c)  $\text{V}_3\text{Ru}$ , (d)  $\text{Cr}_3\text{Ru}$  (e)  $\text{Mn}_3\text{Ru}$ , (f)  $\text{Fe}_3\text{Ru}$ , (g)  $\text{Co}_3\text{Ru}$ , (h)  $\text{Ni}_3\text{Ru}$ , (i)  $\text{Co}_3\text{Ru}$ , and (j)  $\text{Zn}_3\text{Ru}$  alloys in tP16 phase at high symmetric points in the Brillouin zone. The blue lines indicate the zero-phonon frequency.

#### 4. Conclusions

In summary, we have utilized first-principles calculations to investigate the structural, electronic, and mechanical properties of tetragonal  $\text{X}_3\text{Ru}$  ( $\text{X} = \text{Sc} - \text{Zn}$ ) binary alloys in the tP16 phase. Analysis of the heat of formation results indicates that  $\text{Mn}_3\text{Ru}$ ,  $\text{Sc}_3\text{Ru}$ ,  $\text{Ti}_3\text{Ru}$ ,  $\text{V}_3\text{Ru}$ , and  $\text{Zn}_3\text{Ru}$  alloys are thermodynamically stable. Furthermore, the electronic density

of states reveals a strong overlap between the valence and conduction bands in Sc<sub>3</sub>Ru, Ti<sub>3</sub>Ru, V<sub>3</sub>Ru, Mn<sub>3</sub>Ru, Fe<sub>3</sub>Ru, Co<sub>3</sub>Ru, Ni<sub>3</sub>Ru, Cu<sub>3</sub>Ru, and Zn<sub>3</sub>Ru alloys, which exhibit a metallic character. All alloys except Cr<sub>3</sub>Ru demonstrate ductility, as evidenced by high B/G ratios exceeding 1.75, along with high melting temperatures. We observe a direct correlation between these elastic moduli (B, G, and E) and the melting temperatures of the proposed structures, consistent with previous theoretical data. Phonon calculations indicate that Cr<sub>3</sub>Ru, Co<sub>3</sub>Ru, Ni<sub>3</sub>Ru, and Cu<sub>3</sub>Ru are dynamically stable. Our findings suggest that tetragonal Ru-based alloys are promising candidates for ultra-high-temperature structural applications.

**Supplementary Materials:** The following supporting information can be downloaded at <https://www.mdpi.com/article/10.3390/alloys3020007/s1>, Table S1: Calculated lattice constants (Å) in pure 3d-transition metal X and ruthenium; Table S2: Calculated lattice constants and magnetic moments in pure chromium, ruthenium and iron for different values of Hubbard U parameter.

**Author Contributions:** B.O.M.: conceptualization, methodology, validation, investigation, writing—original draft, and project administration; M.E.B.: writing—review and editing, resources, and supervision. M.M.T.: writing—review and editing, resources, and supervision. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the National Research Foundation (NRF), grant number 121479, and the Grow Your Own Timber (GYOT) initiative from UNISA.

**Institutional Review Board Statement:** The study was conducted in accordance with the Declaration of University of South Africa approved by the Institutional Review Board (or Ethics Committee) of University of South Africa (from 8 February 2022 to 6 February 2027). This study is a pure computation and does not involve humans or animals.

**Data Availability Statement:** The raw data supporting the conclusions of this article will be made available by the authors on request.

**Acknowledgments:** All DFT calculations were performed at the University of South Africa using high-performance computing (HPC) resources.

**Conflicts of Interest:** The authors declare no conflicts of interest.

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