mathematics

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Fixed-Time Synchronization for Fuzzy-Based Impulsive Complex Networks

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Abstract: This paper mainly deals with the issue of fixed-time synchronization of fuzzy-based impulsive complex networks. By developing fixed-time stability of impulsive systems and proposing a T-S fuzzy control strategy with pure power-law form, some simple criteria are acquired to achieve fixed-time synchronization of fuzzy-based impulsive complex networks and the estimation of the synchronized time is given. Ultimately, the presented control scheme and synchronization criteria are verified by numerical simulation.

Keywords: fixed-time synchronization; complex network; impulse; T-S fuzzy logic

MSC: 34A37; 34D20; 93C42

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Citation: Pang, L.; Hu, C.; Yu, J.; Jiang, H. Fixed-Time Synchronization for Fuzzy-Based Impulsive Complex Networks. *Mathematics* **2022**, *10*, 1533. https://doi.org/10.3390/math10091533

Academic Editors: Xinsong Yang and Ruofeng Rao

Received: 8 April 2022
Accepted: 28 April 2022
Published: 3 May 2022

1. Introduction

Complex networks are ubiquitous and the dynamic behaviors of considerable real systems in society can be simulated by complex network models [1]. Nowadays, complex networks have been acquired wide attention in different branches of science and engineering. In addition, many real network systems, such as power grid [2], communication network [3], image protection [4], and genetic regulatory network [5], are always subject to various instantaneous disturbances caused by burst noises, frequency changes, etc., which lead to abrupt changes of system states at some certain instants and finally form the impulsive phenomenon. In 1960, Miliman et al. initially studied stability of motion with impulse [6]. Afterwards, complex networks with impulse have naturally attracted widespread attention [7–9].

As is well known, many practical systems are sophisticated and nonlinear, it is thus difficult to explore the dynamic characteristics of those systems by means of conventional analysis methods and control strategies [10,11]. For this topic, Takagi and Sugeno proposed a fuzzy model, i.e., T-S fuzzy model [12], which has universal function approximation capability and can approximate any smooth nonlinear function on any convex compact domain [13]. Furthermore, the T-S fuzzy controller is a versatile controller, because a T-S fuzzy controller can always be found to stabilize the nonlinear system [14]. Recently, much attention has been carried out on the dynamic analysis of various T-S fuzzy systems, including filtering [15,16], stability [17,18], stabilization [19,20], synchronization [21–23] and so forth.

As one of the most fundamental subjects in analysis and control for various systems, synchronization has acquired widespread concern in recent decades. Generally, synchronization refers to the consensus feature of nodes' dynamics driven by interaction of nodes or forced by additional control [24–26]. So far, many valuable results about synchronization of deterministic fuzzy complex networks (FCNs) have been reported [27–30]. For instance, the finite-time cluster synchronization for T-S fuzzy complex networks with discontinuous subsystems and probabilistic coupling delays was concerned based on T-S fuzzy interpolation approach and Fillipov solution [27]. Moreover, the synchronization of T-S fuzzy complex networks was analyzed in [28–30] via applying sampling control and adaptive fuzzy control, respectively. Diversely, the influence of impulse on FCNs is considered and excellent achievements have been obtained [31,32]. For example, the exponential pinning synchronization of stochastic T-S fuzzy delayed complex networks with heterogeneous impulsive delays was discussed based on fuzzy memory pinning impulsive control method in [31]. The synchronization conditions for fuzzy-based complex networks with different switching topologies and time-varying delays were established in [32] via applying fuzzy impulsive control strategy. Significantly, these researches mainly focus on the asymptotic synchronization based on the traditional asymptotic stability theory, which indicates that network synchronization only happens when the time goes to infinity.

In many practical problems, such as image encryption and communication security, it is popular to realize performance control in finite time. Considering the fact, various finite-time techniques have been proposed to explore the synchronization about impulsive networks [33,34], where the synchronized time (ST) is associated with the initial states of networks. Regretfully, it is unrealistic to access all system's initial states in many engineering problems, so it is extremely difficult to estimate the settling time. To eliminate the dependence on the initial states, fixed-time (FT) synchronization for impulsive complex networks (ICNs) was proposed [35–39], where the synchronization rate is improved compared with asymptotic synchronization and the relevance between the ST and the initial states is avoided compared with finite-time synchronization. Especially, the synchronization of ICNs was analyzed in [36] via applying fixed-time stability theory and fixed-time control protocols, in which the estimation of the synchronized time is more accurate than the work [35]. The FT synchronization conditions of delayed complex systems with stochastic perturbation were established in [37] by means of novel impulsive pinning control strategy. Recently, by virtue of non-chattering control scheme, and conceiving a novel time-dependent Lyapunov function, the FT stochastic synchronization for multi-weighted impulsive complex dynamical networks was investigated in [38]. The FT synchronization of time-varying delayed impulsive networks with discontinuous activation functions was analyzed in [39] based on generalized variable transformation with suitable tunable variables and differential inclusion theory. Nevertheless, there seems to be no related reports about the FT synchronization of fuzzy-based impulsive complex networks (FICNs). Thus, it is urgent and worthy to be deeply studied to develop some innovative control protocols and analysis techniques to explore the FT synchronization of FICNs.

Enlightened by the above analysis, the FT synchronization of FICNs will be deeply addressed. The innovations of the paper are mainly demonstrated as below.

(1) Since the T-S fuzzy rules and impulsive coupling are all considered, the FICNs in this article complement and extend the models without fuzzy rules in [35–40] and FCNs without impulse [27–30].

(2) An improved fixed-time stability theorem is developed for impulsive systems, where the more general linear term $V(y(t))$ is involved compared with [36]. Furthermore, the subsequent control design can be simplified by using the stability theorem.

(3) A type of pure power-law control scheme is developed and some simple conditions are established based on the FT stability theorem to realize FT synchronization of FICNs. Note that the proposed control scheme here is a pure power-law form, which is simpler in comparison with the previous FT control design given in [35–40], in which both the linear part and the power-law term are involved.

The remainder of the work is arranged as below. The model of FICNs and some related basic results are provided in Section 2. The FT synchronization is investigated for FICNs in Section 3. A numerical example is provided to validate the theoretical results in Section 4. Ultimately, the results of the paper are summarized.

Notations: In this paper, the sets of non-negative integers and real numbers are respectively denoted by $\mathbb{N}_+$ and $\mathbb{R}$. $\mathbb{R}^n$ represents the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $N_\natural = \{1, \ldots, N\}$ for positive integer N. For a matrix A, its transpose is denoted as A^T, its largest eigenvalue is represented as $\lambda_{\max}(A)$ if it is symmetric. diag$\{\cdot\}$ represents a diagonal matrix, and I_m denotes the identity matrix with m dimension. For any $d = (d_1, \ldots, d_n)^T \in \mathbb{R}^n$, $\theta > 0$, $[d]^\theta = \big(\mathrm{sign}(d_1)|d_1|^\theta, \ldots, \mathrm{sign}(d_n)|d_n|^\theta\big)^T$.

2. Problem Description and Preliminaries

Since impulse effect is ubiquitous in network systems, the following fuzzy-based impulsive network is considered:

Rule r: IF $\varphi_1(t)$ is Θ_1^r, ..., $\varphi_G(t)$ is Θ_G^r, THEN

$$\begin{cases} \dot{z}_q(t) = f(z_q(t)) + s^{\langle r\rangle} \sum\limits_{j=1}^{N} c_{qj}\Gamma\hbar(z_j(t)) + \mathrm{u}_q^{\langle r\rangle}(t),\ t \neq t_k,\ t \geq 0, \\ \triangle z_q(t_k) = \varsigma_k^{\langle r\rangle} \sum\limits_{j=1}^{N} c_{qj} z_j(t_k^-),\ k \in \mathbb{N}_+,\ q \in N_\natural,\ r \in M_\natural, \end{cases} \tag{1}$$

where M is the number of fuzzy IF-THEN rules, $\varphi_g(t)$ and Θ_g^r $(g \in G_\natural)$ are premise variables and fuzzy sets, respectively, $z_q(t) = \big(z_{q1}(t), \ldots, z_{qn}(t)\big)^T \in \mathbb{R}^n$ denotes the state variable of the node q at time t, $f(z_q(t)) = \big(f_1(z_q(t)), \ldots, f_n(z_q(t))\big)^T$ represents nonlinear dynamical function, function $\hbar(z_q(t)) = \big(\bar{\hbar}(z_{q1}(t)), \ldots, \bar{\hbar}(z_{qn}(t))\big)^T$, $\Gamma \in \mathbb{R}^{n\times n}$ is the internal coupling matrix, $s^{\langle r\rangle} > 0$ is the coupling strength, $\mathrm{u}_q^{\langle r\rangle}(t)$ is the imposed control input on the qth node, $\{t_k, k \in \mathbb{N}_+\}$ is a sequence of strictly increasing impulsive instants, $\varsigma_k^{\langle r\rangle}$ is the strength of impulse, $\triangle z_q(t_k) = z_q(t_k^+) - z_q(t_k^-)$, $z_q(t_k^+) = \lim_{t\to t_k^+} z_q(t)$, $z_q(t_k^-) = \lim_{t\to t_k^-} z_q(t)$, without loss of generality, we assume that $z_q(t)$ is right continuous at $t = t_k$, i.e., $z_q(t_k^+) = z_q(t_k)$. $C = (c_{qj})_{N\times N}$ is the adjacency matrix, here $c_{qj} > 0$ if there has an edge between node j and node q $(q \neq j)$, otherwise, $c_{qj} = 0$, and $c_{qq} = -\sum_{j=1,j\neq q}^{N} c_{qj}, q \in N_\natural$. Furthermore, the corresponding Laplacian matrix $\mathcal{L} = (l_{qj})_{N\times N}$ is defined as $l_{qj} = -c_{qj}$, $j \neq q$, the diagonal elements $l_{qq} = \sum_{j=1,j\neq q}^{N} c_{qj}$. The initial condition is given by $z_q(0) = \varphi_q(0) \in \mathbb{R}^n, q \in N_\natural$.

Using the singleton fuzzifier, product fuzzy inference, and a weighted average defuzzifier [12,13], T-S fuzzy network (1) is transformed into the following form

$$\begin{cases} \dot{z}_q(t) = f(z_q(t)) + \sum\limits_{r=1}^{M} \Upsilon_r(\varphi(t)) \Big[s^{\langle r\rangle} \sum\limits_{j=1}^{N} c_{qj}\Gamma\hbar(z_j(t)) + \mathrm{u}_q^{\langle r\rangle}(t)\Big], \\ \qquad t \neq t_k,\ t \geq 0, \\ \triangle z_q(t_k) = \sum\limits_{r=1}^{M} \Upsilon_r(\varphi(t))\varsigma_k^{\langle r\rangle} \sum\limits_{j=1}^{N} c_{qj} z_j(t_k^-),\ k \in \mathbb{N}_+,\ q \in N_\natural,\ r \in M_\natural, \end{cases} \tag{2}$$

where $\varphi(t) = (\varphi_1(t), \ldots, \varphi_G(t))^T$, $\Upsilon_r(\varphi(t)) = \frac{\varpi_r(\varphi(t))}{\sum_{r=1}^{M} \varpi_r(\varphi(t))}$, $\varpi_r(\varphi(t)) = \prod\limits_{g=1}^{G} \Theta_g^r(\varphi_g(t))$, and $\Theta_g^r(\varphi_g(t))$ denotes the grade of membership of $\varphi_g(t)$ in Θ_g^r.

In addition, since $\varpi_r(\varphi(t)) \geq 0, r \in M_\natural$ and $\sum\limits_{r=1}^{M} \varpi_r(\varphi(t)) > 0$, then $\Upsilon_r(\varphi(t)) \geq 0$ and $\sum\limits_{r=1}^{M} \Upsilon_r(\varphi(t)) = 1$.

Remark 1. *At present, linear systems have been widely studied, but considerable practical systems are nonlinear, which makes the analysis of control problems extremely difficult [13]. However, Takagi and Sugeno [12] found that by using the qualitative and linguistic information of nonlinear dynamic systems, a set of fuzzy IF-THEN languages can be introduced to describe the local input-output relationship of nonlinear systems. Meanwhile, these local subsystems are fuzzy mixed through appropriate membership function, so that the whole fuzzy system model can be established. In view of this advantage, a kind of fuzzy-based impulsive coupling network (2) is considered in this paper.*

Consider the synchronous state satisfying the following equation

$$\dot{\nu}(t) = f(\nu(t)),\ t \geq 0, \tag{3}$$

in which $\nu(t) = \big(\nu_1(t), \ldots, \nu_n(t)\big)^T \in \mathbb{R}^n$ *is the state variable. The initial value is provided as* $\nu(0) = \psi(0) \in \mathbb{R}^n$.

By setting $\epsilon_q(t) = z_q(t) - \nu(t)$, *the error dynamic model is described by*

$$\begin{cases} \dot{\epsilon}_q(t) = \sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \Big[s^{\langle r \rangle} \sum_{j=1}^{N} c_{qj} \Gamma \hat{\hbar}(\epsilon_j(t)) + \mathrm{u}_q^{\langle r \rangle}(t) \Big] \\ \qquad\qquad + f(z_q(t)) - f(\nu(t)),\ t \neq t_k,\ t \geq 0, \\ \triangle \epsilon_q(t_k) = \sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \varsigma_k^{\langle r \rangle} \sum_{j=1}^{N} c_{qj} \epsilon_j(t_k^-),\ k \in \mathbb{N}_+, \\ \qquad q \in \mathrm{N}_\natural,\ r \in \mathrm{M}_\natural, \end{cases} \tag{4}$$

where $\hat{\hbar}(\epsilon_j(t)) = \hbar(z_j(t)) - \hbar(\nu(t)) = \big(\bar{\hat{\hbar}}(\epsilon_{q1}(t)), \ldots, \bar{\hat{\hbar}}(\epsilon_{qn}(t))\big)^T$.

Assumption 1. *There has a positive real number* ω *such that for any* $\mu, v \in \mathbb{R}^n$,

$$\|f(\mu) - f(v)\| \leq \omega \|\mu - v\|.$$

Assumption 2. *There exists a real number* $\alpha > 0$, *such that for any* $a \neq b \in \mathbb{R}$,

$$\frac{\bar{\hbar}(a) - \bar{\hbar}(b)}{a - b} \geq \alpha.$$

Definition 1 ([41]). *The fuzzy-based impulsive network (2) is said to reach FT synchronization, provided that there exist two real numbers* $T^*(\epsilon(0)) \geq 0$ *and* $T_{\max} > 0$ *such that*

$$\lim_{t \to T^*(\epsilon(0))} \|\epsilon(t)\| = 0, \quad \epsilon(t) = 0 \text{ for all } t \geq T^*(\epsilon(0)),$$

and $T^*(\epsilon(0)) \leq T_{\max}$ *for any* $\epsilon(0) \in \mathbb{R}^{nN}$, *and*

$$T(\epsilon(0)) = \inf\{T^*(\epsilon(0)) \geq 0 : \|\epsilon(t)\| = 0, t \geq T^*(\epsilon(0))\}$$

is said to the synchronized time of network (2), where $\epsilon(t) = (\epsilon_1^T(t), \ldots, \epsilon_N^T(t))^T$.

Lemma 1 ([42]). *If* $x_i \geq 0$ *for* $i \in n_\natural$, $0 \leq v \leq 1$, $p > 1$, *then*

$$\sum_{i \in n_\natural} x_i^v \geq (\sum_{i \in n_\natural} x_i)^v, \quad \sum_{i \in n_\natural} x_i^p \geq n^{1-p} (\sum_{i \in n_\natural} x_i)^p.$$

Consider the following impulsive system

$$\begin{cases} \dot{y}(t) = h\big(y(t)\big), & t \neq t_k, \\ \Delta y(t_k) = \Phi(y(t_k^-)), & k \in \mathbb{N}_+, \end{cases} \tag{5}$$

in which $y \in \mathbb{R}^n$, $h : \mathbb{R}^n \to \mathbb{R}^n$ *and* $\Phi : \mathbb{R}^n \to \mathbb{R}^n$ *are continuous functions,* $h(0) = 0$ *and* $\Phi(0) = 0$.

Definition 2 ([36]). *The scalar function* $V(y(t))$ *defined on* $\mathbb{R}^n$ *is called to belong to class* $\mathcal{V}$ *if*
(i) It is locally Lipschitzian with respect to y *and* $V(0) = 0$;

(ii) It is positive definite and radially unbounded;
(iii) It is continuous on the interval $[t_{k-1}, t_k)$ with $k \in \mathbb{N}_+$, and $\lim_{t\to t_k^-} V(y(t)) = V(y(t_k^-))$.

Assumption 3 ([35,36])**.** *The impulsive time set $\{t_s, s \in \mathbb{N}_+\}$, satisfying $0 = t_0 < t_1 < \ldots < t_{s-1} < t_s < \ldots$ and $\lim_{s\to+\infty} t_s = +\infty$, belongs to $I(\tau_{\min}, \tau_{\max}) \triangleq \{\tau_{\min} \le t_s - t_{s-1} \le \tau_{\max}\}$, in which $\tau_{\min}$, $\tau_{\max}$ are positive constants.*

By Assumption 3, for any time interval $(\flat, t)$, the number of impulsive jumps denoted by $\iota(\flat, t)$ satisfies $\frac{t-\flat}{\tau_{\max}} - 1 \le \iota(\flat, t) \le \frac{t-\flat}{\tau_{\min}}$.

Lemma 2 ([36])**.** *Considering system (5), assume that there exist a function $V(y(t)) \in \mathcal{V}$ and numbers $\xi, \pi > 0$, $\varrho > 1$, $\sigma, \eta \in (0, 1)$, such that*

$$\begin{cases} \dfrac{d}{dt}V(y(t)) \le -\xi V^{\varrho}(y(t)) - \pi V^{\sigma}(y(t)),\ t \ne t_k,\ t \ge 0, \\ V(y(t_k)) \le \eta V(y(t_k^-)),\ k \in \mathbb{N}_+, \end{cases} \tag{6}$$

then $y(t) = 0$ for $t \ge T_1$, here

$$\begin{aligned} T_1 =& \frac{\tau_{\max}}{(1-\sigma)\ln\eta} \ln\Big(\frac{\pi\tau_{\min}}{\pi\tau_{\min} - \eta^{\sigma-1}\ln\eta}\Big) \\ &+ \frac{\tau_{\max}}{(1-\varrho)\ln\eta} \ln\Big(1 - \frac{\eta^{1-\varrho}\ln\eta}{\xi\tau_{\max}}\Big). \end{aligned}$$

Lemma 3. *Considering system (5), if there exist a function $V(y(t)) \in \mathcal{V}$, and several constants $0 < \kappa < \min\{\xi, \pi\}$, $\varrho > 1$, $\sigma, \eta \in (0, 1)$, such that*

$$\begin{cases} \dfrac{d}{dt}V(y(t)) \le \kappa V(y(t)) - \xi V^{\varrho}(y(t)) - \pi V^{\sigma}(y(t)),\ t \ne t_k,\ t \ge 0, \\ V(y(t_k)) \le \eta V(y(t_k^-)),\ k \in \mathbb{N}_+, \end{cases} \tag{7}$$

then $y(t) = 0$ for $t \ge T_2$, here

$$\begin{aligned} T_2 =& \frac{\tau_{\max}}{(1-\sigma)\ln\eta} \ln\Big(\frac{(\pi-\kappa)\tau_{\min}}{(\pi-\kappa)\tau_{\min} - \eta^{\sigma-1}\ln\eta}\Big) \\ &+ \frac{\tau_{\max}}{(1-\varrho)\ln\eta} \ln\Big(1 - \frac{\eta^{1-\varrho}\ln\eta}{(\xi-\kappa)\tau_{\max}}\Big). \end{aligned}$$

Proof. When $t \ne t_k$, let $\Xi(V) = -\kappa V + \xi V^{\varrho} + \pi V^{\sigma}$, from the condition $0 < \kappa < \min\{\xi, \pi\}$,

$$\begin{cases} \Xi(V) \ge \xi V^{\varrho} + (\pi-\kappa)V^{\sigma} > 0, \quad 0 < V \le 1, \\ \Xi(V) \ge (\xi-\kappa)V^{\varrho} + \pi V^{\sigma} > 0, \quad V > 1, \end{cases}$$

which indicates that $\Xi(V) > 0$ for all $V > 0$. So, the following impulsive comparison system is considered

$$\begin{cases} \dot{m}(t) = \begin{cases} -(\pi-\kappa)(m(t))^{\sigma},\ 0 < m(t) \le 1,\ t \ne t_k, \\ -(\xi-\kappa)(m(t))^{\varrho},\ m(t) > 1,\ t \ne t_k, \end{cases} \\ m(t_k) = \theta m(t_k^-), \\ m(0) = m_0 \ge V(y(0)). \end{cases}$$

Similar to the proof of Lemma 2 given in [36], $V(y(t)) = 0$ and $y(t) = 0$ for any $t \ge T_2$. □

Remark 2. *The FT stability and the estimation of settling time are given in Lemma 2 when* $0 < \eta < 1$*. Note that if system* (5) *has no impulse, i.e.,* $\eta = 1$*, we can get* $\tau_{\min} = \tau_{\max}$*, and by using the equivalent replacement,*

$$\begin{aligned}
&\lim_{\eta\to 1}\Big(\frac{\tau_{\max}}{(1-\sigma)\ln\eta}\ln\big(\frac{\pi\tau_{\min}}{\pi\tau_{\min}-\eta^{\sigma-1}\ln\eta}\big)+\frac{\tau_{\max}}{(1-\varrho)\ln\eta}\ln\big(1-\frac{\eta^{1-\varrho}\ln\eta}{\xi\tau_{\max}}\big)\Big)\\
&=\lim_{\eta\to 1}\Big(\frac{\tau_{\max}}{(1-\sigma)\ln\eta}\cdot\frac{\eta^{\sigma-1}\ln\eta}{\pi\tau_{\min}-\eta^{\sigma-1}\ln\eta}+\frac{\tau_{\max}}{(1-\varrho)\ln\eta}\cdot\frac{\eta^{1-\varrho}-\ln\eta}{\xi\tau_{\max}}\Big)\\
&=\frac{1}{\pi(1-\sigma)}+\frac{1}{\xi(\varrho-1)}.
\end{aligned}$$

So, when $\eta = 1$*, the ST is estimated as*

$$T_3 = \frac{1}{\pi(1-\sigma)}+\frac{1}{\xi(\varrho-1)}.$$

In Lemma 3, similarly, when $\eta = 1$*, the ST is evaluated by*

$$T_4 = \frac{1}{(\pi-\kappa)(1-\sigma)}+\frac{1}{(\xi-\kappa)(\varrho-1)}.$$

Remark 3. *Compared with Lemma 2 given in [36], a general condition is provided in Lemma 3, in which the linear term* $V(y(t))$ *is involved. It makes Lemma 3 more flexible and the subsequent control design more pithy.*

3. Fixed-Time Synchronization

In this section, a T-S fuzzy control scheme is proposed to ensure FT synchronization of impulsive network (2).

The following fixed-time fuzzy control strategy is designed:

Rule r: IF $\varphi_1(t)$ is $\Theta_1^r, \ldots, \varphi_G(t)$ is Θ_G^r, THEN

$$u_q^{\langle r\rangle}(t) = -\beta_1^{\langle r\rangle}[\epsilon_q(t)]^{\theta_1} - \beta_2^{\langle r\rangle}[\epsilon_q(t)]^{\theta_2},\ q \in N_\natural,\ r \in M_\natural, \tag{8}$$

where $\beta_1^{\langle r\rangle}, \beta_2^{\langle r\rangle} > 0$, $0 < \theta_1 < 1$, $\theta_2 > 1$.

For any $r \in M_\natural$, $\ell \in n_\natural$ and $\ni = 1, 2$, denote

$\rho^{\langle r\rangle} = \lambda_{\max}(\omega I_N - s^{\langle r\rangle}\alpha\gamma_\ell\mathcal{L})$,

$\widetilde{\lambda} = \max_{r\in R_\natural}\{\rho^{\langle r\rangle}\}$, $\quad \widetilde{\beta}_\ni = \min_{r\in R_\natural}\{\beta_\ni^{\langle r\rangle}\}$,

$\widetilde{\xi} = \widetilde{\beta}_2(nN)^{\frac{1-\theta_2}{2}}$, $\quad \widetilde{\varsigma} = \min_{k\in\mathbb{N}_+, r\in R_\natural}\{\varsigma_k^{\langle r\rangle}\}$.

Theorem 1. *Based on Assumptions 1–3 and the control protocol* (8)*, system* (2) *is fixed-time synchronized if there exists* $\widetilde{\eta} \in (0, 1)$ *such that*

$$\begin{pmatrix} -\widetilde{\eta} I_N & (I_N - \widetilde{\varsigma}\mathcal{L})^T \\ * & -I_N \end{pmatrix} \le 0. \tag{9}$$

Moreover, the settling-time is evaluated by

$$T(\epsilon(0)) \leq \begin{cases} \hat{T}_1 = \dfrac{2\tau_{\max}}{(1-\theta_1)\ln\widetilde{\eta}} \ln\Big(\dfrac{2\widetilde{\beta}_1\tau_{\min}}{2\widetilde{\beta}_1\tau_{\min} - \widetilde{\eta}^{\frac{\theta_1-1}{2}}\ln\widetilde{\eta}}\Big) \\ \qquad + \dfrac{2\tau_{\max}}{(1-\theta_2)\ln\widetilde{\eta}} \ln\Big(1 - \dfrac{\widetilde{\eta}^{\frac{1-\theta_2}{2}}\ln\widetilde{\eta}}{2\widetilde{\xi}\tau_{\max}}\Big),\ \widetilde{\lambda} \leq 0, \\ \hat{T}_2 = \dfrac{2\tau_{\max}}{(1-\theta_1)\ln\widetilde{\eta}} \ln\Big(\dfrac{2(\widetilde{\beta}_1-\widetilde{\lambda})\tau_{\min}}{2(\widetilde{\beta}_1-\widetilde{\lambda})\tau_{\min} - \widetilde{\eta}^{\frac{\theta_1-1}{2}}\ln\widetilde{\eta}}\Big) \\ \qquad + \dfrac{2\tau_{\max}}{(1-\theta_2)\ln\widetilde{\eta}} \ln\Big(1 - \dfrac{\widetilde{\eta}^{\frac{1-\theta_2}{2}}\ln\widetilde{\eta}}{2(\widetilde{\xi}-\widetilde{\lambda})\tau_{\max}}\Big),\ 0 < \widetilde{\lambda} < \min\{\widetilde{\xi}, \widetilde{\beta}_1\}. \end{cases}$$

Proof. Select the following Lyapunov function

$$V(\epsilon(t)) = \Big(\sum_{q=1}^{N} \epsilon_q^T(t)\epsilon_q(t)\Big)^{\frac{1}{2}}.$$

When $t \neq t_k$, for any $\epsilon(t) \in \mathbb{R}^{nN}\backslash\{0\}$,

$$\begin{aligned} \frac{d}{dt}V(\epsilon(t)) =& \frac{1}{V(\epsilon(t))} \sum_{q=1}^{N} \epsilon_q^T(t)\Big[\big(f(z_q(t)) - f(\nu(t))\big) \\ &+ \sum_{r=1}^{M} \Upsilon_r(\varphi(t))\Big(s^{\langle r\rangle} \sum_{j=1}^{N} c_{qj}\Gamma\hat{\hbar}(\epsilon_j(t)) \\ &- \beta_1^{\langle r\rangle}[\epsilon_q(t)]^{\theta_1} - \beta_2^{\langle r\rangle}[\epsilon_q(t)]^{\theta_2}\Big)\Big]. \end{aligned} \tag{10}$$

Firstly, denote $\epsilon_\ell(t) = (\epsilon_{1\ell}(t), \ldots, \epsilon_{N\ell}(t))^T$ with $\ell \in n_\natural$, and according to Assumption 1, one has

$$\begin{aligned} &\sum_{q=1}^{N} \epsilon_q^T(t)\big(f(z_q(t)) - f(\nu(t))\big) \\ \leq& \sum_{q=1}^{N} \|\epsilon_q(t)\|\omega\|\epsilon_q(t)\| \\ =&\omega \sum_{q=1}^{N} \epsilon_q^T(t)\epsilon_q(t) \\ =& \sum_{\ell=1}^{n} \epsilon_\ell^T(t)\omega I_N\epsilon_\ell(t). \end{aligned} \tag{11}$$

In addition,

$$
\begin{aligned}
&\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) s^{\langle r\rangle} \sum_{q=1}^{N}\sum_{j=1}^{N} \epsilon_q^T(t) c_{qj} \Gamma \hat{\hbar}(\epsilon_j(t)) \\
&= \sum_{r=1}^{M} \Upsilon_r(\varphi(t)) s^{\langle r\rangle} \Big[-\frac{1}{2} \sum_{q=1}^{N}\sum_{j=1}^{N} \big(\epsilon_q(t) - \epsilon_j(t)\big)^T c_{qj} \Gamma \\
&\quad \times \big(\hat{\hbar}(\epsilon_q(t)) - \hat{\hbar}(\epsilon_j(t))\big)\Big] \\
&= \sum_{r=1}^{M} \Upsilon_r(\varphi(t)) s^{\langle r\rangle} \Big[-\frac{1}{2} \sum_{\ell=1}^{n} \gamma_\ell \sum_{q=1}^{N}\sum_{j=1}^{N} c_{qj} \big(\epsilon_{q\ell}(t) - \epsilon_{j\ell}(t)\big) \\
&\quad \times \big(\bar{\hat{\hbar}}(\epsilon_{q\ell}(t)) - \bar{\hat{\hbar}}(\epsilon_{j\ell}(t))\big)\Big] \\
&\leq \sum_{r=1}^{M} \Upsilon_r(\varphi(t)) s^{\langle r\rangle} \Big[-\frac{\alpha}{2} \sum_{\ell=1}^{n} \gamma_\ell \sum_{q=1}^{N}\sum_{j=1}^{N} c_{qj} \big(\epsilon_{q\ell}(t) - \epsilon_{j\ell}(t)\big)^2\Big] \\
&= -\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) s^{\langle r\rangle} \alpha \sum_{\ell=1}^{n} \gamma_\ell \epsilon_\ell^T(t) \mathcal{L} \epsilon_\ell(t). \qquad (12)
\end{aligned}
$$

Using Lemma 1,

$$
\begin{aligned}
&-\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \beta_1^{\langle r\rangle} \sum_{q=1}^{N} \epsilon_q^T(t) [\epsilon_q(t)]^{\theta_1} \\
&= -\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \beta_1^{\langle r\rangle} \sum_{q=1}^{N}\sum_{\ell=1}^{n} |\epsilon_{q\ell}(t)|^{\theta_1+1} \\
&\leq -\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \beta_1^{\langle r\rangle} \Big(\sum_{q=1}^{N}\sum_{\ell=1}^{n} \epsilon_{q\ell}^2(t) \Big)^{\frac{\theta_1+1}{2}} \\
&= -\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \beta_1^{\langle r\rangle} V^{\theta_1+1}(\epsilon(t)) \\
&\leq -\widetilde{\beta}_1 V^{\theta_1+1}(\epsilon(t)), \qquad (13)
\end{aligned}
$$

and

$$
\begin{aligned}
&-\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \beta_2^{\langle r\rangle} \sum_{q=1}^{N} \epsilon_q^T(t) [\epsilon_q(t)]^{\theta_2} \\
&= -\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \beta_2^{\langle r\rangle} \sum_{q=1}^{N}\sum_{\ell=1}^{n} |\epsilon_{q\ell}(t)|^{\theta_2+1} \\
&\leq -\sum_{r=1}^{M} \Upsilon_r(\varphi(t)) \beta_2^{\langle r\rangle} (nN)^{\frac{1-\theta_2}{2}} \Big(\sum_{q=1}^{N}\sum_{\ell=1}^{n} \epsilon_{q\ell}^2(t) \Big)^{\frac{\theta_2+1}{2}} \\
&\leq -\widetilde{\beta}_2 (nN)^{\frac{1-\theta_2}{2}} V^{\theta_2+1}(\epsilon(t)). \qquad (14)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{d}{dt}V(\epsilon(t)) \leq & \frac{1}{V(\epsilon(t))}\sum_{r=1}^{M}\Upsilon_r(\varphi(t))\sum_{\ell=1}^{n}\epsilon_\ell^T(t)(\omega I_N - s^{\langle r\rangle}\alpha\gamma_\ell\mathcal{L})\epsilon_\ell(t) \\
& - \frac{1}{V(\epsilon(t))}\widetilde{\beta}_1 V^{\theta_1+1}(\epsilon(t)) - \frac{1}{V(\epsilon(t))}\widetilde{\beta}_2(nN)^{\frac{1-\theta_2}{2}}V^{\theta_2+1}(\epsilon(t)) \\
\leq & \widetilde{\lambda}V(\epsilon(t)) - \widetilde{\beta}_1 V^{\theta_1}(\epsilon(t)) - \widetilde{\xi}V^{\theta_2}(\epsilon(t)).
\end{aligned}
$$

On the other hand, for $t = t_k$, it has

$$
\begin{aligned}
\triangle\epsilon_q(t_k) &= \epsilon_q(t_k^+) - \epsilon_q(t_k^-) \\
&= \sum_{r=1}^{M}\Upsilon_r(\varphi(t_k^-))\varsigma_k^{\langle r\rangle}\sum_{j=1}^{N}c_{qj}\epsilon_j(t_k^-) \quad q \in N_\natural,
\end{aligned}
$$

therefore,

$$
\triangle\epsilon(t_k) = -\sum_{r=1}^{M}\Upsilon_r(\varphi(t_k^-))\varsigma_k^{\langle r\rangle}(\mathcal{L}\otimes I_n)\epsilon(t_k^-).
$$

According to (9), one gets

$$
\begin{aligned}
V(\epsilon(t_k)) &= V(\epsilon(t_k^+)) \\
&= \Big(\epsilon^T(t_k^+)\epsilon(t_k^+)\Big)^{\frac{1}{2}} \\
&\leq \Big(\epsilon^T(t_k^-)(((I_N - \widetilde{\varsigma}\mathcal{L})^T(I_N - \widetilde{\varsigma}\mathcal{L}))\otimes I_n)\epsilon(t_k^-)\Big)^{\frac{1}{2}} \\
&\leq \widetilde{\eta}^{\frac{1}{2}}V(\epsilon(t_k^-)).
\end{aligned}
$$

When $\widetilde{\lambda} \leq 0$, in view of Lemma 2, system (2) achieves synchronization in a fixed time $\hat{T}_1$. If $0 < \widetilde{\lambda} < \min\{\widetilde{\xi}, \widetilde{\beta}_1\}$, system (2) achieves synchronization within a fixed time $\hat{T}_2$ by utilizing Lemma 3. $\square$

Especially, if impulsive dynamic network (2) without fuzzy rules [43,44], the network model is changed into the following network

$$
\begin{cases}
\dot{z}_q(t) = f(z_q(t)) + s\sum_{j=1}^{N}c_{qj}\Gamma\hbar(z_j(t)) + \mathfrak{u}_q(t),\ t \neq t_k,\ t \geq 0, \\
\triangle z_q(t_k) = \varsigma_k\sum_{j=1}^{N}c_{qj}z_j(t_k^-),\ k \in \mathbb{N}_+,\ q \in N_\natural.
\end{cases} \tag{15}
$$

Correspondingly, the error dynamic model is described by

$$
\begin{cases}
\dot{\epsilon}_q(t) = f(z_q(t)) - f(\nu(t)) + s\sum_{j=1}^{N}c_{qj}\Gamma\hat{\hbar}(\epsilon_j(t)) \\
\qquad + \mathfrak{u}_q(t),\ t \neq t_k,\ t \geq 0, \\
\triangle\epsilon_q(t_k) = \varsigma_k\sum_{j=1}^{N}c_{qj}\epsilon_j(t_k^-),\ k \in \mathbb{N}_+,\ q \in N_\natural,
\end{cases}
$$

where $\hat{\hbar}(\epsilon_j(t)) = \hbar(z_j(t)) - \hbar(\nu(t))$. At this point, the control strategy (8) is reduced to the following form

$$u_q(t) = -\beta_1[\epsilon_q(t)]^{\theta_1} - \beta_2[\epsilon_q(t)]^{\theta_2}, \quad q \in N_\natural, \tag{16}$$

with $\beta_1, \beta_2 > 0$, $0 < \theta_1 < 1$, $\theta_2 > 1$.

Corollary 1. *Under Assumptions 1–3 and the control protocol (16), if there exists $\hat{\eta} \in (0,1)$ such that*

$$\begin{pmatrix} -\hat{\eta} I_N & (I_N - \hat{\varsigma}\mathcal{L})^T \\ * & -I_N \end{pmatrix} \leq 0, \tag{17}$$

then system (15) is FT synchronized. Besides, the ST is evaluated by

$$T(\epsilon(0)) \leq \begin{cases} \hat{T}_3 = \dfrac{2\tau_{\max}}{(1-\theta_1)\ln\hat{\eta}} \ln\Big(\dfrac{2\beta_1\tau_{\min}}{2\beta_1\tau_{\min} - \hat{\eta}^{\frac{\theta_1-1}{2}}\ln\hat{\eta}}\Big) \\ \qquad + \dfrac{2\tau_{\max}}{(1-\theta_2)\ln\hat{\eta}} \ln\Big(1 - \dfrac{\hat{\eta}^{\frac{1-\theta_2}{2}}\ln\hat{\eta}}{2\hat{\xi}\tau_{\max}}\Big), \ \hat{\lambda} \leq 0, \\ \hat{T}_4 = \dfrac{2\tau_{\max}}{(1-\theta_1)\ln\hat{\eta}} \ln\Big(\dfrac{2(\beta_1-\hat{\lambda})\tau_{\min}}{2(\beta_1-\hat{\lambda})\tau_{\min} - \hat{\eta}^{\frac{\theta_1-1}{2}}\ln\hat{\eta}}\Big) \\ \qquad + \dfrac{2\tau_{\max}}{(1-\theta_2)\ln\hat{\eta}} \ln\Big(1 - \dfrac{\hat{\eta}^{\frac{1-\theta_2}{2}}\ln\hat{\eta}}{2(\hat{\xi}-\hat{\lambda})\tau_{\max}}\Big), \ 0 < \hat{\lambda} < \min\{\hat{\xi}, \beta_1\}. \end{cases}$$

Here $\hat{\lambda} = \lambda_{\max}(\omega I_N - s\alpha\gamma_\ell\mathcal{L})$, $\ell \in n_\natural$, $\hat{\xi} = \beta_2(nN)^{\frac{1-\theta_2}{2}}$, $\hat{\varsigma} = \min_{k\in\mathbb{N}_+}\{\varsigma_k\}$.

If any two nodes in fuzzy-based network (2) are interconnected only at the impulsive time, then (2) is converted to the following form

$$\begin{cases} \dot{z}_q(t) = f(z_q(t)) + \sum_{r=1}^{M} \Upsilon_r(\varphi(t)) u_q^{\langle r\rangle}(t), \ t \neq t_k, \ t \geq 0, \\ \triangle z_q(t_k) = \sum_{r=1}^{M} \Upsilon_r(\varphi(t))\varsigma_k^{\langle r\rangle} \sum_{j=1}^{N} c_{qj} z_j(t_k^-), \\ \qquad k \in \mathbb{N}_+, \ q \in N_\natural, \ r \in M_\natural. \end{cases} \tag{18}$$

Remark 4. *Notably, system (2) is a kind of network with mixed continuous coupling and impulse coupling. In this kind of network, the state information of a large number of nodes needs to be continuously transmitted, so there is a great possibility of communication congestion. However, the impulsive coupling mode is different, which requires less information to be transmitted through the network and occupies fewer communication channels. Considering this fact, the network model (18) with impulsive coupling only is introduced in this paper.*

According to Theorem 1, the following Corollary 2 can be easily derived.

Corollary 2. *Based on Assumptions 1–3 and the control protocol (8), if there exists* $\widetilde{\eta} \in (0,1)$ *satisfying (9), then system (18) is FT synchronized at the fixed time* $\hat{T}_5$*, where*

$$\begin{aligned}\hat{T}_5 = &\frac{2\tau_{\max}}{(1-\theta_1)\ln\widetilde{\eta}}\ln\big(\frac{2(\widetilde{\beta}_1-\omega)\tau_{\min}}{2(\widetilde{\beta}_1-\omega)\tau_{\min}-\widetilde{\eta}^{\frac{\theta_1-1}{2}}\ln\widetilde{\eta}}\big)\\ &+\frac{2\tau_{\max}}{(1-\theta_2)\ln\widetilde{\eta}}\ln\big(1-\frac{\widetilde{\eta}^{\frac{1-\theta_2}{2}}\ln\widetilde{\eta}}{2(\widetilde{\xi}-\omega)\tau_{\max}}\big),\ 0<\omega<\min\{\widetilde{\xi},\widetilde{\beta}_1\}.\end{aligned}$$

When there is no impulsive interference, system (2) is changed into the following form

$$\begin{aligned}\dot{z}_q(t) =&f(z_q(t))+\sum_{r=1}^{M}\Upsilon_r(\varphi(t))\Big[s^{\langle r\rangle}\sum_{j=1}^{N}c_{qj}\Gamma\hbar(z_j(t))\\ &+u_q^{\langle r\rangle}(t)\Big],\ q\in N_\natural,\ r\in M_\natural.\end{aligned}\tag{19}$$

Corollary 3. *Under Assumptions 1 and 2 and the control protocol (8), systems (19) achieves FT synchronization, the ST is evaluated by*

$$T(\epsilon(0))\leq\begin{cases}\hat{T}_6=\dfrac{1}{\widetilde{\beta}_1(1-\theta_1)}+\dfrac{1}{\widetilde{\xi}(\theta_2-1)},\ \widetilde{\lambda}\leq 0,\\ \hat{T}_7=\dfrac{1}{(\widetilde{\beta}_1-\widetilde{\lambda})(1-\theta_1)}+\dfrac{1}{(\widetilde{\xi}-\widetilde{\lambda})(\theta_2-1)},\ 0<\widetilde{\lambda}<\min\{\widetilde{\xi},\widetilde{\beta}_1\}.\end{cases}$$

Remark 5. *In the existing results [27–30], the synchronization for fuzzy-based complex networks was investigated. Differently, Yang et al. [31] and Behinfaraz et al. [32] considered the impact of impulse on fuzzy-based complex networks. However, the above results are about the asymptotic and finite-time synchronization of fuzzy complex networks, in which the initial values need to be obtained in advance. Different from the above results, a fixed-time fuzzy control strategy is designed, and the synchronization of impulsive coupled network is deeply explored based on T-S fuzzy logic theory in this paper, in which the synchronization time is independent of the initial state of the system.*

Remark 6. *In [36,37,39], FT synchronization has been discussed for complex networks with impulsive effect by designing the following hybrid control strategy composed of linear part and power-law term:*

$$u_q(t) = -\beta_0\epsilon_q(t)-\beta_1[\epsilon_q(t)]^{\theta_1}-\beta_2[\epsilon_q(t)]^{\theta_2},\ q\in N_\natural,$$

where $\beta_0,\beta_1,\beta_2>0$, $0<\theta_1<1$, $\theta_2>1$. *Actually, the linear part* $-\beta_0\epsilon_q(t)$ *is only to ensure asymptotic synchronization, while the power-law term* $-\beta_1[\epsilon_q(t)]^{\theta_1}-\beta_2[\epsilon_q(t)]^{\theta_2}$ *is the core to achieve FT synchronization. Based on this idea, a new FT stability theorem is proposed for impulsive system in Lemma 3 to facilitate the design of pure power-law control scheme (8), and the FT synchronization of fuzzy-based impulsive coupled networks is also ensured under the new design in this paper. Evidently, our control design is simpler and effective.*

Remark 7. *Over the years, there have been many excellent results on the synchronization of complex networks [33–41,45,46]. Specially, the influences of actuator saturation and stochastic coupling strength on the network were considered in [45]. Besides, the passivity-based synchronization for complex networks under Markovian jump parameters, actuator fault and random coupling delay was considered in [46]. In these articles, the effects of random perturbations on network dynamics are covered, which is more consistent with many real-world applications, since systems are often subject to unpredictable sudden changes and sudden environmental disturbances. Therefore, it is of practical significance to apply the methods of control design and theoretical analysis proposed in this paper to further study the fixed-time control of complex networks with random disturbances, which will be one of the directions of future research.*

4. Numerical Example

To verify the above stability and synchronization results, two numerical examples and several related simulations are provided by using Matlab software in this part.

Example 1. *The following system is considered*

$$\begin{cases} \dot{y}_i(t) = \kappa_i y_i(t) - \xi_i \operatorname{sign}(y_i(t))|y_i(t)|^{\varrho} \\ \qquad\qquad - \pi_i \operatorname{sign}(y_i(t))|y_i(t)|^{\sigma},\ t \neq t_k,\ t \geq 0, \\ y_i(t_k) = \wp_i y_i(t_k^-),\ k \in \mathbb{N}_+,\ i = 1,2,3, \end{cases} \tag{20}$$

where $y = (y_1, y_2, y_3)^T \in \mathbb{R}^3$, $\varrho > 1$, $\sigma \in (0,1)$, $\kappa_i \geq 0$, $\xi_i, \pi_i > 0$, $\wp_i \in (-1,1)$.

Similarly, by defining $V(y(t)) = \|y(t)\|_1$, *for any* $y(t) \in \mathbb{R}^3 \backslash \{0\}$, *one has*

$$\begin{cases} \dfrac{d}{dt} V(y(t)) \leq \kappa V(y(t)) - \xi V^{\varrho}(y(t)) - \pi V^{\sigma}(y(t)),\ t \neq t_k,\ t \geq 0, \\ V(y(t_k)) \leq \wp V(y(t_k^-)),\ k \in \mathbb{N}_+, \end{cases}$$

where $\kappa = \max\{\kappa_i, i = 1,2,3\}$, $\xi = 3^{1-\varrho} \min\{\xi_i, i = 1,2,3\}$, $\pi = \min\{\pi_i, i = 1,2,3\}$, $\wp = \max\{|\wp_i|, i = 1,2,3\}$. *Choose* $\kappa_1 = 0.8$, $\kappa_2 = 0.7$, $\kappa_3 = 0.6$, $\xi_1 = \xi_3 = 2.1$, $\xi_2 = 2$, $\pi_1 = 1.6, \pi_2 = 1, \pi_3 = 1.3, \wp_1 = -0.6$, $\wp_2 = 0.4$, $\wp_3 = 0.5$, $\tau_{\min} = 0.08$, $\tau_{\max} = 0.14$, $\varrho = 1.6$ *and* $\sigma = 0.5$. *It can be obtained that system (20) is stable within the time* $T_2 = 3.4663$ *through Lemma 3, and its numerical simulation is shown in Figure 1. However, the origin of system (20) can not be ensured according to Lemma 2, which shows that the stability condition of Lemma 3 proposed in this article is less conservative and more applicable than [36].*

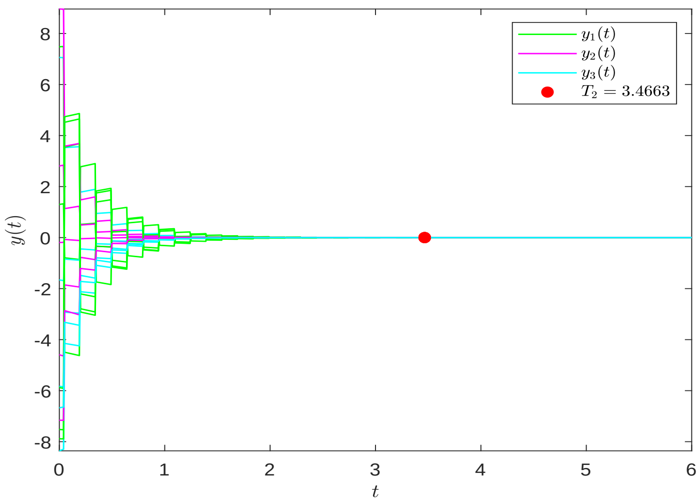

Figure 1. FT stability of system (20).

Example 2. *Consider a class of fuzzy-based impulsive networks composed of 9 nodes, its dynamic model is written as*

$$\begin{cases} \dot{z}_q(t) = f(z_q(t)) + \sum\limits_{r=1}^{2} \Upsilon_r(\varphi(t)) \Big[s^{\langle r \rangle} \sum\limits_{j=1}^{9} c_{qj} \Gamma \hbar(z_j(t)) \\ \qquad\qquad + \mathrm{u}_q^{\langle r \rangle}(t) \Big],\ t \neq t_k,\ t \geq 0, \\ \triangle z_q(t_k) = \sum\limits_{r=1}^{2} \Upsilon_r(\varphi(t)) \varsigma_k^{\langle r \rangle} \sum\limits_{j=1}^{9} c_{qj} z_j(t_k^-),\ k \in \mathbb{N}_+, \\ \qquad q = 1, \ldots, 9, \end{cases} \tag{21}$$

where $z_q(t) = \big(z_{q1}(t), z_{q2}(t), z_{q3}(t)\big)^T$, $s^{\langle 1\rangle} = 0.5$, $s^{\langle 2\rangle} = 1$, $\Gamma = 2I_3$, $\Upsilon_1(z) = 1 - 0.1\sin^4(z)$, $\Upsilon_2(z) = 0.1\sin^4(z)$, $\hbar(z_q(t)) = \big(\hbar(z_{q1}(t)), \hbar(z_{q2}(t)), \hbar(z_{q3}(t))\big)^T$, $\hbar(\phi) = 5\phi + 2\sin\phi$, $f(z_q(t)) = Wz_q(t) + \Psi\chi(z_q(t))$, $\chi(z_q(t)) = \big(\chi_1(z_{q1}(t)), \chi_2(z_{q2}(t)), \chi_3(z_{q3}(t))\big)^T$, $\chi_\ell(\phi) = 0.5(|\phi+1| - |\phi-1|)$, $\ell = 1,2,3$, $W = -I_3$, *the impulsive control gains* $\varsigma_k^{\langle 1\rangle}$ *and* $\varsigma_k^{\langle 2\rangle}$ *are arbitrarily taken from* $(-2,-1)\cup(-1,0)$*, and*

$$\Psi = \begin{pmatrix} 1.25 & -3.2 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1.0 \end{pmatrix}.$$

The topology of model (21) *is expressed in Figure* 2.

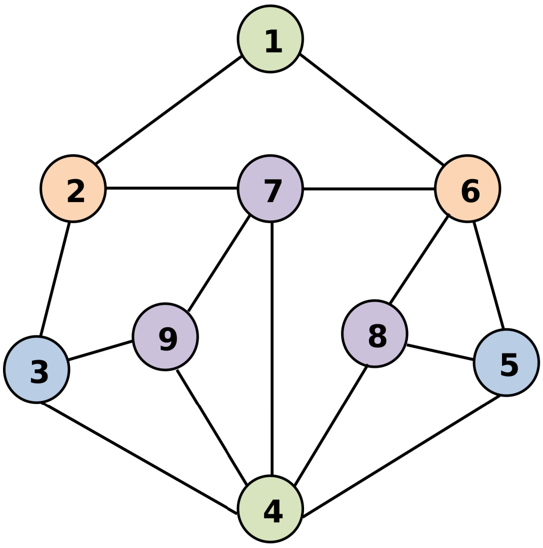

Figure 2. Topology structure of impulsive network (21).

The dynamic behavior of synchronous state is described as

$$\dot{\nu}(t) = W\nu(t) + \Psi\chi(\nu(t)), \tag{22}$$

and the dynamical behavior of system (22) is illustrated by Figure 3, in which the initial value is chosen as $(\psi_1(0), \psi_2(0), \psi_3(0))^T = (1.8, 1, -1.3)^T$.

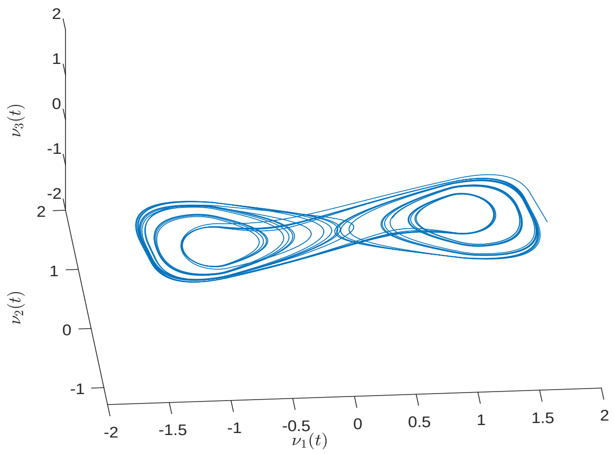

Figure 3. Phase diagram of system (22).

Consider the FT synchronization of system (21) based on the control strategy (8). By simple computation, $\omega = 7.3391$, $\alpha = 4$, $\widetilde{\lambda} = 7.3391$. Choose $\beta_1^{\langle 1 \rangle} = 8.6$, $\beta_1^{\langle 2 \rangle} = 9$, $\beta_2^{\langle 1 \rangle} = 14$, $\beta_2^{\langle 2 \rangle} = 13.5$, $\theta_1 = 0.4$, $\theta_2 = 1.3$, $\widetilde{\eta} = 0.45$, $\tau_{\min} = 0.08$, $\tau_{\max} = 0.1$, then conditions $0 < \widetilde{\lambda} < \min\{\widetilde{\xi}, \widetilde{\beta}_1\}$ and (9) are satisfied. By Theorem 1, the FT synchronization between system (21) and system (22) is ensured within the time $\hat{T}_2 = 2.2497$. The corresponding simulation result is revealed in Figure 4 by picking random initial values. Distinctly, in contrast to the work in [27] about finite-time synchronization, the proposed FT synchronization control has a better robust performance since they allow the FICNs possess arbitrary initial states.

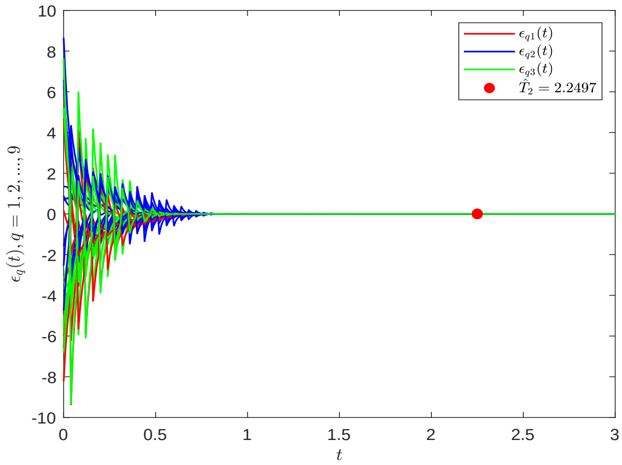

Figure 4. FT synchronization error of network (21).

In addition, since $\widetilde{\lambda} > 0$, Lemma 2 given in [36] cannot be used to judge that system (21) is fixed-time synchronized, but the fixed-time synchronization is ensured by Lemma 3 proposed in this paper, so Lemma 3 can be applied more flexibly.

5. Conclusions

In this paper, the FT synchronization of impulsive coupling networks based on T-S fuzzy is discussed. Firstly, by proposing a fuzzy control strategy and using Lyapunov method, some conditions of FT synchronization of fuzzy impulsive network are obtained, and the upper bound of synchronization time is estimated. It is worth noting that the controller designed in this paper is in the form of pure power-law, which is simpler and more effective than the traditional design composed of linear part and power-law part. Finally, a numerical example is given to illustrate the feasibility of the proposed control scheme and criteria.

When studying the synchronization of fuzzy-based impulsive coupled networks in this paper, the model considered has the same membership function and fuzzy rules as the designed control strategy. However, if the membership function and fuzzy rules between the model and the control strategy are different, how to design a simple and effective control strategy to study the synchronization control problem of impulsive complex networks remains to be discussed. In addition, it is noted that although fuzzy is considered in the network model and fuzzy control strategies are designed, the synchronization conditions established are independent of fuzziness. Therefore, the discussion to establish synchronization conditions depending on fuzzy rules should be considered in the future research.

Moreover, note that the synchronization can be solved for a fixed-time under the FT control protocol (8) by adjusting the control parameters. However, the adjustment may be seriously troublesome since the relation between the convergence time and the parameters are not clear. Besides, the faster convergence time may result in larger control costs. Therefore, how to design an effective control strategy to achieve synchronization in the specified-time remains to be further explored.

Author Contributions: Conceptualization, Formal analysis, Writing—original draft preparation, Writing—review and editing, L.P.; Methodology, Funding acquisition, Validation, C.H.; Software, Supervision, Funding acquisition, J.Y.; Visualization, Funding acquisition, H.J. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded partially by National Natural Science Foundation of China (Grant Nos. 61866036, 61963033), partially by the Key Project of Natural Science Foundation of Xinjiang (Grant No. 2021D01D10), partially by Xinjiang Key Laboratory of Applied Mathematics (Grant No. XJDX1401), partially by Tianshan Innovation Team Program (Grant No. 2020D14017).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: This work was supported partially by National Natural Science Foundation of China (Grant Nos. 61866036, 61963033), partially by the Key Project of Natural Science Foundation of Xinjiang (2021D01D10), partially by Xinjiang Key Laboratory of Applied Mathematics (XJDX1401), partially by Tianshan Innovation Team Program (Grant No. 2020D14017). Authors are thankful to the handling editor and reviewers for their valuable comments and suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

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