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Significant Location Accuracy Changes Resulting from Lightning Detection Networks Deployed on Inclined Terrains

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Abstract: This study investigates the location accuracy distribution of the lightning detection networks (LDNs) deployed on inclined terrains, an aspect frequently encountered in complex terrains but hitherto disregarded in previous studies. By designing eight substation LDNs deployed on slope-type (ST), mountain-type (MT) and basin-type (BT) terrains, respectively, we employed Monte Carlo simulations to analyze their spatial location accuracy distribution based on time-of-arrival technology. Significant differences among the LDNs on inclined terrains and between them and the LDN on plain-type (PT) terrain were revealed. Compared to the PT LDN, LDNs on inclined terrains exhibited a reduction in high-precision location regions and a shift in the distribution pattern of location accuracy. The ST LDN showed marked deviations in the high-precision vertical location toward the lower slope side with increases in the elevation angle and consistently smaller high-precision vertical location areas compared to MT and BT LDNs. The variations in elevation angles of MT and BT LDNs had a substantial impact on the spatial distribution patterns of both horizontal and vertical location accuracy, with BT LDNs featuring larger vertical high-precision areas than MT LDNs. Our conclusions were further corroborated through an analysis of an actual LDN, which combined characteristics from both ST and MT terrain patterns.

Keywords: inclined terrain; lightning detection network; lightning location accuracy; time-of-arrival technique; Monte Carlo simulation



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1. Introduction

A lightning discharge is characterized by high voltage, large current and strong electromagnetic radiation, often causing significant human casualties and property damage [1–4]. Lightning location systems (LLSs) play a vital role in providing essential data on lightning events, including precise times, locations, and intensities. This information is critical for early warning, prevention, and investigation of lightning disasters.

Most LLSs detect electromagnetic signals generated by lightning discharges using a multi-station network, and time-of-arrival (TOA) location technology is now one of the most commonly used localization methods. Ground-based lightning detection networks (LDNs) can be broadly categorized into three categories based on their observation scope: The first is the global LDN, such as the Worldwide Lightning Location Network [5,6], Global Lightning Detection Network [7,8], Earth Networks Total Lightning Network [9,10], etc. The second is the national or intercontinental LDNs, such as the National Lightning Detection Network in the United States [11–13], Advanced TOA and Direction lightning

location system in China [14], VLF/LF Lightning Detection Network in Europe [15,16], etc. The third is the regional LDN, which typically pays more attention to total lightning detection and the precise and detailed location of the lightning discharge processes, such as the Lightning Mapping Array [17,18], Low-frequency E-field Detection Array [19–21], Beijing Lightning Network [22,23], Fast Antenna Lightning Mapping Array [24], Lightning Position and Tracking System [25,26], etc. The baselines in such LDNs tend to be relatively short; therefore, the layout of LDNs may be significantly impacted by the regional terrain, especially in complicated mountainous (or hilly) areas, and junction areas between plains and mountains. Such terrain-related factors may result in certain tilting characteristics in the overall LDN layout.

There are various factors contributing to the location errors in an LDN, such as time error in the detection system itself, antenna position error, the number of substations, signal acquisition accuracy and processing method, the effects of atmosphere and terrain on signal propagation, and so on [27–30]. The layout of an LDN plays a crucial role in determining the spatial distribution of lightning location accuracy. Prior studies predominantly emphasized the location accuracy distribution of LDNs with varying geometric configurations, typically deployed over flat terrains [27,31]. No studies have delved into the impact of LDNs featuring a consistent vertical layout on location accuracy. Nevertheless, in the real-world scenario of deploying a regional LDN in complex terrains, where the network aligns with the terrain contours, such circumstances are frequently encountered.

In this study, we first theoretically defined several scenarios for LDN layouts in complex terrains. They include slope-type (ST), mountain-type (MT) and basin-type (BT) layout patterns. All these layout patterns encompass inclined surfaces, collectively referred to as ‘inclined terrains’ in this study. The spatial location accuracy distribution of LDNs deployed on these inclined terrains was meticulously examined and compared with LDNs deployed on plain-type (PT) terrain, unveiling significant disparities in our analysis. Subsequently, the model results were subjected to further validation using an actual LDN situated over the Tibetan Plateau (TP), exhibiting an inclined configuration due to the local topography. To the best of our knowledge, this study may be a pioneering exploration of location accuracy for LDNs featuring an inclined layout in a vertical direction. It offers valuable insights and references for optimizing the design and using the data of LDNs deployed in intricate topographical settings.

2. Methodology

Four representative terrains, PT, ST, MT, and BT terrains were designed in this study. Substations of LDNs were deployed on these terrains. Previous studies primarily focused on the location accuracy distribution of an LDN located in PT terrain; therefore, PT LDN will serve as the main reference for comparison. The latter three inclined terrains and relevant layout patterns of the substations of LDNs are shown in Figure 1a–c, where the MT terrain is idealized as a normal pyramid, while the BT terrain is idealized as an inverted pyramid, both containing four inclined surfaces.

Eight regularly distributed detection substations are placed in the study area. In the horizontal projection (Figure 1d), the four inner substations form a square with a side length of 10 km. The square’s sides are parallel to the x -axis and y -axis, with the center position being located at the coordinate origin. The four outer substations are located on the x -axis and y -axis, each at a distance of 30 km away from the coordinate origin. For the ST LDN (Figure 1a), all eight substations are pre-arranged on the inclined surface. In the case of MT (Figure 1b) and BT (Figure 1c) LDNs, the four inner substations are arranged on the ridges of the pyramids, while the four outer substations are positioned at the middle of the bottom edges of the four triangular slopes forming the pyramids. In each layout pattern, the pre-arranged position for each substation a random height adjustment value is added, which is taken as a random number from a normal distribution with a range of 0 to 10 m. To investigate the influence of the inclination degree of the inclined surface on the location accuracy of LDN, the elevation angle of the inclined surface (angle with the

horizontal surface) is set to vary at an interval of 0.1° between 0.1° and 1° . Figure 1a–c illustrates the network layouts at an elevation angle of 1° .

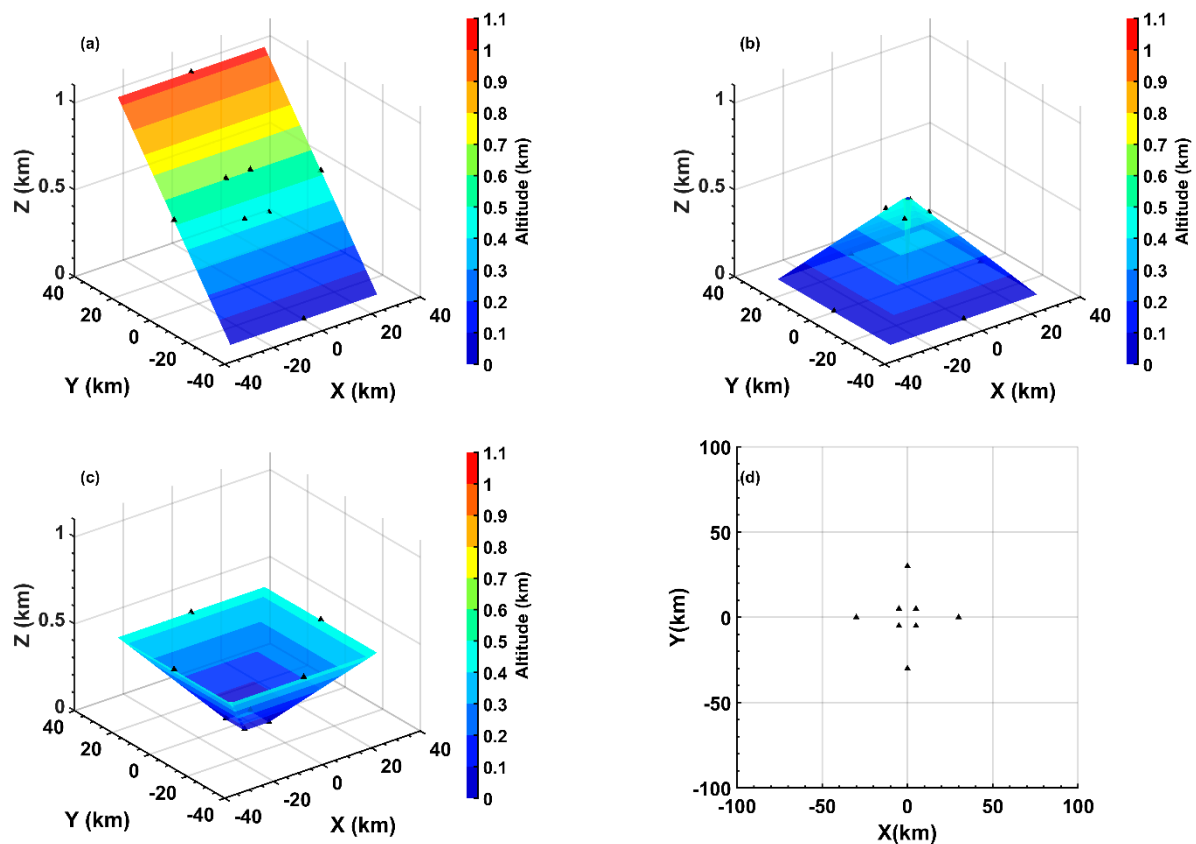


Figure 1. Layouts of lightning detection networks (LDNs) deployed on: (a) slope-type (ST) terrain; (b) mountain-type (MT) terrain; (c) basin-type (BT) terrain with inclined surfaces having the elevation angle of 1° ; and (d) horizontal projection of these LDNs. The black triangles in the figure indicated the positions of substations.

The Monte Carlo simulation is a commonly used method for studying the accuracy of LDN [17,21,23]. In this study, the simulation covered a horizontal spatial domain of $200 \times 200 \text{ km}^2$ with a spatial resolution of 10 km horizontally and 2 km vertically. The center position is located at the horizontal center of the simulated network (Figure 1d). The simulation first assumed a lightning discharge event occurring at the center of a specific grid, and then calculated the time taken for this event to propagate in a straight line at the speed of light to each substation. Then, a Gaussian random error, with a mean of 0 ns and a root mean square of 100 ns, was superimposed on this time to simulate the real measured arrival time at the substation. Notably, considering that this study mainly focuses on the effect of the inclined layout of the LDN, the influence of time errors of signal acquisition on location accuracy has not been tested, because varying the time errors would alter the location accuracy on the same grid point, but typically would not change the spatial relative distribution pattern of the location error [21]. To ensure statistical robustness, two hundred simulations were performed for each grid point in space to obtain the average location errors in the horizontal and vertical directions for the corresponding grid.

The TOA lightning location algorithm was used in the simulation [17,21]. The arrival time t_i of the electromagnetic pulse from a radiation source occurring at (X, Y, Z) to the i -th substation at (X_i, Y_i, Z_i) can be expressed as:

$$t_i = t + \left[(X_i - X)^2 + (Y_i - Y)^2 + (Z_i - Z)^2 \right] / c \quad (1)$$

where t is the occurring time, and c is the speed of light. When the number of substations in the location system is not less than five, multiple equations similar to Equation (1) can be obtained based on the observations from these five substations, constituting a set of nonlinear equations. The numerical solution is optimized using the Levenberg-Marquardt algorithm, which can be found in [17,21].

3. Results and Analysis

3.1. Spatial Distribution Pattern of Lightning Location Accuracy

For the purpose of this presentation, this section focuses on the spatial distribution of horizontal and vertical location errors, as well as their evolutionary trends for the ST, MT, and BT LDNs at elevation angles of 0.1, 0.5, and 1.0° and heights of 2, 10, and 18 km, respectively. As a point of comparison, the PT LDN at the same heights (2, 10, and 18 km) is also utilized. While we have not displayed data for all elevations and heights, it is important to note that the evolution trend of lightning location accuracy, corresponding to the elevations and heights shown, aligns consistently with the overall trend.

3.1.1. PT LDN

The lightning location accuracy of the PT LDN has been extensively studied in previous research. Here, we utilize the PT LDN layout illustrated in Figure 1d to analyze the spatial distribution of lightning positioning accuracy, employing it as a reference for comparison.

Figure 2a–c illustrates that the horizontal location error distribution pattern of the PT LDN remains relatively consistent with varying heights. Regions with horizontal location errors below 0.2 km are nearly circular, while the regions with errors greater than 0.2 km display a “◇” pattern associated with the PT LDN layout. As shown in Figure 2d–f, the low-value vertical location error regions of the PT LDN exhibit a “+” pattern associated with the LDN layout. Moreover, the high-precision location regions demonstrate a clear increasing trend with increases in height.

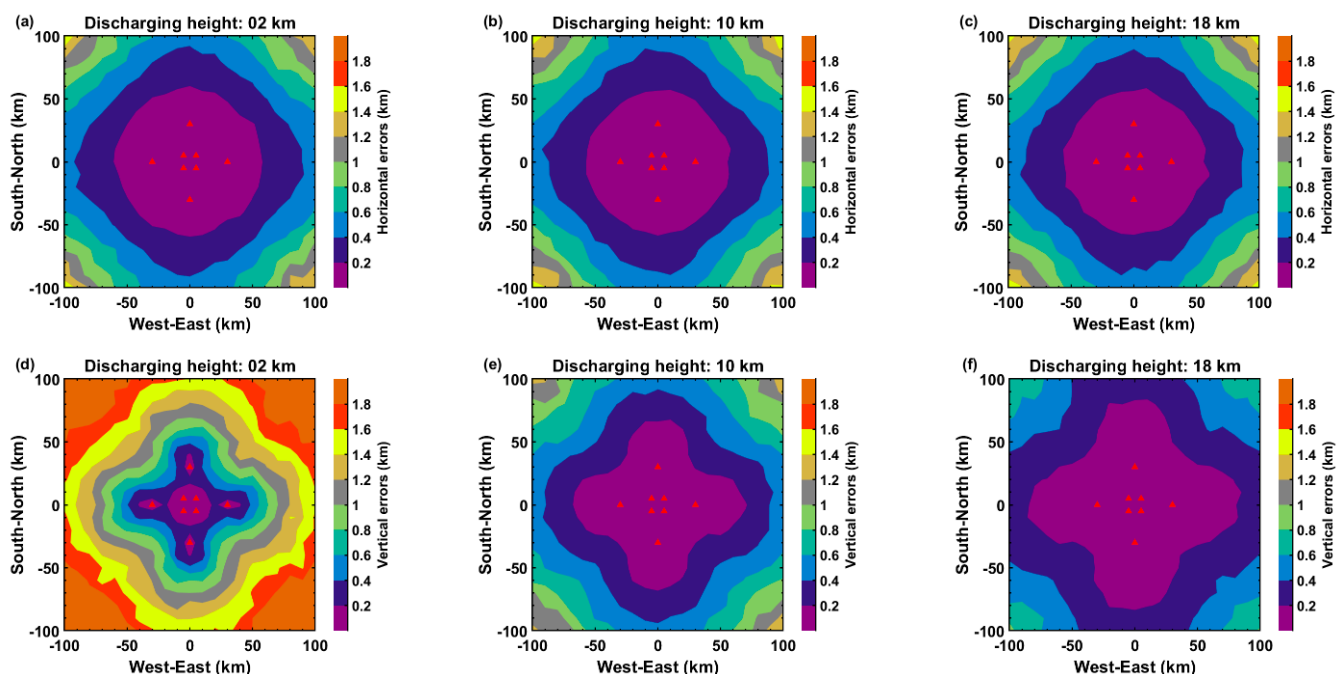


Figure 2. Horizontal (a–c); and vertical errors (d–f) of lightning locations for the PT LDN. The heights of the simulated lightning discharge events are 2 km (a,d); 10 km (b,e); and 18 km (e,f). The red triangles in the figure indicated the positions of substations.

3.1.2. ST LDN

Figure 3 presents the horizontal error distribution of lightning location for the ST LDN at elevation angles of 0.1° , 0.5° , and 1.0° corresponding to heights of 2, 10, and 18 km, respectively. Similar to the PT LDN, the distributions of regions with horizontal errors greater than 0.2 km here still tend to exhibit the “ $\diamond$ ” pattern associated with the network layout. For the case of an elevation angle of 0.1° (Figure 3a–c), the distributions of horizontal errors show minimal variations with changes in height, and the overall distribution pattern of location errors closely resembles that of the PT LDN. However, as the elevation angle increases to 0.5° (Figure 3d–f) and 1° (Figure 3g–i), the high-precision location regions shrink significantly with increases in height. In terms of different elevation angles for the same height, when discharging events occur at a lower level, such as 2 km (Figure 3a,d,g), the distribution pattern of horizontal errors and the regions of different location errors remain relatively stable, regardless of varying elevation angles. In contrast, at higher heights, such as 10 (Figure 3b,e,h) and 18 km (Figure 3c,f,i), the increase in the elevation angle leads to a significant decrease in the region of low-value horizontal errors.

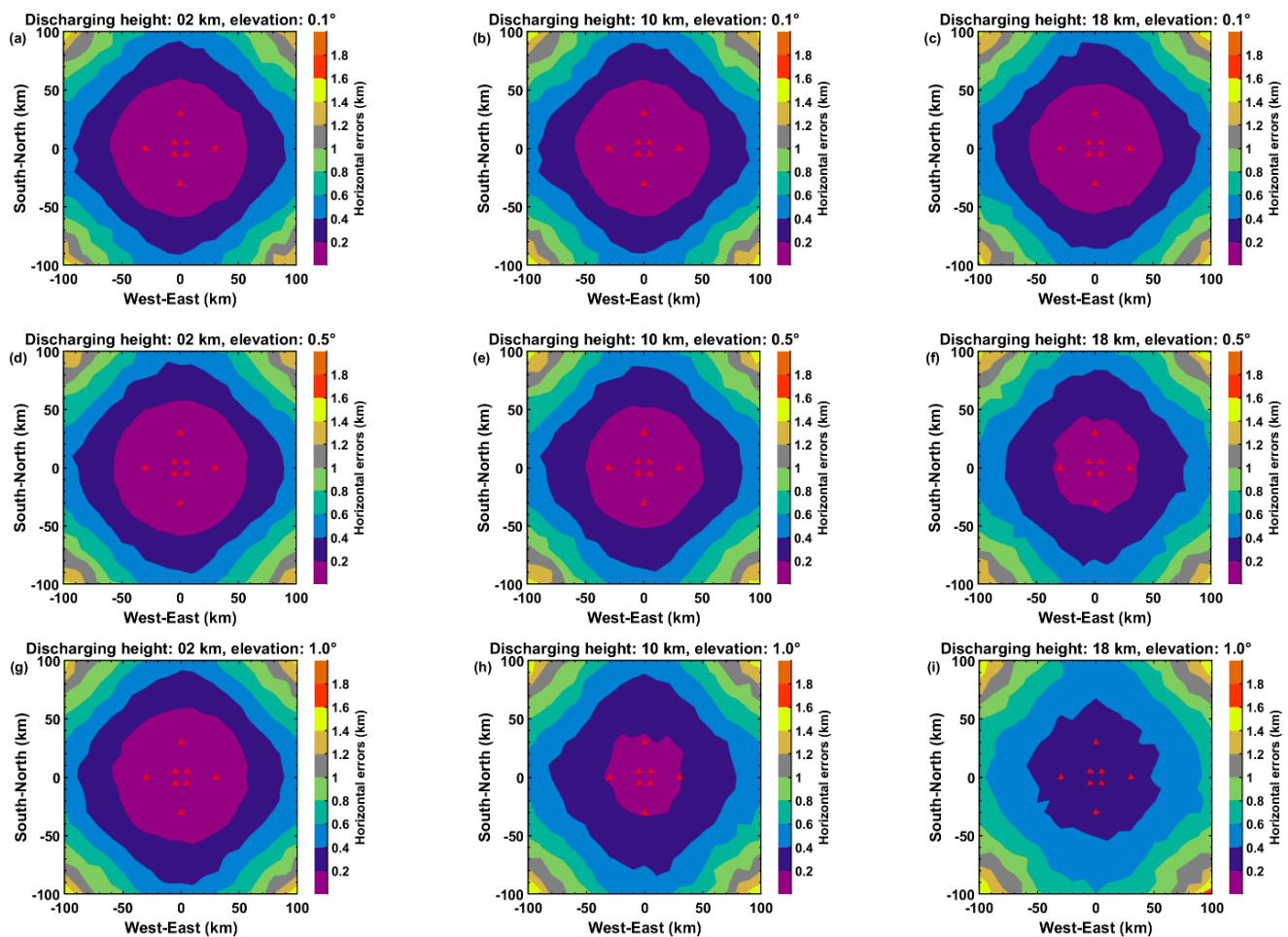


Figure 3. Horizontal errors of lightning location for the ST LDN. The elevation angles are 0.1° (a–c); 0.5° (d–f); and 1.0° (g–i). The heights are 2 km (a,d,g); 10 km (b,e,h); and 18 km (c,f,i), respectively. The red triangles in the figure indicated the positions of substations.

Figure 4 illustrates the spatial distribution of vertical location errors in the ST LDN. Unlike the PT LDN, the vertical location error distribution pattern for the ST LDN exhibits significant variations with increases in height and elevation angle. When the lightning discharge event is located at a height of 2 km and the elevation angle of the inclined surface is 0.1° (Figure 4a), there is no significant difference between the distribution pattern of the

vertical errors for the ST and PT LDNs. However, at an elevation angle of 0.5° (Figure 4d), the low-value vertical error regions shrink significantly. In addition, the original low-error convex region on the north side of the network (the region with vertical error less than 0.6 km in Figure 4a) disappears, resulting in the overall distribution of the high-precision location region changing from the “+” shape at the elevation angle of 0.1° to the flat “T” shape at the elevation angle of 0.5° . As the elevation angle is further increased to 1° (Figure 4g), the area of low-value vertical errors further reduces in size, with its core region distributed around the southernmost substation. It is worth noting that the vertical error of 0.8–1 km forms a relatively independent region on the north side of the northernmost substation, almost separated from the corresponding vertical error region in the shape of the flat “T” on the south side (Figure 4g). At 10 (Figure 4b,e,h) and 18 km heights (Figure 4c,f,i), the centers of the low-value vertical error regions divert obviously to the south side, and their area tends to decrease with the increasing elevation angle. Furthermore, the centers of the high-precision location regions move near the southernmost substation at elevation angles of 0.5° and 1° , resulting in the overall vertical error distribution pattern evolving from a “+” shape to a flat “T” shape first, and then to the “–” shape. In brief, for the ST LDN, the increase in elevation angle results in the overall vertical location having greater accuracy in the lower regions of the inclined surface than in the upper regions, with the high-precision regions shrinking towards the substation in the lower part of the inclined surface.

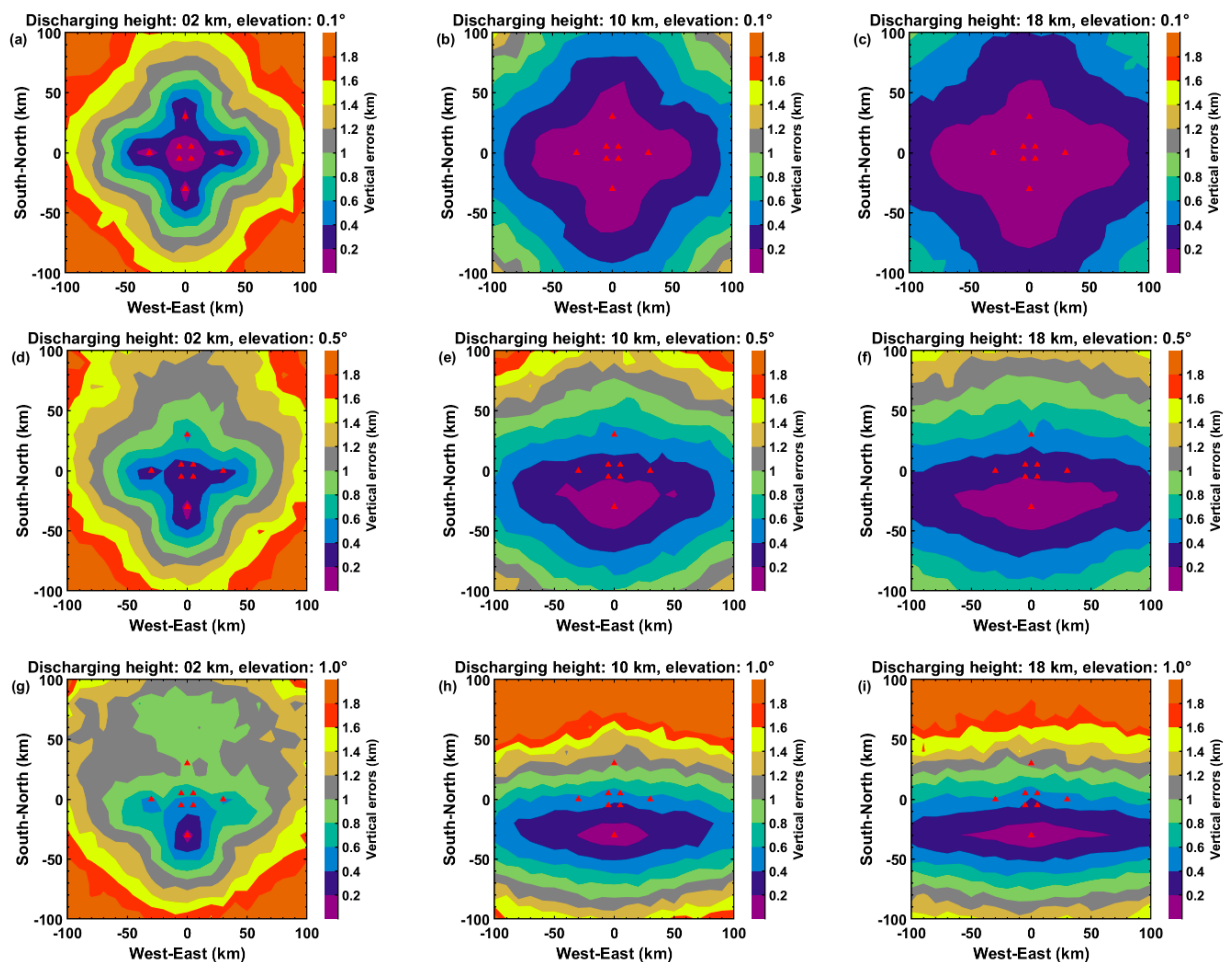


Figure 4. Vertical errors of lightning location for the ST LDN. The elevation angles are 0.1° (a–c); 0.5° (d–f); and 1.0° (g–i). The heights are 2 km (a,d,g); 10 km (b,e,h); and 18 km (c,f,i), respectively. The red triangles in the figure indicated the positions of substations.

3.1.3. MT LDN

Figure 5 depicts the horizontal error distribution of the lightning location for the MT LDN. The spatial distribution patterns of horizontal errors, corresponding to all heights at the elevation angle of 0.1° (Figure 5a–c) and all elevation angles at the height of 2 km (Figure 5a,d,g), are similar to those of the PT LDN. However, when the heights are 10 km (Figure 5b,e,h) and 18 km (Figure 5c,f,i), the distribution pattern of the horizontal error changes from the approximate “ $\diamond$ ” shape at 0.1° to the “+” shape at 0.5° and 1.0° with the increasing elevation angle of the inclined surface in the MT LDN. A comparison of the several elevation angles shown in Figure 5 reveals that regions with high-value vertical errors stretch closer to the center of the network in the northeast, southwest, southeast, and northwest directions (i.e., the direction of the four ridges of the pyramid) as the elevation angle increases to heights of 10 and 18 km. Meanwhile, the high-precision horizontal location regions extend along the direction of the four inclined planes.

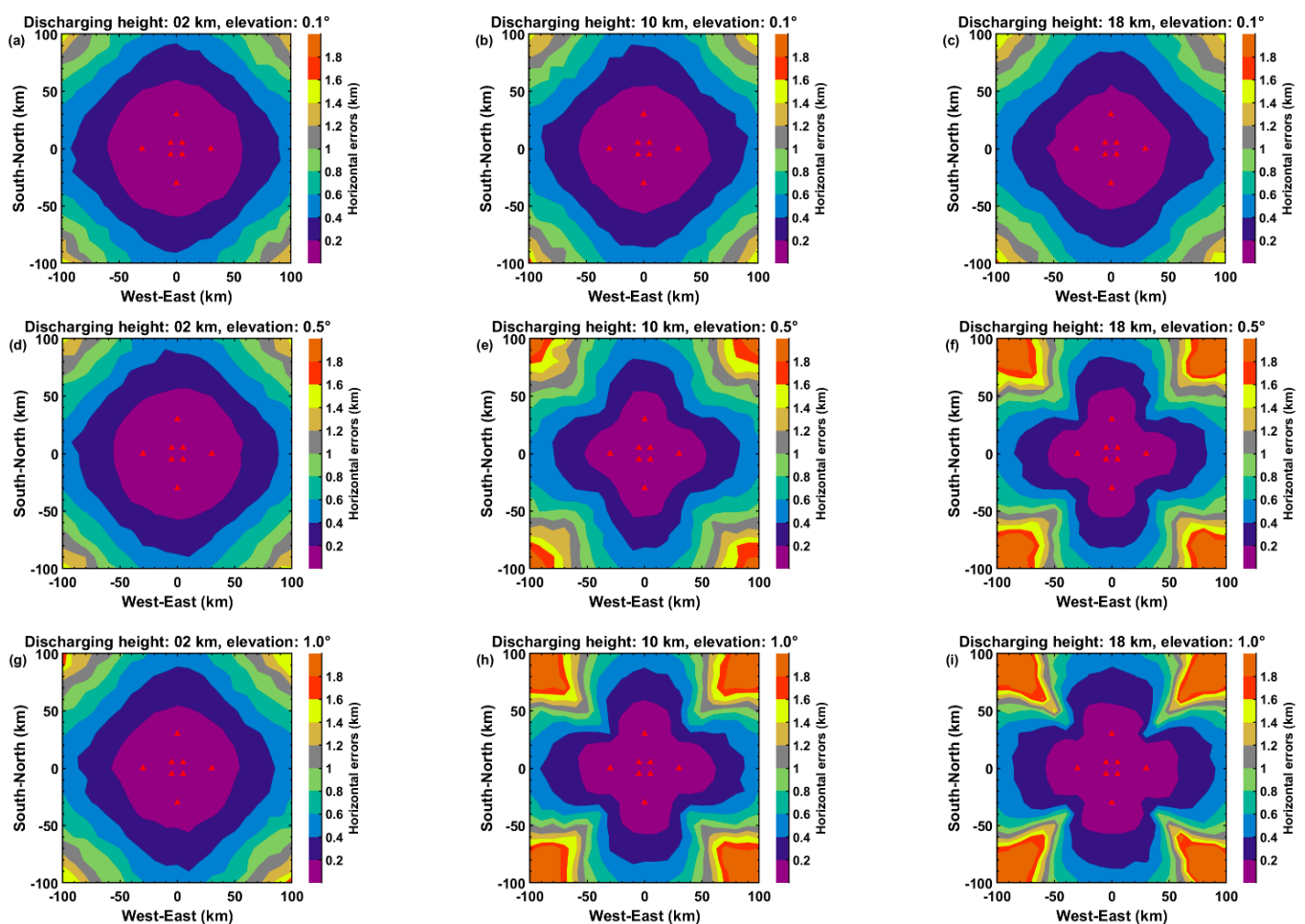


Figure 5. Horizontal errors of lightning location for the MT LDN. The elevation angles are 0.1° (a–c); 0.5° (d–f); and 1.0° (g–i). The heights are 2 km (a,d,g); 10 km (b,e,h); and 18 km (c,f,i), respectively. The red triangles in the figure indicated the positions of substations.

Figure 6 illustrates the vertical error distribution of the lightning location for the MT LDN. The overall distribution patterns are somewhat similar to those of the PT LDN, characterized by a “+” shape in the low-value error regions. At the height of 2 km (Figure 6a,d,g), an increase in the elevation angle results in significant shrinkage of the relatively low-value error regions surrounding the four short baseline substations in the center, and the average vertical errors near the center of the network increase, while regions near the four outer substations still correspond to the low-value vertical errors. If the elevation angle remains

constant, the decrease in the vertical errors near the center of network occurs with increases in height. When the discharging heights are at 10 (Figure 6b,e,h) and 18 km (Figure 6c,f,i), an increase in the elevation angle leads to an increase in the extent of high-value error regions in northeast, southwest, southeast, and northwest directions (corresponding to the directions of the pyramid ridges), extending towards the center of the network. The high-precision regions are mainly located in the east, west, south, and north directions corresponding to the inclined planes.

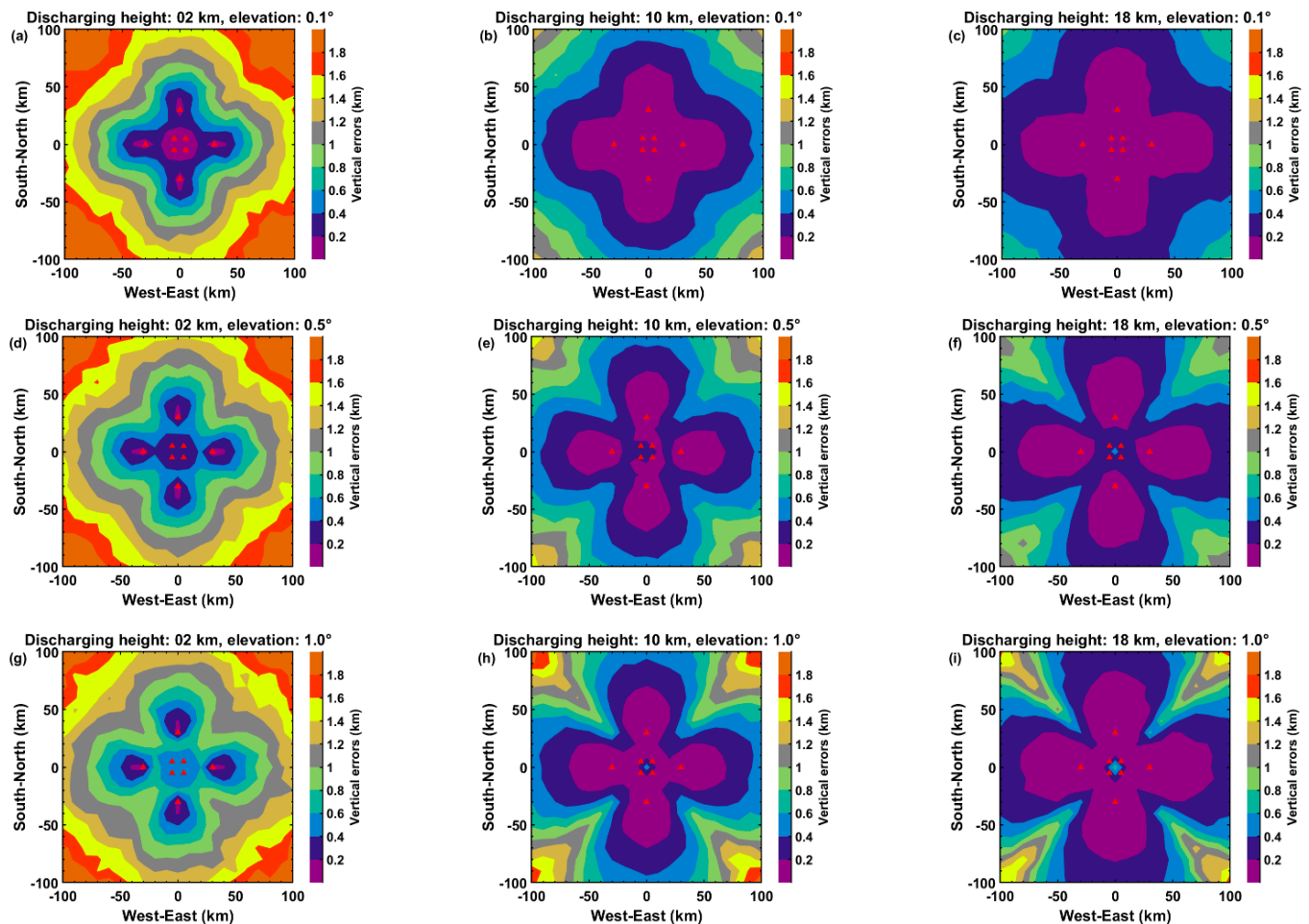


Figure 6. Vertical errors of lightning location for the MT LDN. The elevation angles are 0.1° (a–c); 0.5° (d–f); and 1.0° (g–i). The heights are 2 km (a,d,g); 10 km (b,e,h); and 18 km (c,f,i), respectively. The red triangles in the figure indicated the positions of substations.

3.1.4. BT LDN

The spatial distribution pattern of horizontal errors in lightning localization for the BT LDN is very similar to that of the MT LDN. The distribution patterns of horizontal errors, corresponding to all analyzed heights at the elevation angle of 0.1° (Figure 7a–c) and all analyzed elevation angles at the height of 2 km (Figure 7a,d,g), are close to the PT LDN. At heights of 10 (Figure 7b,e,h) and 18 km (Figure 7c,f,i), as the elevation angle increases, the distribution of the horizontal error changes from the “◇” shape corresponding to the elevation angle of 0.1° to the “+” shape at 0.5° and 1° , and the high-precision region is oriented towards the direction of the inclined planes. It can also be found that increasing the elevation angle for a fixed height or increasing the height for a fixed elevation angle results in relatively high-error regions in the northeast, southwest, southeast, and northwest directions (corresponding to the pyramid ridges), extended towards the center of the network.

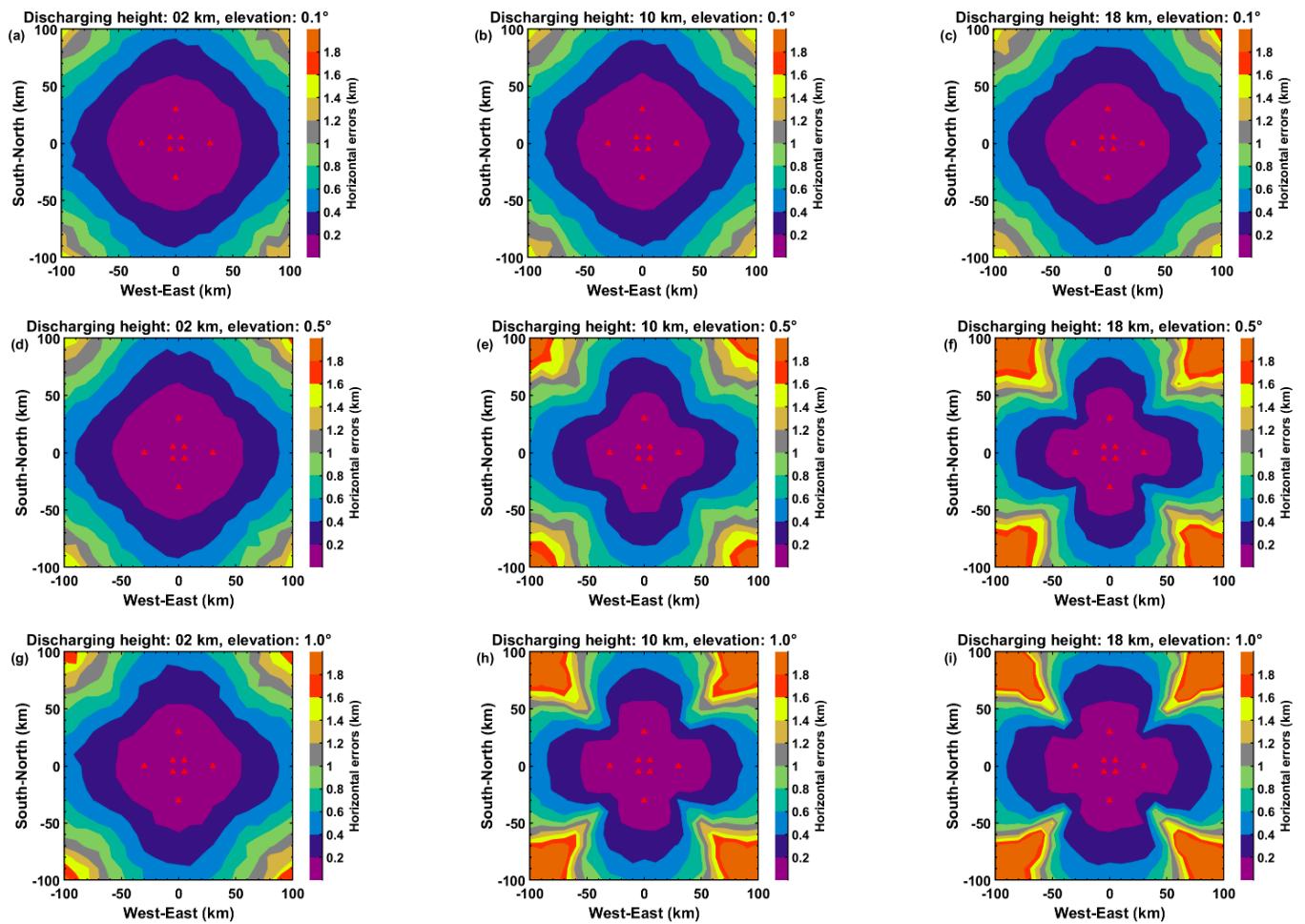


Figure 7. Horizontal errors of lightning location for the BT LDN. The elevation angles are 0.1° (a–c); 0.5° (d–f); and 1.0° (g–i). The heights are 2 km (a,d,g); 10 km (b,e,h); and 18 km (c,f,i), respectively. The red triangles in the figure indicated the positions of substations.

The distribution of vertical errors in the lightning location for the BT LDN is shown in Figure 8. At an elevation angle of 0.1° , the patterns are similar to those for the PT LDN. At a height of 2 km (Figure 8a,d,g), the low-error regions of the vertical location are clustered towards the substation locations. However, as the elevation angle increases, the low-error regions around the four short baseline substations in the center are almost unchanged, while the low-error regions of the four substations in the periphery shrink significantly. This characteristic distinguishes it from the MT LDN. Similar to the MT LDN, at 10 (Figure 8b,e,h) and 18 km (Figure 8c,f,i), the “+” shaped distribution of the vertical location errors tends to become more pronounced with increases in the elevation angle, and relatively high-error regions extend to the center in the northeast, southwest, southeast and northwest directions. The distinction is that the low-error regions of the BT LDN are “plumper” near the center of network, whereas the MT LDN exhibits a more significant characteristic of shrinking towards the center in the corresponding regions.

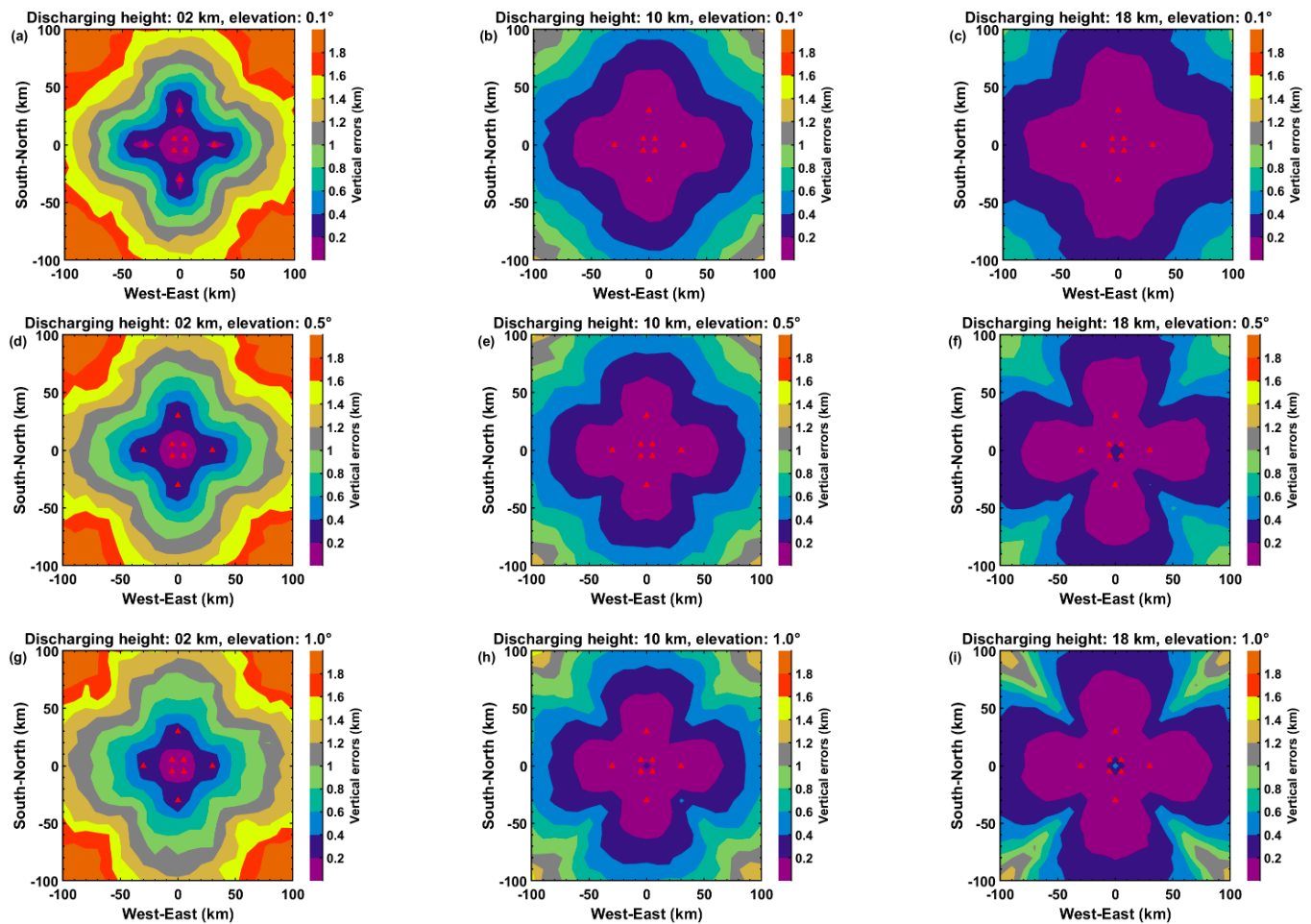


Figure 8. Vertical errors of lightning locations for the BT LDN. The elevation angles are: 0.1° (a–c); 0.5° (d–f); and 1.0° (g–i). The heights are 2 km (a,d,g), 10 km (b,e,h), and 18 km (c,f,i), respectively. The red triangles in the figure indicated the positions of substations.

3.2. Quantitative Analysis of Area Variation in High-Precision Location Region

This section will primarily focus on analyzing the variation characteristics in areas of high-precision regions with the elevation angles and heights for LDNs with different layout patterns. It can be found from the above analysis that regions with a location error of less than 0.2 km consistently remained entirely within the simulated spatial domain of $200 \times 200 \text{ km}^2$, thus representing the referenced high-precision location regions; the variation of its area ratio ($AR_{0.2\text{km}}$) over the $200 \times 200 \text{ km}^2$ area is used to characterize the variation of the high-precision regions.

Figure 9 illustrates the characteristics of the horizontal and vertical $AR_{0.2\text{km}}$ of the ST LDN with the elevation angle and height. As shown in Figure 9a, the horizontal $AR_{0.2\text{km}}$ demonstrates an overall decreasing trend with increases in the elevation angle (black dashed arrows) and height (black solid arrows). The numbers in the top row in Figure 9a are the decrease rate of $AR_{0.2\text{km}}$ with the increases in height for each column (corresponding to a certain fixed elevation angle). Similarly, the numbers in the right column represent the decrease rate of $AR_{0.2\text{km}}$ with increases in the elevation angle for each row (corresponding to a certain fixed height). These decrease rates are obtained from a linear fitting using the values on the corresponding rows or columns. The color of the numbers in Figure 9a is correlated with the coefficient of determination (R^2) corresponding to the linear fitting. Red indicates that R^2 is greater than 0.9, blue indicates that R^2 is between 0.8 and 0.9, and black indicates that R^2 is between 0.75 and 0.8. Please note that, for PT LDN, and when the height is 2 km, although there is still a general decreasing trend for $AR_{0.2\text{km}}$ with increases

in height and in the elevation angle, the specific decrease rate is not provided due to the small numerical differences between them and the poor performance of the linear fitting. It can be observed that the overall decrease rate increases with the increases in elevation angle and height. In other words, the area of the high-precision location region shows an accelerating contraction trend as the elevation angle and height increase.

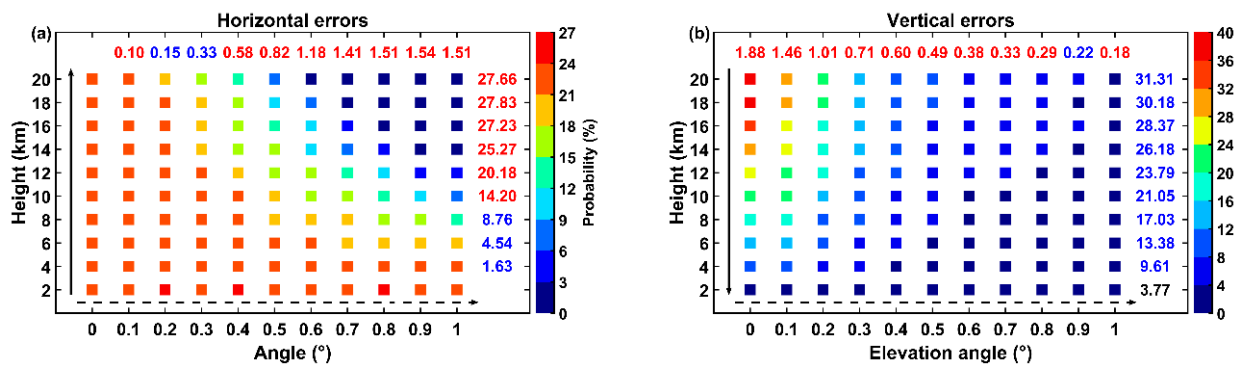


Figure 9. Variation of $AR_{0.2km}$ corresponding to the horizontal location error (a); and vertical location error (b) of ST LDN with respect to the changes in elevation angle and height. Note that the elevation angle of 0° represents the PT LDN. The numbers in the top row in (a,b) indicate the decrease and increase rates (see direction of arrows which indicate the descent direction) of each column (corresponding to a certain fixed elevation angle) with increasing height, respectively, and have the unit of $\% \cdot km^{-1}$. The numbers in the right column represent the decrease rate of each row (corresponding to a certain fixed height) with increasing elevation angle, and have the unit of $\%$ per degree. These rates are obtained by linear fitting. The color of numbers in the figure is correlated with R^2 (determined coefficient) of the linear fitting, with red indicating R^2 greater than 0.9, blue indicating R^2 between 0.8 and 0.9, and black indicating R^2 between 0.75 and 0.8.

As shown in Figure 9b, the $AR_{0.2km}$ corresponding to the vertical errors of the ST LDN decreases with increases in the elevation angle but increases with increases in height. When the elevation angle is at a lower value, the ST LDN exhibits a larger area in the high-precision region at a higher height, and vice versa. In terms of the decrease rate in the variation of $AR_{0.2km}$ shown in Figure 9b, it can be seen that the increase in the elevation angle of the ST LDN leads to a faster decrease in $AR_{0.2km}$ for higher heights than for lower heights. On the other hand, $AR_{0.2km}$ increases more quickly with increases in height for smaller elevation angles than with larger elevation angles. In other words, the increase rate of the areas in the high-precision region with increases in height decreases with increases in the elevation angle, while the decrease rate with an increasing elevation angle increases with increases in height.

The characteristics of the horizontal and vertical $AR_{0.2km}$ of the MT LDN with the elevation angle and the height are shown in Figure 10. On the one hand, when the elevation angle is smaller than 0.4° , the $AR_{0.2km}$ corresponding to the horizontal location decreases with increases in height (Figure 10a). However, when the elevation angle exceeds 0.4° , the horizontal $AR_{0.2km}$ shows a decrease and then an increase with increases in height. Additionally, the height corresponding to the minimum $AR_{0.2km}$ decreases with an increasing elevation angle. On the other hand, when the discharging height is below 4 km, the horizontal $AR_{0.2km}$ decreases with an increasing elevation angle. When the height exceeds 4 km, the horizontal $AR_{0.2km}$ decreases first and then increases with the increasing elevation angle. Furthermore, the elevation angle corresponding to the minimum value decreases with an increase in height.

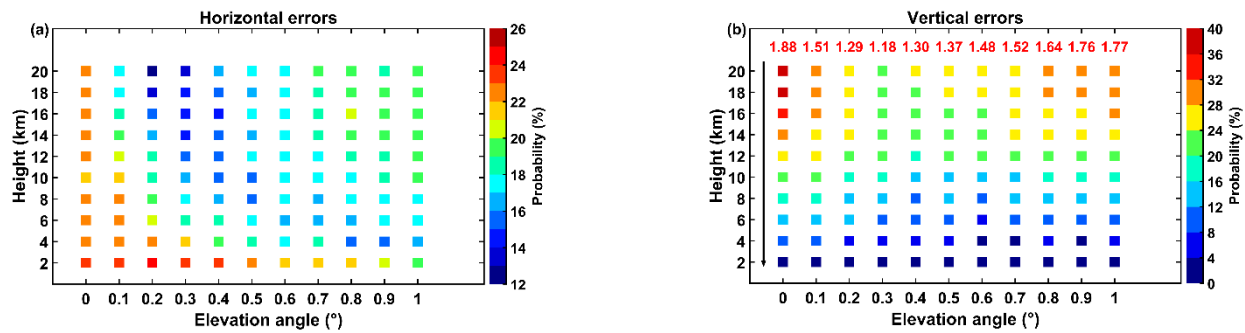


Figure 10. Variation of horizontal (a); and vertical errors (b) corresponding to $AR_{0.2km}$ versus elevation angle and height for the MT LDN. Note that the elevation angle of 0° corresponds to the PT LDN. The numbers in the top row in (b) indicate the increase rates of each column (corresponding to a certain fixed elevation angle) with increasing height, and the unit is $\% \text{ km}^{-1}$.

As shown in Figure 10b, the vertical $AR_{0.2km}$ exhibits an increasing trend with increases in height, and its corresponding increasing rates (numbers in the top row) decrease first and then increase with increases in the elevation angle, with a minimum rate at an approximately 0.3° elevation angle. When the discharging height is below 4 km, $AR_{0.2km}$ decreases with increases in the elevation angle. However, when the discharging height exceeds 4 km, $AR_{0.2km}$ decreases first and then increases with the increasing elevation angle, and the elevation angle corresponding to the minimum value decreases with increases in height.

Figure 11 demonstrates the characteristics of the horizontal and vertical $AR_{0.2km}$ of the BT LDN with the elevation angle and the height. Overall, it shows a similar pattern to the MT LDN. As shown in Figure 11a, the horizontal $AR_{0.2km}$ of the BT LDN decreases with increases in height at elevation angles below 0.4° but decreases first and then increases with increases in height when the elevation angle exceeds 0.4° . When the discharging height is below 4 km, the horizontal $AR_{0.2km}$ decreases with the increasing elevation angle. However, when the discharging height exceeds 4 km, the $AR_{0.2km}$ decreases first and then increases with the increasing elevation angle. The elevation angle corresponding to the minimum $AR_{0.2km}$ also decreases with increases in height.

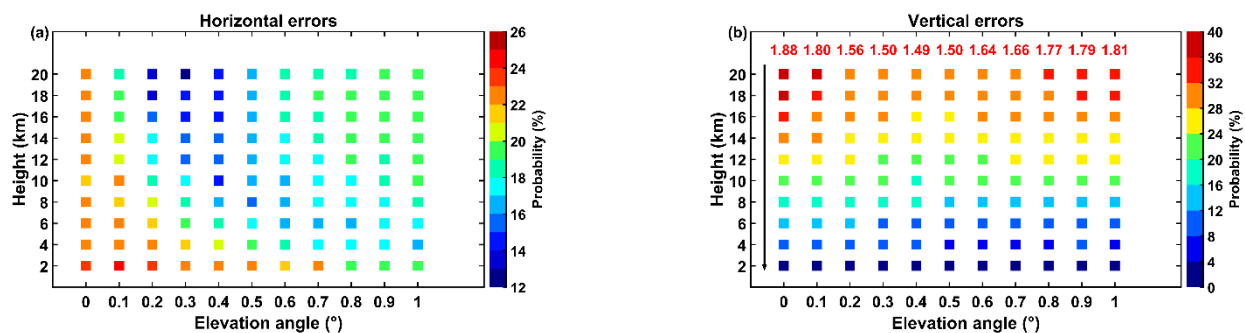


Figure 11. Variation of horizontal (a) and vertical errors (b) corresponding to $AR_{0.2km}$ versus elevation angle and height for the BT LDN. Note that the elevation angle of 0° corresponds to the PT LDN. The numbers in the top row in (b) indicate the increase rates of each column (corresponding to a certain fixed elevation angle) with increasing height, and the unit is $\% \cdot \text{km}^{-1}$.

As shown in Figure 11b, the vertical $AR_{0.2km}$ increases with increases in the discharging height, and these increase rates (numbers in top row) decrease first and then increase with the increasing elevation angle, with a minimum rate at an approximately 0.4° elevation angle. When the discharging height is lower than 4 km, $AR_{0.2km}$ tends to decrease with increases in the elevation angle. However, when the height exceeds approximately 4 km, $AR_{0.2km}$ shows the characteristic that decreases first and then increases with the increasing

elevation angle, and the elevation angle corresponding to the minimum value decreases with increases in height.

In order to further compare the discrepancy in the degree of influence among networks with different types on the location accuracy, the differences in $AR_{0.2km}$ at each elevation angle and the height between the LDN shaped by different terrains are calculated, as shown in Figure 12.

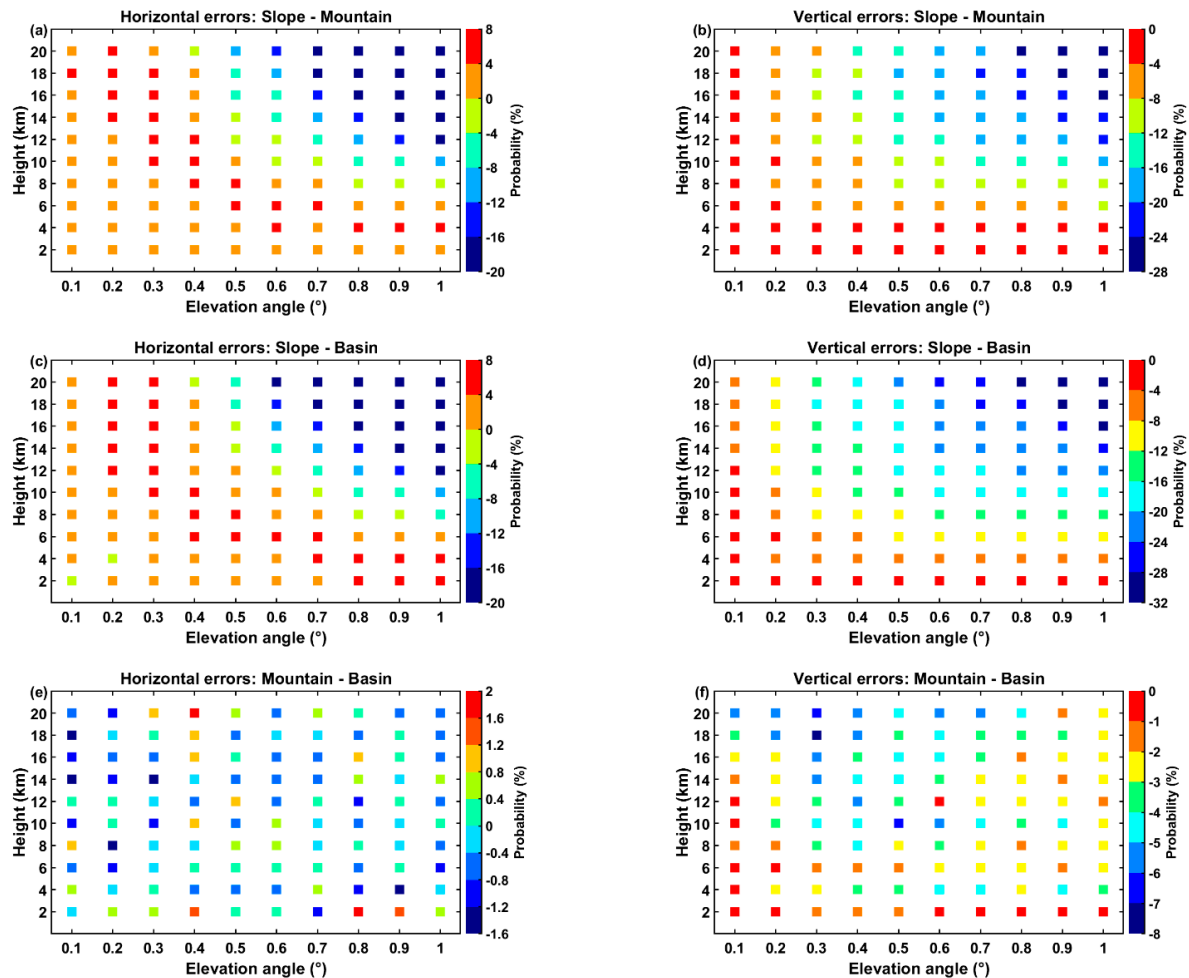


Figure 12. Differences in $AR_{0.2km}$ between different type networks at changing elevation angles and heights, (a,c,e) show the differences in horizontal errors; (b,d,f) show the differences in vertical errors; (a,b) correspond to the $AR_{0.2km}$ of ST LDN minus that of MT LDN; (c,d) correspond to the $AR_{0.2km}$ of ST LDN minus that of BT LDN; and (e,f) correspond to the $AR_{0.2km}$ of MT LDN minus that of BT LDN.

In the context of horizontal $AR_{0.2km}$, the differences between the MT LDN and BT LDN remain relatively small, spanning from -1.6 to 2% , as illustrated in Figure 12e. However, the ST LDN exhibited distinct deviations from the other two types, and these disparities can be roughly categorized into three phases, correlating with changes in the elevation angle and height, as depicted in Figure 12a,c. At lower elevation angles or heights, the differences in $AR_{0.2km}$ between the ST LDN and the other two LDN types hover between approximately 0 and 4% (represented by the orange blocks in the bottom-left direction in Figure 12a,c), while the ST LDN demonstrates more extensive high-precision location areas than the MT and BT LDNs. The $AR_{0.2km}$ of the ST LDN is significantly greater than the other two LDN types with increasing elevation angles and heights, with differences between 4% and 8% (red squares near the diagonal position in Figure 12a,c). However, as the elevation angle and height increase further, the $AR_{0.2km}$ of the ST LDN becomes

smaller than the other two LDN types (blue area on the top right of Figure 12a,c), and these differences increase rapidly with increases in height and in the elevation angle.

In relation to vertical location, $AR_{0.2km}$ corresponding to the ST LDN remains consistently smaller than that of both MT and BT LDNs, regardless of the various elevation angles and heights investigated in this analysis. Moreover, the differences in $AR_{0.2km}$ between the ST LDN and the other two LDN types increase with increases in the elevation angle and in height (Figure 12b,d). As shown in Figure 12f, $AR_{0.2km}$ corresponding to the MT LDN proves to be consistently smaller than that of BT LDN across all investigated elevation angles and heights.

4. Simulation of Lightning Location Accuracy Based on an Actual Network

In 2021, an LDN was built in Naqu, a city located in the central TP of China. This network comprises 10 substations strategically positioned in an area with an average altitude exceeding 4500 m (Figure 13). The primary objective of this initiative is to comprehensively investigate the distinctive plateau thunderstorms and associated lightning phenomena. The foundational technology for this network draws from the Lightning Low-Frequency E-Field Detection Array (LFEDA) [21]. However, significant enhancements were made by upgrading key components such as the data acquisition system, power supply system, communication control system, and so on, which adapted the network to the environmental conditions of the TP. Referring to Figure 13, where the altitude of each substation is indicated in parentheses, the network first exhibits an inclination pattern, characterized by higher altitudes in the north and lower altitudes in the south. A planar fitting was made to this inclination, with an elevation angle of approximately 0.38° and a determination coefficient of fitting, R^2 , of 0.71. Meanwhile, the station at the highest altitude is PQC. When observing from this point in various directions, a noticeable trend emerges: the altitudes of the stations gradually decrease as they move towards the periphery. This configuration somewhat mirrors the distribution of MT LDN. Therefore, this network is suggested to be a combination of ST and MT LDN layouts. Utilizing this existing network as a foundation, we further conducted a Monte Carlo simulation to derive the spatial distribution of location errors. These outcomes were subsequently juxtaposed with those from the theoretical ST and MT LDNs, as outlined in Section 3. This comparative analysis serves to authenticate and elucidate the significant impact of topography on the LDN's performance.

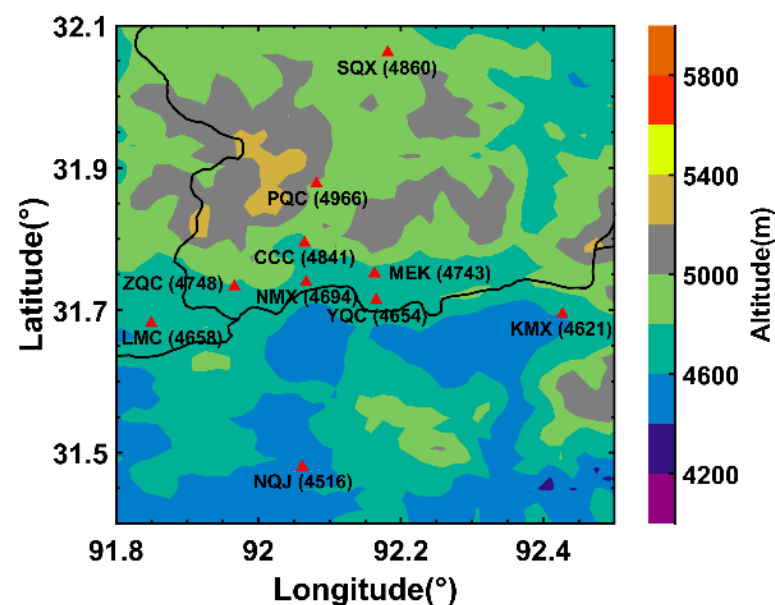


Figure 13. The layout of the substations of LFEDA in Naqu, Tibetan Plateau. The red triangles indicate the positions of substations and the numbers in parentheses indicate the altitudes of the corresponding substations.

Taking into account that the lowest altitude of the substation within the Naqu LFEDA is approximately 4.5 km, Figure 14 presents a depiction of both the horizontal and vertical location error distributions of this network. These distributions are showcased under conditions where the discharging heights are set at 6 km, 12 km, and 18 km (above sea level). It can be found that, with increases in height, the distribution pattern of horizontal error remains relatively stable, meanwhile, the area of the regions with a low-value horizontal location error shows a decreasing tendency (Figure 14a–c), which agrees with the characteristics of the ST LDN (Figure 3). On the contrary, the distribution of the vertical location error distinctly demonstrates a noticeable trend: the high-precision region exhibits an inclination towards the southern direction (the direction of the lower substations). Particularly, the vertical error distribution at 6 km (approximately 1.5 km above the ground) agrees well with the distribution patterns illustrated in Figure 4d,g.

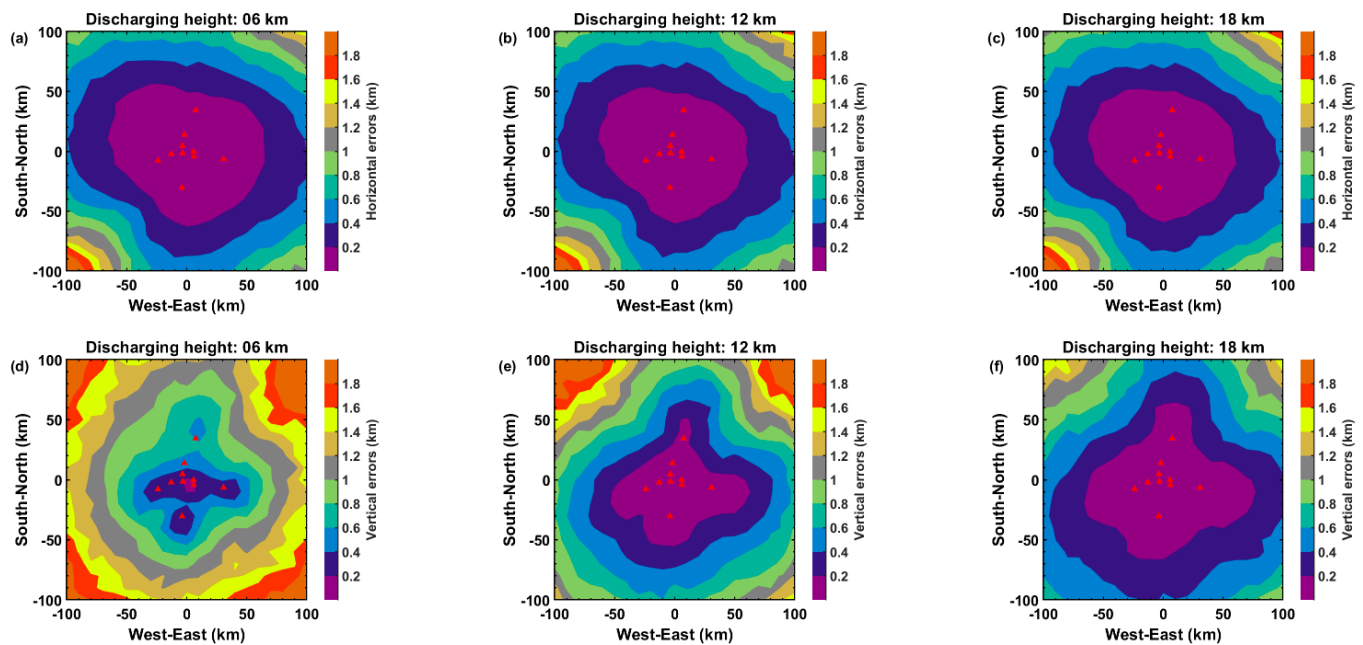


Figure 14. Horizontal (a–c); and vertical location errors (d–f) for the Naqu LFEDA. The heights are 2 (a,d); 10 (b,e); and 18 km (d,f), respectively. The red triangles in the figure indicated the positions of substations.

Figure 15 exhibits the variation of horizontal and vertical $AR_{0.2km}$ with increases in height corresponding to the Naqu LFEDA. The horizontal $AR_{0.2km}$ initially decreases and subsequently increases as the altitude increases. This pattern aligns consistently with the corresponding trend of $AR_{0.2km}$ associated with the MT LDN (Figure 10a). The vertical $AR_{0.2km}$ shows a continuous increase with increases in height, spanning approximately from 1.13% at 6 km to 26.53% at 20 km. This observed trend is consistent with the variation trend of $AR_{0.2km}$ evident in the ST and MT LDNs (Figures 9b and 10b). The relevant results underscore that the simulated lightning location accuracy of the Naqu LFEDA indeed mirrors the distribution pattern observed in both the ST and MT LDNs.

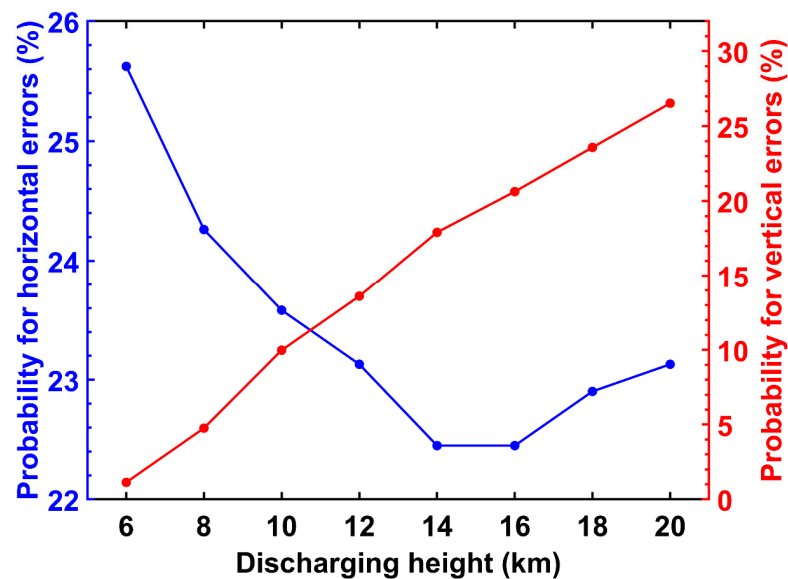


Figure 15. Variation in horizontal and vertical $AR_{0.2km}$ with height for Naqu LFEDA.

5. Conclusions and Remarks

5.1. Conclusions

In this study, LDNs with regular layouts comprising eight substations were established across three configurations: ST, MT, and BT types. The Monte Carlo method was used to simulate and analyze the influence of the LDN configuration on the location accuracy within the framework of the TOA location technique. Noteworthy conclusions emerged from this analysis, as outlined below.

5.1.1. ST LDN

An increase in the elevation angle of the inclined surface leads to a rapid reduction in the extent of the high-precision horizontal location region at mid- to high-level positions. Furthermore, the rate at which this region decreases with increasing height amplifies in conjunction with the rise in elevation angle. Similarly, the rate at which it decreases with an increase in the elevation angle intensifies with higher altitudes. The increase in the elevation angle of the inclined surface causes a significant shift of the region, with low vertical location errors towards the lower part of the slope, and the shape of this region progressively takes on a “—” pattern. For a fixed height, the region characterized by high vertical accuracy diminishes as the slope elevation angle increases. Moreover, as the height increases, the rate of decrease becomes more pronounced. For a fixed elevation angle, the region with high-precision vertical accuracy expands with increasing height; simultaneously, a larger elevation angle is associated with a slower rate of expansion.

5.1.2. MT LDN

Increases in the elevation angle of mountainous slopes and the height both lead to significant variations in the spatial distribution patterns of horizontal location errors. Within the context of the prescribed station network layout, an increase in elevation angle causes the distribution pattern of horizontal location errors to shift from a “◇” shape to a “+” shape, while the vertical location errors persistently exhibit a “+” shape. Both high-precision regions for horizontal and vertical locations extend in the direction of the pyramid slopes. The region characterized by a large vertical location error prominently expands along the ridge direction toward the central position of the network. Additionally, the vertical location error at the network’s central position is relatively large.

The area characterized by high horizontal location accuracy experiences different trends with height as the elevation angle of the inclined surface changes. When the

elevation angle is relatively small (specifically, less than 0.4° for the network layout in this paper), the area diminishes as height increases. Conversely, for larger elevation angles (greater than 0.4°), the area first decreases and then increases with increases in height. At lower heights (below 4 km), this area decreases as the elevation angle increases. However, at higher altitudes (above 4 km), the area follows a pattern of decreasing initially, then increasing as the elevation angle increases. The elevation angle at which the minimum area is achieved decreases as height increases, and the height at which the minimum area occurs decreases with an increase in the elevation angle.

The area of the high vertical location accuracy displays an increasing trend with increases in height. However, the increase rate varies with the elevation angle, initially decreasing and then increasing as the elevation angle becomes larger. At low heights (approximately below 4 km), the area decreases with increases in the elevation angle. In contrast, at higher altitudes (approximately above about 4 km), the area follows a pattern of initially decreasing and then increasing as the elevation angle increases.

5.1.3. BT LDN

The distribution patterns of horizontal and vertical location accuracy in the BT LDN generally exhibit similar characteristics to those in the MT LDN, in terms of their evolution with the elevation angle of the inclined surface and height. The primary distinction arises when the height is relatively low (approximately around 2 km for the analyzed configuration). In this case, an increase in the elevation angle does not significantly affect the high location accuracy at the center position (corresponding to the center four sub-stations in the LDN), whereas in the MT LDN, the location error in this area increased. In addition, the “+” shape of the low-value vertical error area appears “plumper” compared to the MT LDN.

5.1.4. Comparison of Location Accuracy of LDNs with Different Configurations

The analysis indicates that, compared to the PT LDN, all three LDNs characterized by inclined terrains led to a reduction in the extent of high-precision horizontal and vertical location regions. The differences in the area of the high-precision horizontal location region between MT and BT LDNs are relatively minor, but the ST LDN differs significantly from both. Taking horizontal $AR_{0.2km}$ as an example, when the elevation angle of the inclined surface or height is low, the ST LDN exhibits larger $AR_{0.2km}$ values. However, under conditions of relatively large elevation angles and heights, the ST LDN displays a relatively smaller $AR_{0.2km}$. In terms of vertical location, the $AR_{0.2km}$ values for the ST LDN are consistently smaller than that for MT and BT LDNs across various elevation angles and heights, with the difference increasing as the elevation angle and height increase. Moreover, across all elevation angles and heights, the $AR_{0.2km}$ values corresponding to the vertical location of the MT LDN are smaller than those of the BT LDN.

5.2. Remarks

The simulation analysis has unveiled significant distinctions in the spatial location accuracy distribution among LDNs deployed on different terrains. This finding implies that established knowledge concerning location accuracy, which does not encompass LDNs with regular vertical layouts, may not be applicable to LDNs deployed in complex terrains. While we artificially configured the layout of LDNs with a fixed geometric structure in their horizontal projections (as depicted in Figure 1d), this arrangement undeniably influenced the specific spatial distribution patterns of location accuracy. Nevertheless, certain trends in location accuracy corresponding to LDNs deployed on different terrains are expected to be universal. These trends should encompass, at a minimum, a decrease in the size of high-precision location areas in LDNs on inclined terrains compared to those on PT terrains, along with fluctuations in the extent of high-precision areas in both horizontal and vertical locations, as the elevation of the inclined surface varies. Hence, this study offers valuable insights for the design and practical data application of LDNs in regions with

complex terrains. In the future, we plan to explore various LDN layouts to deepen our understanding of their impact on location accuracy.

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