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Ride-Hailing Matching with Uncertain Travel Time: A Novel Interval-Valued Fuzzy Multi-Objective Linear Programming Approach

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Abstract: This study introduces an innovative approach to tackle multi-objective linear programming (MOLP) problems amidst uncertainty, employing interval-valued fuzzy numbers. The method is tailored to resolve ride-hailing matching challenges encompassing uncertain travel times. Findings reveal that managing uncertainty parameters within interval-valued fuzzy MOLP is achieved through strategic reformulations, focusing on constraint coefficients, resulting in streamlined linear programming formulations conducive to solution simplicity. The efficacy of the proposed model in efficiently handling ride-hailing matching quandaries is demonstrated. Moreover, this study delves into the prospective applications of the developed method, including its potential for generalization to address non-linear programming (NLP) issues pertinent to the ride-hailing domain. This research advances decision-making processes under uncertainty and paves the way for broader applications beyond ride-hailing.

Keywords: multi-objective linear programming; interval-valued fuzzy numbers; uncertainty; ride-hailing matching

MSC: 90C29; 90C70; 90C90

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1. Introduction

Transportation supported by science and technology creates a creative innovation known as ride-hailing. Ride-hailing is a transportation breakthrough that must continue to be developed [1]. Ride-hailing, referring to taxi and motorcycle taxi services that can be accessed via the Internet, is a transportation service developed by utilizing Internet technology to make it easier for people to access services via smartphones in a short time, which is more effective than traditional transportation [2–6]. Ride-hailing services are very convenient for supporting community activities [7–9]. Grab and Gojek are Southeast Asia's two largest ride-hailing companies, with 183 million and 170 million users utilizing their services, respectively, and 2.8 million and 2 million driver partners [10–12]. Ride-hailing makes it easy for people to make reservations, search for transportation costs, and identify drivers [13]. Ride-hailing can access information regarding the location of drivers and passengers in real-time and precisely [14]. The people of Bandung, Indonesia, have responded well to the ride-hailing platform because it has low prices and improves the mobility and economy of the community [15]. According to Flores and Rayle [16], ride-hailing opens up new jobs worldwide for people in big cities.

The emergence and growth of the ride-hailing industry have brought profound impacts on both the traditional taxi sector and transportation regulations. Ride-hailing services have swiftly gained global prominence [17]. Nevertheless, concerns persist regarding the profitability and sustainability of ride-hailing platforms [18]. Consequently, researchers have extensively investigated the growth mechanisms and performance indicators of ride-hailing platforms, providing valuable insights into their operational and managerial practices [19]. Furthermore, studies have delved into planning aspects within ride-hailing systems, such as vehicle matching and repositioning, leveraging advanced machine learning methods [20].

Additionally, the socioeconomic impacts of ride-hailing services have been closely examined, including their effects on travel time, accessibility, and overall productivity. Overall, the emergence and expansion of the ride-hailing industry have reshaped the transportation landscape, underscoring the necessity for further research and policy considerations. This transformation has been particularly notable in urban transport, with China emerging as a trailblazer in this domain [21]. However, the industry's rapid growth has sparked concerns about exacerbating environmental issues and equity disparities [22]. Regulatory challenges in governing ride-hailing services in emerging markets have been emphasized, with proposed frameworks aimed at addressing these issues [23]. Additionally, factors such as education, income, and neighborhood characteristics play significant roles in influencing the adoption of ride-hailing services [24].

Efficient matching algorithms play a pivotal role in ride-hailing platforms for several compelling reasons. Firstly, they optimize profitability by determining the most suitable pricing policies and matching rates, considering driver preferences, network dynamics, and fleet size [25]. Secondly, these algorithms facilitate the platform in achieving a balance across multiple objectives, including revenue generation, driver service quality, pick-up distance, and the number of successful matches, by devising algorithms that carefully weigh the trade-offs between these metrics [26]. Thirdly, efficient matching algorithms enhance the overall user experience by incorporating the individual preferences of passengers and drivers, leading to stable matches and reduced total pick-up distances [27]. Lastly, they play a crucial role in driving the long-term sustainable growth of ride-hailing platforms by enabling informed decisions in batch processing and optimizing supply and market efficiency [28]. In essence, efficient matching algorithms are indispensable for optimizing system performance and elevating user satisfaction within ride-hailing platforms.

The challenge of travel time uncertainty in ride-hailing platforms has spurred various studies to propose solutions for its management. Reference [29] introduces a dynamic equilibrium framework tailored to address morning commute issues within the ride-sourcing market amidst travel time uncertainties. Reference [30] presents a robust optimization framework rooted in machine learning aimed at predicting sets of travel time uncertainties and enhancing overall reductions in travel time. Reference [31] conceptualizes the ridesharing matching problem as a robust vehicle routing dilemma incorporating time windows, providing a data-driven solution through deep learning to estimate sets of travel time uncertainties. Reference [32] tackles this issue by employing a fuzzy linear programming approach, while [33] addresses it using a fuzzy quadratic programming approach.

Ride-hailing matching algorithms have undergone advancements to enhance efficiency and maximize benefits for drivers and passengers. Integration of algorithms has demonstrated improvements in decision-making efficiency within ridesharing systems [34]. Pricing strategies and matching rates have been fine-tuned to optimize profitability, considering drivers' choices, network dynamics, and fluctuating fleet sizes [25]. A distributed approach to matching problems, employing a multi-queue model and network-flow theory, has been proposed to amplify service revenue and enhance efficiency and user satisfaction across diverse geographical regions [35]. The utilization of Simulated Annealing has effectively balanced stable matching and total pick-up distance, resulting in reduced overall distance traveled while maintaining control over the proportion of unstable matches [27]. Tabu Search and Greedy Matching algorithms have been devised for multi-driver, multirider ride-matching scenarios, with system-optimized centralized ride-matching yielding more significant cost savings for all involved parties [36].

The conventional linear programming (LP) problem aims to determine a linear function's minimum or maximum values within constraints defined by linear inequalities or equations. However, in many practical scenarios, expecting precise definitions for these constraints or the objective function is often impractical. In such cases, employing fuzzy linear programming (FLP) becomes desirable. FLP models found in the literature typically account for imprecision in the coefficients of the objective function, the right-hand side values, and the coefficient matrix elements. Over the past four decades, researchers have investigated various FLP problems and proposed diverse models for addressing LP problems with fuzzy data. While it would be challenging to reference all of these contributions, we will focus our discussion on those closely related to the topic explored in this paper.

Interval-valued fuzzy linear programming has garnered considerable attention in recent years, prompting researchers to explore various methodologies for addressing this challenge. One approach involves utilizing interval-valued intuitionistic fuzzy (IVIF) numbers as both parameters and decision variables, with researchers opting to decompose the IVIFLP problem into smaller crisp linear problems (CLPs) for individual resolution [37]. Another strategy entails transforming a bilevel linear programming problem featuring interval type-2 triangular fuzzy numbers (IT2TFNs) into an interval linear programming problem [38]. Moreover, the introduction of interval-valued linear Diophantine fuzzy sets has enhanced decision-making precision in urgent scenarios [39]. A novel concept known as LR-type interval-valued intuitionistic fuzzy numbers has been put forward, along with a corresponding methodology for addressing linear programming problems incorporating LR-type IVIFNs as parameters [40].

This research aims to achieve several objectives. Firstly, it aims to develop an adequate mathematical model for matching passengers with drivers in ride-hailing services while considering uncertainty in travel time estimation. Secondly, it seeks to integrate the concepts of interval-valued fuzzy into mathematical modeling to address the uncertainty associated with travel time estimation in ride-hailing environments. Thirdly, this research aims to develop an approach to handle multi-objective linear programming with interval-valued fuzzy parameters. Lastly, it aims to evaluate the performance and advantages of the proposed approach through simulations and comparisons with conventional methods in the context of uncertain travel time. By setting these objectives, this research aims to significantly contribute to developing more adaptive and efficient ride-hailing matching systems capable of coping with uncertainty in travel time estimation.

The developed interval-valued fuzzy method for solving multi-objective linear programming (MOLP) problems aims to address uncertainty in decision-making. This approach allows for a more complex representation of uncertainty by modeling variables in the form of interval-valued fuzzy sets, thus providing greater flexibility in handling situations where precise values cannot be determined clearly. By employing this method, we intend to apply an innovative approach to solving ride-hailing matching problems, where the uncertainty of travel time between passengers and drivers is a critical factor. By modeling travel time as interval-valued fuzzy sets, we can optimize the matching between passengers and drivers by considering multi-objective criteria such as travel distance, waiting time, and profitability. It is expected that the use of this interval-valued fuzzy method will lead to a significant improvement in efficiency and customer satisfaction in ride-hailing systems, as well as making a valuable contribution to the development of decision-making techniques under uncertainty. To clarify the research contribution, relevant information is presented in Table 1.

Table 1. Author Contributions.

Authors	Considering Uncertainty?	Using Interval-Valued Fuzzy?	Developing Novel Approach for Handle Multi-Objective Optimization Under Uncertainty?
[32]	√	X	X
[33]	√	√	X
[41]	X	X	X
[42]	X	X	X
[43]	X	X	X
[44]	√	X	X
[45]	X	X	X
[46]	√	X	X
[47]	X	X	X
This research	√	√	√

2. Preliminaries

This section reviews essential background information and concepts concerning the level $\left(w^L, w^U\right)$-interval-valued trapezoidal fuzzy numbers. These concepts will be utilized extensively in this paper.

Definition 1 ([48]). *A w-level trapezoidal fuzzy number, denoted as $\widetilde{A} = (a_1,\ a_2,\ a_3,\ a_4; w)$, where $0 < w \leq 1$ represents a fuzzy set defined over $\mathbb{R}$. Its membership function is described as follows:*

$$\mu_{\widetilde{A}}(x) = \begin{cases} w\frac{x-a_1}{a_2-a_1}, a_1 \leq x \leq a_2, \\ w, a_2 \leq x \leq a_3, \\ w\frac{a_4-x}{a_4-a_3},\ a_3 \leq x \leq a_4, \\ 0,\ \text{otherwise}. \end{cases} \tag{1}$$

$F_{TN}(w)$ denote the family of all w-level trapezoidal fuzzy numbers. This set consists of $\widetilde{A} = (a_1,\ a_2,\ a_3,\ a_4; w)$ where the values of a_1, a_2, a_3, and a_4 satisfy the condition $a_1 \leq a_2 \leq a_3 \leq a_4$, and w ranges from 0 to 1 inclusively. That is,

$$F_{TN}(w) = \left\{\widetilde{A} = (a_1,\ a_2,\ a_3,\ a_4; w), a_1 \leq a_2 \leq a_3 \leq a_4\right\}, 0 < w \leq 1. \tag{2}$$

Definition 2 ([49]). *Let $\widetilde{A}^L$ be an element of the set $F_{TN}\left(w^L\right)$, and $\widetilde{A}^U$ be an element of the set $F_{TN}\left(w^U\right)$. An interval-valued fuzzy set on $\mathbb{R}$, denoted as a level $\left(w^L,\ w^U\right)$-interval-valued trapezoidal fuzzy number $\widetilde{\widetilde{A}}$, represented as $\widetilde{\widetilde{A}} = \left[\widetilde{A}^L, \widetilde{A}^U\right] = \left\langle\left(a_1^L,\ a_2^L,\ a_3^L,\ a_4^L; w^L\right), \left(a_1^U,\ a_2^U,\ a_3^U,\ a_4^U; w^U\right)\right\rangle$, is defined by the lower trapezoidal fuzzy number $\widetilde{A}^L$ expressed as:*

$$\mu_{\widetilde{A}^L}(x) = \begin{cases} w^L\frac{x-a_1^L}{a_2^L-a_1^L}, a_1^L \leq x \leq a_2^L, \\ w^L, a_2^L \leq x \leq a_3^L, \\ w^L\frac{a_4^L-x}{a_4^L-a_3^L},\ a_3^L \leq x \leq a_4^L, \\ 0,\ \text{otherwise}, \end{cases} \tag{3}$$

and the upper trapezoidal fuzzy number $\widetilde{A}^U$ expressed as:

$$\mu_{\widetilde{A}^U}(x) = \begin{cases} w^U\frac{x-a_1^U}{a_2^U-a_1^U}, a_1^U \leq x \leq a_2^U, \\ w^U, a_2^U \leq x \leq a_3^U, \\ w^U\frac{a_4^U-x}{a_4^U-a_3^U},\ a_3^U \leq x \leq a_4^U, \\ 0,\ \text{otherwise}, \end{cases} \tag{4}$$

where $a_1^L \leq a_2^L \leq a_3^L \leq a_4^L, a_1^U \leq a_2^U \leq a_3^U \leq a_4^U, 0 < w^L \leq w^U \leq 1, a_1^U \leq a_1^L$ *and* $a_4^L \leq a_4^U$. *Additionally, it holds that,* $\widetilde{A}^L \subseteq \widetilde{A}^U$.

Let $F_{IVTN}(w^L, w^U)$ *denote the family of all* (w^L, w^U)*-level trapezoidal fuzzy numbers,*

$$F_{IVTN}(w^L, w^U) = \left\{ \widetilde{\widetilde{A}} = \left[\widetilde{A}^L, \widetilde{A}^U\right] = \left\langle \left(a_1^L,\ a_2^L,\ a_3^L,\ a_4^L; w^L\right), \left(a_1^U,\ a_2^U,\ a_3^U,\ a_4^U; w^U\right) \right\rangle : \right.$$
$$\left. \widetilde{A}^L \in F_{TN}(w^L), \widetilde{A}^U \in F_{TN}(w^U), a_1^U \leq a_1^L, a_4^L \leq a_4^U \right\}, 0 < w^L \leq w^U \leq 1.$$

Definition 3. *Let* $\widetilde{\widetilde{A}} = \left[\widetilde{A}^L, \widetilde{A}^U\right] = \left\langle \left(a_1^L,\ a_2^L,\ a_3^L,\ a_4^L; w^L\right), \left(a_1^U,\ a_2^U,\ a_3^U,\ a_4^U; w^U\right) \right\rangle$ *and* $\widetilde{\widetilde{B}} = \left[\widetilde{B}^L, \widetilde{B}^U\right] = \left\langle \left(b_1^L,\ b_2^L,\ b_3^L,\ b_4^L; w^L\right), \left(b_1^U,\ b_2^U,\ b_3^U,\ b_4^U; w^U\right) \right\rangle$ *belong to* $F_{IVTN}(w^L,\ w^U)$ *and k be a non-negative real number. Then, the specific expressions for the extended addition and scalar multiplication are defined as follows (refer to* [50]):

$$\begin{aligned} \widetilde{\widetilde{A}} \oplus \widetilde{\widetilde{B}} &= \left[\widetilde{A}^L + \widetilde{B}^L, \widetilde{A}^U + \widetilde{B}^U\right] \\ &= \left\langle \left(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; w^L\right), \left(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; w^U\right) \right\rangle \end{aligned}$$

$$k\widetilde{\widetilde{A}} = \begin{cases} \left\langle \left(ka_1^L,\ ka_2^L,\ ka_3^L,\ ka_4^L; w^L\right), \left(ka_1^U, ka_2^U,\ ka_3^U,\ ka_4^U; w^U\right) \right\rangle, k > 0, \\ \left\langle \left(ka_4^L,\ ka_3^L,\ ka_2^L,\ ka_1^L; w^L\right), \left(ka_4^U, ka_3^U,\ ka_2^U,\ ka_1^U; w^U\right) \right\rangle, k > 0, \\ \left\langle (0,0,0,0; w^L), (0,0,0,0; w^U) \right\rangle = \widetilde{\widetilde{0}}, k = 0. \end{cases}$$

Definition 4 ([48]). *Given* $r, 0 \in \mathbb{R}$, *the signed distance from* r *to* 0 *is defined as* $d(r, 0) = r$.

Definition 5 ([51]). $\widetilde{\widetilde{A}} \in F_{IVTN}(w^L, w^U)$, *the* α*-cut set of* $\widetilde{\widetilde{A}}$, *denoted by* $\widetilde{\widetilde{A}}(\alpha)$, *is defined as follows (see* Figure 1*):*

$$\widetilde{\widetilde{A}}(\alpha) = \left[\widetilde{A}^L(\alpha), \widetilde{A}^U(\alpha)\right] = \begin{cases} \left[\widetilde{A}_l^U(\alpha), \widetilde{A}_l^L(\alpha)\right] \cup \left[\widetilde{A}_r^L(\alpha), \widetilde{A}_r^U(\alpha)\right], 0 \leq \alpha \leq w^L, \\ \left[\widetilde{A}_l^U(\alpha), \widetilde{A}_r^U(\alpha)\right], w^L \leq \alpha \leq w^U, \end{cases}$$

where

$$\widetilde{A}_l^L(\alpha) = a_1^L + \left(a_2^L - a_1^L\right)\frac{\alpha}{w^L}, \widetilde{A}_r^L(\alpha) = a_4^L + \left(a_4^L - a_3^L\right)\frac{\alpha}{w^L},$$
$$\widetilde{A}_l^U(\alpha) = a_1^U + \left(a_2^U - a_1^U\right)\frac{\alpha}{w^U}, \widetilde{A}_r^U(\alpha) = a_4^U + \left(a_4^U - a_3^U\right)\frac{\alpha}{w^U}.$$

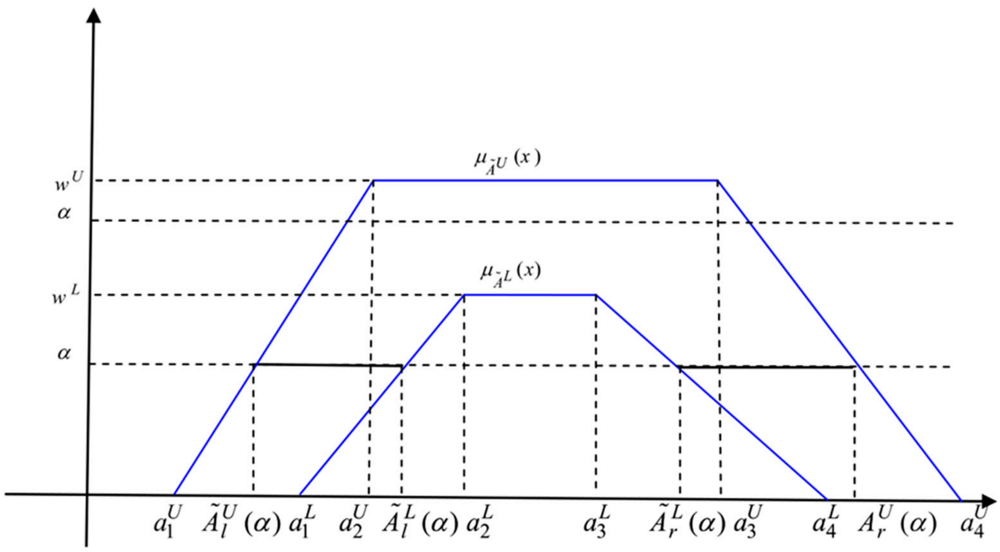

Figure 1. An α-cut of level (w^L, w^U)-interval-valued trapezoidal fuzzy number $\widetilde{\widetilde{A}}$.

Theorem 1 ([51]). *For $\widetilde{\widetilde{A}} \in F_{IVTN}\left(w^L, w^U\right)$, the signed distance of $\widetilde{\widetilde{A}}$ from O_1 (the y-axis) is expressed as follows:*

$$d\left(\widetilde{A}, O_1\right) = \frac{1}{4}[a_1 + a_2 + a_3 + a_4],\ \widetilde{A}^L = \widetilde{A}^U = \widetilde{A}, \tag{5}$$

$$d\left(\widetilde{\widetilde{A}}, O_1\right) = \frac{1}{8}\left[a_1^L + a_2^L + a_3^L + a_4^L + a_1^U + a_2^U + a_3^U + a_4^U\right],\ 0 < w^L = w^U \leq 1, \tag{6}$$

$$d\left(\widetilde{\widetilde{A}}, O_1\right) = \frac{1}{8}\left[a_1^L + a_2^L + a_3^L + a_4^L + 4a_1^U + 2a_2^U + 2a_3^U + 3\left(a_2^U + a_3^U - a_1^U - a_4^U\right)\frac{w^L}{w^U}\right],\ 0 < w^L < w^U \leq 1. \tag{7}$$

Theorem 1 outlines a method for ordering level $\left(w^L, w^U\right)$-interval-valued trapezoidal fuzzy numbers, employing the concept of comparing fuzzy numbers through signed distance ranking.

Definition 6 ([51]). *$\widetilde{\widetilde{A}}, \widetilde{\widetilde{B}} \in F_{IVTN}\left(w^L, w^U\right)$, the ranking of level $\left(w^L, w^U\right)$-interval-valued trapezoidal fuzzy numbers in $F_{IVTN}\left(w^L, w^U\right)$ is established based on the signed distance $d(., O_1)$ as follows:*

$$\widetilde{\widetilde{A}} \preccurlyeq \widetilde{\widetilde{B}} \text{ iff } d\left(\widetilde{\widetilde{A}}, O_1\right) \leq d\left(\widetilde{\widetilde{B}}, O_1\right), \tag{8}$$

$$\widetilde{\widetilde{A}} > \widetilde{\widetilde{B}} \text{ iff } d\left(\widetilde{\widetilde{A}}, O_1\right) > d\left(\widetilde{\widetilde{B}}, O_1\right), \tag{9}$$

$$\widetilde{\widetilde{A}} \approx \widetilde{\widetilde{B}} \text{ iff } d\left(\widetilde{\widetilde{A}}, O_1\right) = d\left(\widetilde{\widetilde{B}}, O_1\right). \tag{10}$$

It is worth noting that the signed distance $d(., O_1)$ provides us with a linear ranking function. That is, for any $\widetilde{\widetilde{A}}, \widetilde{\widetilde{B}} \in F_{IVTN}\left(w^L, w^U\right)$ and $k \in \mathbb{R}$, we have $d\left(k\widetilde{\widetilde{A}} \oplus \widetilde{\widetilde{B}},\ O_1\right) = kd\left(\widetilde{\widetilde{A}}, O_1\right) + kd\left(\widetilde{\widetilde{B}}, O_1.\right)$

Moreover, $(F_{IVTN}\left(w^L, w^U\right), \approx, \prec)$ conforms to the law of trichotomy [48], wherein we either have $\widetilde{\widetilde{A}} \prec \widetilde{\widetilde{B}}$, or $\widetilde{\widetilde{A}} \approx \widetilde{\widetilde{B}}$, or $\widetilde{\widetilde{B}} \prec \widetilde{\widetilde{A}}$.

3. Results

In this section, uncertainty parameters are formulated using IVTN. The interval-valued fuzzy multi-objective linear programming model is discussed. The multi-objective problem is solved using the weighted sum method.

3.1. Novel Approach for Handling Interval-Valued Fuzzy Multi-Objective Linear Programming

A multi-objective optimization problem refers to an optimization challenge encompassing multiple objective functions. Mathematically, it can be written as:

$$\begin{gathered} \min(f_1(x), f_2(x),\ f_3(x), \ldots, f_o(x)), \\ s.t \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_{ij} \geq b_l, \forall l, \\ x_{ij} \geq 0, \forall i, j. \end{gathered} \tag{11}$$

Here, f_k represents the k-th objective function, a_{ij} denotes the constraint coefficients, b_l denotes the right-hand side of the constraints, and x_{ij} represents the decision variables.

Let $f_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ijk}x_{ij}$, where c_{ijk} represent parameters of the objective function. Hence, this problem is multi-objective linear programming. Mathematically, it can be written as:

$$\begin{gathered}\min\left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij1}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij2}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij3}x_{ij}, \ldots, \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ijk}x_{ij}\right),\\ s.t \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_{ij} \geq b_l, \forall l,\\ x_{ij} \geq 0, \forall i, j.\end{gathered} \tag{12}$$

Note: Maximizing the function f_k is equivalent to minimizing the function $-f_k$.

Fuzzy linear programming, a technique introduced in [52] and rooted in the principles of fuzzy sets pioneered by [53], offers a means to manage uncertain data within linear programming models. This method can be categorized into three distinct types based on the nature of uncertainty present in the model, as delineated in [54]: (i) fuzzy variables linear programming, (ii) fuzzy parameters linear programming, and (iii) fuzzy variables and parameters linear programming. Multi-objective linear programming with fuzzy parameters involves incorporating uncertainty parameters within both the objectives and the constraints, as illustrated below:

$$\begin{gathered}\min\left(\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij1}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij2}x_{ij}, \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ij3}x_{ij}, \ldots, \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ijk}x_{ij}\right),\\ s.t \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{a}}_{ij}x_{ij} \geq \tilde{\tilde{b}}_l, \forall l,\\ x_{ij} \geq 0, \forall i, j.\end{gathered} \tag{13}$$

$\tilde{\tilde{c}}_{ijk}$ represent fuzzy parameters in the objective function, $\tilde{\tilde{a}}_{ij}$ denotes fuzzy parameters in the constraints, $\tilde{\tilde{b}}_l$ represents the right-hand side l-th constraint, and x_{ij} signifies crisp decision variables.

The weighted sum method aggregates all the multi-objective functions into a single scalar composite objective function by applying weights. That is

$$\begin{gathered}\min \sum_{k=1}^{o}\sum_{i=1}^{m}\sum_{j=1}^{n} w_k\tilde{\tilde{c}}_{ijk}x_{ij},\\ s.t \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{a}}_{ij}x_{ij} \geq \tilde{\tilde{b}}_l, \forall l,\\ x_{ij} \geq 0, \forall i, j.\end{gathered} \tag{14}$$

where w_k is the weight of k-th objective function.

Introducing simplifying assumptions for clarity, we assume: (A1) the objective functions are crisp, and (A2) the right-hand side parameters are crisp. Let us introduce additional non-negative crisp decision variables t_k, which represents the supremum of the objective function such that $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{\tilde{c}}_{ijk}x_{ij} \leq t_k$. Consequently, it can be written as:

$$
\begin{aligned}
&\min \sum_{k=1}^{o} w_k t_k, \\
&s.t \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\tilde{a}}_{ij} x_{ij} \geq \tilde{\tilde{b}}_l, \forall l, \\
&\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\tilde{c}}_{ijk} x_{ij} \leq t_k, \forall k, \\
&t_k, x_{ij} \geq 0, \forall i, j, k.
\end{aligned} \tag{15}
$$

Hence, assumption (A1) holds. We can now introduce additional variables, denoted as y_l, representing the crisp decision variables, where all values are set to one. Consequently, it can be written as:

$$
\begin{aligned}
&\min \sum_{k=1}^{o} w_k t_k, \\
&s.t \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\tilde{a}}_{ij} x_{ij} - \tilde{\tilde{b}}_l y_l \geq 0, \forall l, \\
&\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{\tilde{c}}_{ijk} x_{ij} \leq t_k, \forall k, \\
&y_l = 1, t_k, x_{ij} \geq 0, \forall i, j, k, l.
\end{aligned} \tag{16}
$$

Hence, the assumption (A2) holds.

During solving the optimization problem, the membership grade of $\tilde{\tilde{c}}_{ijk}$ may not necessarily be equal to 1. We allow the membership grade of $\tilde{\tilde{c}}_{ijk}$ to range within the interval $[\lambda, 1]$, where $0 < \lambda < 1$. We define $\tilde{\tilde{c}}_{ijk}$ as a level $(\lambda, 1)$ interval-valued fuzzy number, with $0 < \lambda < 1$ (see Figure 2), that is

$$
\tilde{\tilde{c}}_{ijk} = \left\langle \left(c_{ijk} - \delta_{ijk3}, c_{ijk} - \delta_{ijk4}, c_{ijk} + \delta_{ijk5}, c_{ijk} + \delta_{ijk6}; \lambda \right), \left(c_{ijk} - \delta_{ijk1}, c_{ijk} - \delta_{ijk2}, c_{ijk} + \delta_{ijk7}, c_{ijk} + \delta_{ijk8}; 1 \right) \right\rangle, \tag{17}
$$

where $0 < \delta_{ijk1} < \delta_{ijk2} < \delta_{ijk3} < \delta_{ijk4} < c_{ijk}$, $0 < \delta_{ijk5} < \delta_{ijk6} < \delta_{ijk7} < \delta_{ijk8}, \forall k$.

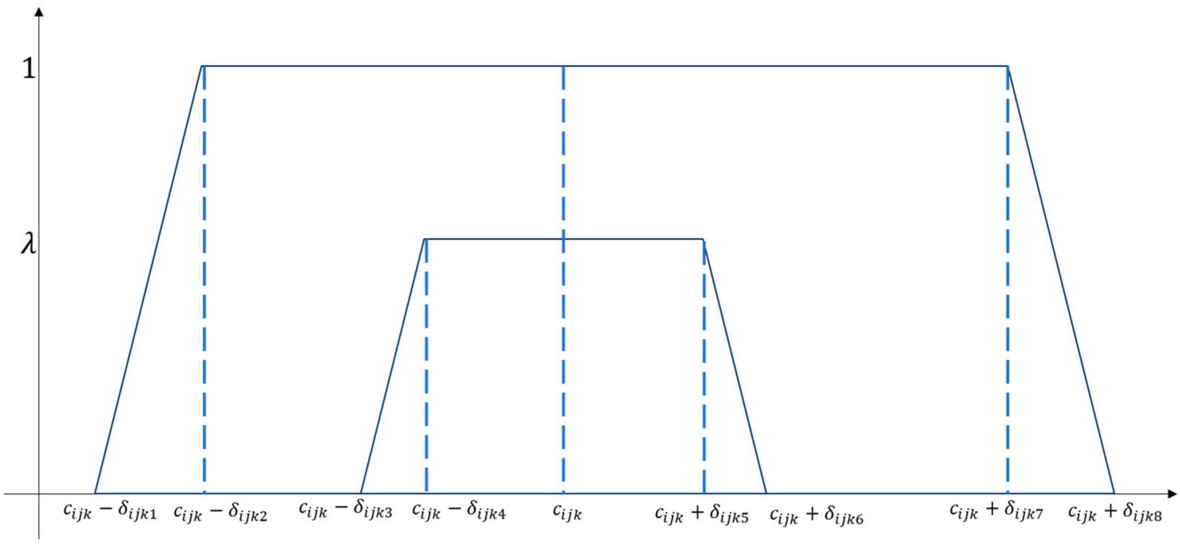

Figure 2. Level $(\lambda, 1)$ interval-valued fuzzy numbers $\tilde{\tilde{c}}_{ijk}$.

Similarly to $\tilde{\tilde{c}}_{ijk}$, we define $\tilde{\tilde{a}}_{ij}$ and $\tilde{\tilde{b}}_l$ as a level $(\lambda, 1)$ interval-valued fuzzy number, with $0 < \lambda < 1$, that is

$$
\tilde{\tilde{a}}_{ij} = \langle (a_{ij} - \epsilon_{ij3}, a_{ij} - \epsilon_{ij4}, a_{ij} + \epsilon_{ij5}, a_{ij} + \epsilon_{ij6}; \lambda), (a_{ij} - \epsilon_{ij1}, a_{ij} - \epsilon_{ij2}, a_{ij} + \epsilon_{ij7}, a_{ij} + \epsilon_{ij8}; 1) \rangle, \tag{18}
$$

where $0 < \epsilon_{ij1} < \epsilon_{ij2} < \epsilon_{ij3} < \epsilon_{ij4} < a_{ij}$, $0 < \epsilon_{ij5} < \epsilon_{ij6} < \epsilon_{ij7} < \epsilon_{ij8}, \forall i, j$,

$$\tilde{\tilde{b}}_l = \langle (b_l - \omega_{l3}, b_l - \omega_{l4}, b_l + \omega_{l5}, b_l + \omega_{l6}; \lambda), (b_l - \omega_{l1}, b_l - \omega_{l2}, b_l + \omega_{l7}, b_l + \omega_{l8}; 1) \rangle, \tag{19}$$

where $0 < \omega_{l1} < \omega_{l2} < \omega_{l3} < \omega_{l4} < b_l$, $0 < \omega_{l5} < \omega_{l6} < \omega_{l7} < \omega_{l8}, \forall l$.

Corresponding to (16), since $y_l, x_{ij} \geq 0, \forall i, j, l$, by Definition 3, Definition 6, and Equations (17)–(19), we obtain

$$\begin{aligned}
&\min \sum_{k=1}^{o} w_k t_k, \\
s.t \Bigg\langle \Bigg(&\sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} - \epsilon_{ij3}) x_{ij} - (b_l - \omega_{l3}) y_l, \sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} - \epsilon_{ij4}) x_{ij} \\
&-(b_l - \omega_{l4}) y_l, \sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} + \epsilon_{ij5}) x_{ij} \\
&-(b_l + \omega_{l5}) y_l, \sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} + \epsilon_{ij6}) x_{ij} \\
&-(b_l + \omega_{l6}) y_l; \lambda), (\sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} - \epsilon_{ij1}) x_{ij} \\
&-(b_l - \omega_{l1}) y_l, \sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} - \epsilon_{ij2}) x_{ij} \\
&-(b_l - \omega_{l2}) y_l, \sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} + \epsilon_{ij7}) x_{ij} \\
&-(b_l + \omega_{l7}) y_l, \sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij} + \epsilon_{ij8}) x_{ij} - (b_l + \omega_{l8}) y_l; 1) \rangle \geq 0, \forall l, \\
\Bigg\langle \Bigg(\sum_{i=1}^{m} \sum_{j=1}^{n} &\Big(c_{ijk} - \delta_{ijk3}\Big) x_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(c_{ijk} - \delta_{ijk4}\Big) x_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(c_{ijk} + \delta_{ijk5}\Big) x_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(c_{ijk} \\
&+\delta_{ijk6}\Big) x_{ij}; \lambda), \Bigg(\sum_{i=1}^{m} \sum_{j=1}^{n} \Big(c_{ijk} - \delta_{ijk1}\Big) x_{ij}, \Big(c_{ijk} \\
&-\delta_{ijk2}\Big) x_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(c_{ijk} + \delta_{ijk7}\Big) x_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(c_{ijk} + \delta_{ijk8}\Big) x_{ij} \Bigg) \rangle \leq t_k, \forall k, \\
&\quad y_l = 1, t_k, x_{ij} \geq 0, \forall i, j, k, l.
\end{aligned} \tag{20}$$

Since there are no fuzzy parameters, we can evaluate $y_l = 1, \forall l$. Through Theorem 1, we obtain

$$\begin{aligned}
&\min \sum_{k=1}^{o} w_k t_k, \\
s.t\ \tfrac{1}{8}[12 \sum_{i=1}^{m} \sum_{j=1}^{n} &a_{ij} x_{ij} - 12 b_l \\
&+ \sum_{i=1}^{m} \sum_{j=1}^{n} (-4\epsilon_{ij1} - 2\epsilon_{ij2} - \epsilon_{ij3} - \epsilon_{ij4} + \epsilon_{ij5} + \epsilon_{ij6} + 2\epsilon_{ij7}) x_{ij} + 4\omega_{l1} \\
&+2\omega_{l2} + \omega_{l3} + \omega_{l4} - \omega_{l5} - \omega_{l6} - 2\omega_{l7} \\
&+3\lambda \Bigg(\sum_{i=1}^{m} \sum_{j=1}^{n} (\epsilon_{ij1} - \epsilon_{ij2} + \epsilon_{ij7} - \epsilon_{ij8}) x_{ij} - \omega_{l1} + \omega_{l2} - \omega_{l7} + \omega_{l8} \Bigg)] \\
&\geq 0, \forall l, \\
\tfrac{1}{8}[12 \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ijk} x_{ij}& \\
&+ \sum_{i=1}^{m} \sum_{j=1}^{n} \Big(-4\delta_{ijk1} - 2\delta_{ijk2} - \delta_{ijk3} - \delta_{ijk4} + \delta_{ijk5} + \delta_{ijk6} + 2\delta_{ijk7}\Big) x_{ij} \\
&+3\lambda \Bigg(\sum_{i=1}^{m} \sum_{j=1}^{n} \Big(\delta_{ijk1} - \delta_{ijk2} + \delta_{ijk7} - \delta_{ijk8}\Big) x_{ij} \Bigg)] \leq t_k, \forall k, \\
&\qquad t_k, x_{ij} \geq 0, \forall i, j, k.
\end{aligned} \tag{21}$$

3.2. Ride-Hailing Matching with Interval-Valued Fuzzy Travel Time Uncertainty

The matching problem, also known as the assignment problem, is a fundamental combinatorial optimization problem. In general, an assignment problem has some workers and some tasks. The main problem discussed in the matching problem is determining the optimum match from each possible match to minimize costs. The model of the matching problem (MP) in this paper uses two objectives: (i) minimize total waiting time and (ii) minimize abandoned requests. The sets used in the model are I and J, a set of requests and available vehicles, respectively. Parameters used in the model are t_{ij}, waiting time when request i is matched with vehicle j. Decision variables in the model are $x_{ij} \in \{0, 1\}$, which answers the question, "Is a request i matched with vehicle j?". Also, $y_i \in \{0, 1\}$, answer the question, "Is a request i abandoned?".

Interval-valued fuzzy linear programming is used to deal with the uncertain problem. In this study, we assume that waiting time $\tilde{\tilde{t}}_{ij}$ are uncertain parameters since drivers have different speeds when they drive, also due to unpredictable traffic conditions.

The objective function that minimizes the total waiting time is given by:

$$\min \sum_{i \in I} \sum_{j \in J} \tilde{\tilde{t}}_{ij} x_{ij}, \tag{22}$$

The objective function that minimizes abandoned requests is given by:

$$\min \sum_{i \in I} y_i, \tag{23}$$

The objective functions (22) and (23) serve distinct purposes in the optimization process. While objective function (22) aims to minimize the overall waiting time, objective function (23) is geared towards reducing the occurrence of abandoned requests. These objectives encapsulate the dual focus of the optimization problem, addressing both passenger waiting times and service reliability.

To ensure the efficiency and fairness of the matching process, constraints (24) and (25) play a pivotal role. Constraint (24) stipulates that each vehicle can be assigned to at most one request, preventing overcommitment and optimizing resource allocation.

$$\sum_{i \in I} x_{ij} \leq 1, \forall j \in J, \tag{24}$$

Similarly, constraint (25) ensures that each request is matched with only one vehicle, maintaining the integrity of the matching system.

$$\sum_{j \in J} x_{ij} \leq 1, \forall i \in I, \tag{25}$$

Additionally, constraint (26) serves to regulate the outcome of the matching process. By enforcing that each request must either be matched with a vehicle or abandoned, constraint (26) promotes a comprehensive resolution for every request, leaving every request to be addressed.

$$\sum_{j \in J} x_{ij} = 1 - y_i, \ \forall i \in I, \tag{26}$$

Decision variables are given by:

$$x_{ij}, y_i \in \{0, 1\}, \ \forall i \in I, \forall j \in J. \tag{27}$$

With similar steps as described before, we have ride-hailing matching with uncertain travel time in the fuzzy sense as follows:

$$\min \frac{1}{8}\left[12\sum_{i\in I}\sum_{j\in J} t_{ij}x_{ij} + \sum_{i\in I}\sum_{j\in J}(-4\delta_{ij1} - 2\delta_{ij2} - \delta_{ij3} - \delta_{ij4} + \delta_{ij5} + \delta_{ij6} + 2\delta_{ij7})x_{ij} + 3\lambda\sum_{i\in I}\sum_{j\in J}(\delta_{ij1} - \delta_{ij2} + \delta_{ij7} - \delta_{ij8})x_{ij}\right] + M\sum_{i\in I} y_i,$$
$$s.t \sum_{i\in I} x_{ij} \le 1, \forall j \in J,$$
$$\sum_{j\in J} x_{ij} \le 1, \forall i \in I, \tag{28}$$
$$\sum_{j\in J} x_{ij} = 1 - y_i,\ \forall i \in I,$$
$$x_{ij}, y_i \in \{0,1\},\ \forall i \in I, \forall j \in J.$$

where M is a big number (big M).

3.3. Numerical Experiments

3.3.1. Case Study

The model proposed in this study was tested on a publicly accessible dataset of taxi trips in Manhattan (https://www.nyc.gov/site/tlc/about/tlc-trip-record-data.page, accessed on 15 April 2024). This dataset contains several pieces of information, including pick-up and drop-off locations and request times. We chose data from initial orders at noon on 15 January 2024 as input data, where pick-up and drop-off locations lie in Manhattan. The distance and travel time between two locations were estimated using OSMnx as developed by [55]. The basic experimental scenario was set as follows: 1000 drivers were deployed at random locations, and the maximum tolerable waiting time was 5 min. A vehicle could accept another request if it had finished the ride service. The number of vehicles could not decrease or increase during the simulation. Vehicles could continuously pick up and drop off passengers based on the model decision. Requests were grouped in a batch in a 0.5 min time window, and abandoned requests joined the next batch; optimization ran in every batch. Every abandoned request was removed from the queue if the waiting time exceeded 5 min. The requested data can be seen in Figure 3.

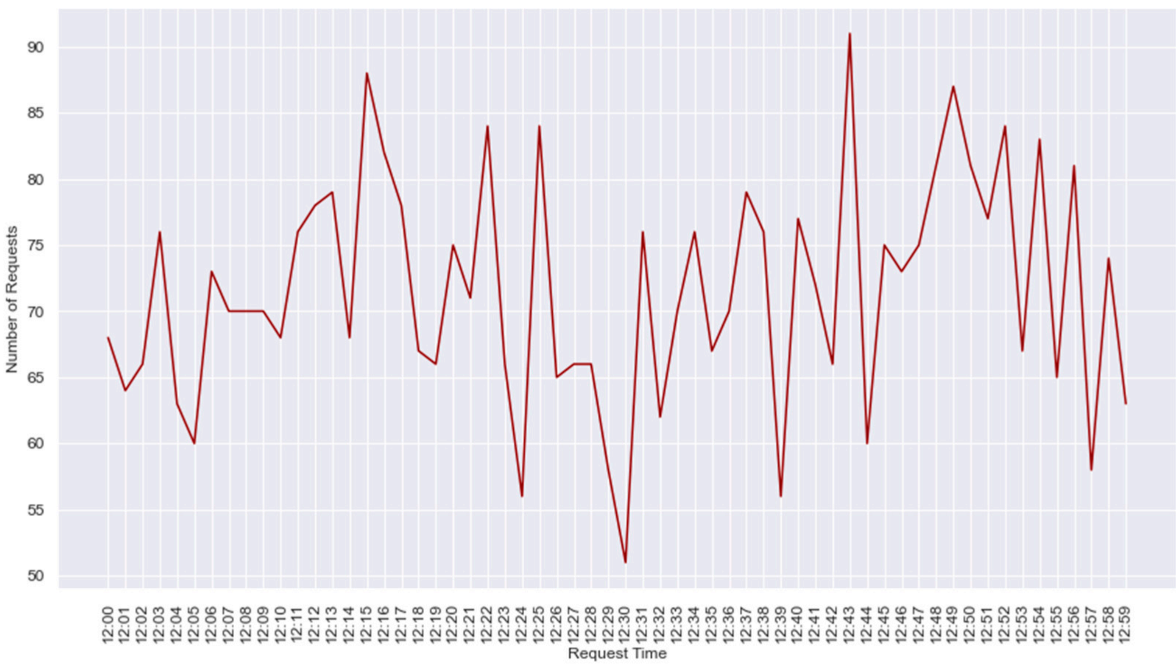

Figure 3. Number of requests.

We assume that travel time data are uncertain and influenced by historical data. Due to the lack of information about the error of the estimates made by the OSMnx calculations, we generated several travel time scheme calculations. The following are several travel time schemes used: calculations based on travel time weights when the free flow (OSMnx calculation results) and when the flow is at levels 50%, 60%, 70%, 80%, and 90% (which is represented the congestions); calculations based on the shortest distance weight when the vehicle is running at a constant 20, 30 and 40 km/h. These nine data are then used to predict travel time estimates using the average method. The travel time data used are stored in seconds. Directly, travel pick-up time impacts wait time. The uncertain waiting time, represented by an interval-valued fuzzy number, can be seen in Figure 4, where ϵ is the residual from forecasting.

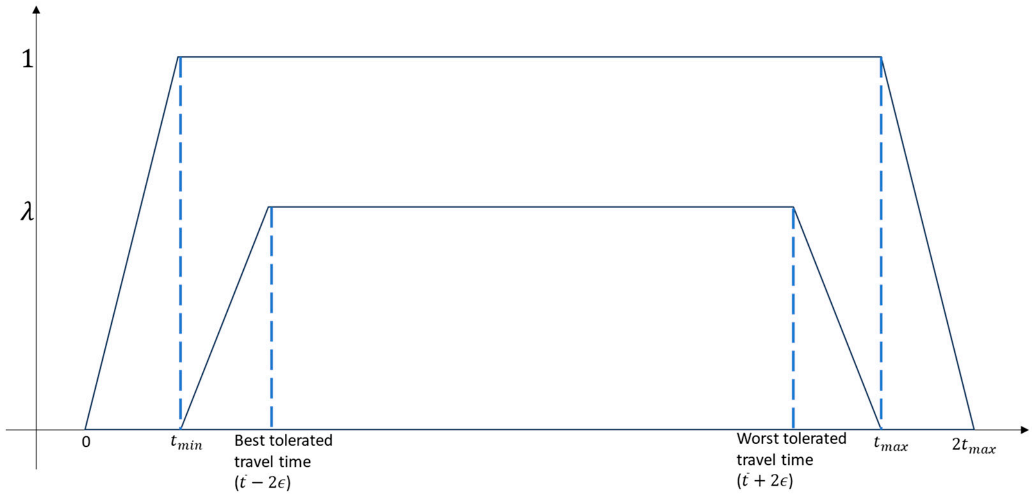

Figure 4. The interval-valued fuzzy trapezoidal number for waiting time uncertainty.

To clarify the settings of the case study conducted, we present essential information closely related to the parameter settings. Parameter settings for the ride-hailing matching problem can be seen in Table 2.

Table 2. Parameter settings for ride-hailing matching problem.

Parameter	Value	Parameter	Values
Number of passengers in a request	1 request	λ	0.8
Number of vehicles in a day	1000 vehicles	M	99,999
The tolerable maximum waiting times	5 min	ϵ	Forecasting errors
Batching time window	0.5 min	$\bar{t}$	Forecasted waiting time
Number of batches	20 batch	t_{min}, t_{max}	Minimum and maximum possible waiting time (based on nine data)

3.3.2. Experimental Results

In the next part of this section, we show the numerical simulation results for deterministic and interval-valued fuzzy linear programming models. Pick-up travel time is uncertain; thus, waiting time is also uncertain. We use best-case travel time as a parameter used in deterministic model calculations. It should be noted that the resulting solution is not guaranteed to remain feasible after the assignment is carried out because traffic conditions can influence it.

The waiting time for each parameter setting based on simulation results using the deterministic model and the interval-valued fuzzy linear programming model can be seen in Figure 5. It can be seen that the deterministic model has an average waiting time of around 0.80 min, while the interval-valued fuzzy model shows an average of about 0.85 min. The variance in the deterministic model reaches around 0.62, which is slightly smaller than the interval-valued fuzzy model, which is around 0.74.

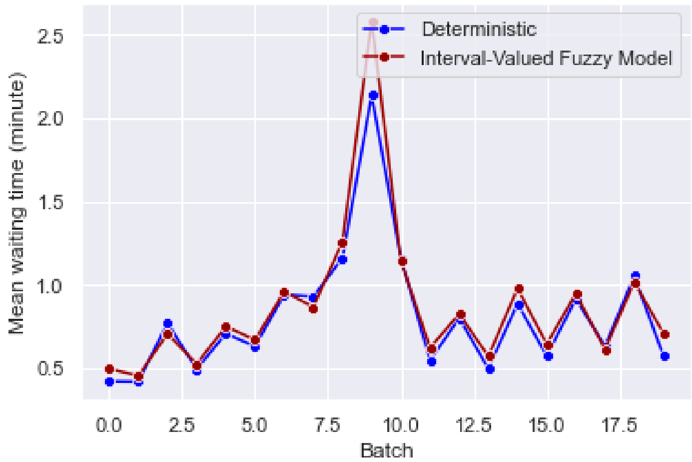

Figure 5. Mean waiting time.

The numerical simulation results reveal a not-so-significant comparison between the deterministic model and the interval-valued fuzzy model in serving requests, as seen in Figure 6. The deterministic model serves about 1509 requests, while the interval-valued fuzzy model serves about 1480 requests. The average number of requests served per batch in the deterministic model is about 75, which is not very different from the interval-valued fuzzy model, which is about 74. The variance in the number of requests served per batch in the deterministic model is about 2496, lower than the 2527 in the interval-valued fuzzy model.

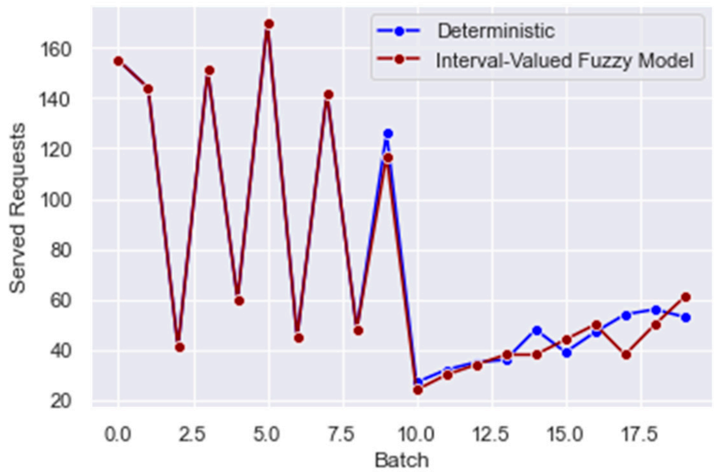

Figure 6. Service level.

The numerical simulation results show a not-so-significant comparison in the number of jobs obtained by drivers between the deterministic model and the fuzzy interval-valued model, as shown in Figure 7. The deterministic model obtains about 1509 jobs for drivers, while the fuzzy interval-valued model obtains about 1480 jobs for drivers. The variance in the number of jobs drivers receive in the deterministic model is around 0.34, slightly higher than the interval-valued fuzzy model, which has a variance of around 0.32.

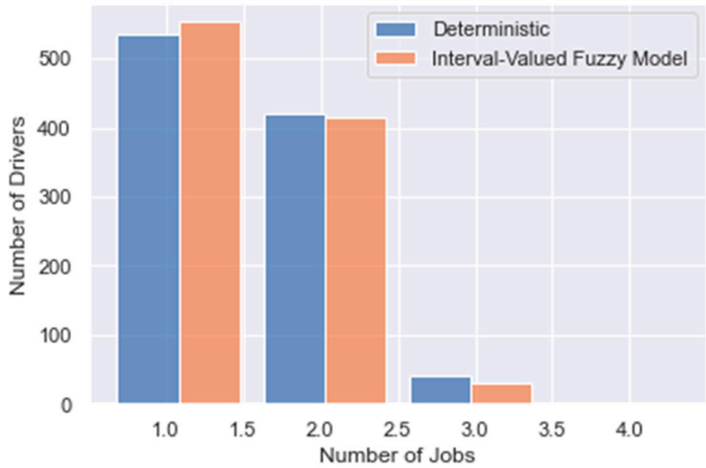

Figure 7. Distribution of the number of driver jobs.

The results obtained from the numerical simulations show that the quality of the two models in handling real-world problems is not significantly different. However, it should be noted that the deterministic model cannot handle changes in traffic conditions. In contrast, the interval-valued fuzzy model can reliably deal with changing traffic conditions or other changes that affect waiting time changes.

4. Discussion

This study developed a novel method to address multi-objective linear programming problems while considering uncertainty represented by interval-valued fuzzy numbers. This method is then applied to solve ride-hailing matching problems with uncertain travel times. The results indicate that addressing uncertainty parameters in interval-valued fuzzy multi-objective linear programming can be achieved by focusing on constraint coefficients through a series of reformulations. Additionally, the reformulated results of this method exhibit a highly simplified linear programming form, facilitating ease of solution. Furthermore, the proposed model demonstrates proficiency in handling ride-hailing matching problems effectively.

The potential of the developed method can be expanded by generalizing interval-valued fuzzy techniques to tackle non-linear programming (NLP) problems and their application within the ride-hailing context. Selecting algorithms, such as simplex, interior point methods, and greedy algorithms, among others, can enhance the method's quality. Moreover, the development of interval-valued fuzzy methods for solving NLP problems presents both research challenges and opportunities, given the current plethora of tools available for solving NLP problems.

Future research may explore further advancements in algorithm selection, model refinement, and applying interval-valued fuzzy techniques to broader problem domains beyond ride-hailing, potentially revolutionizing decision-making processes in uncertain environments.

5. Conclusions

In this study, we have successfully developed an effective interval-valued fuzzy method to solve multi-objective linear programming (MOLP) problems while considering uncertainty, particularly in the context of travel time uncertainty in ride-hailing services. By integrating interval-valued fuzzy concepts into mathematical modeling, we have created a more adaptive and efficient approach to selecting matches between passengers and drivers. Simulation results and comparisons with conventional methods demonstrate that the proposed approach better addresses travel time uncertainty and achieves the desired multi-objective criteria. Thus, this research contributes significantly to developing more adaptive and efficient ride-hailing matching systems under uncertainty, paving the way for further research in advancing decision-making techniques in uncertain environments.

In the future, the research can be expanded to address non-linear programming problems and their specific application to ride-hailing. Developing interval-valued fuzzy methods for non-linear programming problems will broaden the scope of applications and enhance the system's capability to handle more complex and realistic situations. Additionally, future research can explore applying interval-valued fuzzy concepts in other contexts beyond ride-hailing, such as logistics, manufacturing, or supply chain management. Thus, future research is expected to deepen our understanding of using interval-valued fuzzy methods in decision-making in uncertain and dynamic environments.

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