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Optimal Preventive Maintenance Policy for Equipment Rented under Free Leasing as a Contributor to Sustainable Development

Lazar Tlili [1,2], Anis Chelbi [3,4,*], Rim Gharyani [3] and Wajdi Trabelsi [5]

1. National School of Advanced Sciences and Technologies of Borj Cedria, University of Carthage, Borj Cedria 2084, Tunisia; tlililazhar@gmail.com
2. Laboratoire de Recherche en Robotique Intelligente, Laboratory: LR20ES02 RIFTSI, Fiabilité et Traitement de Signal et d'Images, University of Tunis, Montfleury, Tunis 1008, Tunisia
3. Ecole Nationale Supérieure d'Ingénieurs de Tunis (ENSIT), Laboratory: LR20ES02 RIFTSI, University of Tunis, Montfleury, Tunis 1008, Tunisia; rimagharyani28@gmail.com
4. Mediterranean School of Business, South Mediterranean University, Lac 2, Tunis 1053, Tunisia
5. Laboratoire de Génie Informatique, de Production et de Maintenance (LGIPM), ICN Business School, 57073 Metz, France; wajdi.trabelsi@univ-lorraine.fr or wajdi.trabelsi@icn-artem.com

* Correspondence: anis.chelbi@planet.tn

Abstract: Leasing has proven to be a business model that is perfectly suited to the circular economy. It significantly contributes to sustainable development by enabling the reuse of machinery and equipment after each lease period and by including preventive maintenance and overhauls within and between lease terms. This helps to extend the life cycle of equipment, promote value recovery, and reduce waste. This paper examines an imperfect preventive maintenance (PM) strategy applied to equipment rented under the terms of "free leasing". In free leasing, the lessor makes the equipment available to the customer for a specified period of time without charging rent. In return, the customer is required to purchase the equipment's consumables exclusively from the lessor. The lessor is also responsible for the maintenance of the equipment at the customer's premises. The greater the quantity of consumables used by the customer, the more the equipment will deteriorate. Consequently, the lessor must be able to determine the most effective approach to preventive maintenance, ensuring that it aligns with the customer's planned usage rate while maximizing profit. This work proposes a PM strategy to be adopted by the lessor during the free lease period. This strategy involves the performance of imperfect PM actions just before the start of the lease period and then periodically. Different packages of preventive actions can be applied each time, with each package having a different cost depending on the level of effectiveness in terms of rejuvenating the equipment. Minimal repairs are performed in the event of equipment failure. The decision variables are the PM period to be adopted and the maintenance efficiency level to be chosen for each preventive intervention. The objective is to determine, for a given customer with an estimated consumption rate profile of consumables, the optimal values of these decision variables so that the lessor maximizes their profit. A mathematical model is developed to express the lessor's average profit over each lease period. A solution procedure is developed for small instances of the problem, and an Artificial Bee Colony algorithm is implemented for larger instances. A numerical example and a sensitivity analysis are presented.

Keywords: closed-loop supply chains; circular business models; leasing; imperfect preventive maintenance; virtual age; Artificial Bee Colony algorithm

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1. Introduction

Leasing is a financing method that allows a company or individual to rent an asset for a fixed period of time. The asset can be of various types, such as equipment, a vehicle, or a building. During the lease term, the lessee uses the leased asset and makes regular payments to the lessor. At the end of the lease term, the lessee has several options: either return the leased asset, purchase it at a pre-agreed price, or renew the lease for another term [1]. Leasing has become a widely used business strategy, especially for industrial equipment [2]. It is also becoming increasingly popular because of its significant contribution to reducing waste and environmental impact. Leasing was first introduced in the United States in 1977 [3] and has since been adopted in many other countries. It is an attractive investment financing technique because it avoids the need for companies to raise or borrow large amounts of money to acquire production, transportation, or other equipment.

There are three main types of leasing [4]: financial leasing, also known as lease with purchase option (LPO); operating leasing, and sale and leaseback, also known as cession bail. In addition to these three well-known types of leasing, there is an increasing number of situations involving a rather new type of leasing known as "free leasing", where the supplier of a particular piece of equipment provides it to the customer free of charge for a certain period in return for a commitment to purchase a certain number of consumables for the equipment during the lease period. In other respects, according to [5], leasing can be one-dimensional or two-dimensional. The former is characterized by a single dimension, either time or equipment usage. It is generally used for cars, IT equipment, medical equipment, etc. Two-dimensional leasing requires both time and use restrictions for the leased equipment. In addition, leasing can be carried out for a single period or for multiple consecutive periods, such as for construction and industrial equipment, which are usually leased several times to different customers during their life cycles. In many situations, given the complexity of the leased equipment, which requires highly skilled technicians, and given the relatively short lease terms, lessors offer to provide maintenance for the leased equipment and sometimes even commit to a minimum level of availability of their equipment while it is being operated by the lessee. In practice, the lessor must pay a penalty if the leased equipment fails and is not repaired in time, resulting in a loss for the lessee [6,7].

2. Maintenance Strategies for Leased Equipment

The first studies on the maintenance of leased equipment were published by [8,9]. They studied the problem from the perspective of optimizing a sequential imperfect preventive maintenance strategy for the leased equipment. The authors of [9] assumed that the lessor incurs a penalty whenever the equipment fails and the time to repair a failure exceeds a specified time limit. The authors of [10] proposed a strategy for performing minimal repairs whenever a failure occurs. The authors of [11] extended their work by proposing a sequential preventive maintenance strategy. They investigated both the number of PM actions to be performed and the time intervals between these actions during the lease period. They showed that the proposed maintenance strategy is equivalent or even superior in effectiveness to those previously developed in [8,9]. Still, in the same context of equipment being leased for a fixed period, the authors of [12] proposed two strategies. The first one consists of implementing preventive actions once the reliability of the equipment reaches a certain threshold. These actions aim to reduce the age of the equipment. The second strategy proposes to periodically perform imperfect preventive actions with the aim of minimizing maintenance costs.

The authors of [13] developed a multi-phase imperfect preventive maintenance strategy for leased equipment. This strategy uses the age reduction method. It consists of several preventive maintenance phases during the lease period, where the interval between consecutive PM actions within each phase remains constant. Ref. [14] proposed a maintenance strategy for leased equipment during consecutive periods in the oil and gas industry. The strategy includes imperfect upgrades at the end of each lease period and minimal repair in the case of failure. The authors determined the optimal upgrade efficiency level to be adopted at the end of each lease period in order to maximize the lessor's expected total profit over the equipment's lifecycle. The authors of [15] considered leased equipment during a finite period with imperfect preventive maintenance and repair actions, with penalties related to unavailability after failures. The objective is to find the optimal PM period during the lease period that minimizes the expected total cost. The authors of [16] developed a strategy to upgrade and maintain leased industrial equipment based on utilization, taking into account the warranty period.

Lessees may overuse the leased equipment or not use it properly during the lease term because they do not feel concerned about its maintenance if the lessor takes care of it. This often results in additional maintenance costs for the lessor. In this context, the authors of [17] developed a cost-sharing approach that takes into account the lessees' efforts to protect the leased equipment during the lease term. In this strategy, the equipment lessor offers a rent reduction to the lessee, and the costs associated with preventive maintenance (PM) and the lessee's efforts are incurred according to a cost-sharing coefficient.

There are also customized leasing strategies. These strategies classify lessees into three categories based on their use of the leased equipment: high, moderate, and low. In this way, the lessor can provide specific services to each category. The authors of [18] proposed a framework for studying these customized rental services. Their model considers different customer characteristics, warranties, and preventive maintenance policies to create customized rental contracts. The goal of this study is to help renters find the rental contract that best suits their needs, taking into account their diversity.

The authors of [19] developed another maintenance strategy that considers capacity balancing. It proposes dynamic maintenance decisions based on the state of the leased machines. These machines operate according to a series-parallel structure. This strategy not only allows the lessor to achieve a higher overall profit, but also reduces the time and capacity lost due to separate maintenance actions. Considering the variation of usage rate, especially for companies producing seasonal goods, the authors of [7] proposed a multi-phase preventive maintenance strategy to determine the PM times and degrees at different phases with different usage rates. Recently, the authors of [20] presented a preventive maintenance strategy for leased equipment and studied the impacts of different financing modes (advanced payment and bank loan financing) on the lessor's decision-making process regarding pricing. Another form of rental called "free leasing" has recently been discussed [21]. In this case, the equipment is made available to the user free of charge for a pre-determined period of time, while the user is responsible for its maintenance. In contrast to traditional leasing, where the lessee pays periodic rental and service charges for maintenance, in this case, the lessee must purchase a minimum number of consumables of the leased equipment.

There are several examples of this type of lease. The first is medical equipment, such as X-ray machines. In some countries, this equipment is provided free of charge to healthcare institutions on the condition that they purchase a predetermined minimum quantity of certain films during the lease period. A second example relates to the packaging industry, where machines called "case formers" are purchased by corrugated board manufacturers and made available free of charge to some of their customers. The latter use the machine to produce boxes of various designs from sheets of corrugated board that they purchase exclusively from the corrugated board supplier. Another example is the many types of beverage dispensers provided to hotels and restaurants in exchange for the purchase of beverages in the form of pressurized kegs. There is also the example of photocopier manufacturers who lend their machines free of charge to certain large professional customers, such as reprographic centers. In return, the customer agrees to purchase a minimum number of ink cartridges. Generally, in this free leasing context, the maintenance of the equipment is offered by the lessor during the lease period.

In addressing free leasing in situations where maintenance is performed and paid for by the lessor, the authors of [21] developed a mathematical model to determine the optimal preventive maintenance period that minimizes the average total cost of maintenance per time unit for the lessor. Based on this, they also determined the minimum quantity of consumables to be sold to the customer for each lease over a given period to ensure a profit.

However, the lessor's profit model did not take into account the relationship between the lessee's intended use of the equipment (production rate) and its degradation rate. If the leased equipment is used more intensively by the lessee, it will degrade faster and therefore require more PM effort by the lessor. In addition, a key assumption was that overhauls between successive lease periods would restore the equipment to a like-new condition, which is not very realistic.

The present work aims to extend this model of free leasing to more realistic settings by overcoming the above limitations. The first extension is designed to consider the relationship between the degradation profile of the equipment and the customer's estimated equipment usage profile (i.e., the usage profile for consumables estimated by the customer according to their demand forecasts for the products produced by the equipment in question). The second extension is to reflect the fact that, in practice, the overhaul of equipment between successive leases does not restore it to a like-new condition. Rather, the overhaul is an imperfect maintenance operation that may have different effects on the condition of the equipment depending on the operations performed and their degree of effectiveness. Finally, the third extension deals with situations where the supplier must pay a penalty if it contractually exceeds the specified downtime limits in repairing breakdowns.

The remainder of this paper is organized as follows: Section 3 presents the proposed problem. Section 4 details the profit model to optimize the maintenance service strategy. In Section 5, we present the numerical procedure used to solve the optimization problem. Section 6 provides numerical examples to analyze the effects of some input parameters. Section 7 concludes the paper and discusses future research.

3. Problem Description and Analytical Framework

3.1. Problem Description

Let us consider a lessor who manufactures or acquires a piece of equipment (industrial, medical, or other) in order to lease it for free to a certain number of lessees for successive periods of duration T_j, where $j = 1, 2, \ldots, N$ during its life cycle of duration L. In return for this free lease, each customer will purchase a certain quantity of consumables to be used by the equipment, according to a consumption profile that they must estimate a priori according to their demand forecast. Obviously, the lessor will try to make a profit over each lease period. The problem is that as the customer produces and uses more and more consumables, the equipment will continue to deteriorate, increasing the risk of breakdowns and downtime, and thus reducing the potential number of consumables to be purchased. To reduce this risk of loss of revenue, the lessor offers to take over the maintenance of its equipment at the customer's site to enable it to meet the expected demand. In this way, the lessor must know how to dose the preventive maintenance (PM) effort to best match the lessee's expected usage rate while ensuring maximum profit. If the expected usage profile is relatively low, the lessor should not devote significant effort to PM and could even decide to focus only on repairs following breakdowns. On the other hand, if the expected usage profile is rather high, the lessor will have to perform substantial PM to reduce the number of breakdowns and thus reduce the loss of revenue related to unused consumables due to stoppages for repairs.

The maintenance strategy that will be proposed in the contract for the jth lease period is to carry out an imperfect PM action just before the start of the period and carry out other actions periodically thereafter. This is achieved by applying a package of maintenance actions, such as cleaning, lubrication, inspection, the replacement of certain parts, overhaul, etc., from a set of K predetermined packages. Each package has an efficiency level m_k and a corresponding cost depending on the actions it contains.

A total of n_j imperfect preventive maintenance actions are performed during the jth free lease period. They restore the equipment to an intermediate state between "as good as new" and "as bad as old". These PM actions are performed before the beginning of each lease period and periodically thereafter at predetermined times, $\tau_{j,i}$, $i = 1, 2, \ldots, n_j$, regardless of the age and condition of the equipment. In addition, corrective maintenance (CM) is carried out in the event of breakdowns. This consists of carrying out minimal repair that restores the equipment to operation without affecting its failure rate. Each of the two types of maintenance (PM and CM) obviously involves a certain cost for the lessor.

Figure 1 below shows an example of how the proposed strategy might be deployed.

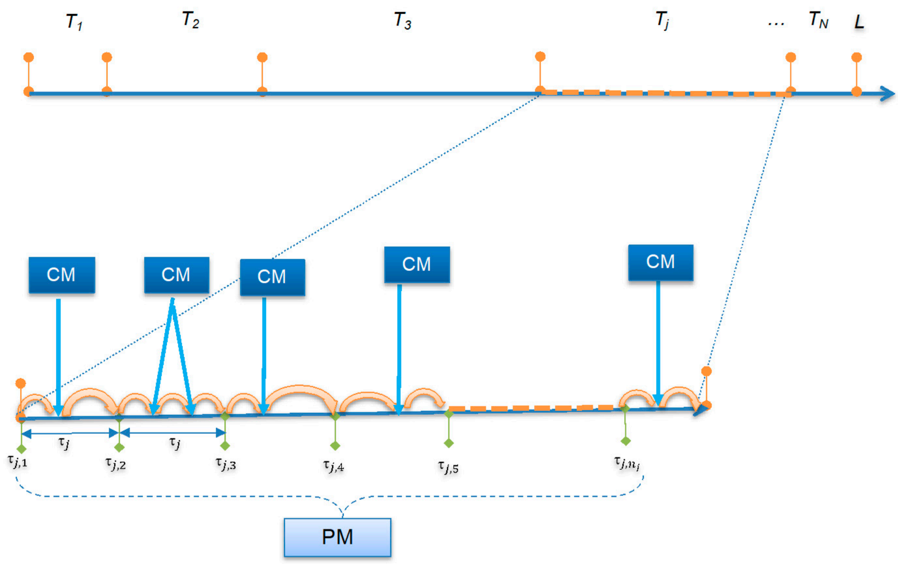

Figure 1. A scenario of the maintenance strategy during a free lease period T_j.

The more consumables the customer uses, the more the machine will deteriorate, and the more maintenance will be required. Before drawing up the contract for the jth lease period, the lessor will ask the customer to give them an estimate of the distribution of their expected consumption rate, $Gj(u)$. This distribution is, in some way, related to the demand profile they have to satisfy. Once the customer has given the lessor this information $Gj(u)$, the lessor will have to simultaneously determine the number of preventive maintenance operations to be performed (n_j) and the maintenance efficiency level $(m_k)_{\mathrm{j,i}}$ of the PM to be adopted at each PM instant of the jth period in order to obtain a maximum profit. Thus, depending on the consumption profile for consumables $Gj(u)$ provided by the customer, the lessor will be able to determine the optimal maintenance strategy (n_j* and $(m_k)_{j,i}$*) to adopt in order to maximize their expected profit over the lease period.

3.2. Analytical Framework

3.2.1. Modeling the Probability Distribution Function of Equipment Failure Times as a Function of the Usage Rate

In the context of this work, equipment failures can be considered as random events occurring during the lease period and modeled by a counting process characterized by a failure intensity function that depends on both age and usage. We use a one-dimensional approach that treats the random usage rate as a covariate. Let u be the non-negative random usage rate, and let $g(u)$ the probability density function of u. For a given consumable usage rate u, the total quantity used over a period t is estimated as a linear function of time ($Q = ut$).

Following [22], we assume that the equipment breakdown process can be modeled using a Non-Homogeneous Poisson Process (NHPP) with an intensity function, $\lambda(t|u)$, which is given by the following:

$$\lambda(t|u) = \theta_0 + \theta_1 u + \theta_2 T(t) + \theta_3 U(u) \tag{1}$$

where θ_0, θ_1, θ_2, and θ_3 are the coefficients of the intensity function. Conventionally, the accumulated usage time is assumed to be the lease period, i.e., $T(t) = t$, so the usage amount is thus

$$(u) = u \quad T(t) = ut \tag{2}$$

Therefore, the intensity function can be simplified as follows [23]:

$$\lambda(t|u) = \theta_0 + \theta_1 u + (\theta_2 + \theta_3 u)t \tag{3}$$

The parameters of θ_0, θ_1, θ_2, and θ_3 can be estimated from historical data.

From a practical point of view, the usage rate for consumables, which corresponds to the equipment's usage rate, depends on the lessee's activity and the demand they must satisfy. Consequently, each lessee will use the equipment at a different rate during the lease term. This usage rate will of course be limited by the capacity of the machine u_{max}.

3.2.2. The Adopted Working Assumptions and the Used Notation

The assumptions considered in this study are as follows:

- The usage rate for consumables reflects the equipment's usage rate.
- The probability distribution function, $G_j(u)$, associated with the lessee's usage of consumables in the jth period is known.
- The first lease period does not require PM at the beginning because the equipment is new.
- PM actions reduce the virtual age of the equipment by a certain amount depending on the effectiveness of the maintenance action package applied.
- The durations of preventive maintenance actions are considered negligible in relation to the duration T_j of the free lease period.
- The average duration of a minimal repair is negligible in relation to the time between failures.
- The lessor reserves the right to adjust the maintenance policy at any time if they finds that the customer's consumption of consumables deviates significantly from the profile, $g(u)$, considered when the contract was drawn up. This will be easy for both parties to verify, since the consumables are supplied exclusively by the lessor, who will therefore have full knowledge of the lessee's actual consumption profile and will be able to compare it with the profile, $g(u)$, initially considered. If a different consumption profile, $g'(u)$, is identified, the lessor will use this new profile to update their maintenance policy using the mathematical model proposed in this work.

The notations used to express the mathematical model are as follows:

L	:	Equipment lifecycle duration
T_j	:	Duration of the jth period (j = 1, 2, ..., N)
τ_j	:	Preventive maintenance (PM) period to be adopted during the jth period ($\tau_j < T_j$)
$\tau_{j,i}$	:	Instant of the ith PM during the jth period
u_{max}	:	Maximum usage rate of consumables (equipment capacity)
u	:	Random variable associated with the rate of use of consumables
θ_i	:	Coefficients of the intensity function, i = 0, 1, 2, 3
$A^v_{j,i}$	:	Virtual age of the equipment after the ith PM action in the jth period
C_{mr}	:	Average cost of minimal repair
Q_{lj}	:	Average quantity of consumables not sold during repairs within the jth period
Pn_j	:	Average penalty incurred by the lessor for unsold consumables due to unavailability of equipment during breakdown repairs
D_r	:	Average duration of minimal repair

K	:	Number of available PM action packages
n_j	:	Number of PM actions to be carried out during the jth period; ***Decision variable***
m_k	:	Maintenance efficiency level of package k of PM actions (k = 1, 2, . . ., K)
$(m_k)_{j,i}$	:	Maintenance efficiency level of package k of PM actions (k = 1, 2, . . ., K) *carried out at the ith PM during the jth lease period;* ***Decision variable***
$\delta(m_k)$	:	Efficiency factor of PM to be performed by applying maintenance package k with maintenance level m_k
$C_{m,k}$	:	Cost of PM carried out using maintenance package k with maintenance level m_k
P_j	:	Average profit to be realized by the lessor over the jth period
K_0	:	Depreciation rate of the value of equipment
C_m	:	Purchase price of new equipment (at the beginning of the life cycle)
V_j	:	Estimated value of the equipment at the end of jth period
S_s	:	Selling price per unit of consumables
S_p	:	Purchase price/cost of one unit of consumables (purchased by the lessor)
R_j	:	The lessor's average revenue from the sale of consumables during the jth period
$\lambda_0(t \mid u_0)$	:	Function of equipment nominal failure rate under the nominal usage rate u_0
$\lambda(t \mid u)$	:	Function of equipment failure rate under usage rate u
$h(x)$	:	Density probability function associated with repair time
$G_j(u)$	:	Probability distribution function for consumable usage rate during the jth leasing period.

4. The Mathematical Model

Below, we develop the mathematical expression of the expected total profit that will be made by the lessor over the jth lease period for an estimated equipment and usage rate profile for consumables. This profit is the difference between the revenue generated by the sale of consumables and the residual value of the equipment, and the costs associated with maintenance and penalties for losses due to downtime during repairs.

4.1. Average Cost of Preventive Maintenance

During the jth lease period, a number n_j of PM actions will have to be performed every τ_j units of time.

$$n_j = \begin{cases} \left\lfloor \frac{T_j}{\tau_j} \right\rfloor & for\ j = 1 \\ \left\lfloor \frac{T_j}{\tau_j} \right\rfloor + 1 & for\ j > 1 \end{cases} \tag{4}$$

$\lfloor y \rfloor$ represents the integer part of y.

Furthermore, the instants at which PM will be carried out during the jth period are given by

$$\tau_{j,i} = \begin{cases} i\tau_1 & for\ j = 1 \\ \sum_{k=1}^{j-1} T_k + (i-1)\tau_j & for\ j > 1 \end{cases} \tag{5}$$

where T_0 = 0 and i = 1, 2, . . ., n_j.

Let us introduce the binary variable $X_{k,i}^{j}$ as follows:

$$X_{k,i}^{j} = \begin{cases} 1\ :\ PM\ level\ m_k\ is\ applied\ in\ the\ ith\ PM\ action\ during\ the\ jth\ period \\ 0\ :\ Otherwise \end{cases} \tag{6}$$

Since for any given PM action, only one PM level is adopted, we have

$$\sum_{k=1}^{K} X_{k,i}^{j} = 1 \quad for\ i = 1, 2, \ldots, n_j \tag{7}$$

Consequently, the average total cost of PM actions during the *j*th period is given by the following equation:

$$E\left[C_p(j)\right] = \sum_{i=1}^{n_j} \sum_{k=1}^{K} X_{k,i}^{j} C_{m,k} \quad (8)$$

The lessor will perform PM actions on the equipment to improve its reliability by reducing its age. After PM, the age, t, of the equipment decreases. Its virtual age becomes t^v ($t^v < t$).

To express the virtual age after a PM action, we adopt the same modeling method as that used in [24,25]. Thus, the virtual age $A_{j,i}^{v}$ following the *i*th PM during the *j*th period is expressed as

$$A_{j,i}^{v} = \sum_{q=1}^{j-1} \sum_{z=1}^{n_q} \sum_{k=1}^{K} \delta(m_k)_{q,z} X_{k,z}^{q} \left(\tau_{q,z} - \tau_{q,z-1}\right) + \sum_{z=1}^{i} \sum_{k=1}^{K} \delta(m_k)_{J,z} X_{k,z}^{j} \left(\tau_{J,z} - \tau_{J,z-1}\right) \quad (9)$$

where

$\tau_{j,i}$: the instant of the *i*th PM period during the *j*th period, and $\tau_{1,0} = 0$ and $\tau_{j,0} = \tau_{j-1,n_{j-1}}$ for $j > 1$.

$\delta(m_k) \in [0, 1]$. It represents the efficiency factor of a PM action by applying maintenance package *k* with maintenance level m_k.

According to [24], $\delta(mk) = (1 + m_k)\, e^{-m_k}$ with $0 \leq \delta(m_k) \leq 1$.

$\delta(0) = 1$ corresponds to no action (no PM)

$\delta(\infty) = 0$ corresponds to an infinite maintenance effort (impossible in practice); the age of the equipment is reduced to the age it had at the time of the last PM action.

Some works, such as [26,27], provide guidance on how to estimate the efficiency factor of maintenance activities. Ref. [28] uses a logarithmic regression model for this purpose. Thus, considering the efficiency factor of the *i*th PM action performed during the *j*th lease period to obtain the virtual age of the equipment (just after the *i*th PM of the *j*th period), it is possible to express the equipment failure rate after the *i*th PM during the *j*th period as follows:

$$\lambda_{j,i}(t|u) = \lambda_0\left(A_{j,i}^{v} + t\middle|u\right) t < T_j \quad (10)$$

4.2. Average Cost of Minimal Repairs

During the *j*th period, the lessor will periodically perform preventive maintenance at predetermined times. $\tau_{j,i}\ i = 1, 2, \ldots, n_j$ regardless of the age and condition of the equipment. Failures that may occur between these PM actions are subject to minimal repairs. Based on the partial renewal model, it is possible to express the average number of minimal repairs $E[N_{r,j}]$ in the *j*th period as follows:

$$E\left[N_{r,j}\right] = \sum_{i=1}^{n_j} \int_0^{\tau_j} \int_0^{u_{max}} \lambda_0\left(A_{j,i}^{v} + t\middle|u\right) dG_j(u)dt + \int_0^{T_j - n_j\tau_j} \int_0^{u_{max}} \lambda_0\left(A_{j,n_j}^{v} + t\middle|u\right) dG_j(u)dt \quad (11)$$

The first term of the sum represents the average number of failures (minimal repairs) during the n_j PM periods, while the second term represents the average number of failures (minimal repairs) during the period between the last PM performed at $n_j\tau_j$ and the end of the *j*th period at time T_j.

It is clear from Equation (9) that the average number of breakdowns is directly related to the probability distribution $G_j(u)$ of the equipment usage rate (usage rate for consumables).

The average total cost of minimal repairs during the *j*th lease period is given by

$$E[C_c(j)] = E\left[N_{r,j}\right] C_{mr} \quad (12)$$

4.3. The Average Penalty for Unsold Consumables due to Failures during the jth Period

During periods when the equipment is shut down for repairs due to breakdowns, consumables that would normally be used are not used because the equipment is being repaired. This results in a loss of revenue for the lessor.

Considering an average repair time, D_r, the average quantity not sold is given by

$$Q_{lj} = E\left[N_{r,j}\right] D_r \int_0^{u_{max}} u dG_j(u) \tag{13}$$

where $D_r = \int_0^{+\infty} th(t)dt$.

Therefore, the average penalty incurred is expressed as the loss of profit that would have been earned had there been no failures:

$$Pn_j = \left(S_s - S_p\right) Q_{lj} \tag{14}$$

4.4. The Estimated Residual Value of the Equipment at the End of the jth Period

Considering the depreciation rate of the equipment value, K_0, it is possible to estimate the residual value V_j of the equipment at the end of the *j*th lease period. This value can be given by the following expression [28,29]:

$$V_j = K_0 C_m \left(1 - \frac{\sum_{i=0}^{j} T_i}{L}\right) \tag{15}$$

where $0 < K_0 < 1$.

4.5. The Lessor's Average Income during the jth Period

During the *j*th lease period, the lessor will sell the customer an average number of consumables Q_j. This quantity is expressed by the following equation:

$$Q_j = T_j \int_0^{u_{max}} u dG_j(u) \tag{16}$$

Thus, the lessor's expected income R_j for the *j*th lease period is given by

$$R_j = S_s T_j \int_0^{u_{max}} u dG_j(u) \tag{17}$$

Similarly, the average cost C_j corresponding to the purchase of consumables by the lessor for sale to the lessee is

$$C_j = S_p T_j \int_0^{u_{max}} u dG_j(u) \tag{18}$$

4.6. The Lessor's Average Profit over the jth Lease Period

The average profit to be realized by the lessor over the *j*th lease period is equal to the average revenue minus the sum of the average costs incurred in connection with maintenance, penalties due to breakdowns, and the purchase of consumables.

$$P_j = \sum Revenue - \sum \text{Costs} \tag{19}$$

Thus, we obtain the following expression for the lessor's average profit as a function of the two decision variables $\left(n_j, (m_k)_{j,i}\right)$ related to the maintenance strategy to be adopted by the lessor:

$$\mathrm{P_j}\left(n_j, (m_k)_{j,i}\right) = \mathrm{R_j} + \mathrm{V_j} - C_j - C_m - \mathrm{E}[\mathrm{C_c}(j)] - \mathrm{E}\left[\mathrm{C_p}(j)\right] - Pn_j \tag{20}$$

The terms regarding this expression (Equation (20)) are as follows:

- The first term corresponds to the average revenue generated by consumables sold to the lessee (Equation (17)).

- The second term corresponds to the residual value at the end of the *j*th period (the income generated by the eventual sale of the equipment) (Equation (15)).
- The third term corresponds to the average purchase (or production) cost of the consumables used (Equation (18)).
- The fourth term corresponds to the acquisition cost of the new equipment.
- The fifth term corresponds to the average total cost of minimal repairs following breakdowns (Equation (12)).
- The sixth term is the average cost of preventive maintenance (Equation (8)).
- The seventh term represents the average penalty incurred by the lessor for unsold consumables due to the unavailability of equipment during breakdown repairs (Equation (14)).

Thus, we obtain the following maximization problem (P1):

$$\max_{n_j,(m_k)_{j,i}} \mathrm{P_j} = \max_{n_j,(m_k)_{j,i}} \left[(S_s - S_p) T_j \int_0^{u_{max}} u dG_j(u) + K_0 C_m \left(1 - \frac{\sum_{i=0}^{j} T_i}{L} \right) - C_m - \left(\sum_{i=1}^{n_j} \int_0^{\tau_j} \int_0^{u_{max}} \lambda_0 \left(A_{j,i}^v + t | u \right) dG_j(u) dt + \int_0^{T_j - n_j \tau_j} \int_0^{u_{max}} \lambda_0 \left(A_{j,n_j}^v + t \middle| u \right) dG_j(u) dt \right) \left(C_{mr} + (S_s - S_p) D_r \int_0^{u_{max}} u dG_j(u) \right) - \sum_{i=1}^{n_j} \sum_{k=1}^{K} X_{k,i}^j C_{m,k} \right] \tag{21}$$

Subject to

$$\sum_{l=1}^{j} T_l \leq \mathrm{L} \tag{22}$$

$$G_j(u) \ such\ that\ 0 \leq u \leq u_{max}$$

$$\sum_{k=1}^{K} X_{k,i}^j = 1\ for\ all\ j\ and\ i = 1, 2, \ldots, n_j$$

5. Numerical Procedure

Note that, for a given number K of possible PM efficiencies and for n_j PM actions, there are Kn_j possibilities to consider. Due to this complexity of the analytical model, a numerical procedure, shown in Figure 2 below, was established to obtain the optimal preventive maintenance strategy. This procedure performs a systematic search between different values of $\tau_j \in [\tau_0, T_j]$ and $(m_k)_{j,i}$ to simultaneously determine the optimal MP period τ_j* and the PM actions' efficiency levels $(m_k)_{j,i}$* that maximize the lessor's average profit for a given lease period.

The logic behind this procedure is based on the fact that it must be used systematically for all lease periods, sequentially from the first to the *j*th period. This is because certain input parameters for period *j* are relative to period $(j - 1)$. These parameters are the time of the last PM in period $(j - 1)$, the virtual age just after this action, and the efficiency level used.

The steps of the numerical procedure we implemented are described below for each period $j = 1, 2, \ldots$ (see also Figure 2):

- **Step 1**: Enter the input parameters (L, C_m, C_{mr}, S_p, S_s, K_0, D_r, u_{max}, $\Delta\tau$, τ_0, μ_d, σ_d, θ_0, θ_1, θ_2, θ_3, m_k, $\delta(m_k)$, and C_{mk}) and the history of previous lease periods (T_{j-1}, $\tau_{j-1,n_{j-1}}$, $A_{j-1,i}^v$).
- **Step 2**: Enter the data for the new lease period, *j*, characterized by the lease term T_j and the consumption profile for consumables $G_j(u)$.
- **Step 3**: Check whether the equipment will reach the end of its useful life cycle at the end of this lease period, *j*.
- **Step 4**: Calculate the average profit for each combination of $\tau_j \in [\tau_0, T_j]$ and $(m_k)_{j,i}$ values of the PM packages.
- **Step 5**: Select the maximum average profit and the corresponding τ_j* and $(m_k)_{j,i}$* values.

It is important to note that, due to the iterative nature of this numerical procedure, in which the interval $[\tau_0, T_j]$ is traversed with an increment $\Delta\tau$ to be set beforehand by the user, the final solution obtained depends on the degree of accuracy required. The smaller the increment Δτ, the more accurate the solution. Thus, in all that follows, whenever we speak of an optimal solution, it will be, in all rigor, a near-optimal solution.

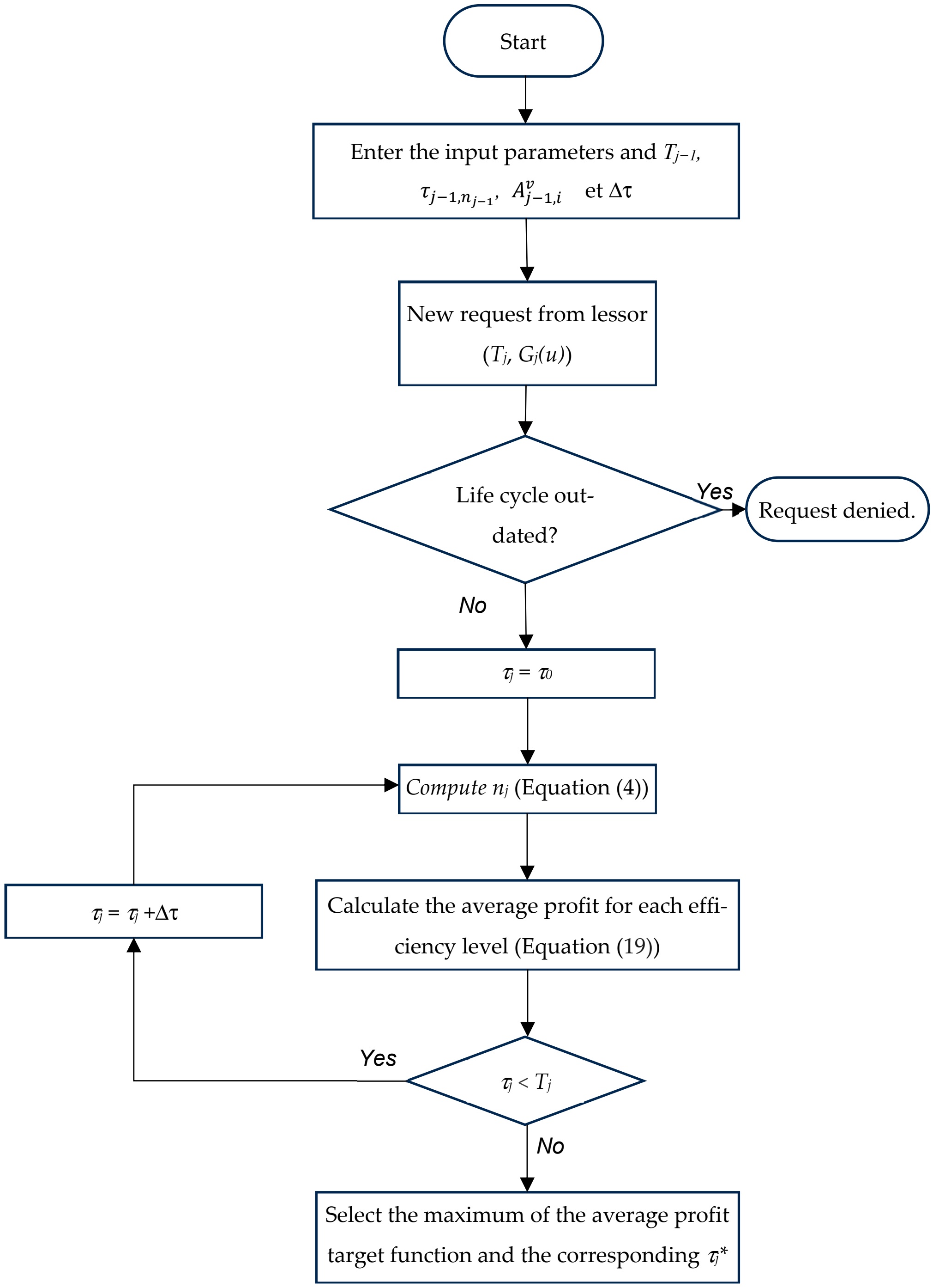

Figure 2. Numerical procedure.

6. Numerical Application

6.1. Numerical Example

Let us consider a lessor who provides their customers with equipment with a life cycle of L = 60 months. The following input parameters of the problem were chosen arbitrarily while ensuring a certain consistency based on practical experience. The lease periods are assumed to be equal periods of 12 months each, and the repair time follows a Normal distribution with parameters μ_d = 0.05 months and σ_d = 0.01 months.

In this example, we limit ourselves to the first two periods of T_1 = 12 months and T_2 = 12 months, respectively, assuming that the lessor only has visibility for these two periods at the moment, knowing the customers who will be leasing, the duration of the lease period agreed with each one, and the estimated consumption profile provided by each one. This, knowing that subsequent periods can be treated in the same way by the proposed numerical procedure. The consumption functions $G_1(u)$ and $G_2(u)$ of the two customers are represented by the Normal distribution with parameters μ_1 = 300 and σ_1 = 50 consumables per month and μ_2 = 500 and σ_2 = 75 consumables per month, respectively.

Tables 1 and 2 below show the other input parameters considered.

Table 1. Input parameters corresponding to the considered example.

C_m	USD 80,000	u_{max}	1000 items/month
C_{mr}	USD 5000	K_0	0.5
S_p	USD 70	S_s	USD 125
$\Delta\tau$	2 months	T_1	12 months
τ_0	3 months	T_2	12 months
$\theta_0 = 10^{-5}$; $\theta_1 = 2 \times 10^{-6}$; $\theta_2 = 3 \times 10^{-6}$; $\theta_3 = 5 \times 10^{-5}$			

Table 2. PM efficiency levels and costs corresponding to the considered example.

k	*PM Level* (m_k)	Δ (m_k)	C_{mk} (*USD*)
1	0	1	0
2	1	0.74	500
3	2	0.41	1000
4	3	0.20	2000
5	4	0.09	3000

The solution procedure was programmed using MATLAB® software run on a PC (i3 processor with 8 GB of RAM).

Based on the above input parameters, the optimal solution obtained is shown in Table 3 and Figure 3.

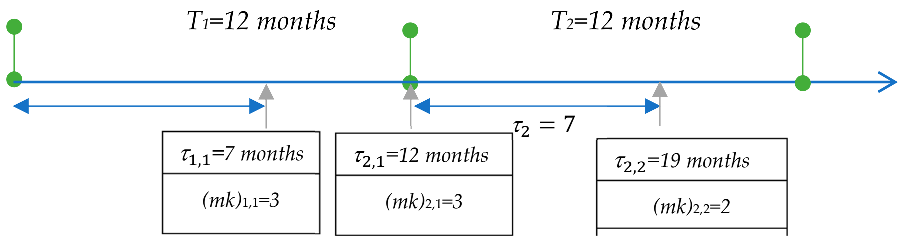

Figure 3. The obtained optimal PM strategy.

For the first period, the recommended optimal strategy is to perform a single PM action after a period of τ_j = 7 months, applying the preventive maintenance package (k = 4) with an efficiency $(m_k)_{1,1}$ = 3. For the second period, on the other hand, the optimal strategy is to execute two PM actions as follows: the first before the start of the period with an efficiency level $(m_k)_{2,1}$ = 3 (package # 4), and then after 7 months with an efficiency $(m_k)_{2,2}$ = 2 (package # 3). This maintenance strategy would allow the lessor to maximize their average profit over each of the successive lease periods. The total average profit is equal to the sum of the maximum average profits expected for each of the two periods, i.e., 143,260 + 259,950 = USD 403,210.

Table 3. The obtained results: the optimal solution and the corresponding average costs and profit.

τ_j*	j = 1	**7 months**
	j = 2	**7 months**
$(m_k)_{j,i}$*	j = 1	**$(m_k)_{1,1}$ = 3**
	j = 2	**$(m_k)_{2,1}$ = 3** **$(m_k)_{2,2}$ = 2**
n_j	j = 1	1 PM
	j = 2	2 PM
Average income R_j (USD)	j = 1	450,000
	j = 2	750,000
C_j (USD)	j = 1	252,000
	j = 2	420,000
V_j (USD)	j = 1	32,000
	j = 2	24,000
Average cost of CM (USD)	j = 1	4 072.4
	j = 2	8 664.6
Average cost of PM (USD)	j = 1	2 000
	j = 2	3 000
Penalty Pn_j (USD)	j = 1	671.95
	j = 2	2 382.8
Average profit P_j (USD)	j = 1	143,260
	j = 2	259,950

Program runtime = 5 min 36 s.

6.2. Sensitivity Study

In this section, we examine how variations in input parameters affect the best strategy to adopt and the corresponding total profit.

6.2.1. The Influence of the Variation in the Average Cost of a Minimal Repair (C_{mr})

We varied the average cost of a minimal repair (C_{mr}). The results obtained are shown below (Table 4 and Figure 4).

Table 4. The effect of changes in the average repair cost on the optimal solution.

		C_{mr} = USD 1500	**C_{mr} = USD 3500**	**C_{mr} = USD 5000**	**C_{mr} = USD 8000**	**C_{mr} = USD 10,000**
τ_j*	j = 1	7	7	7	7	7
	j = 2	7	7	7	5	5
$(m_k)_{j,i}$*	j = 1	**$(mk)_{1,1}$ = 2**	**$(m_k)_{1,1}$ = 3**	**$(m_k)_{1,1}$ = 3**	**$(m_k)_{1,1}$ = 4**	**$(m_k)_{1,1}$ = 4**
	j = 2	**$(m_k)_{2,1}$ = 2** **$(m_k)_{2,2}$ = 2**	**$(m_k)_{2,1}$ = 2** **$(m_k)_{2,2}$ = 2**	**$(m_k)_{2,1}$ = 3** **$(m_k)_{2,2}$ = 2**	**$(m_k)_{2,1}$ = 4** **$(m_k)_{2,2}$ = 2** **$(m_k)_{2,3}$ = 1**	**$(m_k)_{2,1}$ = 4** **$(m_k)_{2,2}$ = 2** **$(m_k)_{2,3}$ = 2**
n_j	j = 1	**1**	**1**	**1**	**1**	**1**
	j = 2	**2**	**2**	**2**	**3**	**3**
Average income R_j (USD)	j = 1	450,000	450,000	450,000	450,000	450,000
	j = 2	750,000	750,000	750,000	750,000	750,000

Table 4. *Cont.*

		C_{mr} = USD 1500	C_{mr} = USD 3500	C_{mr} = USD 5000	C_{mr} = USD 8000	C_{mr} = USD 10,000
C_j (USD)	j = 1	252,000	252,000	252,000	252,000	252,000
	j = 2	420,000	420,000	420,000	420,000	420,000
V_j (USD)	j = 1	32,000	32,000	32,000	32,000	32,000
	j = 2	24,000	24,000	24,000	24,000	24,000
Average cost of CM (USD)	j = 1	1618.7	2850.7	4072.4	5406.8	6758.5
	j = 2	3301.3	6708.4	8664.6	11,693	13,653
Average cost of PM (USD)	j = 1	1000	2000	2000	3000	3000
	j = 2	2000	2000	3000	4000	5000
Penalty Pn_j (USD)	j = 1	890.29	671.95	671.95	557.58	557.58
	j = 2	3026.2	2635.5	2382.8	2009.7	1877.3
Average profit P_j (USD)	j = 1	146,490	144,480	143,260	141,040	139,680
	j = 2	265,670	262,660	259,950	256,300	253,470

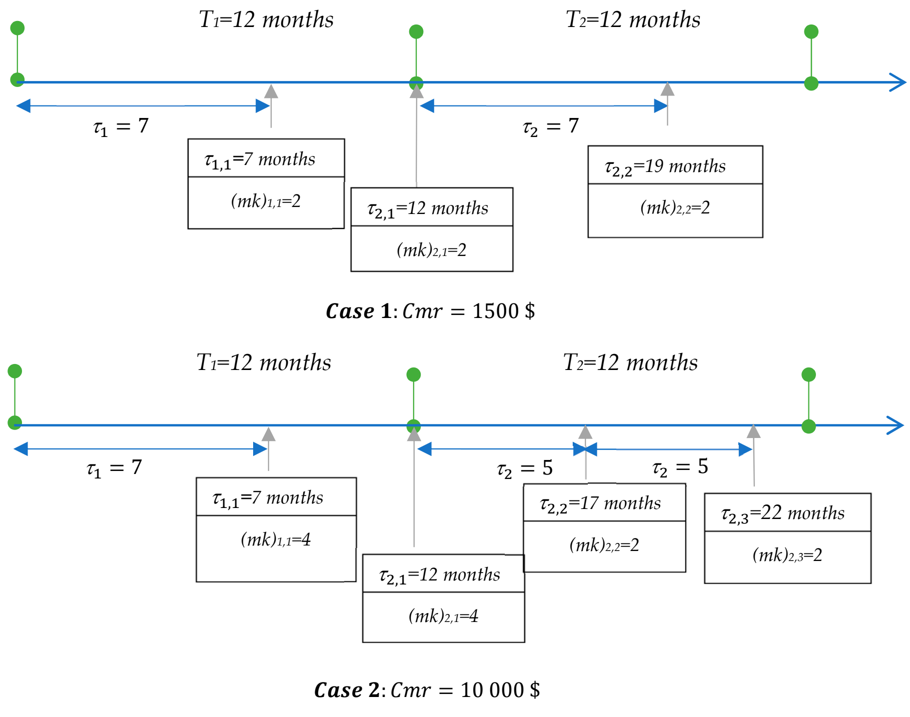

Figure 4. The obtained optimal PM strategies for the lower and upper bounds of minimal repair costs (C_{mr} = USD 1500 and C_{mr} = USD 10,000).

It is interesting to note from these results that for higher minimal repair costs, the optimal maintenance strategy is to shorten the PM period and perform relatively high efficiency PM actions to reduce penalties.

6.2.2. The Effect of Varying the Average Consumption of the Second Customer (for the Second Lease Period)

Calculations were made by varying the second customer's average consumption of consumables. Table 5 and Figure 5 below illustrate the obtained results.

Table 5. The effect of the variation in the customer's average consumption of consumables during the second period (μ_2).

		μ_2: Normal Distribution (200, 75)	μ_2: Normal Distribution (300, 75)	μ_2: Normal Distribution (500, 75)	μ_2: Normal Distribution (650, 75)	μ_2: Normal Distribution (800, 75)
τ_j^*	j = 1	7	7	7	7	7
	j = 2	7	7	7	5	5
$(m_k)_{j,i}^*$	j = 1	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$
	j = 2	$(m_k)_{2,1} = 1$ $(m_k)_{2,2} = 1$	$(m_k)_{2,1} = 2$ $(m_k)_{2,2} = 2$	$(m_k)_{2,1} = 3$ $(m_k)_{2,2} = 2$	$(m_k)_{2,1} = 4$ $(m_k)_{2,2} = 2$ $(m_k)_{2,3} = 2$	$(m_k)_{2,1} = 4$ $(m_k)_{2,2} = 3$ $(m_k)_{2,3} = 2$
n_j	j = 1	1	1	1	1	1
	j = 2	2	2	2	3	3
Average income $R_{j.}$ (USD)	j = 1	450,000	450,000	450,000	450,000	450,000
	j = 2	300,130	450,000	750,000	975,000	1,194,100
C_j (USD)	j = 1	252,000	252,000	252,000	252,000	252,000
	j = 2	168,070	252,000	420,000	546,000	668,710
V_j (USD)	j = 1	32,000	32,000	32,000	32,000	32,000
	j = 2	24,000	24,000	24,000	24,000	24,000
Average cost of CM (USD)	j = 1	4072.4	4072.4	4072.4	4072.4	4072.4
	j = 2	4615.5	5750,8	8664.6	9566.2	10,950
Average cost of PM (USD)	j = 1	2000	2000	2000	2000	2000
	j = 2	1000	2000	3000	5000	6000
Penalty Pn_j (USD)	j = 1	671.95	671.95	671.95	671.95	671.95
	j = 2	507.93	948,89	2382.8	3419.9	4794.2
Average profit P_j (USD)	j = 1	143,260	143,260	143,260	143,260	143,260
	j = 2	69,935	133,300	259,950	355,010	447,670

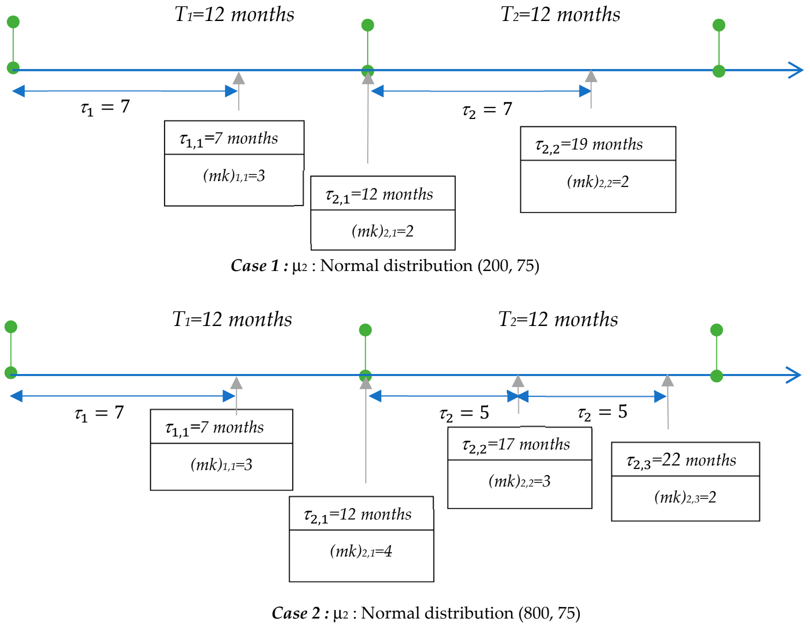

Figure 5. The obtained optimal PM strategies for the lower and upper bounds of the customer's average consumption of consumables during the second period (μ_2: N(200, 75) and μ_2: N(800, 75)).

The higher the average rate of consumption (μ_2), the more the equipment is used, the more it degrades, and the more likely it is to fail. As a result, the optimal strategy is to perform preventive maintenance more frequently (by shortening the optimal period) and to improve the effectiveness of the PM actions to be performed.

6.2.3. The Influence of Varying the Selling Price of a Unit of Consumables, Ss

We varied the selling price of a unit of consumables *Ss*. The results obtained are shown below (Table 6 and Figure 6).

Table 6. The effect of the variation in the selling price of a unit of consumables, Ss, on the optimal solution.

		Ss = USD 90	**Ss = USD 100**	**Ss = USD 125**	**Ss = USD 150**	**Ss = USD 175**
τ_j*	j = 1	7	7	7	7	7
	j = 2	7	7	7	7	5
$(m_k)_{j,i}$*	j = 1	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$	$(m_k)_{1,1} = 3$
	j = 2	$(m_k)_{2,1} = 3$ $(m_k)_{2,2} = 2$	$(m_k)_{2,1} = 3$ $(m_k)_{2,2} = 2$	$(m_k)_{2,1} = 3$ $(m_k)_{2,2} = 2$	$(m_k)_{2,1} = 4$ $(m_k)_{2,2} = 2$	$(m_k)_{2,1} = 4$ $(m_k)_{2,2} = 2$ $(m_k)_{2,3} = 2$
n_j	j = 1	**1**	**1**	**1**	**1**	**1**
	j = 2	**2**	**2**	**2**	**2**	**3**
Average income R_j (USD)	j = 1	324,000	360,000	450,000	630,000	720,000
	j = 2	520,000	600,000	750,000	1,050,000	1,200,000
C_j (USD)	j = 1	252,000	252,000	252,000	252,000	252,000
	j = 2	420,000	420,000	420,000	420,000	420,000
V_j (USD)	j = 1	32,000	32,000	32,000	32,000	32,000
	j = 2	24,000	24,000	24,000	24,000	24,000
Average cost of CM (USD)	j = 1	4072.4	4072.4	4072.4	4072.4	4072.4
	j = 2	8664.6	8664.6	8664.6	7971.5	7359
Average cost of PM (USD)	j = 1	2000	2000	2000	2000	2000
	j = 2	3000	3000	3000	4000	5000
Penalty Pn_j (USD)	j = 1	244,34	366.52	671.95	1282.80	1588.2
	j = 2	866.5	1299.7	2382.8	4185.1	4783.3
Average profit P_j (USD)	j = 1	17,683	53,561	143,260	322,640	412,340
	j = 2	51,469	111,040	259,950	557,840	706,860

As the selling price of a unit of consumables (*Ss*) increases, the lessor's unit profit increases. In such a situation, the optimal maintenance strategy advocates increasing the PM effort to improve equipment availability and consequently increase the quantity of consumables to be sold.

We can observe that this increase in the number and efficiency of PM actions leads to a reduction in the average cost of maintenance, which is induced by the reduction in the number of minimal repairs and, consequently, in the average cost of CM.

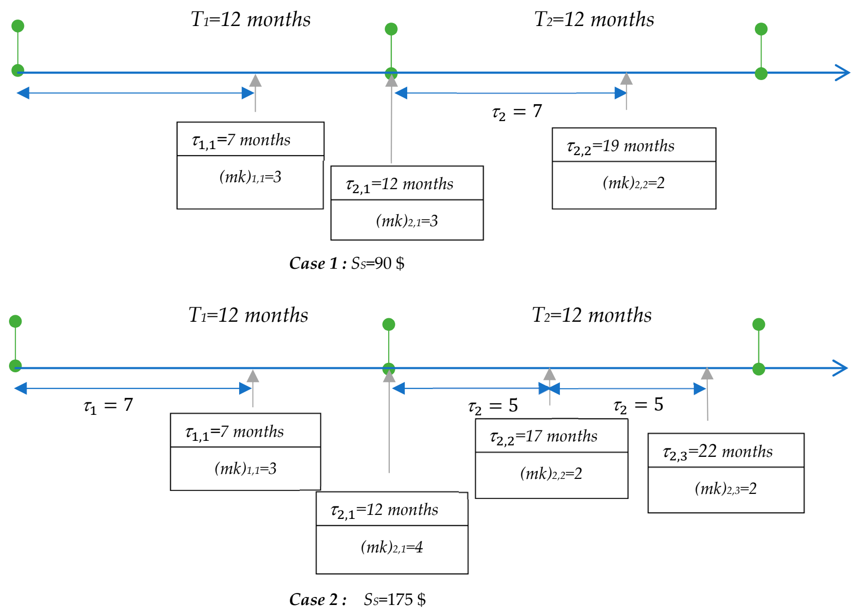

Figure 6. The obtained optimal PM strategies for the lower and upper bounds of the selling prices of a unit of consumables (Ss = USD 90 and Ss = USD 175).

6.2.4. The Influence of the Variation in the Duration of the Second Free Lease Period, T_2

Table 7 below shows the results obtained for the optimal values of the decision variables, as well as the various average costs and the average profit, for different values of the duration of the second lease period.

Table 7. The effect of the variation in the duration of the 2nd lease period T_2 on the optimal solution.

		T_2 = 8 Months	**T_2 = 12 Months**	**T_2 = 18 Months**
τ_j*	j = 1	**7**	**7**	**7**
	j = 2	**5**	**7**	**5**
$(m_k)_{j,i}$*	j = 1	**$(m_k)_{1,1} = 3$**	**$(m_k)_{1,1} = 3$**	**$(m_k)_{1,1} = 3$**
	j = 2	**$(m_k)_{2,1} = 2$ $(m_k)_{2,2} = 2$**	**$(m_k)_{2,1} = 3$ $(m_k)_{2,2} = 2$**	**$(m_k)_{2,1} = 4$ $(m_k)_{2,2} = 3$ $(m_k)_{2,3} = 2$ $(m_k)_{2,4} = 2$**
n_j	j = 1	**1**	**1**	**1**
	j = 2	**2**	**2**	**4**
Average income R_j (USD)	j = 1	450,000	450,000	450,000
	j = 2	500,000	750,000	1,125,000
C_j (USD)	j = 1	252,000	252,000	252,000
	j = 2	280,000	420,000	630,000
V_j (USD)	j = 1	32,000	32,000	32,000
	j = 2	26,667	24,000	20,000
Average cost of CM (USD)	j = 1	4072.4	4072.4	4072.4
	j = 2	5086.8	8664.6	10,082

Table 7. *Cont.*

		T_2 = 8 Months	T_2 = 12 Months	T_2 = 18 Months
Average cost of PM (USD)	j = 1	2000	2000	2000
	j = 2	2000	3000	8000
Penalty Pn_j (USD)	j = 1	671.95	671.95	671.95
	j = 2	1398.9	2382.8	2772.6
Average profit P_j (USD)	j = 1	143,260	143,260	143,260
	j = 2	158,180	259,950	414,150

It is worth noting that if the lessor decides to lease the machine for a relatively long period of time per customer, they will have to perform more preventive maintenance actions (n_j increases) with relatively high levels of efficiency at the beginning of the period.

Finally, it is important to note that as the number n_j of PM actions increases over longer periods, the program runtime, in turn, increases significantly. This is mainly due to the increase in the number of possible scenarios (K^{n_j}, K being equal to five possible PM levels in the example considered). Table 8 gives examples of the number of possible scenarios and the execution time of the program when it is not blocked (without saturation).

Table 8. Possible scenarios of the application of PM efficiency levels and their durations.

n_j	*Possible Scenarios*	*Program Runtime*
1	5	1 min 11 s
2	25	2 min 6 s
3	125	3 min 12 s
4	625	6 min 27 s
5	3125	22 min 5 s
6	15,625	16 h 27 min
7	78,125	Saturation
8	390,625	Saturation
9	1,953,125	Saturation
10	9,765,625	Saturation
11	48,828,125	Saturation
12	244,140,625	Saturation

The blockage could also be caused by the increment value $\Delta\tau$ being too small.

Thus, it is clear that the use of other heuristic-based algorithms will be necessary to reduce the runtime and to allow for good solutions (avoiding saturation) to be found for large instances. Therefore, we developed an Artificial Bee Colony algorithm (ABC) to solve any instance of the problem.

6.3. Implementation of Artificial Bee Colony Algorithm (ABC)

6.3.1. The ABC Algorithm

In order to deal with large-sized instances of the problem, we implemented an Artificial Bee Colony algorithm (ABC), which allowed us to obtain good solutions (not necessarily optimal) for any given number of lease periods and maintenance task packages.

The ABC algorithm was developed in [30]. This algorithm is widely used to solve various numerical optimization problems. It is inspired by the foraging behavior of bee colonies. It consists of three groups: employed bees, onlooker bees, and scout bees. It generates an initial population of randomly distributed solutions. The employed bees modify the positions in memory according to the amount of food. The onlooker bees evaluate the food information from the employed bees and choose a food source. The scout bees are responsible for finding new food sources when the current ones are exhausted. The algorithm has control parameters, such as food sources (solutions), numbers of employed and scout bees, thresholds, etc. Its advantages include fast convergence and efficient exploration and exploitation.

Figure 7 below illustrates the global framework of the ABC algorithm, which was adapted and implemented in this study. The framework comprises four phases: initialization, employed bees, onlooker bees, and scout bees. Details about each phase are provided below.

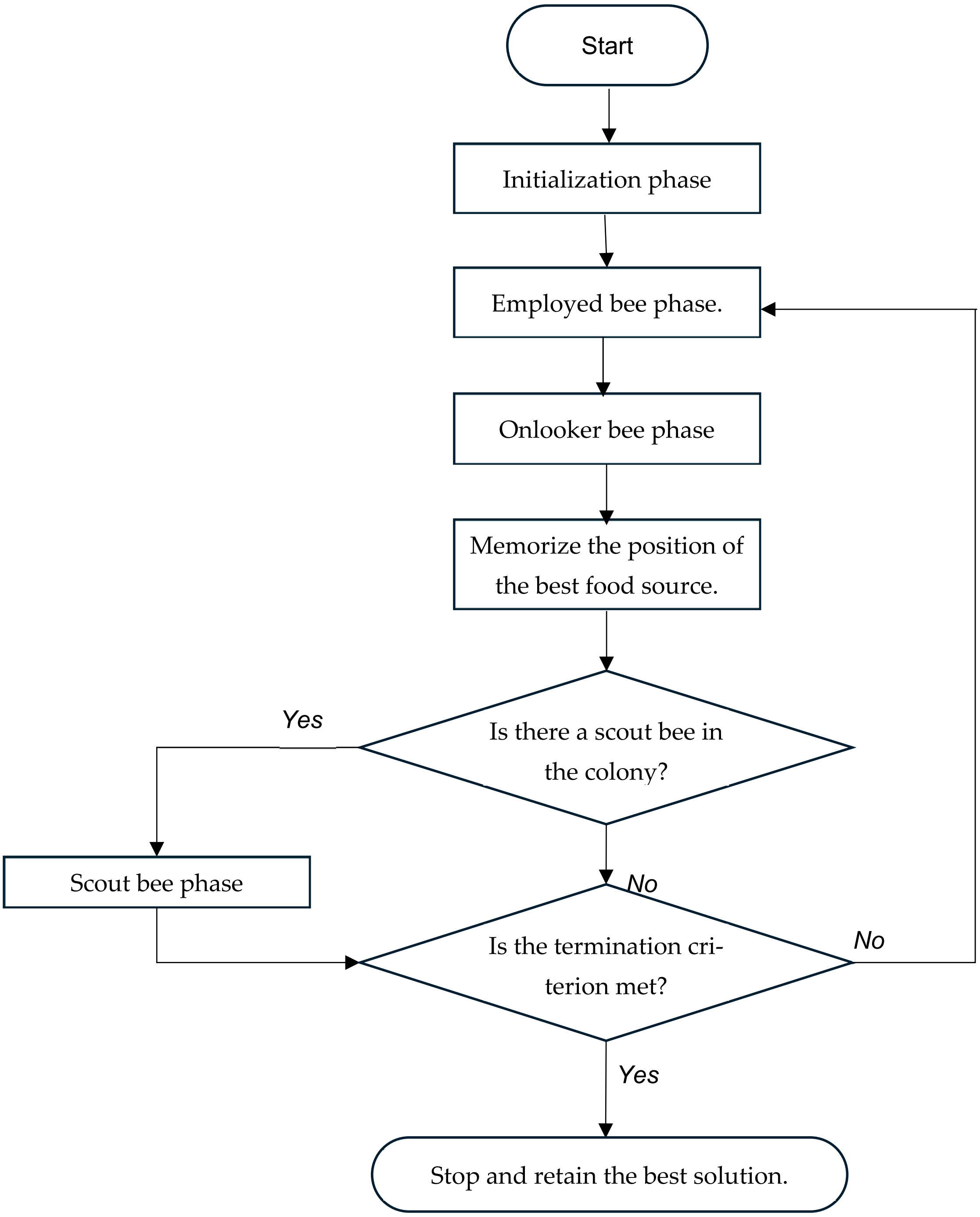

Figure 7. A flowchart of the used ABC algorithm.

(a) Initialization phase:

At the initialization phase, a number of food sources (initial solutions) N_f is selected. N_f will be equal to the number of the employed bees and onlooker bees. The position information of each food source l, $l \in (1, \ldots, N_f)$ is described by a vector $w_l = (\tau_j^l, (m_k)_{j,i}^l)$, $i = 1, 2, \ldots, n_j$, which corresponds to the two decision variables. Here, $(m_k)_{j,i}^l$ represents the maintenance level applied for each of the n_j PM actions for the lth food source.

The PM period τ_j^l is generated randomly as follows:

$$\tau_j^l = \tau_0 + rand(0,1)\left(T_j - \tau_0\right) \tag{23}$$

where τ_0 is the lower bound of the maintenance period and rand (0, 1) is a random number distributed uniformly over the interval [0, 1].

An additional operation is required for the random selection of the maintenance levels $(m_k)_{j,i}^l$ ($i = 1, 2, \ldots, n_j$) among the set of K available PM action packages, with n_j given in Equation (4).

$$(m_k)_{j,i}^l = randsample(K)\ \text{for}\left(i = 1, 2, \ldots, n_j\right) \tag{24}$$

In this step, each food source is assigned a trial variable trl, $l = 1, 2, \ldots, N_f$; trl is initialized at 0 for each food source.

(b) Employed bee phase:

The employed bee explores each food source w_l and a new temporary food source w_l^{new} around it. A new position for a food source is generated, and its fitness is then evaluated $fit\left(w_l^{new}\right) = \frac{1}{\left(1+P_j\left(w_l^{new}\right)\right)}$. If the fitness of the new position is better than that of the old one $fit\left(w_l^{new}\right) < fit(w_l)$, then this new food source replaces the old one, and the trial variable trl for this new food source is set to zero. Otherwise, trl for the old food source is incremented by 1 [31].

(c) Onlooker bee phase:

At this step, the onlooker bees choose a food source with a higher quality that will have a larger probability, expressed as follows: $p_l = \frac{fit_l}{\sum_{q=1}^{Nf} fit_q}$. Based on p_l and a generated random number, r, in the interval (0, 1) using the roulette wheel selection [32], the onlooker bees select a solution to update using Equation (25), and a better solution is selected with respect to its fitness. The new solution is updated as follows:

$$\tau_j^{l,new} = \tau_j^l + \psi\left(\tau_j^l - \tau_j^q\right),\ (m_k)_{j,i}^{l,new} = (m_k)_{j,i}^l + round\left(\psi\left((m_k)_{j,i}^l - (m_k)_{j,i}^q\right)\right) \tag{25}$$

where ψ is a uniformly distributed real number within [−1, 1]; round(X) rounds X to the nearest integer, and $q \in (1, \ldots, N_f)$, $q \neq l$, $\tau_j^{l,new} \in \left[\tau_0, T_j\right]$ and $(m_k)_{j,i}^{l,new} \in \{m_k,\ k = 1,\ 2,\ \ldots,\ K\}$.

(d) Scout bee phase:

If the number of trials for a given food source reaches a predetermined threshold (tr_{th}) without improvement, this food source is rejected. The employed bee corresponding to this rejected food source is transformed into a scout bee, and the food source is replaced with a food source that is generated using the same method used in the initialization phase.

Once the maximum predetermined number of cycles (N_{max}) is reached, the algorithm is stopped, and the solution corresponding to the highest obtained average profit is selected.

6.3.2. Obtained Results

The ABC algorithm has been tested using many numerical examples. One of them consists of taking the same input parameters of the numerical example presented in the previous section but considers cases of a larger second lease period T_2 (18, 24, and 36 months) instead of the initial second period, T_2 = 12 months.

Table 9 shows the values we used for the control parameters of the ABC algorithm.

Table 9. Input parameters used for the ABC algorithm.

N_f	tr_{th}	N_{max}	τ_0
7	3	70	1 month

Table 10 below shows the results obtained using the ABC algorithm. The time needed to obtain the solution for each instance is also given. It is worth noting that the solutions obtained using the ABC algorithm are very close to those obtained with the first algorithm (when possible). Also, as the second period becomes longer with a greater number n_j of PM actions, the ABC algorithm takes much less time than the first algorithm, and it is the only one that yields a solution for the cases of T_2 = 24 months and T_2 = 36 months (the first algorithm saturates in these cases).

Table 10. The effect of the variation in the duration of the 2nd lease period T_2 using the ABC algorithm.

		T_2 = 12 months	**T_2 = 18 months**	**T_2 = 24 months**	**T_2 = 36 months**
Runtime		**2 min**	**9 min**	**28 min**	**44 min**
τ_j*	j = 1	**6.65**	**6.65**	**6.65**	**6.65**
	j = 2	**6.43**	**4.94**	**5.32**	**5.75**
$(m_k)_{j,i}$*	j = 1	**$(m_k)_{1,1}$ = 3**	**$(m_k)_{1,1}$ = 3**	**$(m_k)_{1,1}$ = 3**	**$(m_k)_{1,1}$ = 3**
	j = 2	**$(m_k)_{2,1}$ = 3 $(m_k)_{2,2}$ = 2**	**$(m_k)_{2,1}$ = 4 $(m_k)_{2,2}$ = 4 $(m_k)_{2,3}$ = 3 $(m_k)_{2,4}$ = 2**	**$(m_k)_{2,1}$ = 4 $(m_k)_{2,2}$ = 4 $(m_k)_{2,3}$ = 4 $(m_k)_{2,4}$ = 3 $(m_k)_{2,5}$ = 2**	**$(m_k)_{2,1}$ = 4 $(m_k)_{2,2}$ = 4 $(m_k)_{2,3}$ = 4 $(m_k)_{2,4}$ = 4 $(m_k)_{2,5}$ = 4 $(m_k)_{2,6}$ = 3 $(m_k)_{2,7}$ = 2**
n_j	j = 1	**1**	**1**	**1**	**1**
	j = 2	**2**	**4**	**5**	**7**
Average income R_j (USD)	j = 1	450,000	450,000	450,000	450,000
	j = 2	750,000	1,125,000	3,000,000	9,000,000
C_j (USD)	j = 1	252,000	252,000	252,000	252,000
	j = 2	420,000	630,000	1,680,000	5,040,000
V_j (USD)	j = 1	32,000	32,000	32,000	32,000
	j = 2	24,000	20,000	16,000	8000
Average cost of CM (USD)	j = 1	4001	4001	4001	4001
	j = 2	8522	9973	12780	14801
Average cost of PM (USD)	j = 1	2000	2000	2000	2000
	j = 2	3000	8000	12,000	18,000
Penalty Pn_j (USD)	j = 1	647	647	647	647
	j = 2	2195	2553	2971	3207
Average profit P_j (USD)	j = 1	143,352	143,352	143,352	143,352
	j = 2	260,283	414,474	1,228,249	3,851,992

7. Conclusions

In this paper, we focused on the development and optimization of a preventive maintenance strategy for equipment freely leased by a lessor in exchange for the customer's purchase of a quantity of equipment consumables during the lease period.

The lessor acquires a given piece of equipment and then makes it available to some of their customers for several successive periods along its life cycle. Each time, the lessor is responsible for the corrective and preventive maintenance of the equipment. The lessor's revenue comes from the sale of equipment consumables to each customer. The quantities sold are random. An estimate of their probability distribution function is provided by the customer before the lease period begins.

The more consumables the customer uses, the more they degrade the equipment, and the greater the risk of equipment failure and unavailability. To reduce the risk of loss of income, the lessor offers to maintain the equipment at their own expense. In this way, the latter must carefully weigh the preventive maintenance effort to be deployed in order to adapt, as closely as possible, to the usage rate forecasted by the customer while guaranteeing maximum profit.

In this context, we developed an expression of the average profit to be obtained by the lessor over a given lease period. The aim was to express this profit as a function of decision variables, namely the period of preventive maintenance actions to be performed and their degree of effectiveness. We then developed a numerical procedure for determining, for any instance of the problem, the optimal values of the decision variables that maximize the lessor's profit over each of the lease periods in a sequential manner (starting from the first to the last period along the equipment's life cycle).

To show the consistency and robustness of the mathematical model, we presented a numerical example describing an arbitrary instance of the problem, taking care to choose input values that are consistent from a practical point of view.

The sensitivity analysis performed identified the effects of variations in certain input parameters on the optimal maintenance strategy to be adopted by the lessor during each lease period. The robustness of the model was verified.

It was observed that the size of the problem can quickly become very large, especially if the lease period is long enough to contain many PM actions. In such situations, the computation time becomes extremely long, leading to saturation. For this reason, another algorithm of the Artificial Bee Colony type was proposed to solve large instances of the problem in a reasonable time, obtaining good solutions.

An important extension of this work would be to consider the lessee's revenues generated using the equipment. This could be carried out by developing a framework that considers both the lessor's and lessee's revenues. This framework should result in a win–win situation for the lessee and the lessor. It would also be interesting to consider associating estimated average execution times with each package of preventive maintenance actions, rather than just average costs, as was carried out in this work.

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