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Estimating the Duration of Construction Works Using Fuzzy Modeling to Assess the Impact of Risk Factors

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Abstract: One of the most pressing issues in the implementation of construction projects is the extension of planned deadlines, significantly impacting project costs. This situation often arises due to inaccurate estimation of construction durations, which rely on normative values without accounting for factors hindering construction progress. Consequently, this article aims to develop an innovative approach for assessing construction durations, considering specific risk factors and their influence on construction activities. Given the difficulty of determining risk factors and their effects during the design phase using classical probability theory, characterized by unknown probability distributions, it is highlighted that this scenario represents planning and implementation under conditions of non-statistical uncertainty. Therefore, the article proposes an approach utilizing elements of fuzzy set theory, particularly fuzzy rules and linguistic variables, to determine delays in individual construction tasks. The proposed approach involves estimating extensions of construction timelines based on a specified probability level of occurrence for risk events and their impact. Additionally, the article provides a theoretical description of the proposed approach and practical calculation examples, demonstrating that the authors' approach significantly enhances the accuracy of construction timeline forecasts, providing more reliable data for project planning and management.

Keywords: construction scheduling; fuzzy sets; time contingency; construction project planning; risk management; risk matrix

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1. Introduction

One of the pertinent issues in construction is exceeding project construction deadlines [1–5], which is a contributing factor to the increase in building and infrastructure construction costs [6–8]. Asiedu and Frempong [9] noted that cost overrun is an inherent characteristic of construction projects regardless of size and complexity. Johnson and Babu identify five reasons for time overrun in construction projects such as design variation from client and consultant, unrealistic schedules and completion dates projected by clients, delays in obtaining government permits and approvals, inaccurate time estimation by consultants, and changes in orders from clients [8]. In works [9,10], there are four major causes of cost overruns such as poor contract planning and supervision; a change of orders; a weak institutional and economic environment for projects, and a lack of effective coordination among the contracting parties.

Famiyeh, Amoatey, Adaku, and Agbenohevi considered that there are key factors that impact construction time overrun, such as financial problems, unrealistic contract durations imposed by clients, poorly defined project scope, client-initiated variations, underestimation of project cost by consultants, and poor inspection/supervision of projects by consultants. There are additional factors such as the underestimation of project complexity by contractors, poor site management, inappropriate construction methods used by contractors, and delays in the issuance of permits by government agencies. [11,12]. The findings of work [13] show that the fundamental group causing delays was the contractor category, and the most critical cause was the contractor's financial problems. Research [14,15] demonstrated that inefficient site management is certainly another key factor affecting the time performance of most construction projects in India. In work [16], there are 28 causes of budget overrun identified, and several significant causative factors are unveiled, which include improper planning, variation in material prices, poor site management, lack of communication among project parties, and frequent design changes. Research [17] noted that 'poor planning' is a pervasive issue not only in higher education building projects but also in all construction projects on a global scale and has an impact on cost and time overrun. The findings of work [18] are that, as a result of delays in the realization of work, additional resources may need to be hired to help make up for the lost time. Most often, delays with some work propagate throughout the schedule as subsequent work cannot start unless their predecessors are completed. In such cases, the schedule developed at the planning stage is an appropriate starting point for verification of the assumptions adopted in the plan.

Thus, many studies highlight issues with budget and schedule overruns in construction projects, partly attributable to poor planning. Construction project planning begins with the development of the technological and organizational solution (TOS), which forms the basis for determining the construction schedule and budget. Therefore, one pressing task is determining the construction duration during the organizational and technological design phase, which would be equal to or close to the actual construction duration. In practice, the normative duration is determined based on averaged data of individual construction works, for example, in Poland, using the Construction Norms and Regulations (KNR) [19]. Normative construction duration often represents a very approximate estimate, which significantly differs from the actual construction duration in most cases, as each construction project is unique in its implementation, and the normative duration is determined as the average value across thousands of projects without considering implementation specifics.

The primary trend of the 21st century is the application of best practice methodologies such as the Project Management Body of Knowledge (PMBOK) and Projects IN Controlled Environments (PRINCE2) [20–28], as well as the implementation of various management systems [29,30]. These regulatory documents provide procedures for assessing additional factors that may negatively impact project implementation duration.

The article aims to present an algorithm for determining the actual duration of a construction project, considering additional risk lag time.

The main contributions of this paper are the following:

- Development of a novel algorithm for estimating construction duration considering additional risk lag time. The risk lag time is determined utilizing a risk matrix and fuzzy set theory. Fuzzy set theory allows for the use of linguistic terms for estimating the level of risk;
- Validation through a practical case study of construction project implementation;
- Analysis of defuzzification methods and their impact on the overall construction duration value;
- Examination of the influence of various risk matrices on the overall construction duration value.

The remaining part of this article is structured as follows: Section 2 provides a literature review. The proposed approach for estimating the duration of a construction project considering a risk matrix using fuzzy set theory is outlined in Section 3.1, with practical calculation examples presented in Section 3.2. Section 4 presents the results of the practical example calculations and ensuing discussion, followed by Section 5, which concludes the paper with a summary.

2. Literature Review

The estimation of construction duration is based on the assessment of the TOS for the construction project, for example, using the critical path method (CPM) [31,32]. Considering the overall process of estimating construction duration based on the TOS, it is necessary to identify the components of this process [33–35]. The TOS is a set of activities with technological and organizational dependencies between each other and is located on the time scale of the project duration. The TOS also describes the possible risks that may affect the performance of this or that work. Figure 1 provides a graphical representation of the TOS.

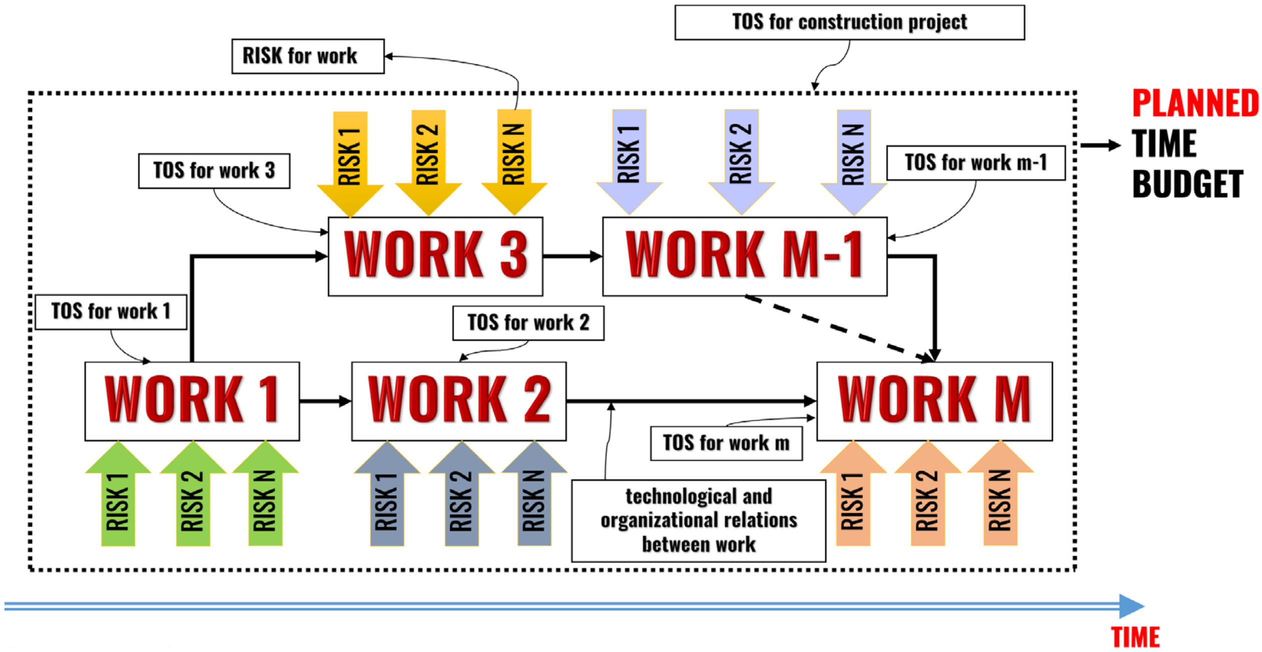

Figure 1. TOS model of the construction project.

The smallest unit of any TOS is an individual construction activity—characterized by the following indicators: TOS, normative duration, and normative cost. In practice, the normative duration is determined based on averaged data of individual construction activities, for instance, in Poland, using KNR [19]. Additionally, external factors, such as the occurrence of risk events, can influence the duration and cost of each construction activity, either increasing or decreasing it [36–44]. The authors of this article consider risk events solely as negative occurrences that may prolong construction duration. Thus, the actual work duration or total activity duration is the sum of the normal (average) duration and risk lag time. A graphical representation of the impact of risks on individual activities and the actual work duration is shown in Figure 2.

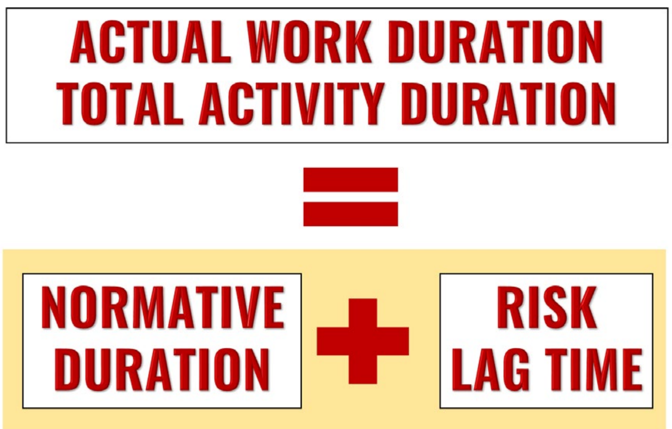

Figure 2. The impact of a risk event on an individual construction activity upon occurrence.

From Figure 2, it can be concluded that the duration of each activity comprises two components: the duration determined based on averaged indicators (for example, based on estimates from KNR [19]) and an additional risk lag time [45,46], which considers the probability and impact level of a risk event. Therefore, to accurately address the issue of actual construction duration, it is necessary to precisely determine the additional time by which the duration of an individual task may increase upon the occurrence of risk events associated with that specific task.

Most construction projects are implemented using best management practices, for instance, employing methodologies such as PMBOK [20] or PRINCE2 [21]. Such management methods describe various approaches and techniques for handling risks and incorporating them into project implementation, including construction projects. For example, research [47,48] indicates that an effective risk management tool is the use of a probability and impact matrix. However, there is also the problem of choosing a risk management methodology, illustrated by the example of railway construction [49]. A probability and impact matrix is a basis for rules and dependencies between probability and impact, as well as cumulative effects on threats. This matrix allows for ranking the level of risk threat based on the likelihood of occurrence of a risk event and its impact on work or the project as a whole. For instance, in PMBOK guidance [20] and related works, numerical values are used, which can be multiplied to provide a likelihood-impact score for each risk, enabling the evaluation of relative priority levels among individual risks within each priority level. By employing this approach, it is necessary to define two input parameters—the likehood and impact score for each risk. A probability and impact matrix is a basis for rules and dependencies between probability and impact, as well as cumulative effects on threats of a risk event occurring and the level of impact of this risk event on either the project as a whole or a specific project element. Additionally, a relationship is established between the input parameters and the output parameter—the threat level, which may occur upon the incidence of a risk event.

A simplified approach exists for determining the threat level—which involves multiplying the probability of occurrence by the level of impact. For example, such an approach is utilized in works [45,46]. There is also an approach to selecting a construction project that considers risks or various TOS using fuzzy sets [50–52]. In this article, the probability of occurrence of each risk is determined, and the impact of risk events is assessed in terms of lag time. A probability-impact score is a numerical value representing the increase in the duration of each task without considering the threat level typically determined by an expert based on the risk matrix. However, this simplified method has significant drawbacks: it does not account for the threat level, which represents the probability of risk occurrence and impact. Currently, there is a lack of an approach in the literature for assessing the overall construction duration considering additional time, such as risk lag time, based on the fuzzy estimation of the risk matrix. The approach proposed in this article addresses this gap.

3. Fuzzy Approach to Modeling Construction Duration Using Risk Matrix

3.1. Theoretical Aspects of the Proposed Fuzzy Approach

3.1.1. Critical Path Method (CPM)

The Critical Path Method (CPM) is a tool for project planning and schedule management [53]. The central aspect of the CPM method is the critical path. The critical path is the longest sequence of tasks upon which the entire project depends. It is a chain of activities where the next task can only be started once the previous one is completed. Using the CPM algorithm, a minimum planned duration project can be determined, and for individual project activity, both the earliest start and the latest finish time are then calculated. In this study, the CPM has been applied with the precedence diagramming method (PDM), a strategy for developing a project schedule network diagram that utilizes nodes to represent activities and associates them with projectiles that illustrate the dependencies. This method is the activity-on-node (AON). The term AON pertains to a methodology in project management known as precedence diagramming, wherein rectangular nodes represent scheduled activities. These nodes are interconnected by arrows, delineating the sequential dependencies among the activities. Each node is assigned a unique identifier, typically a letter or number, corresponding to a specific activity in the project schedule. Primarily, an activity-on-node diagram illustrates the prerequisites for commencing subsequent activities. This arrangement is commonly known as the "finish-to-start" precedence, implying that an activity must conclude before the succeeding one can begin. The PDM can be described into four basic types of dependencies or logical relationships between activities: finish-to-start (FS), start-to-start (SS), finish-to-finish (FF), and start-to-finish (SF). FS is an activity that cannot start before a previous activity has ended. SS is a defined relationship between the start of activities. FF is a defined relationship between the end dates of activities. SF is a defined relationship between the start of one activity and the end date of a successor activity. Every activity has four states: early start (ES) is the earliest time an activity can be started; late start (LS) is the latest time an activity can be started. If the activity is started beyond this time, it will affect the critical path; early finish (EF) is the earliest time an activity is completed; late finish (LF) is the latest time the activity can be completed. If the activity crosses this time, the project will be delayed. An example of a precedence diagram is presented in Figure 3.

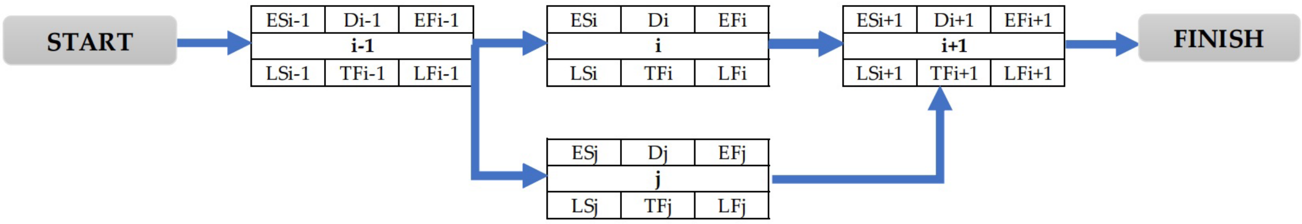

Figure 3. Example of precedence diagram.

The ES for the first activity after the start is always equal to 0. The ES, EF, LS, and LF for subsequent activities are determined using the following Equations (1)–(7):

$$ES_i = EF_{i-1}, \tag{1}$$

where ES_i—early start for activity i (in days), EF_{i-1}—early finish for activity i−1 (in days).

$$EF_i = ES_i + D_i, \tag{2}$$

where EF_i—early finish for activity i (in days), ES_i—early start for activity i (in days), D_i—duration for activity i (in days).

$$ES_{i+1} = \max\{EF_i; EF_j\}, \tag{3}$$

where ES_{i+1}—early start for activity i+1 (in days), EF_i—early finish for activity i (in days), EF_j—early finish for activity j (in days).

For the last activity in the sequence before the finish, for example, for activity i+1 as shown in Figure 3, Equation (4) is used.

$$EF_{i+1} = LF_{i+1}, \tag{4}$$

where EF_{i+1}—early finish for activity i+1 (in days), LF_{i+1}—late finish for activity i+1 (in days).

After determining all the values for EF, we proceed to determine LF. The PDM backward pass calculation determines the latest dates by which each activity can be performed without increasing the project's minimum duration using Equations (5)–(7).

$$LS_{i+1} = LF_{i+1} - D_{i+1}, \tag{5}$$

where LS_{i+1}—late start for activity i+1 (in days), LF_{i+1}—late finish for activity i+1 (in days), D_{i+1}—duration for activity i+1 (in days).

$$LF_i = LF_j = LS_{i+1}, \quad (6)$$

where LF_i—late finish for activity i (in days), LF_j—late finish for activity j (in days), LS_{i+1}—late start for activity i+1 (in days).

$$LF_{i-1} = \min\{LS_j; LS_i\}, \quad (7)$$

where LF_{i-1}—late finish for activity i−1 (in days), LS_j—late finish for activity j (in days), LS_i–late start for activity i (in days).

Total float for every activity is determined using Equation (8).

$$TF_{i+1} = LF_{i+1} - EF_{i+1}, \quad (8)$$

where TF_{i+1}—total float for activity i+1 (in days), EF_{i+1}—early finish for activity i+1 (in days), LF_{i+1}—late finish for activity i+1 (in days).

In this study, it was previously described that actual work duration is the sum of normal duration and risk lag time. Figure 4 illustrates the example of a precedence diagram with risk lag time for activity j.

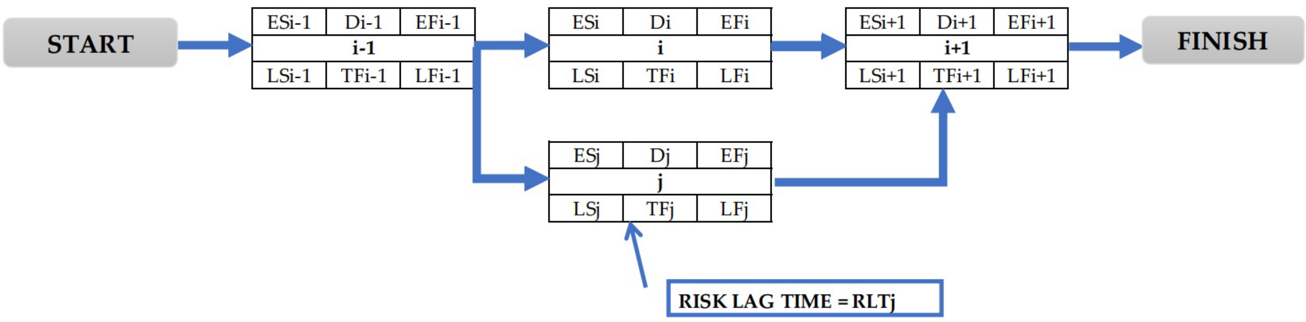

Figure 4. Example of input risk lag time into precedence diagram.

The EF and LS for activities with risk lag such as j on Figure 4 are determined using Equations (9) and (10) instead of Equations (2) and (5).

$$EF_j = ES_j + D_j + RLT_j, \quad (9)$$

where EF_j—early finish for activity j (in days), ES_j—early start for activity j (in days), D_j—duration for activity j (in days), and risk lag time for activity j (in days).

$$LS_j = LF_j - D_j - RLT_j, \quad (10)$$

where LS_j—late start for activity j (in days), LF_j—early finish for activity j (in days), D_j—duration for activity j (in days), risk lag time for activity j (in days).

3.1.2. Basic Concepts of the Fuzzy Sets Theory

Let X be a non-empty set considered to be the universe of discourse. The fuzzy set A is a pair (X, μ_A), where $\mu_A{:}X \to I$ and $I = [0, 1]$. The notion of the fuzzy set has been introduced by L.A. Zadeh [54,55].

$$A = \{(x, \mu_A(x)); x \in X\}, \quad (11)$$

$\mu_A(x)$ is the membership function degree of x to A. It may also be interpreted as the plausibility degree of the affirmation "x belongs to A". If $\mu_A(x) = 0$, x is definitely not in A, and if $\mu_A(x) = 1$, x is definitely in A. The intermediate cases are fuzzy.

Operations on fuzzy sets A and B, such as the standard intersection (∩) and standard union (∪), can be displayed in the following Equations (12) and (13), [54,55]:

$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x)), \forall x\in X, \tag{12}$$

$$\mu_{A\cup B}(x) = \max(\mu_A(x), \mu_B(x)), \forall x\in X, \tag{13}$$

Formulating the mapping from a given input to an output using fuzzy logic is a fuzzy inference system (FIS). The general scheme of the FIS, recorded in the form of fuzzy rules or control rules is as follows in Equation (14).

$$\text{IF } u = A_i \text{ THEN } v = C_i, i = 1, \ldots, \text{n}, \tag{14}$$

There are different methods of the FIS such as the Mamdani FIS and Takagi-Sugeno Fuzzy Model. In this study, the Mamdani FIS determines the consequent of rule by combining the rule strength and the output membership function. The Mamdani FIS block diagram is presented in Figure 5 [56].

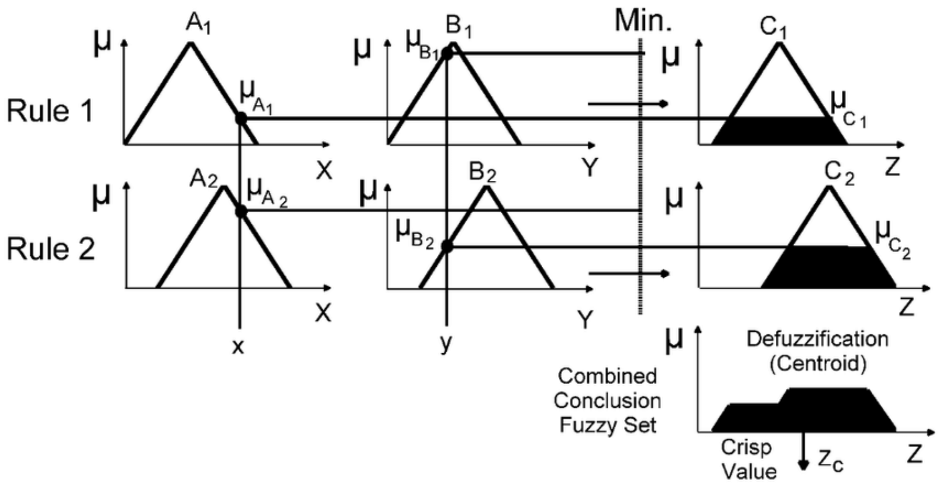

Figure 5. Schematic illustration of the Mamdani fuzzy inference system [56].

In cases in the Mamdani FIS approach, the fuzzy relation R is used, coded by (14), and additionally refers to the compositional rule of inference. Hence, having given as an input the value assignment $u{:=}A$ to the output variable v the value using Equation (15):

$$v{:} = A \circ R, \tag{15}$$

And it has a form of two-dimensional fuzzy set membership function:

$$\mu_{A\ \cdot\ R}\ (z) = \max_{x\in X}\{\min\{\mu_A(x), \mu_R(x,z)\}\},\ \text{for all} z \in Z \tag{16}$$

After getting the output distribution, combine all the consequents. The next step is the defuzzified output distribution. Several defuzzification methods for MAMDANI composing rules are used in MATLAB (ver. R2023) software such as centroid, bisector, middle of maximum, smallest of maximum, and largest of maximum [57].

- Centroid method. The crisp solution is obtained by taking the center point of the fuzzy area and can be written as Equation (17) and presented in Figure 6a:

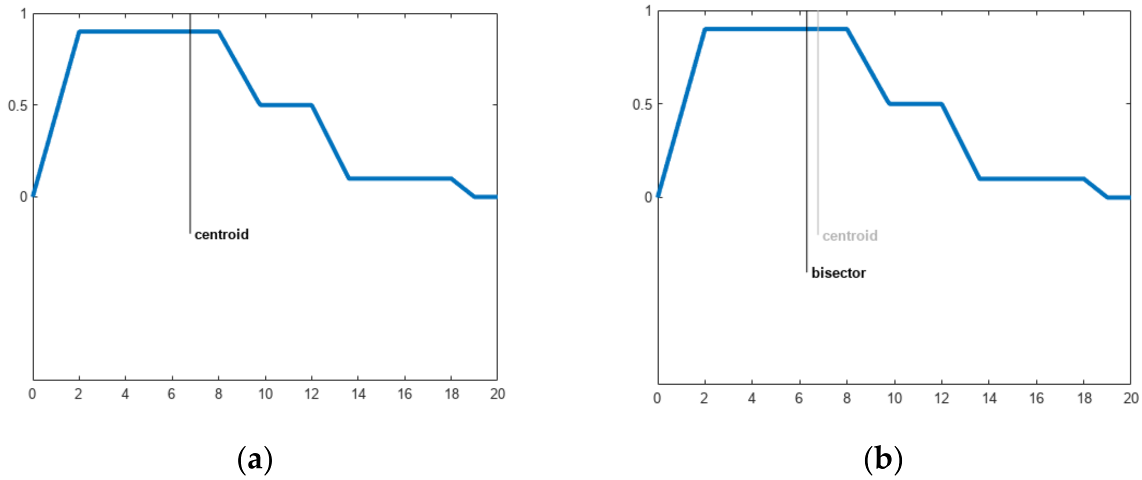

Figure 6. The graphical explanation of the defuzzification methods [57]: (**a**) Centroid; (**b**) Bisector.

$$z_c = (\Sigma z_i \cdot \mu(z_i))/\Sigma\mu(z_i), \tag{17}$$

- Bisector method. The crisp solution is obtained by taking the domain which has a value from the number of membership values in the fuzzy area and can be written as and presented in Figure 6b:

$$z_c = 0.5 \cdot (\Sigma z_i \cdot \mu(z_i)), \tag{18}$$

- Middle of Maximum (MOM) method. The crisp solution is obtained by taking the average value of the domain that has the maximum membership value and can be presented in Figure 7.

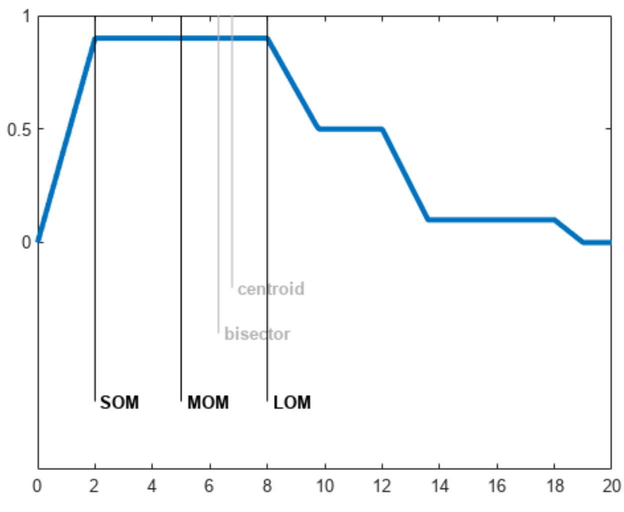

Figure 7. The graphical explanation of the defuzzification methods, such as SOM, MOM, and LOM [57].

- Largest of Maximum (LOM) method. The crisp solution is obtained by taking the largest value from the domain that has the maximum membership value and can be presented in Figure 7.
- Smallest of Maximum (SOM) method. The crisp solution is obtained by taking the smallest value from the domain that has the maximum membership value and can be presented in Figure 7.

3.1.3. Fuzzy Approach to Estimate the Risk Lag Time Using the Level of Influence of the Risk Factors

The conceptual framework of the model was developed to establish a reliable estimate of the total duration of a construction project. Figure 8 illustrates the model to estimate the total duration of a construction project.

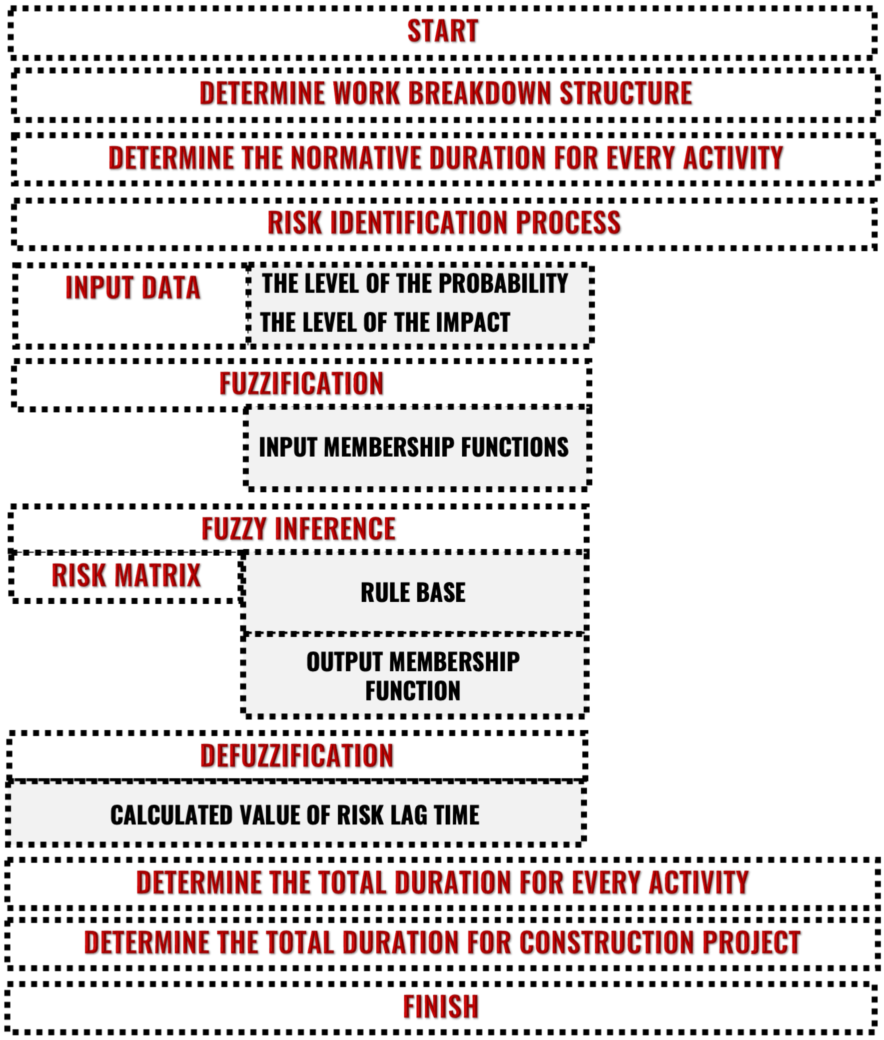

Figure 8. Approach flowchart for modeling construction duration using a risk matrix.

For the new construction project, the construction project manager or planner begins by generating a work breakdown structure, which is a systematic division of projects into smaller sub-projects in a hierarchical order to achieve the project objectives [20]. The work breakdown structure consists of work packages, work units, and activities. Then, relationships between activities are established, and the normative duration of each activity is determined. The ES, EF, LS, and LF are determined for every activity, critical path, and total duration of the construction project using PDM and Equations (1)–(8). The next step is a risk identification for each construction activity. After that, the construction project manager or planners should analyze and assess the identified risks for each activity. The risk assessment for each activity will be divided into two major parts: part A and part B. In part A, the project manager should estimate the level of the probability for each activity based on their own experience or collected data from experts. The impact assessment should indicate the severity or damage that will occur to the project. If the risk event influences the duration of the activity, the impact of this risk event may be estimated in days. The impact of a risk event as a percentage is determined by Equation (19):

$$IoRE_{\%} = IoRE_d \times 100/ND_d, \quad (19)$$

where $IoRE_{\%}$—the impact of a risk event (in percentage), $IoRE_d$—the impact of a risk event (in days), ND_d—normal (average) duration (in days).

The next step is fuzzification. Fuzzification is the process of decomposing a system input and/or output into one or more fuzzy sets using Equations (11)–(13). Finally, the construction project manager should establish the dependencies between input data, such as probability and impact, and output data, such as the level of threat for activity, and represent them in the risk matrix form. An example of a risk matrix is presented in Table 1. Thus, the risk matrix dependencies will be used to build the rule base using Equation (14).

Table 1. An example of risk matrix.

	The Level of Impact Low	**The Level of Impact Medium**	**The Level of Impact High**
The level of probability Unlikely	The level of threat Low	The level of threat Low	The level of threat Medium
The level of probability Unlikely	The level of threat Low	The level of threat Medium	The level of threat High
The level of probability Unlikely	The level of threat Medium	The level of threat High	The level of threat Very High

The authors' approach applies the Mamdani FIS (Figure 5) [50] and Equations (15) and (16) for output fuzzy sets. After that, the crisp value of the risk lag time is determined using a defuzzification method such as centroid, bisector, middle of maximum, smallest of maximum, and largest of maximum. A description of each defuzzification method is presented in Figures 6 and 7 and Equations (17) and (18).

The output obtained after defuzzification is the calculated value of the risk lag time in a percentage ($RLT^c{}_{\%}$). The calculated value of the risk lag time is an additional reserve time that depends on the probability of the risk, the magnitude of its impact, and the level of threat determined based on the risk matrix. To convert the risk lag time from percentage to days, Equation (20) is applicable:

$$RLT^c{}_d = IoRE_d \times RLT^c{}_{\%}/100, \quad (20)$$

where $RLT^c{}_d$—the calculated value of the risk lag time (in days), $IoRE_d$—the impact of risk event (in days), $RLT^c{}_{\%}$—the calculated value of the risk lag time (in percentage).

3.2. Case Study

3.2.1. Input Data

The proposed approach was applied to assess risk in the construction of an apartment building with one underground and four above-ground stories in Warsaw. The total normative duration of construction is 230 days, excluding the occurrence and impact of risks on the work. The TOS for the construction of the building involves dividing it into two sections (equal in volume of work performed) for the execution of the foundation and monolithic works. These works are planned to be carried out simultaneously by two different companies. The remaining work will be performed by only one team. The list of works and their normative duration is determined based on the Construction Norms and Regulations (KNR). It is presented in aggregated groups in Table 2 and Figure 9 as a CPM diagram. The ES, EF, LS, LF, and TF for this construction project are determined using Equations (1)–(8).

Table 2. Work activities and their duration.

Activity	Description of Activity	Normal Duration, Days	Previous Task
A	Site Investigation and Preparation	24	-
B	Foundation Works on Section 1	45	A
C	Foundation Works on Section 2	45	A
D	Construction of Monolithic Building Frame on Section 1	39	B
E	Construction of Monolithic Building Frame on Section 2	39	C
F	Masonry Works	30	D, E
G	Finishing Works	72	F
I	Landscaping	20	G

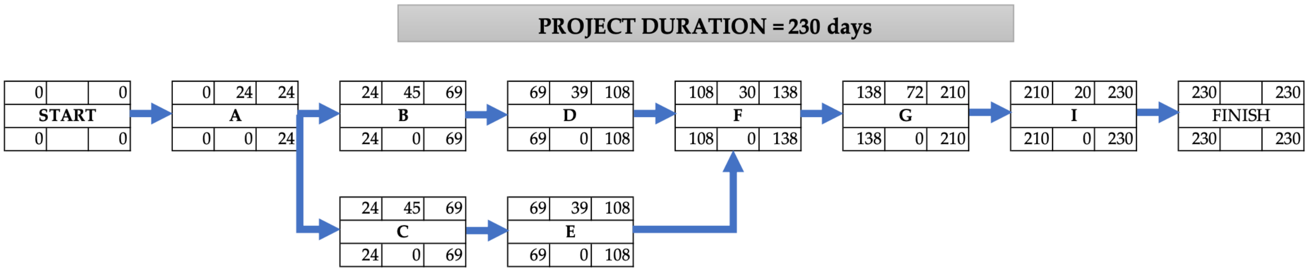

Figure 9. Application of work activities to the CPM diagram.

3.2.2. Risk Identification

Considering the selected organizational and technological solution, risk identification is conducted, which may affect the duration of construction. Risk identification takes part in this process by considering risk events that may occur during construction. For this example project, project planners identified six activities that are prone to risks:

- Site Investigation and Preparation;
- Foundation Works on Section 1;
- Foundation Works on Section 2;
- Construction of Monolithic Building Frame on Section 1;
- Construction of Monolithic Building Frame on Section 2;
- Masonry Works.

The CPM diagram calculated the project duration at 230 days of project duration and identified the risk events in the activities as illustrated in Figure 10 and Table 3.

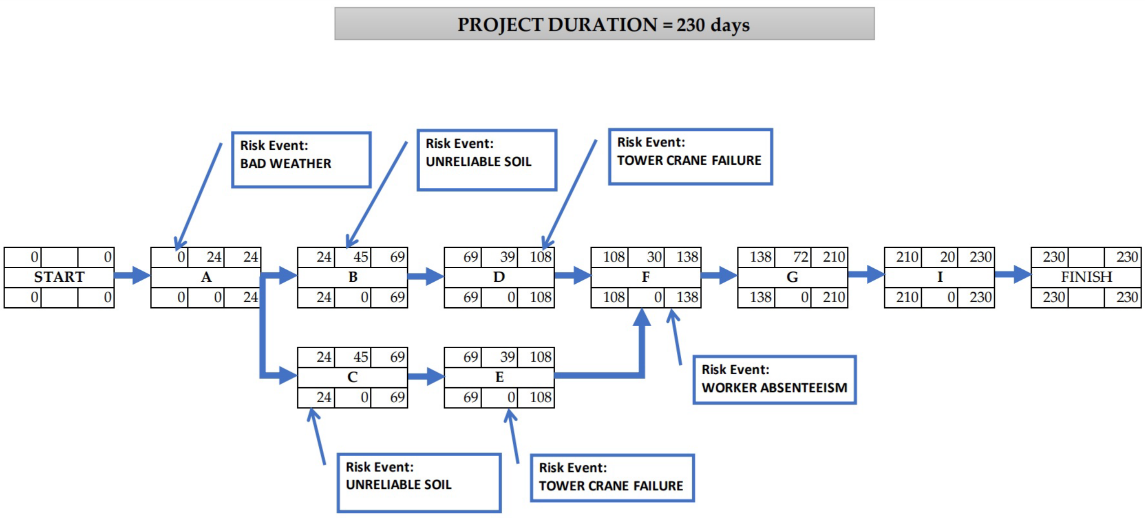

Figure 10. CPM diagram and activities exposed to risk.

Table 3. The activities and their possible risk events and factors.

Description of Activity	Risk Event	Risk Factor
(A) Site Investigation and Preparation	Bad weather	The amount of rainfall
(B) Foundation Works on Section 1	Unreliable soil information	Error from survey team Unexpected underground objects
(C) Foundation Works on Section 2	Unreliable soil information	Error from survey team Unexpected underground objects
(D) Construction of Monolithic Building Frame on Section 1	Tower crane failure	Lack of maintenance Carry overload
(E) Construction of Monolithic Building Frame on Section 2	Misunderstanding of technical documentation by workers	Low language proficiency Lack of technical education
(F) Masonry Works	Worker absenteeism	Worker illness Rule and regulation

3.2.3. Risk Analysis and Assessment

The construction project managers or planners analyzed and estimated the possibility of risk occurrence by using their own experience, data from subcontractors, and other methods of expert assessments for every risk event. The possibility of a risk event is shown in Table 4. In addition, the impact of risk events has been estimated. The level of impact is calculated in days and shows how many days of work can be increased if a risk event occurs. The level of impact for this example and the determined level of impact as a percentage using Equation (19) are summarized in Table 5. The project duration is determined using PDM methods and is presented in Figure 11.

Table 4. The activities and their possible risk events with the level of probability of occurrence.

Activity	Risk Event	Probability, %
(A) Site Investigation and Preparation	Bad weather	38
(B) Foundation Works on Section 1	Unreliable soil information	46
(C) Foundation Works on Section 2	Unreliable soil information	29
(D) Construction of Monolithic Building Frame on Section 1	Tower crane failure	84
(E) Construction of Monolithic Building Frame on Section 2	Misunderstanding of technical documentation by workers	70.5
(F) Masonry Works	Worker absenteeism	50

Table 5. Lag Time is associated with activities and risks.

Activity	Normal Duration, Days	Impact of Risk Event (Lag Time), Days	Impact of Risk Event ($IoRE_{\%}$), %
(A) Site Investigation and Preparation	24	12	50
(B) Foundation Works on Section 1	45	13	29

Table 5. *Cont.*

Activity	Normal Duration, Days	Impact of Risk Event (Lag Time), Days	Impact of Risk Event ($IoRE_{\%}$), %
(C) Foundation Works on Section 2	45	6	13
(D) Construction of Monolithic Building Frame on Section 1	39	8	20
(E) Construction of Monolithic Building Frame on Section 2	39	14	36
(F) Masonry Works	30	15	50

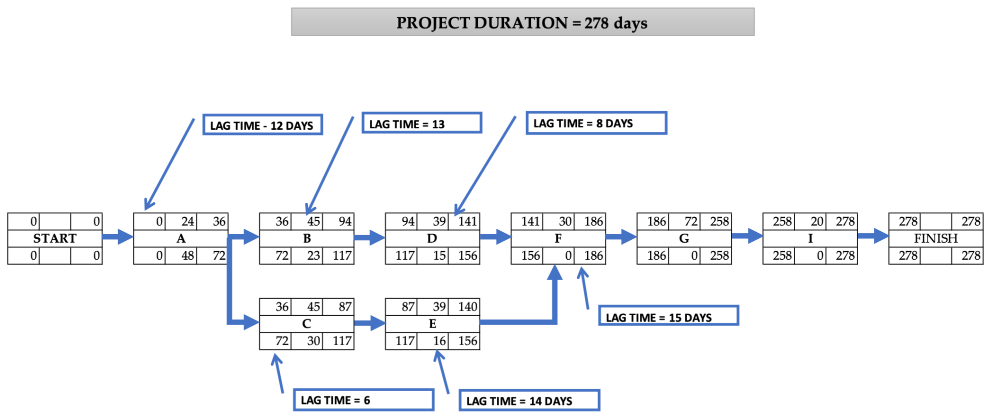

Figure 11. Input Lag time into CPM and calculate project duration.

3.2.4. Risk Response Approach

In this step, the construction project manager or planner establishes dependencies between the probability of risk event, the level of impact, and the level of output threat using a risk matrix. A risk matrix allows for prioritizing the level of risk. The scale for the level of probability consists of three states: Likely, Possible, and Unlikely. The scale for the level of impact consists of three states: Low, Medium, and High. The scale for the level of threat consists of four states: Low, Medium, High, and Very High. For this case study, the two types of risk matrix have been selected, which are shown in Tables 6 and 7. The two types of risk matrices will allow us to obtain more experimental data and compare the values obtained under the same conditions, but using different risk matrices.

Table 6. Risk matrix—type 1.

	The Level of Impact Low B_1	**The Level of Impact Medium B_2**	**The Level of Impact High B_3**
The level of probability Unlikely A_1	The level of threat is Low D_1	The level of threat is Medium D_2	The level of threat is High D_3
The level of probability Unlikely A_2	The level of threat is Medium D_2	The level of threat is High D_3	The level of threat is High D_4
The level of probability Unlikely A_3	The level of threat is High D_3	The level of threat is High D_4	The level of threat is High D_4

Table 7. Risk matrix—type 2.

	The Level of Impact Low B_1	**The Level of Impact Medium B_2**	**The Level of Impact High B_3**
The level of probability Unlikely A_1	The level of threat is Low D_1	The level of threat is Low D_1	The level of threat is Medium D_2
The level of probability Unlikely A_2	The level of threat is Low D_1	The level of threat is Medium D_2	The level of threat is High D_3
The level of probability Unlikely A_3	The level of threat is Medium D_2	The level of threat is High D_3	The level of threat is High D_4

The next step is the fuzzification of the input data sets, such as the probability of risk events and the level of impact. The degree of the membership of the individual fuzzy sets defined by linguistic variables is determined. The processes of fuzzification, calculation, and defuzzificaton have utilized MATLAB (ver. R2023) software. The membership functions are shown in Figures 12 and 13.

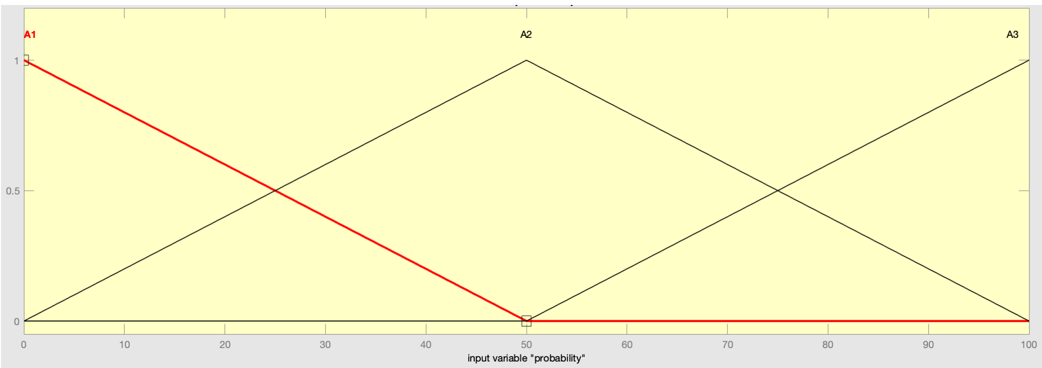

Figure 12. Membership function of the input data set for the level of probability.

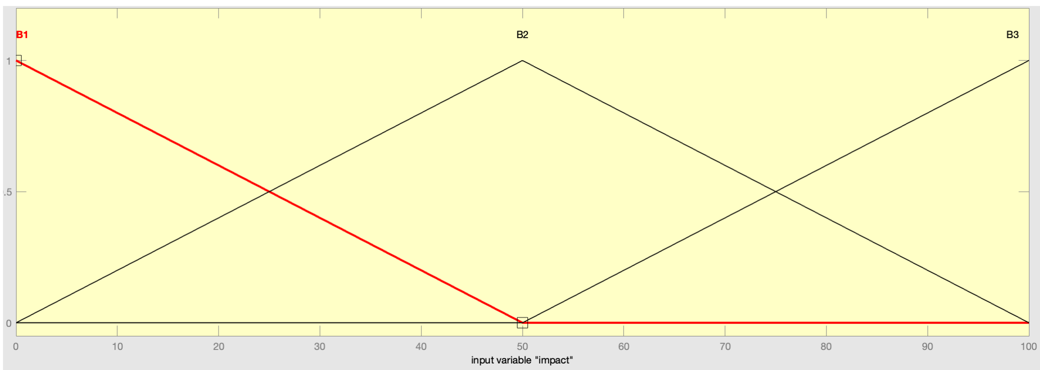

Figure 13. Membership function of the input data set for the level of impact.

The input and output data sets are related based on risk matrices. Using these relations creates the rule base. The rule base contains logical rules defining cause–effect relationships between the fuzzy input and output sets. The developed base includes nine rules for each risk matrix. The basic rule for matrix type 1 is as follows: If the level of probability is unlikely and the level of impact is low, then the level of threat is low.

Mamdani fuzzy inference is used as a method to create a control system by synthesizing a set of linguistic control rules obtained from experienced operators. The output of each rule is a fuzzy set. The output of each rule is a fuzzy set derived from the output membership function. The output data set is the level of output threat. The threat indicates as a percentage how much the duration of an activity can be increased, taking into account the probability of occurrence and impact, based on the rule base. The membership function of the output data set, such as the level of threat, is shown in Figures 14 and 15.

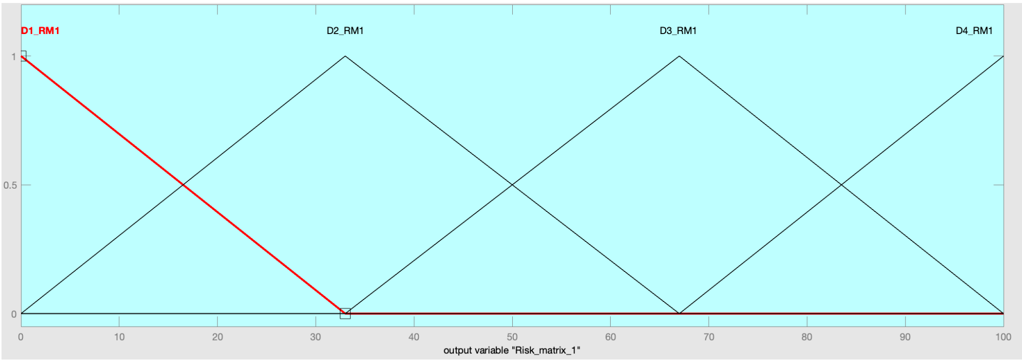

Figure 14. Membership function of the output data set for risk matrix type I.

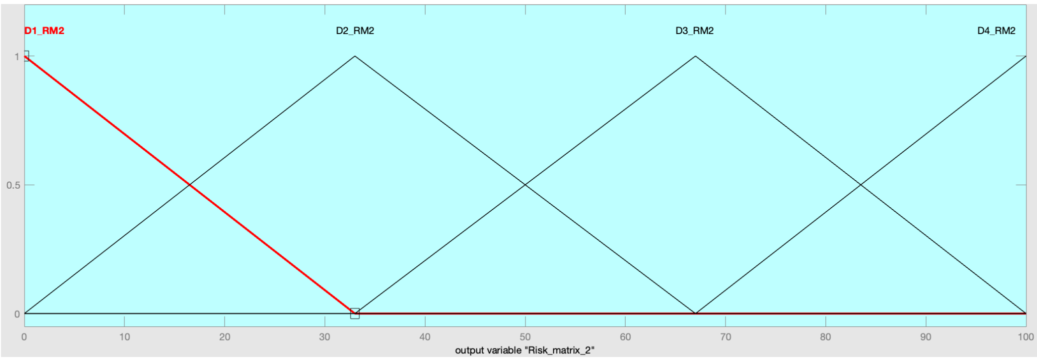

Figure 15. Membership function of the output data set for risk matrix type II.

The next step is defuzzification. There are five methods for defuzzification in MATLAB (ver. R2023) software: centroid, bisector, middle of maximum, smallest of maximum, and largest of maximum. Using all these defuzzification methods, the results are obtained. The Rule Viewer obtained from MATLAB software is shown in Figure 16.

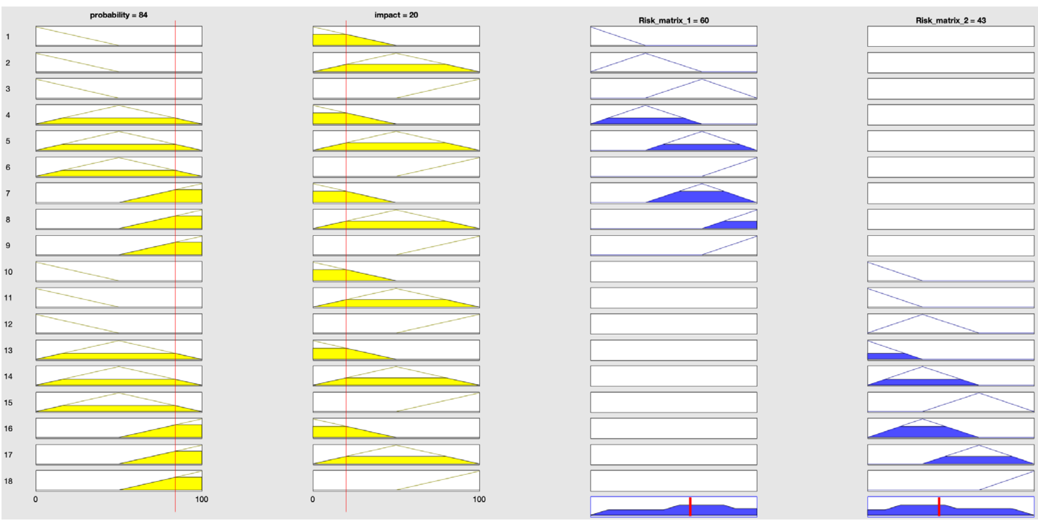

Figure 16. The example of testing data using Rule Viewer. The defuzzification method is bisector for activity D.

4. Results and Discussion

First, let us present the results of the threat level obtained in MATLAB software for two different matrices using various defuzzification methods in Tables 8 and 9.

Table 8. The results of $RLT^{c}_{\%}$ for risk matrix type I with the different defuzzification methods.

Activity	RLT$^{c}_{\%}$, % The Defuzzification Method Centroid	Bisector	Middle of Maximum (MOM)	Smallest of Maximum (SOM)	Largest of Maximum (LOM)
(A) Site Investigation and Preparation	57.4	61.0	66.5	59.0	74.0
(B) Foundation Works on Section 1	52.0	54.0	66.5	53.0	80.0
(C) Foundation Works on Section 2	41.1	38.0	33.5	20.0	47.0
(D) Construction of Monolithic Building Frame on Section 1	57.2	60.0	67.0	54.0	80.0
(E) Construction of Monolithic Building Frame on Section 2	58.3	61.0	67.0	54.0	80.0
(F) Masonry Works	66.7	67.0	67.0	67.0	67.0

Table 9. The results of RLTc% for risk matrix type II with the different defuzzification methods.

Activity	RLT$^{c}_{\%}$, % The Defuzzification Method Centroid	Bisector	Middle of Maximum (MOM)	Smallest of Maximum (SOM)	Largest of Maximum (LOM)
(A) Site Investigation and Preparation	32.3	33.0	33.5	59.0	41.0
(B) Foundation Works on Section 1	30.4	31.0	33.5	53.0	47.0
(C) Foundation Works on Section 2	25.1	20.0	6.5	20.0	13.0
(D) Construction of Monolithic Building Frame on Section 1	45.4	43.0	33.0	54.0	46.0
(E) Construction of Monolithic Building Frame on Section 2	46.1	44.0	33.0	54.0	46.0
(F) Masonry Works	33.3	33.0	33.0	67.0	33.0

Based on the data in Tables 8 and 9, calculate the value of the risk lag time in days (RLT^{c}_{d}) using Equation (2) and the total activity duration considering risk lag time. The results are presented in Tables 10 and 11.

Table 10. The results of $RLT^c{}_d$ for risk matrix type I with the different defuzzification methods.

Activity	$RLT^c{}_d$, Days The Defuzzification Method Centroid	Bisector	Middle of Maximum (MOM)	Smallest of Maximum (SOM)	Largest of Maximum (LOM)
(A) Site Investigation and Preparation	7	7	8	7	9
(B) Foundation Works on Section 1	7	7	9	7	10
(C) Foundation Works on Section 2	2	2	2	1	3
(D) Construction of Monolithic Building Frame on Section 1	5	5	5	4	6
(E) Construction of Monolithic Building Frame on Section 2	8	9	9	8	11
(F) Masonry Works	10	10	10	10	10

Table 11. The results of $RLT^c{}_d$ for risk matrix type II with the different defuzzification methods.

Activity	$RLT^c{}_d$, Days The Defuzzification Method Centroid	Bisector	Middle of Maximum (MOM)	Smallest of Maximum (SOM)	Largest of Maximum (LOM)
(A) Site Investigation and Preparation	4	4	4	7	5
(B) Foundation Works on Section 1	4	4	4	7	6
(C) Foundation Works on Section 2	2	1	0	1	1
(D) Construction of Monolithic Building Frame on Section 1	4	3	3	4	4
(E) Construction of Monolithic Building Frame on Section 2	6	6	5	8	6
(F) Masonry Works	5	5	5	10	5

Let us consider the results for risk matrix type 1, presented in Table 8. Firstly, it should be noted that the maximum values of $RLT^c{}_d$ were obtained using the defuzzification method such as largest of maximum, while the minimum value $RLT^c{}_d$ was calculated using the defuzzification methods such as centroid, bisectors, and smallest of maximum (SOM). The relative difference between the maximal and minimal values of $RLT^c{}_d$ for activities ranges from 0% to 67%, with a mean value of 29.9% for every activity.

Let's consider the results $RLT^c{}_d$ for risk matrix type II, presented in Table 11. The maximal values of $RLT^c{}_d$ were obtained using the defuzzification methods such as largest of maximum (LOM), smallest of maximum (SOM), and centroid. The minimal value of $RLT^c{}_d$ was calculated using the defuzzification methods such as centroid, bisectors, smallest of maximum (SOM), middle of maximum (MOM), and largest of maximum (LOM). The relative difference between the maximal and minimal values of $RLT^{c}{}_{d}$ for activities ranges from 25% to 50%, with a mean value of the relative difference between the maximal and minimal values of $RLT^{c}{}_{d}$ for every activity being 41.3%.

These results show that the defuzzification methods influence the output value, but they should not mark out minimal and maximal levels. The relative difference between the maximal and minimal values of $RLT^{c}{}_{d}$ for two types of risk matrices ranges from 29.9% to 41.3%.

Let's consider the total project duration, taking into account the $RLT^{c}{}_{d}$. Firstly, determine the activity duration considering the $RLT^{c}{}_{d}$ for different risk matrices and the defuzzification method. Then, calculate the project duration for each case using the CPM method and Equations (1)–(10). The results are presented in Tables 12 and 13.

Table 12. The total duration for each activity and project duration for risk matrix type I with the different defuzzification methods.

Activity	Normal Duration, Days	The Total Duration for Each Activity and Project, Days The Defuzzification Method				
		Centroid	Bisector	Middle of Maximum (MOM)	Smallest of Maximum (SOM)	Largest of Maximum (LOM)
(A) Site Investigation and Preparation	24	31	31	32	31	33
(B) Foundation Works on Section 1	45	52	52	54	52	55
(C) Foundation Works on Section 2	45	47	47	47	46	48
(D) Construction of Monolithic Building Frame on Section 1	39	44	44	44	43	45
(E) Construction of Monolithic Building Frame on Section 2	39	47	48	48	47	50
(F) Masonry Works	30	40	40	40	40	40
(G) Finishing Works	72	72	72	72	72	72
(I) Landscaping	20	20	20	20	20	20
Total project duration	230	259	259	262	258	265

Table 13. The total duration for each activity and project duration for risk matrix type II with the different defuzzification methods.

Activity	Normal Duration, Days	The Total Duration for Each Activity and Project, Days The Defuzzification Method				
		Centroid	Bisector	Middle of Maximum	Smallest of Maximum	Largest of Maximum
(A) Site Investigation and Preparation	24	28	28	28	31	29
(B) Foundation Works on Section 1	45	49	49	49	52	51

Table 13. *Cont.*

Activity	Normal Duration, Days	The Total Duration for Each Activity and Project, Days The Defuzzification Method				
		Centroid	Bisector	Middle of Maximum	Smallest of Maximum	Largest of Maximum
(C) Foundation Works on Section 2	45	47	46	45	46	46
(D) Construction of Monolithic Building Frame on Section 1	39	43	42	42	43	43
(E) Construction of Monolithic Building Frame on Section 2	39	45	45	44	47	45
(F) Masonry Works	30	35	35	35	40	35
(G) Finishing Works	72	72	72	72	72	72
(I) Landscaping	20	20	20	20	20	20
Total project duration	230	247	246	246	258	250

Analyzing the data from Table 12, the maximal value of the total project duration is calculated using the largest of maximum defuzzification method, and the minimal value of the total project duration is calculated using the centroid and bisector defuzzification methods. The relative difference between the maximal and minimal project duration is 2.6%. The maximal value of the total project duration is calculated using the smallest of maximum (SOM) defuzzification method, and the minimal value of the total project duration is calculated using the bisector and middle of maximum (MOM) defuzzification methods for the data from Table 13. The relative difference between the maximal and minimal project duration is 4.6%.

The relative difference between the maximal and minimal values of total project duration for two risk matrices ranges from 2.6% to 4.6%. Despite the significant difference between the maximal and minimal values of $RLT^{c}{}_{d}$, the relative difference between the maximal and minimal values of the total project duration amounts to less than 5%. This suggests that the defuzzification method affects the difference between the maximal and minimal level of $RLT^{c}{}_{d}$. Still, it doesn't influence the total project duration, and consequently, any defuzzification method can be chosen.

Then, determine the additional risk lag time as multiple probability and lag time (RLT_{PxI}) and the project duration, considering the additional risk lag time RLT_{PxI}. It will be 255 days. Draw the column charts with the date of the project duration for two types of risk matrices and present them in Figures 17 and 18. The average of the project durations for different defuzzification methods and two kinds of risk matrices is calculated and presented in Figure 19.

Note that using the different defuzzification method, the RLT_{PxI} value is less than the determined $RLT^{c}{}_{d}$ for matrix type 1. But the opposite is true for the risk matrix type 2. The RLT_{PxI} value is more than was determined by $RLT^{c}{}_{d}$ using the different defuzzification method. Therefore, using a risk matrix has allowed construction project managers or planners to range the risks and consider them in the calculation according to the construction project managers' or planners' experience.

Evaluating the differences between the average of the project durations for each risk matrix and the RLT_{PxI} value, consider that the differences between the average of the project durations with $RLT^{c}{}_{d}$ for risk matrix type I and with the RLT_{PxI} value are the same as the differences between the average of the project durations with $RLT^{c}{}_{d}$ for risk matrix type II and with the RLT_{PxI} value and amounts 2.2%. This value is confirmed in Figure 20.

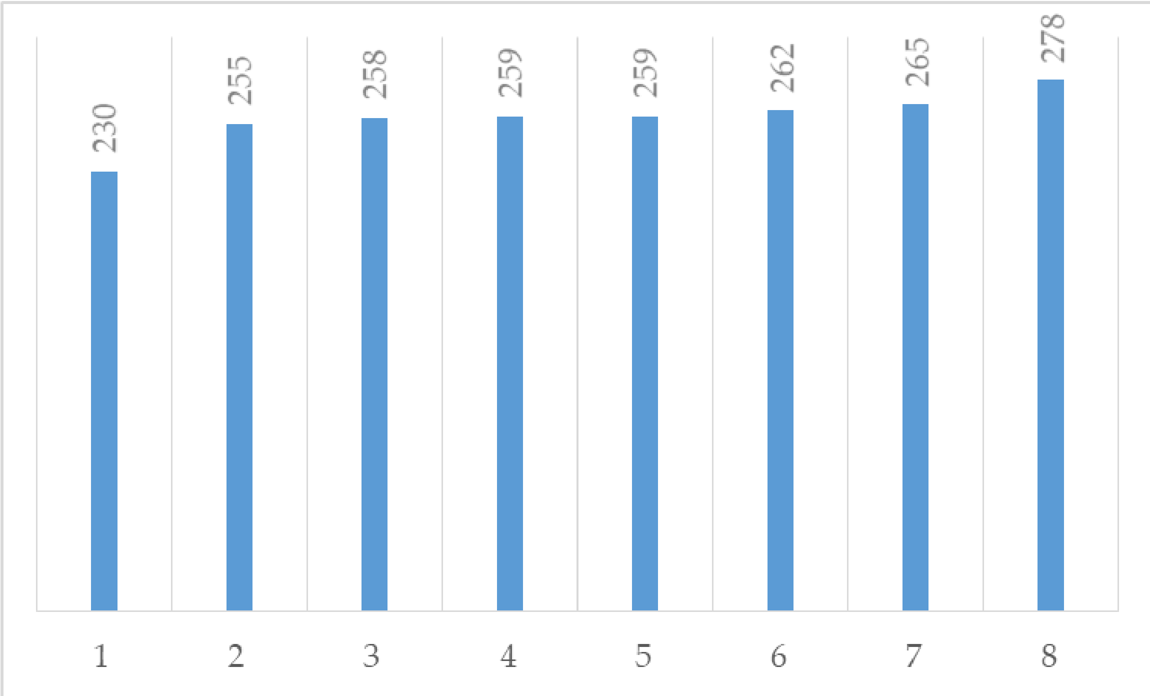

Figure 17. The project duration (with data RLT^c_d from risk matrix type I): 1—normal project duration; 2—project duration with RLT_{PxI}; 3—project duration with RLT^c_d (SOM); 4—project duration with RLT^c_d (bisector); 5—project duration with RLT^c_d (centroid); 6—project duration RLT^c_d (MOM); 7—project duration with RLT^c_d (LOM); 8—project duration with lag time.

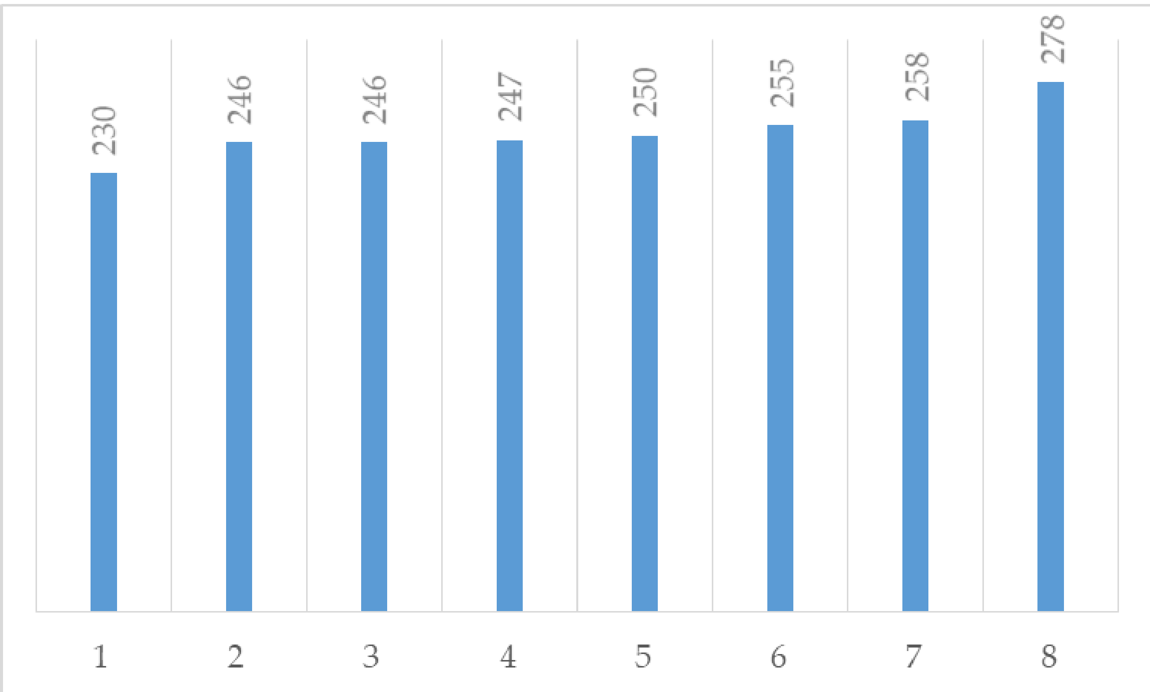

Figure 18. The project duration (with data RLT^c_d from risk matrix type I): 1—normal project duration; 2—project duration with RLT^c_d (bisector); 3—project duration with RLT^c_d (MOM); 4—project duration with RLT^c_d (centroid); 5—project duration with RLT^c_d (LOM); 6—project duration with RLT_{PxI}; 7—project duration with RLT^c_d (SOM); 8—project duration with lag time.

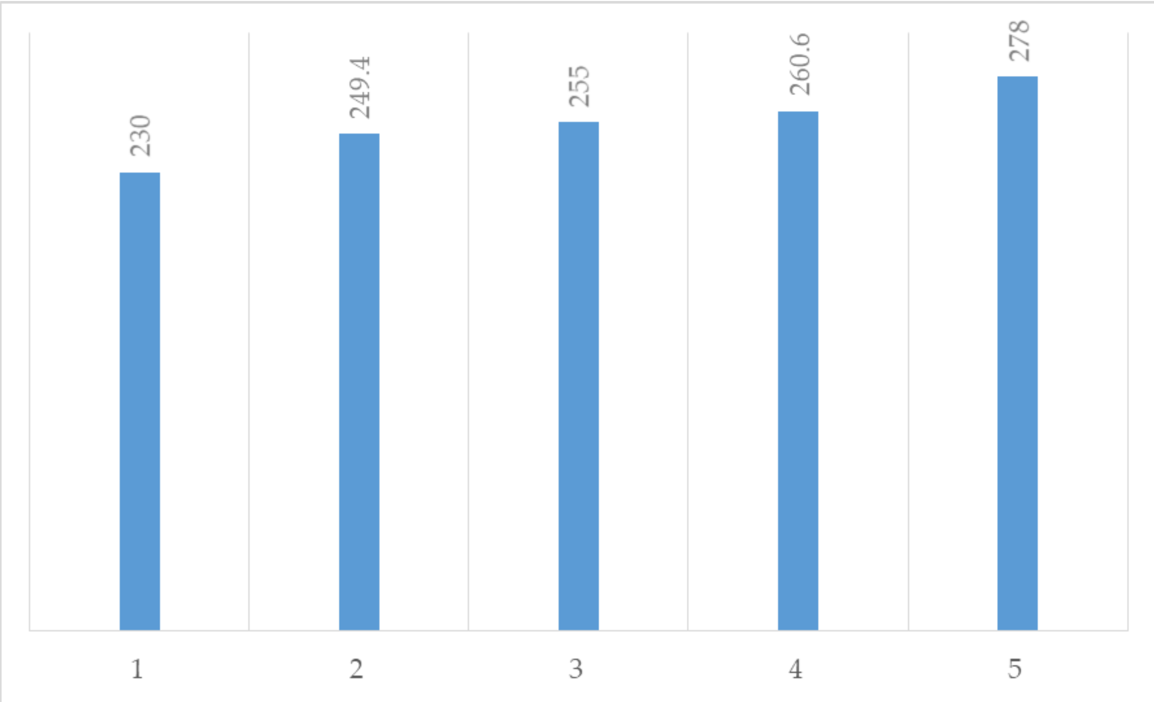

Figure 19. The project duration (with data $RLT^c{}_d$ from risk matrix type I and type II): 1—normal project duration; 2—average of the project duration with $RLT^c{}_d$ from risk matrix type II; 3—project duration with RLT_{PxI}; 4—average of the project duration with $RLT^c{}_d$ from risk matrix type I; 5—project duration with lag time.

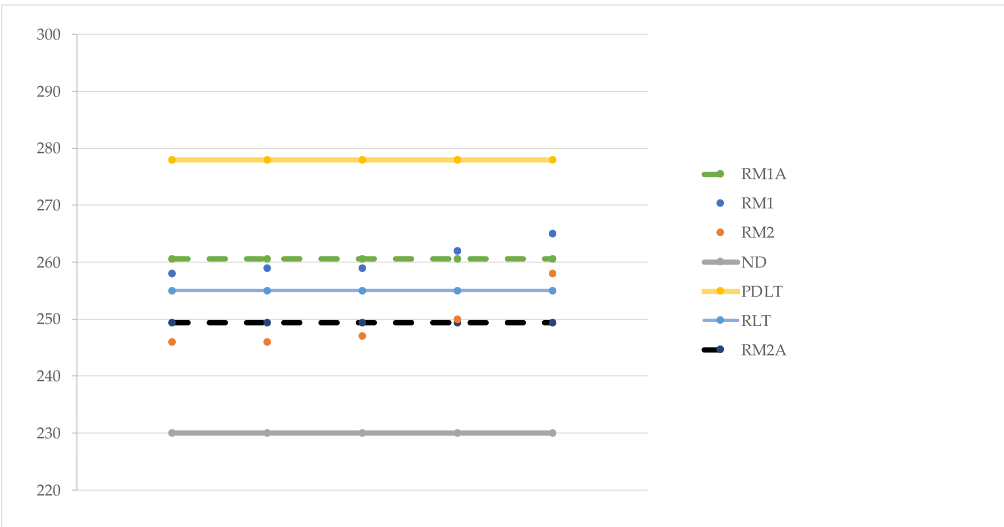

Figure 20. Comparative chart of the project duration (risk matrix type 2): normal project duration (ND); the average of the project duration with $RLT^c{}_d$ from risk matrix type II (RM2A); the project duration with RLT_{PxI} (RLT); the average of the project duration with $RLT^c{}_d$ from risk matrix type I (RM1A); the project duration with lag time (PDLT); the data of the project duration with data $RLT^c{}_d$ from risk matrix type I (RM1); data of the project duration with data $RLT^c{}_d$ from risk matrix type II (RM2).

This experimental data proves that project duration with risk lag time is an RLT_{PxI} value with uncertainty, which is equal to the deviation of the differences between the average of the project durations with $RLT^c{}_d$ for risk matrix type 1 and the RLT_{PxI} value. One could argue that the risk matrix allows one to take into account the threat level based on the construction project manager's or planner's experience and allows the threat level to be increased or reduced relative to the RLT_{PxI} value.

Let us consider a similar approach for each task. For example, let's take task (A), Site Investigation and Preparation. The differences between the average of the activity duration (A) for risk matrix type I and the RLT_{PxI} of the activity (A) value amounts to 5.1% and the differences between the average of the activity duration (A) for risk matrix type 2 and the RLT_{PxI} of the activity (A) value amounts to 4.0%. The relative difference between the maximal and minimal values of the differences between the average activity duration with risk lag time for two types of risk matrices and the RLT_{PxI} of the activity lies in the range from 0.0% to 9.1%. The differences between the average of the other activity for risk matrix type I and type II and the RLT_{PxI} are presented in Table 14. The uncertain data and differences from Table 1 could be considered when defining the membership function using fuzzy set type 2.

Table 14. The results of the differences between output data.

Activity	Normal Duration, Days	The Average of the Activity Duration with Risk Lag Time		The Activity Duration with RLT_{PxI}, Days (3)	The Activity Duration with LT, Days	The Differences between (1) and (3), %	The Differences between (2) and (3), %
		For Risk Matrix 1, Days (1)	For Risk Matrix 2, Days (2)				
(A) Site Investigation and Preparation	24	31.6	28.8	30	36	5.3	4.0
(B) Foundation Works on Section 1	45	53.0	50	49	58	8.2	2.1
(C) Foundation Works on Section 2	45	47.0	46	46	51	2.2	0.0
(D) Construction of Monolithic Building Frame on Section 1	39	44.0	42.6	41	47	7.3	3.9
(E) Construction of Monolithic Building Frame on Section 2	39	48.0	45.2	44	53	9.1	2.7
(F) Masonry Works	30	40.0	36	38	45	5.3	5.3

5. Conclusions

Discrepancies between normative or planned construction durations are globally universal. The primary source of the construction duration increase is the occurrence of risk events that were not accounted for in the project. This paper proposes an algorithm that allows for the assessment of the risk lag time. The algorithm enables the consideration not only of the probability level of risk occurrence and its impact on the work but also of the level of threat it may pose using risk matrices and fuzzy set theory.

As a result of the research, the following has been established:

1. The use of a risk matrix allows for ranking and considering the level of threat, taking into account the experience of the construction project manager or planner. This enables the reduction or increase in the magnitude of the risk lag time relative to the risk lag time determined as a product of probability and impact.
2. The defuzzification method influences the output value of the risk lag time for individual tasks, and the difference between the maximum and minimum values can reach 67%. However, the defuzzification method has little significant impact on the output

value of the risk lag time, reaching less than 5%. This suggests that any convenient defuzzification method can be chosen to simplify calculations.

3. It has been established that the relative deviation between the risk lag time (RxI) and the mean value of the risk lag time is less than 10% for individual tasks. Therefore, to obtain more accurate calculations of the risk lag time using risk matrices, calculations should be performed using five types of defuzzification methods.
4. The relative deviation between the risk lag time (RxI) and the mean value of the risk lag time for individual tasks may represent the level of uncertainty with which each task can be implemented. The value of the uncertainty level can be used in constructing the membership function for fuzzy set type 2.

In further research, the authors plan to enhance the presented approach for estimating the duration of individual construction works, considering the impact of risk events, to enable the application of fuzzy set type 2 based on the data obtained in this study.

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