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Statistical Reliability Assessment with Generalized Intuitionistic Fuzzy Burr XII Distribution

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Abstract: Intuitionistic fuzzy sets provide a viable framework for modelling lifetime distribution characteristics, particularly in scenarios with measurement imprecision. This is accomplished by utilizing membership and non-membership degrees to accurately express the complexities of data uncertainty. Nonetheless, the complexities of some cases necessitate a more advanced approach of imprecise data, motivating the use of generalized intuitionistic fuzzy sets (GenIFSs). The use of GenIFSs represents a flexible modeling strategy that is characterized by the careful incorporation of an extra level of hesitancy, which effectively clarifies the underlying ambiguity and uncertainty present in reliability evaluations. The study employs a methodology based on generalized intuitionistic fuzzy distributions to thoroughly examine the uncertainty related to the parameters and reliability characteristics present in the Burr XII distribution. The goal is to provide a more accurate evaluation of reliability measurements by addressing the inherent ambiguity in the distribution's shape parameter. Various reliability measurements, such as reliability, hazard rate, and conditional reliability functions, are derived for the Burr XII distribution. This extensive analysis is carried out within the context of the generalized intuitionistic fuzzy sets paradigm, improving the understanding of the Burr XII distribution's reliability measurements and providing important insights into its performance for the study of various types of systems. To facilitate understanding and point to practical application, the findings are shown graphically and contrasted across various cut-set values using a valuable numerical example.

Keywords: new type generalized intuitionistic fuzzy set (GenIFS); α,β-cut sets; generalized intuitionistic fuzzy probability (GenIFP); generalized intuitionistic fuzzy reliability characteristics (GenIFRCs); Burr XII distribution

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1. Introduction

Reliability analysis has always depended on specific data and parameters, making it essential for evaluating lifetime data. Nevertheless, real-world situations frequently result in inaccurate and partial information, which raises questions about the precision of these evaluations. This requires the development of approaches that can reliably interpret ambiguous data and maintain the integrity of reliability assessments in the context of incomplete data. In order to meet the obstacles presented by uncertain circumstances, a customized approach to reliability evaluation is required, particularly when dealing with ambiguous lifetime data. In response to the need for a more flexible framework, Zadeh [1] pioneered the development of fuzzy set theory. This theory offers a framework for a sophisticated and flexible reliability analysis in circumstances that are uncertain and complex. Fuzzy set theory is a pragmatic method for handling uncertain data, which utilizes a membership degree to express the probability of an event. This degree quantifies the level of association between an element and a fuzzy set. Through successful research efforts, the concept of fuzzy sets has evolved significantly, leading to the development of different expansions. Type 1, type 2, and intuitionistic fuzzy sets (IFSs) are examples of extensions that increase the utility of fuzzy set theory. Each extension corresponds to a specific situation, providing a versatile framework for collecting and expressing various aspects of uncertainty in many different kinds of situations. Atanassov's fundamental work [2] pioneered the definition of intuitionistic fuzzy sets, while his recent work [3] provided novel operations and bridged conceptual gaps between intuitionistic and type 1 fuzzy sets theories. After that, Zeng et al. [4] presented a new multi-attribute decision-making method using intuitionistic fuzzy values and a modified VIKOR method. In a different case, experienced specialists were limited to provide options within the scope of IFS. To address this problem, Yager [5–7] proposed Pythagorean and generalized orthopair fuzzy sets. The concept of IFSs has been utilized to address various decision-making [8–10] and reliability evaluation issues [11–13].

Several influential research articles have significantly contributed to the advancement of system reliability analysis through the application of fuzzy set theory. One notable approach involves employing fuzzy probability for reliability analysis, by utilizing fuzzy random numbers to represent uncertain probabilities of system events [14]. To enhance the system reliability engineering, fuzzy set theory is also expanded to address numerous multi-objective system reliability optimization scenarios [15–17]. These techniques optimize system reliability under conflicting objectives, considering both probabilistic and imprecise reliability measures [18]. Furthermore, Refs. [19–21] employed intuitionistic fuzzy sets to construct and systematically evaluate the reliability of both time-dependent and blended systems, meticulously accounting for the inherent uncertainties associated with lifetime data. Moreover, the authors also presented the studies (Refs. [22–24]) that apply simulation-based techniques to analyze complex systems, highlighting how these methods enhance understanding and reliability assessments across various contexts. Recently, q-rung orthopair fuzzy sets have been utilized in system reliability analysis to sophisticatedly handle and model the inherent uncertainties in system performance predictions [25–28].

The Burr XII distribution is one of the twelve distributions in the Burr family. It is defined by two positive shape parameters, referred to as η and γ. This distribution is critical in survival analysis, especially when studying heavy-tailed survival time data [29]. Burr XII's probability density function can have a declining or unimodal structure, giving it versatility in representing many events. The curve of hazard rate function of the Burr distribution is flexible, as it can exhibit either diminishing trends or an upside-down bathtub shape. Its versatility enables it to accurately simulate diverse risk patterns [30]. The application of fuzzy approaches has improved the reliability study of the Burr family distribution. One study presents a compound class of the unit Burr XII model, analyzing parameters and fuzzy reliability using both nonfuzzy and fuzzy estimators [31]. In another study, the Burr XII distribution is utilized to apply sophisticated statistical techniques for modeling two competing failure modes [32]. Furthermore, a comparison of classical and Bayesian estimates of a Burr type XII distribution using fuzzy technique is explored in Ref. [33]. Moreover, Burr XII is used in several excellent works pertaining to lifetime and fuzzy analysis [34–39].

Jamkhaneh and Shabani [40,41] presented a new type of generalized intuitionistic fuzzy numbers (GenIFNs) based on Mondal's [42] fundamental generalized intuitionistic fuzzy sets (GenIFSs) for the reliability study of life distribution. They investigated several reliability measures and derived different system reliability for life distribution using this new fuzzy set [43–45]. In a recent work, Kalam, A. et al. [46] also used this new fuzzy set to study the reliability characteristics for Lomax life distribution. The existing literature clearly emphasizes the utmost importance of comprehending fuzzy reliability in the context of fuzzy life distribution, both in theoretical and practical domains.

To achieve this, we used the Burr XII distribution to study the dynamics of life distribution, concentrating on the uncertainty in one of its shape parameters, γ. By fixing the first shape parameter η and fuzzifying the other into a GenIFN, we developed a fuzzy probability framework for the Burr XII distribution. Extending our exploration, we provided generalized intuitionistic fuzzy probability and reliability for this life distribution by applying new (α, β)-cut sets. This approach effectively unraveled the inherent fuzziness in the form of reliability characteristic bands. Our study also involved a comprehensive analysis of the nature of these reliability characteristics over time, shedding light on their temporal evolution and dynamics. This new methodology aims to significantly improve the precision and comprehension of reliability evaluations in the domain of system reliability engineering. Its emphasis point is the intricate analysis of complex systems where the inherent challenges of uncertainty and imprecision play a critical role. By addressing these critical variables, the approach attempts to raise the standard for reliability evaluation, contributing to a finer understanding necessary for the construction and maintenance of highly reliable and resilient systems.

The subsequent parts of the paper are methodically arranged as follows: Section 2 provides a thorough examination of the fundamental ideas of generalized intuitionistic fuzzy set theory. Sections 3 and 4 elaborate on the methodological framework used to derive GIFSs probability, illuminating the intricate process underlying the development of key reliability features, namely, reliability, conditional reliability, and hazard functions modified for the Burr XII life distribution. The subsequent Section 5 systematically gives a numerical example, complemented by a graphical representation, deliberately utilized to support and strengthen the theoretical constructions presented in the preceding sections.

2. Definitions

2.1. Generalized Intuitionistic Fuzzy Set (GenIFS)

A GenIFS [40] within a non-empty universal set X is defined as

$$\overline{A} = \left\{ \left\langle x, \mu_{\overline{A}}(x),\, \lambda_{\overline{A}}(x) : x \in X \right\rangle \right\}, \tag{1}$$

where $\mu_{\overline{A}}: X \to [0,1]$, $\lambda_{\overline{A}}: X \to [0,1]$ are the extent of membership and non-membership functions of X in $\overline{A}$, respectively. Also, $0 \leq {\mu^{\varepsilon}}_{\overline{A}}(x) + {\lambda^{\varepsilon}}_{\overline{A}}(x) \leq 1$, for each $x \in X$ and $\varepsilon = n \left(\text{or } \frac{1}{n}\right), n = 1, 2, 3, \ldots, \mathbb{N}$.

2.2. Generalized Intuitionistic Fuzzy Number (GenIFN)

A new type of GenIFN, $\overline{A}$ [41], can be given as

$$\begin{aligned}
\mu_{\overline{A}}(x) &= \begin{cases} \mu^L(x) = \left(\frac{(x-a)\mu}{b-a}\right)^{\frac{1}{\varepsilon}}, & x \in [a,\ b], \\ u = \mu^{\frac{1}{\varepsilon}}, & x \in [b,\ c], \\ \mu^R(x) = \left(\frac{(d-x)\mu}{d-c}\right)^{\frac{1}{\varepsilon}}, & x \in [c,\ d], \\ 0, & \text{o.w}, \end{cases} \\
\lambda_{\overline{A}}(x) &= \begin{cases} \lambda^L(x) = \left(1 - \frac{(1-\lambda)\left(x-a'_1\right)}{b-a'_1}\right)^{\frac{1}{\varepsilon}}, & x \in [a'_1,\ b], \\ v = \lambda^{\frac{1}{\varepsilon}}, & x \in [b,\ c], \\ \lambda^R(x) = \left(1 - \frac{(1-\lambda)\left(d'_1-x\right)}{d'_1-c}\right)^{\frac{1}{\varepsilon}}, & x \in [c,\ d'_1], \\ 1, & \text{o.w}, \end{cases}
\end{aligned} \tag{2}$$

where $\mu_{\overline{A}}(x), \lambda_{\overline{A}}(x)$ are the membership and non-membership functions of X, respectively. The GenIFN $\overline{A}$ is represented as $\overline{A} = (a'_1, a, b, c, d, d'_1, \mu, \lambda, \varepsilon)$ if ${a'}_1 \leq a \leq b \leq c \leq d \leq {d'}_1$ and $0 \leq {\mu^{\varepsilon}}_{\overline{A}}(x) + {\lambda^{\varepsilon}}_{\overline{A}}(x) \leq 1,\ \forall\, x \in X$.

The above expression indicates that a generalized intuitionistic fuzzy number (GenIFN) has an equal distribution in its membership and non-membership functions, centered around a main value and showing symmetry in its left and right sides.

2.3. GenIFN Based on (α, β)-Cut Sets

A generalized intuitionistic fuzzy number (GenIFN) based on the α-cut and β-cut sets can be defined by

$$\overline{A}\,(\alpha,\beta,\varepsilon) = \left\{\overline{A}_{\mu}[\alpha,\varepsilon],\, \overline{A}_{\lambda}[\beta,\varepsilon]\right\}. \tag{3}$$

Suppose that the fixed numbers $\alpha,\ \beta\ \in\ [0,1]$ such that $0 \le \alpha \le \mu^{\frac{1}{\varepsilon}},\ \lambda^{\frac{1}{\varepsilon}} \le \beta \le 1$ and $0 \le \alpha^{\varepsilon} \le \beta^{\varepsilon} \le 1$; then the (α, β)-cut sets are derived as

$$\overline{A}\,[\alpha,\beta,\varepsilon] = \left\{\left\langle x, \mu_{\overline{A}}(x) \ge \alpha,\ \lambda_{\overline{A}}(x) \le \beta : x \in X\right\rangle\right\}. \tag{4}$$

Note that the above expression contains two cut-sets, the α-cut set of GenIFN,

$$\begin{aligned} \overline{A_{\mu}}\,[\alpha,\varepsilon] &= \left\{\left\langle x, \mu_{\overline{A}}(x) \ge \alpha : x \in X\right\rangle\right\}, \\ &= [L(\alpha),\, R(\alpha)], \end{aligned} \tag{5}$$

where

$$L(\alpha) = a + \frac{(b-a)\alpha^{\varepsilon}}{\mu}, \quad R(\alpha) = d + \frac{(d-c)\alpha^{\varepsilon}}{\mu},$$

and the β-cut set of GenIFN,

$$\begin{aligned} \overline{A_{\lambda}}\,[\beta,\varepsilon] &= \left\{\left\langle x, \lambda_{\overline{A}}(x) \le \beta : x \in X\right\rangle\right\}, \\ &= [L(\beta),\, R(\beta)], \end{aligned} \tag{6}$$

where

$$L(\beta) = {a'}_1 + \frac{(b-{a'}_1)(1-\alpha^{\varepsilon})}{1-\lambda}, \quad R(\beta) = {d'}_1 + \frac{({d'}_1-c)(1-\beta^{\varepsilon})}{1-\lambda}.$$

The (α, β)-cut sets of a GenIFN are the intersection of these two cut-sets,

$$\overline{A}\,[\alpha,\beta,\varepsilon] = \{x, x \in [L(\alpha), R(\alpha)] \cap [L(\beta), R(\beta)]\}, \tag{7}$$

which can be simplified as

$$\overline{A}\,[\alpha,\beta,\varepsilon] = \left\{\left\langle x, \mu_{\overline{A}}(x) \ge \alpha,\ \lambda_{\overline{A}}(x) \le \beta : x \in X\right\rangle\right\}. \tag{8}$$

2.4. Several Relations and Operations on GenIFNs

Let us consider two GenIFNs, $\overline{B}\,[\alpha,\beta,\varepsilon]$ and $\overline{C}[\alpha,\beta,\varepsilon]$, then.

I. $\overline{B}\,[\alpha,\beta,\varepsilon] \oplus \overline{C}[\alpha,\beta,\varepsilon] = \left\{\overline{B}_{\mu}[\alpha,\varepsilon] \oplus \overline{C}_{\mu}[\alpha,\varepsilon],\, \overline{B}_{\lambda}[\beta,\varepsilon] \oplus \overline{C}_{\lambda}[\beta,\varepsilon]\right\}$,

II. $m \otimes \overline{B}\,[\alpha,\beta,\varepsilon] \oplus k = \left\{m \otimes \overline{B}_{\mu}[\alpha,\varepsilon] \oplus k,\ m \otimes \overline{B}_{\lambda}[\beta,\varepsilon] \oplus k\right\}$,

III. $k \ominus \overline{B}[\alpha,\beta,\varepsilon] = \left\{k \ominus \overline{B}_{\mu}[\alpha,\varepsilon], k \ominus \overline{B}_{\lambda}[\beta,\varepsilon]\right\}$,

IV. $\overline{B}[\alpha,\beta,\varepsilon] \preccurlyeq \overline{C}[\alpha,\beta,\varepsilon]\ iff\ \left\{\overline{B}_{\mu}[\alpha,\varepsilon] \preccurlyeq \overline{C}_{\mu}[\alpha,\varepsilon]\ and\ \overline{B}_{\lambda}[\beta,\varepsilon] \preccurlyeq \overline{C}_{\lambda}[\beta,\varepsilon]\ \right\}$,

V. $\overline{B}[\alpha,\beta,\varepsilon] = \overline{C}[\alpha,\beta,\varepsilon]\ iff\ \overline{B}_{\mu}[\alpha,\ \varepsilon] =\ \overline{C}_{\mu}[\alpha,\ \varepsilon]\ and\ \overline{B}_{\lambda}[\beta,\ \varepsilon] =\ \overline{C}_{\lambda}[\beta,\ \varepsilon]$.

3. Generalized Intuitionistic Fuzzy Probability

Lifetime data uncertainty often arises from random variables or imprecise model parameters. In this context, we specifically address parameters represented as fuzzy numbers, enhancing our approach by employing generalized intuitionistic fuzzy numbers (GenIFNs). This leads to the introduction of a refined concept, fuzzy probability, which accommodates and quantifies the inherent uncertainty in model parameters.

Let us consider a lifetime variable X characterized by a density function $f(x, \overline{\gamma})$, where $\overline{\gamma}$ are represented as generalized intuitionistic fuzzy parameters. So, we can define the α cut for membership and β cut for the non-membership function within the provided fuzzy probability framework for a constant point c as

$$\begin{aligned} P_j(c)[k,\varepsilon] &= [P(c) | \gamma \in \gamma_j\,[k,\varepsilon],\, (j,k) = (\mu,\alpha),(\lambda,\beta)] \\ &= \left[P_j^L(c)\,[k],\ P_j^R(c)\,[k]\right], \end{aligned} \tag{9}$$

for $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$ and $0 \leq \alpha^{\varepsilon} + \beta^{\varepsilon} \leq 1$, where $P(c)$ is the crisp probability defined as $P(c) = \int_c f(x,\gamma)\,dx$, and $P_j^L(c)[k] = \inf_{\gamma\in\gamma_j[k,\,\varepsilon]} P(c)$, $P_j^R(c)[k] = \sup_{\gamma\in\gamma_j[k,\,\varepsilon]} P(c)$. Consequently, the generalized intuitionistic fuzzy probability (GenIFP) can be given by

$$\overline{P}(c) = P(c)\,(\alpha,\beta,\varepsilon) = P_\mu(c)\,[\alpha,\varepsilon],\, P_\lambda(c)\,[\beta,\varepsilon], \tag{10}$$

and the (α,β)-cut sets of GenIFP for c are

$$P(c)\,[\alpha,\beta,\varepsilon] = \left\{\nu \middle| \nu \in P_\mu(c)\,[\alpha,\varepsilon] \cap P_\lambda(c)\,[\beta,\varepsilon]\right\}, \tag{11}$$

with $[0,0] \preccurlyeq P_\mu(c)[\alpha,\varepsilon], P_\lambda(c)[\beta,\varepsilon] \preccurlyeq [1,1]$ and $[0,0] \preccurlyeq P(\varnothing)[\alpha,\beta,\varepsilon], P(c)[\alpha,\beta,\varepsilon] \preccurlyeq P(\Omega)[\alpha,\beta,\varepsilon] \preccurlyeq [1,1]$ for empty set $\varnothing$ and universal set Ω.

To be more specific, we considered a lifetime variable X from the Burr XII distribution with two shape parameters, η and γ, having the cumulative distribution function

$$F(x) = 1 - (1 + x^{\eta})^{-\gamma}, \quad x \geq 0, \tag{12}$$

and the probability density function

$$f(x) = \gamma\eta x^{\eta-1}(1 + x^{\eta})^{-(\gamma+1)}, \quad x \geq 0, \tag{13}$$

Moreover, changing the values of γ leads to different shapes in the probability density function graph while η remains constant, and vice versa. Fuzzification with generalized intuitionistic fuzzy sets (GenIFSs) enhances the modeling of Burr XII distribution's shape parameters, offering a more flexible uncertainty representation by considering membership, non-membership, and hesitancy levels. This might be flexibly applied to one or both shape parameters, accommodating the specific needs of diverse datasets.

Consider the lifetime variable X from the Burr XII distribution with fuzzification to the one shape parameter, γ, into a GenIFN as $\overline{\gamma} = (a_1', a, b, c, d, d_1', \mu, \lambda, \varepsilon)$, while the other shape parameter, $\eta = 1$, remains constant.

Since $1 - (1 + x^{\eta})^{-\gamma}$ is the monotonically decreasing function of γ, the cut-sets of the generalized intuitionistic fuzzy distribution function is given by

$$\begin{aligned} P_\mu(x)[\alpha,\varepsilon] &= \left[1 - (1 + x^{\eta})^{-(\frac{(d-c)\alpha^{\varepsilon}}{\mu} + d)},\, 1 - (1 + x^{\eta})^{-(a + \frac{(b-a)\alpha^{\varepsilon}}{\mu})}\right], \\ P_\lambda(x)[\alpha,\varepsilon] &= \left[1 - (1 + x^{\eta})^{-(\frac{(d_1'-c)\,(1-\beta^{\varepsilon})}{1-\lambda} + d_1')},\, 1 - (1 + x^{\eta})^{-(a_1' + \frac{(b-a_1')\,(1-\beta^{\varepsilon})}{1-\lambda})}\right]. \end{aligned} \tag{14}$$

which leads to the cut-sets for the specific point $x = c$:

$$\begin{aligned} P_\mu(c)[\alpha,\varepsilon] &= \left[1 - (1 + c^{\eta})^{-(\frac{(d-c)\alpha^{\varepsilon}}{\mu} + d)},\, 1 - (1 + c^{\eta})^{-(a + \frac{(b-a)\alpha^{\varepsilon}}{\mu})}\right], \\ P_\lambda(c)[\alpha,\varepsilon] &= \left[1 - (1 + c^{\eta})^{-(\frac{(d_1'-c)\,(1-\beta^{\varepsilon})}{1-\lambda} + d_1')},\, 1 - (1 + c^{\eta})^{-(a_1' + \frac{(b-a_1')\,(1-\beta^{\varepsilon})}{1-\lambda})}\right]. \end{aligned} \tag{15}$$

Corollary 1. *Let us consider the GenIFP $P(c)$, then.*

(I) $P(c^c) = 1 \ominus P(c)(\alpha,\beta,\varepsilon)$

(II) $If\ c_1 \subset c_1\ then\ P(c_1)(\alpha,\beta,\varepsilon) \preccurlyeq P(c_2)(\alpha,\beta,\varepsilon)$

Proof.

(I) From the definition of GenIFP, for $(j,k) = (\mu,\alpha),(\lambda,\beta)$,

$$\begin{aligned} P_j(c^c)[k,\varepsilon] &= \left[1 - P(c) | \gamma \in \gamma_j[k,\varepsilon]\right] \\ &= \left[P_j^L(c^c)[k], P_j^R(c^c)[k]\right] \\ &= \left[inf_{\gamma\in\gamma_j[k,\varepsilon]}(1-P(c)), sup_{\gamma\in\gamma_j[k,\varepsilon]}(1-P(c))\right] \\ &= \left[1 - sup_{\gamma\in\gamma_j[k,\varepsilon]}P(c), 1 - inf_{\gamma\in\gamma_j[k,\varepsilon]}P(c)\right] \\ &= 1 \ominus \left[P_j^L(c)[k], P_j^R(c)[k]\right] \end{aligned}$$

which is substantiated by relation 2.4 (V).

(II) Since $P(c_1) \leq P(c_2)$, it is

$$\begin{aligned} P_j(c_1)[k,\varepsilon] &= \left[inf_{\gamma\in\gamma_j[k,\varepsilon]}P(c_1), sup_{\gamma\in\gamma_j[k,\varepsilon]}P(c_2)\right] \\ &\preceq \left[inf_{\gamma\in\gamma_j[k,\varepsilon]}P(c_1), sup_{\gamma\in\gamma_j[k,\varepsilon]}P(c_2)\right] \\ &= P_j(c_2)[k,\varepsilon] \end{aligned}$$

and based on the relation 2.4 (IV), the proof is completed. □

4. Generalized Intuitionistic Fuzzy Reliability Characteristics

The generalized intuitionistic fuzzy sets method enhances the analysis of reliability parameters by considering membership, non-membership, and hesitancy levels, effectively addressing subjective, uncertain, and vague aspects of information. This sophisticated technique goes beyond the limits of conventional binary logic, adapting to the inherent unpredictability found in intricate systems and thereby improving the accuracy of their representations.

Let a lifetime variable X from the density function $f(x,\overline{\gamma})$, where the parameter $\overline{\gamma}$ is vectorized by a generalized intuitionistic fuzzy number (GenIFN) and the generalized intuitionistic fuzzy reliability characteristics (GenIFRCs) is denoted by $\overline{\omega}(t)$.

Then, the (α,β)-cut sets for GenIFRC for $(j,k) = (\mu,\alpha),(\lambda,\beta)$ can be given by the following equation:

$$\begin{aligned} \omega_j(t)\,[k,\varepsilon] &= \left\{\omega(t) \middle| \gamma \in \gamma_j[k,\varepsilon],\, (j,k) = (\mu,\alpha),(\lambda,\beta)\right\} \\ &= \left[\omega_j^L(t)\,[k],\ \omega_j^R(t)\,[k]\right], \end{aligned} \tag{16}$$

where $\omega_j^L(t)[k] = \inf_{\gamma\in\gamma_j[k,\,\varepsilon]}\omega(t), \omega_j^R(t)[k] = \sup_{\gamma\in\gamma_j[k,\,\varepsilon]}\omega(t)$, with the following restrictions: $0 \leq \alpha \leq \mu^{\frac{1}{\varepsilon}},\ \lambda^{\frac{1}{\varepsilon}} \leq \beta \leq 1$, $0 \leq \alpha^\varepsilon + \beta^\varepsilon \leq 1$ and $t > 0$. The function $\overline{\omega}(t)$ may be considered reliability measures, including hazard rate, reversed hazard rate, or cumulative risk functions.

Finally, the GenIFRC is shown as $\omega(t)(\alpha,\beta,\varepsilon) = \left(\omega_\mu(t)\,[\alpha,\varepsilon],\, \omega_\lambda(t)\,[\beta,\varepsilon]\right)$ and the (α,β)-cut sets of GenIFRC are given by

$$\omega\,[\alpha,\beta,\varepsilon] = \left\{\nu \middle| \nu \in \omega_\mu(t)\,[\alpha,\varepsilon] \cap \omega_\lambda(t)\,[\beta,\varepsilon]\right\}. \tag{17}$$

where $\omega_j(t)\,[k,\varepsilon]$, $k = \alpha,\beta$ is a two-variate function of k and time t. For every special time, $t = t_s$, $\overline{\omega}(t_s)$ is a generalized intuitionistic fuzzy number. In this method, for every special value of $\alpha = \alpha_s$ and $\beta = \beta_s$, curves of $\omega_\mu(t)[\alpha_s,\varepsilon], \omega_\lambda(t)[\beta_s,\varepsilon]$ and $\omega_\mu(t)[\alpha_s,\beta_s,\varepsilon]$ behave as bands with lower and upper limits.

These reliability measures possess some important properties, which can be remarked as follows:

Remark 1. *If $\mu_1 \leq \mu_2$ and $\lambda_1 \leq \lambda_2$, then $\omega_{\mu_1}(t)\,[\alpha,\varepsilon] \subset \omega_{\mu_2}(t)\,[\alpha,\varepsilon]$ and $\omega_{\lambda_2}(t)\,[\beta,\varepsilon] \subset \omega_{\lambda_1}(t)\,[\beta,\varepsilon]$,*

Remark 2. *If $\varepsilon_1 \leq \varepsilon_2$, then $\omega_\mu(t)\,[\alpha,\varepsilon_1] \subset_\mu \omega\,[\alpha,\varepsilon_1]$ and $\omega_\lambda(t)\,[\beta,\varepsilon_2] \subset \omega_\lambda(t)\,[\beta,\varepsilon_2]$.*

4.1. Generalized Intuitionistic Fuzzy Reliability

Using the developed notion of reliability characteristics, the cut-sets for one of the measures of GenIFR are

$$\begin{aligned} R_j(t)\,[k,\varepsilon] &= \{R(t)|\gamma \in \gamma_i[k,\varepsilon],\,(j,k) = (\mu,\alpha),(\lambda,\beta)\} \\ &= \left[R_j^L(t)\,[k],\; R_j^R(t)\,[k]\right]. \end{aligned} \tag{18}$$

where $R(t) = \int_t^\infty f(t,\eta,\gamma)dx$, $R_j^L(t)[k] = \inf_{\gamma\in\gamma_j[k,\,\varepsilon]} R(t)$ and $R_j^R(t)[k] = \sup_{\gamma\in\gamma_j[k,\,\varepsilon]} R(t)$ with constraint $0 \leq \alpha^\varepsilon + \beta^\varepsilon \leq 1$.

The GenIFR is shown as $R(t)(\alpha,\beta,\varepsilon) = \left(R_\mu(t)\,[\alpha,\varepsilon],\, R_\lambda(t)\,[\beta,\varepsilon]\right)$ and the cut-sets of GenIFR are given by the following equation:

$$R(t)\,[\alpha,\beta,\varepsilon] = \left[\nu\middle|\nu \in R_\mu(t)\,[\alpha,\varepsilon] \cap R_\lambda(t)\,[\beta,\varepsilon]\right]. \tag{19}$$

For the Burr XII distribution,

$$R_j(t)[k,\varepsilon] = \left[\frac{1}{(1+t^\eta)^\gamma}\middle|\gamma \in \gamma_j[k,\varepsilon],\,(j,\,k) = (\mu,\alpha),\,(\lambda,\beta)\right] \tag{20}$$

Since $(1+t^\eta)^{-\gamma}$ is a monotonically decreasing function of γ, the α and β cut-sets for GenIFR can be modified as

$$\begin{aligned} R_\mu(t)[\alpha,\varepsilon] &= \left[\frac{1}{\{1+t^\eta\}^{(R(\alpha))}},\, \frac{1}{\{1+t^\eta\}^{(L(\alpha))}}\right], \\ R_\lambda(t)[\beta,\varepsilon] &= \left[\frac{1}{\{1+t^\eta\}^{(R(\beta))}},\, \frac{1}{\{1+t^\eta\}^{(L(\beta))}}\right]. \end{aligned} \tag{21}$$

Moreover, for the Burr XII life distribution,

$$\begin{aligned} R_\mu(t)[\alpha,\varepsilon] &= \left[(1+t^\eta)^{-(\frac{(d-c)\alpha^\varepsilon}{\mu}+d)},\, (1+t^\eta)^{-(a+\frac{(b-a)\alpha^\varepsilon}{\mu})}\right], \\ R_\lambda(t)[\beta,\varepsilon] &= \left[(1+t^\eta)^{-(\frac{(d'_1-c)\,(1-\beta^\varepsilon)}{1-\lambda}+d'_1)},\, (1+t^\eta)^{-(a'_1+\frac{(b-a'_1)\,(1-\beta^\varepsilon)}{1-\lambda})}\right]. \end{aligned} \tag{22}$$

Note that for every set of $(\alpha,\,\beta)$, the reliability bands hold the given relations:

1. $R_j(0)\,[k,\varepsilon] = [1,1]$,
2. $R_j(\infty)\,[k,\varepsilon] = [0,0]$,
3. $R_j(t_1) \geq R_j(t_2)\; iff\; t_1 \leq t_2$.

Since $R_j(t)\,[k,\varepsilon]$, $k=\alpha,\beta$ is a two-variate function of k and time t. For every special time $t=t_s$, the membership $\mu_{R(t_s)}(x)$ and non-membership $\lambda_{R(t_s)}(x)$ functions of GenIFR based on Equation (2) can be derived as

$$\mu_{R(t_s)}(x)=\begin{cases}\left(\dfrac{\left(d+\frac{\log(x)}{\log(1+t_s^{\eta})}\right)\mu}{d-c}\right)^{\frac{1}{\varepsilon}}, & x\in\left[\frac{1}{(1+t_s^{\eta})^{d}},\frac{1}{(1+t_s^{\eta})^{c}}\right],\\ \mu^{\frac{1}{\varepsilon}}, & x\in\left[\frac{1}{(1+t_s^{\eta})^{c}},\frac{1}{(1+t_s^{\eta})^{b}}\right],\\ \left(\dfrac{\left(a+\frac{\log(x)}{\log(1+t_s^{\eta})}\right)\mu}{a-b}\right)^{\frac{1}{\varepsilon}}, & x\in\left[\frac{1}{(1+t_s^{\eta})^{b}},\frac{1}{(1+t_s^{\eta})^{a}}\right],\\ 0, & \text{o.w,}\end{cases}$$
$$\lambda_{R(t_s)}(x)=\begin{cases}\left(\dfrac{d_1'-c+(1-\lambda)\left(\frac{\log(x)}{\log(1+t_s^{\eta})}+d_1'\right)}{d_1'-c}\right)^{\frac{1}{\varepsilon}}, & x\in\left[\frac{1}{(1+t_s^{\eta})^{d_1'}},\frac{1}{(1+t_s^{\eta})^{c}}\right],\\ \lambda^{\frac{1}{\varepsilon}}, & x\in\left[\frac{1}{(1+t_s^{\eta})^{c}},\frac{1}{(1+t_s^{\eta})^{b}}\right],\\ \left(\dfrac{b-a_1'+(1-\lambda)\left(a_1'+\frac{\log(x)}{\log(1+t_s^{\eta})}\right)}{b-a_1'}\right)^{\frac{1}{\varepsilon}}, & x\in\left[\frac{1}{(1+t_s^{\eta})^{b}},\frac{1}{(1+t_s^{\eta})^{a_1'}}\right],\\ 1, & \text{o.w,}\end{cases}\tag{23}$$

4.2. Generalized Intuitionistic Fuzzy Conditional Reliability

The concept of conditional reliability focuses on the probability that an object will continue to function at a given time (t), provided it has remained operational up to that point (T). This concept is particularly crucial in reliability engineering and risk assessment. To accommodate uncertainty and imprecision often found in real-world scenarios, this traditional notion of conditional reliability is explored through the application of fuzzy sets. The cut-sets of GenIFCR are

$$\begin{aligned}R_j(t|T)\,[k,\varepsilon]&=[R(t|T)|\gamma\in\gamma_i[k,\varepsilon],\,(j,k)=(\mu,\alpha),(\lambda,\beta)]\\&=\left[R_j^L(t\big|T)\,[k],\ R_j^R(t\big|T)\,[k]\right].\end{aligned}\tag{24}$$

where $R(t\big|T)=\frac{R(t+T)}{R(T)}$, $R_j^L(t\big|T)[k]=\inf_{\gamma\in\gamma_j[k,\varepsilon]}R(t\big|T), R_j^R(t\big|T)[k]=\sup_{\gamma\in\gamma_i[k,\varepsilon]}R(t\big|T)$ for all $0\leq\alpha^{\varepsilon}+\beta^{\varepsilon}\leq 1$. Hence, GenIFCR can be given as $R(t|T)(\alpha,\beta,\varepsilon)=\left(R_\mu(t|T)[\alpha,\varepsilon],R_\lambda(t|T)[\beta,\varepsilon]\right)$ and the cut-sets of the GenIFCR function is described as

$$R\,(t\big|T)[\alpha,\beta,\varepsilon]=\left[\nu\big|\nu\in R_\mu(t\big|T)\,[\alpha,\varepsilon]\cap R_\lambda(t\big|T)\,[\beta,\varepsilon]\right].\tag{25}$$

Considering the Burr XII distribution, the cut-sets of GenIFCR can be given as

$$R_j(t\Big|T)\,[k,\varepsilon]=\left[\left(\frac{1+T^{\eta}}{1+(t+T)^{\eta}}\right)^{\gamma}\Bigg|\gamma\in\gamma_j[k,\varepsilon],\,(j,k)=(\mu,\alpha),(\lambda,\beta)\right]\tag{26}$$

The function $\left(\frac{1+T^{\eta}}{1+(t+T)^{\eta}}\right)^{\gamma}$ exhibits a monotonic decrease in relation to its parameter γ, leading to the subsequent formulation for GenIFCR bands.

$$\begin{aligned}R_\mu(t\Big|T)\,[k,\varepsilon]&=\left[\left(\frac{1+T^{\eta}}{1+(t+T)^{\eta}}\right)^{\left(\frac{(d-c)\alpha^{\varepsilon}}{\mu}+d\right)},\ \left(\frac{1+T^{\eta}}{1+(t+T)^{\eta}}\right)^{\left(a+\frac{(b-a)\alpha^{\varepsilon}}{\mu}\right)}\right],\\R_\lambda(t\Big|T)\,[k,\varepsilon]&=\left[\left(\frac{1+T^{\eta}}{1+(t+T)^{\eta}}\right)^{\left(\frac{(d_1'-c)\,(1-\beta^{\varepsilon})^{\varepsilon}}{1-\lambda}+d_1'\right)},\ \left(\frac{1+T^{\eta}}{1+(t+T)^{\eta}}\right)^{\left(a_1'+\frac{(b-a_1')\,(1-\beta^{\varepsilon})}{1-\lambda}\right)}\right].\end{aligned}\tag{27}$$

4.3. Generalized Intuitionistic Fuzzy Hazard

The hazard function, also commonly referred to as the instantaneous failure rate, is a critical component of life distribution analysis. It essentially provides an estimate of the likelihood that a component will fail at a given point in time, given that it has survived up to that moment. This function is instrumental in predicting the time-dependent failure rates of components or systems in service. For the generalized intuitionistic fuzzy hazard GenIFH functions, the cut-sets play a significant role. These cut-sets provide a means to evaluate the hazard function under varying degrees of uncertainty and fuzziness, allowing for a more comprehensive and elaborated understanding of the failure dynamics of the system. The cut-sets of GenIFH are

$$\begin{aligned} h_j(t)\,[k,\varepsilon] &= [h(t)|\gamma \in \gamma_i[k,\varepsilon],\,(j,k)=(\mu,\alpha),(\lambda,\beta)], \\ &= \left[h_j^L(t)\,[k],\ h_j^U(t)\,[k]\right]. \end{aligned} \tag{28}$$

where $h(t) = \frac{f(t)}{R(t)}, h_j^L(t)[k] = \inf_{\gamma\in\gamma_j[k,\,\varepsilon]} h(t)$ and $h_j^L(t)[k] = \sup_{\gamma\in\gamma_i[k,\,\varepsilon]} h(t)$ for $\forall\ 0 \le \alpha^\varepsilon + \beta^\varepsilon \le 1$.

The GenIFH can be given as $h(\alpha,\beta,\varepsilon) = \left(h_\mu(t)[\alpha,\varepsilon], h_\mu(t)[\beta,\varepsilon]\right)$ and the (α,β)-cut sets of GenIFH are

$$h\,(t)[\alpha,\beta,\varepsilon] = \left[\nu\middle|\nu \in h_\mu(t)\,[\alpha,\varepsilon] \cap h_\lambda(t)\,[\beta,\varepsilon]\right]. \tag{29}$$

For the Burr XII distribution, the above equations will be adjusted as follows:

$$h_j(t)\,[k,\varepsilon] = \left[\left(\frac{\gamma\,\eta}{(1+t^\eta)t^{(1-\eta)}}\right)\middle|\gamma \in \gamma_j[k,\varepsilon],\,(j,k)=(\mu,\alpha),(\lambda,\beta)\right]. \tag{30}$$

Given that the function $\frac{\gamma\,\eta}{(1+t^\eta)t^{(1-\eta)}}$ is characterized by a monotonic increase with respect to its parameter γ, the hazard bands can be described as follows:

$$\begin{aligned} h_\mu(t)\,[k,\varepsilon] &= \left[\left(\frac{\left(a+\frac{(b-a)\alpha^\varepsilon}{\mu}\right)\eta}{(1+t^\eta)t^{(1-\eta)}}\right),\left(\frac{\left(d-\frac{(d-c)\alpha^\varepsilon}{\mu}\right)\eta}{(1+t^\eta)t^{(1-\eta)}}\right)\right], \\ h_\lambda(t)\,[k,\varepsilon] &= \left[\left(\frac{\left(a_1'+\frac{(b-a_1')(1-\beta^\varepsilon)}{1-\lambda}\right)\eta}{(1+t^\eta)t^{(1-\eta)}}\right),\left(\frac{\left(d_1'-\frac{(d_1'-c)(1-\beta^\varepsilon)}{1-\lambda}\right)\eta}{(1+t^\eta)t^{(1-\eta)}}\right)\right]. \end{aligned} \tag{31}$$

Corollary 2. *Consider the two lifetime random variables T_1 and T_2, from two different GenIF density functions $f_1(x,\overline{\gamma},\eta)$ and $f_2(x,\overline{\gamma},\eta)$, respectively. If the condition $\overline{h}_1(t) \succcurlyeq \overline{h}_2(t)$ and $\overline{R}_1(T) = \overline{R}_2(T)$ hold, then $\overline{R}_1(t|T) \preccurlyeq \overline{R}_2(t|T)$ for each $t > 0$.*

Theorem 1. *The function $\overline{R}(x|t)$ must be increasing, which is a necessary and sufficient condition on $\overline{R}(x|t)$ for $f(x,\overline{\gamma},\eta)$ to belong to a class of distribution with a decreasing failure rate.*

Proof. As stated, for every $t_1 < t_2$, $\overline{R}(x|t_1) \preccurlyeq \overline{R}(x|t_2)$ and $\overline{R}(x|t_1)(\alpha,\beta,\varepsilon) \preccurlyeq \overline{R}(x|t_2)(\alpha,\beta,\varepsilon)$, so, it can be observed that

$$\left(R_{1\mu}(x|t_1)[\alpha,\varepsilon],\ R_{1\lambda}(x|t_1)[\beta,\varepsilon]\right) \preccurlyeq \left(R_{2\mu}(x|t_2)[\alpha,\varepsilon],\ R_{2\lambda}(x|t_2)[\beta,\varepsilon]\right),$$

or

$$R_{1\mu}(x|t_1)[\alpha,\varepsilon] \preccurlyeq R_{2\mu}(x|t_2)[\alpha,\varepsilon], R_{1\lambda}(x|t_1)[\beta,\varepsilon] \preccurlyeq R_{2\lambda}(x|t_2)[\beta,\varepsilon].$$

Equating lower and upper bands to ξ, then

$$R_\mu^\xi(x|t_1)[\alpha,\varepsilon] \preccurlyeq R_\mu^\xi(x|t_2)[\alpha,\varepsilon],$$

$$R_\lambda^\xi(x|t_1)[\beta,\varepsilon] \preccurlyeq R_\lambda^\xi(x|t_2)[\beta,\varepsilon].$$

$$R_\mu^\xi(x\big|t_1)\,[\alpha,\varepsilon] \preccurlyeq R_\mu^\xi(x\big|t_2)\,[\alpha,\varepsilon] \text{ and } R_\lambda^\xi(x\big|t_1)\,[\beta,\varepsilon] \preccurlyeq R_\lambda^\xi(x\big|t_2)\,[\beta,\varepsilon].$$

Hence, R_μ^ξ and R_λ^ξ are increasing functions, and by using the definition of the GenIFCR, we have

$$R_j^\xi(x\Big|t)\,[k,\varepsilon] = \frac{R_j^\xi(x+t)\,[k,\varepsilon]}{R_j^\xi(t)\,[k,\varepsilon]},$$

$$\frac{\partial R_j^\xi(x\big|t)\,[k,\varepsilon]}{\partial t} = \frac{-f_j^\xi(x+t)\,[k,\varepsilon]R_j^\xi(t)\,[k,\varepsilon] + f_j^\xi(t)\,[k,\varepsilon]R_j^\xi(x+t)\,[k,\varepsilon]}{R_j^\xi(t)\,[k,\varepsilon]^2}.$$

Since R_j^ξ is increasing, $\frac{\partial R_j^\xi(x|t)\,[k,\varepsilon]}{\partial t} \geq 0$ and $f_j^\xi(t)\,[k]R_j^\xi(x+t)[k] \geq f_j^\xi(x+t)\,[k]R_j^\xi(t)\,[k]$, then,

$$h_j^\xi(t)\,[k,\varepsilon] \geq h_j^\xi(x+t)\,[k,\varepsilon],$$

or

$$h_\mu^\xi(t)\,[k,\varepsilon] \geq h_\mu^\xi(x+t)\,[k,\varepsilon] \text{ and } h_\lambda^\xi(t)\,[k,\varepsilon] \geq h_\lambda^\xi(x+t)\,[k,\varepsilon],$$

hence proving that $\overline{h}(t) \geq \overline{h}(x+t)$.□

5. Numerical Illustration

The application of a generalized intuitionistic fuzzy Burr XII distribution in traffic control systems offers a sophisticated approach to managing complex traffic dynamics. This distribution, characterized by its ability to represent asymmetric behaviors and adapt to varying data scenarios, can be integrated with intuitionistic fuzzy logic to handle the inherent uncertainties and ambiguities in traffic patterns. This integration allows for better interpretation of traffic data, more efficient traffic flow management, and improved adaptive signal system performance, leading to reduced congestion and enhanced road safety.

Let us consider that a traffic control system is modelled by the generalized intuitionistic fuzzy Burr XII distribution with a fixed shape parameter $\eta = 1$, fuzzified shape parameter $\overline{\gamma} = (0.4,\ 0.5,\ 0.7,\ 0.9,\ 1.1,\ 1.2,\ 1,\ 0,\ 2)$, and special values of $\mu = 1, \lambda = 0, \varepsilon = 2$. The (α,β)-cut sets for GenIFP for $x = 2$ are

$$\begin{aligned} P_\mu(2)[\alpha,2] &= \left[1-(3)^{-0.5-0.2\alpha^2},\,1-(3)^{-1.1+0.2\alpha^2}\right], \\ P_\lambda(2)[\alpha,2] &= \left[1-(3)^{-0.7+0.3\beta^2},\,1-(3)^{-0.9-0.3\beta^2}\right]. \end{aligned} \tag{32}$$

The concept here pertains to the idea of generalized intuitionistic fuzzy probability (GenIFP) functions acting as bands. These bands, referred to as the lower and upper bands, define the range or scope of reliable functioning in a system. The bandwidth, which is the measure of this range, varies depending on specific values set (α,β). This variability implies that the extent of reliable performance, denoted by the bandwidth, can expand or contract based on certain conditions or values. The GenIFP bands for different sets of (α,β) and $x = 2$ are calculated in Table 1.

Table 1. The cut-sets of GenIFP bands for various (α, β).

(α,β)	$P_\mu(2)[\alpha,2]$	$P_\lambda(2)[\beta,2]$	$P(2)[\alpha,\beta,2]$
(0, 1)	[0.2987, 0.5774]	[0.2676, 0.3333]	[0.2987, 0.3333]
(0.3, 0.9)	[0.3046, 0.5660]	[0.2849, 0.3549]	[0.3046, 0.3549]
(0.5, 0.7)	[0.3155, 0.5465]	[0.3166, 0.3943]	[0.3166, 0.3943]
(0.7, 0.5)	[0.3326, 0.5184]	[0.3426, 0.4268]	[0.3426, 0.4268]
(0.9, 0.3)	[0.3568, 0.4832]	[0.3612, 0.4499]	[0.3612, 0.4499]
(1, 0)	[0.3720, 0.4635]	[0.3720, 0.4635]	[0.3720, 0.4635]

Since the GenIFP bands are function of (α, β) and x, then for a special set of $(\alpha = 0, \beta = 1)$, $(\mu = 1,\ \lambda = 0)$ and $x = 2$, the membership and non-membership curves are

$$\begin{aligned} \mu_{P(2)}(x) &= \begin{cases} \left(\frac{\frac{log(1-x)}{log(2)}+0.5}{0.2}\right)^{0.5}, & x \in [0.4226,\ 0.5365], \\ 1, & x \in [0.5365,\ 0.6280], \\ \left(\frac{1.1+\frac{log(1-x)}{log(2)}}{0.2}\right)^{0.5}, & x \in [0.6280,\ 0.7013], \\ 0, & \text{o.w}, \end{cases} \\ \lambda_{P(2)}(x) &= \begin{cases} \left(\frac{0.7+\frac{log(1-x)}{log(2)}}{0.3}\right)^{0.5}, & x \in [0.3556,\ 0.5365], \\ 0, & x \in [0.5365,\ 0.6280], \\ \left(\frac{\frac{log(1-x)}{log(2)}+0.9}{0.3}\right)^{0.5}, & x \in [0.6280,\ 0.7324], \\ 1, & \text{o.w}, \end{cases} \end{aligned} \tag{33}$$

Hence, the functions of membership $\mu_{P(2)}$ and non-membership $\lambda_{P(2)}$ curve of the GenIFP bands are described by the above expressions and graphically shown in Figure 1.

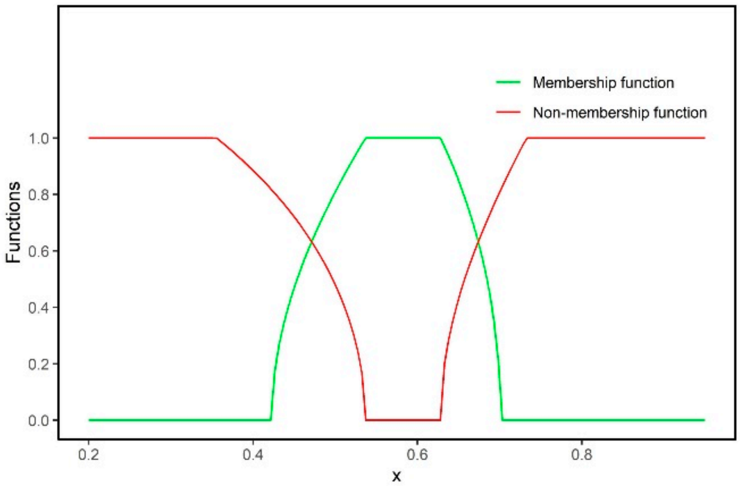

Figure 1. Membership and non-membership curves of GenIFP for $x = 2$.

In the same way, the GenIFR bands for different sets of (α, β) at fixed time $t = 2$ are calculated in Table 2 using the following expressions:

$$\begin{aligned} R_\mu(t)[\alpha, 2] &= \left[(1+t^\eta)^{-(1.1-0.2\alpha^2)},\ (1+t^\eta)^{-(0.5+0.2\alpha^2)}\right], \\ R_\lambda(t)[\beta, 2] &= \left[(1+t^\eta)^{-(0.9+0.3\beta^2)},\ (1+t^\eta)^{-(0.7-0.3\beta^2)}\right]. \end{aligned} \tag{34}$$

Table 2. The cut-sets of GenIFR bands.

(α,β)	$R_{\mu}(t)[\alpha,2]$	$R_{\lambda}(t)[\beta,2]$	$R(t)[\alpha,\beta,2]$
(0, 1)	[0.2987, 0.5774]	[0.2676, 0.3333]	[0.2987, 0.3333]
(0.3, 0.9)	[0.3046, 0.5660]	[0.2849, 0.3549]	[0.3046, 0.3549]
(0.5, 0.7)	[0.3155, 0.5465]	[0.3166, 0.3943]	[0.3166, 0.3943]
(0.7, 0.5)	[0.3326, 0.5184]	[0.3426, 0.4268]	[0.3426, 0.4268]
(0.9, 0.3)	[0.3568, 0.4832]	[0.3612, 0.4499]	[0.3612, 0.4499]
(1, 0)	[0.3720, 0.4635]	[0.3720, 0.4635]	[0.3720, 0.4635]

The functions for membership and non-membership curves of GenIFR bands at special time $t = 2$, $(\mu = 1,\ \lambda = 0)$ and fixed set $(\alpha = 0, \beta = 1)$ are given as

$$\mu_{R(2)}(x) = \begin{cases} \left(\dfrac{1.1+\frac{log(x)}{log(2)}}{0.2}\right)^{0.5}, & x \in [0.2987,\ 0.3720], \\ 1, & x \in [0.3720,\ 0.4635], \\ \left(\dfrac{\frac{log(x)}{log(2)}+0.5}{0.2}\right)^{0.5}, & x \in [0.4635,\ 0.5774], \\ 0, & \text{o.w}, \end{cases}$$
$$\lambda_{R(2)}(x) = \begin{cases} \left(\dfrac{\frac{log(x)}{log(2)}+0.9}{0.3}\right)^{0.5}, & x \in [0.2676,\ 0.3720], \\ 0, & x \in [0.3720,\ 0.4635], \\ \left(\dfrac{0.7+\frac{log(x)}{log(2)}}{0.3}\right)^{0.5}, & x \in [0.4635,\ 0.6444], \\ 1, & \text{o.w}, \end{cases} \tag{35}$$

It can be observed that Table 2 shows the reliability bands for various cut-sets but for a fixed time point $t = 2$. It is necessary to observe the reliability bands for various time points. So, we fixed a cut-set value at $(\alpha = 0, \beta = 1)$ and observed the reliability bands over time t, as shown in Figure 2a.

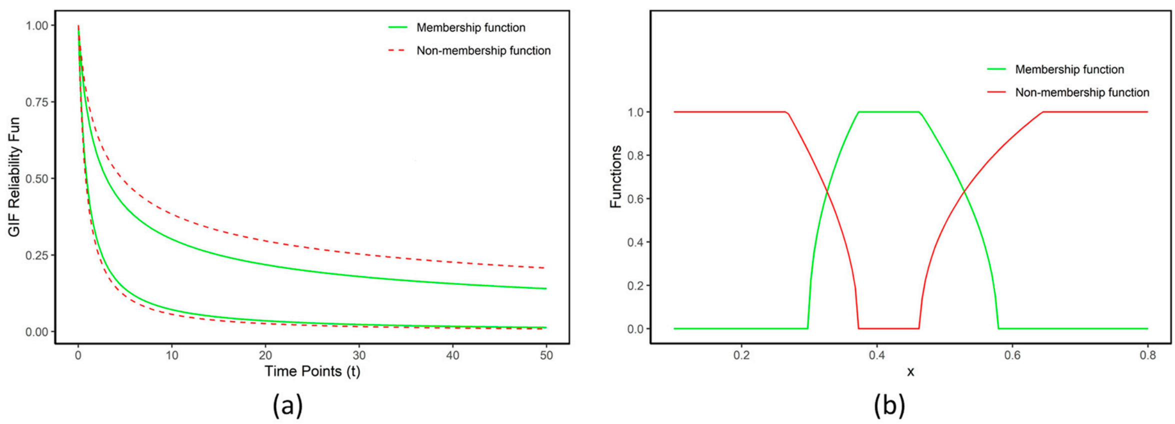

Figure 2. (**a**) The cut-sets of GenIFR bands for $(\alpha = 0, \beta = 1)$ as functions of time t. (**b**) Membership and non-membership curves of GenIFR bands for t = 2.

Similarly, the membership (green solid line) and non-membership (red solid line) functions for GenIFR bands for special time $t = 2$ and cut-set $(\alpha = 0, \beta = 1)$ are shown in Figure 2b.

Figure 3 provides a detailed perspective by presenting additional sets of (α, β) values with respect to time t. These values demonstrate that fuzziness in GenIFR can be reduced by simultaneously increasing α and decreasing β.

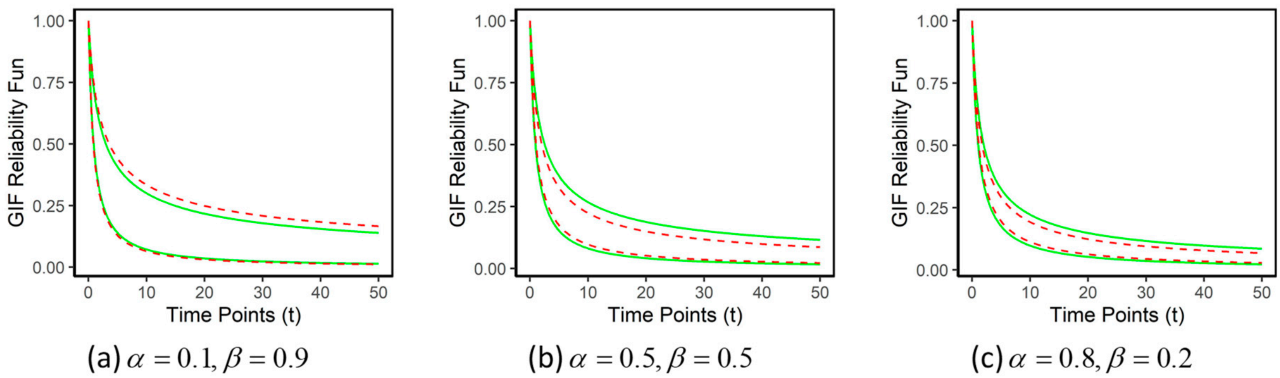

Figure 3. Cut-sets of reliability bands for different (α, β) illustrated with green (membership) and red (non-membership) lines.

The reliability features of generalized intuitionistic fuzzy characteristics (GenIFRC) are influenced by parameter ε. This is illustrated in Figure 4, where the reliability bands for varying ε values are presented with parameters α and β set equal to 0.5. This illustration suggests that as ε increases, the reliability of the system becomes more precise or accurate.

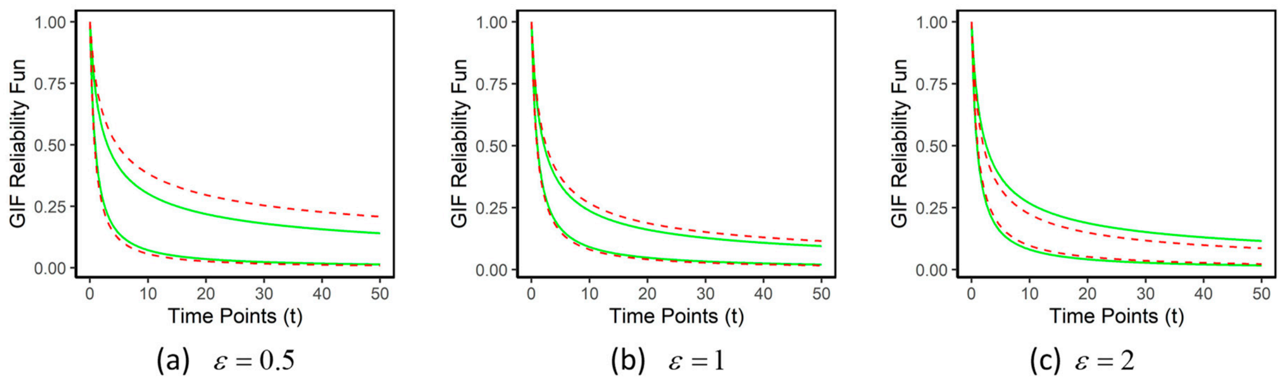

Figure 4. Cut-sets of reliability bands for different ε depicted with green (membership) and red (non-membership) lines.

The cut-sets of GenIFCR bands for various values of (α, β) and time $T = 3$ are obtained using the following relations and presented in Table 3.

$$\begin{aligned} R_\mu\left(t \middle| T\right)[\alpha, 2] &= \left[\left(\frac{1+3^\eta}{1+(t+3)^\eta}\right)^{1.1-0.2\alpha^2}, \left(\frac{1+3^\eta}{1+(t+3)^\eta}\right)^{0.5+0.2\alpha^2}\right], \\ R_\lambda\left(t \middle| T\right)[\beta, 2] &= \left[\left(\frac{1+3^\eta}{1+(t+3)^\eta}\right)^{0.9+0.3\beta^2}, \left(\frac{1+3^\eta}{1+(t+3)^\eta}\right)^{0.7-0.3\beta^2}\right]. \end{aligned} \tag{36}$$

Table 3. The cut-sets of GenIFCR bands.

(α,β)	$R_\mu(t \mid T)[\alpha,2]$	$R_\lambda(t \mid T)[\beta,2]$	$R(t \mid T)[\alpha,\beta,2]$
(0, 1)	[0.6402, 0.8165]	[0.6147, 0.8503]	[0.6147, 0.8503]
(0.3, 0.9)	[0.6449, 0.8106]	[0.6291, 0.8309]	[0.6291, 0.8309]
(0.5, 0.7)	[0.6533, 0.8001]	[0.6541, 0.7991]	[0.6533, 0.8001]
(0.7, 0.5)	[0.6661, 0.7847]	[0.6735, 0.7761]	[0.6661, 0.7847]
(0.9, 0.3)	[0.6836, 0.7646]	[0.6867, 0.7612]	[0.6836, 0.7646]
(1, 0)	[0.6943, 0.7529]	[0.6943, 0.7529]	[0.6943, 0.7529]

Also, the functions of membership and non-membership curves for GenIFCR at $t = 5$, $T = 3$ are

$$\mu_{R(2|3)}(x) = \begin{cases} \left(\frac{1.1+\frac{log(x)}{log(2.25)}}{0.2}\right)^{0.5}, & x \in [0.4098, 0.4820], \\ 1, & x \in [0.4820, 0.5669], \\ \left(\frac{\frac{log(x)}{log(2.25)}+0.5}{0.2}\right)^{0.5}, & x \in [0.5669, 0.6667], \\ 0, & \text{o.w}, \end{cases}$$
$$\lambda_{R(2|3)}(x) = \begin{cases} \left(\frac{\frac{log(x)}{log(2.25)}+0.9}{0.3}\right)^{0.5}, & x \in [0.3779, 0.4820], \\ 0, & x \in [0.4820, 0.5669], \\ \left(\frac{0.7+\frac{log(x)}{log(2.25)}}{0.3}\right)^{0.5}, & x \in [0.5669, 0.7230], \\ 1, & \text{o.w}, \end{cases} \tag{37}$$

The generalized intuitionistic fuzzy conditional reliability (GenIFCR) bands varying with time t are shown in Figure 5a. Also, their membership and non-memberships curves are presented in Figure 5b.

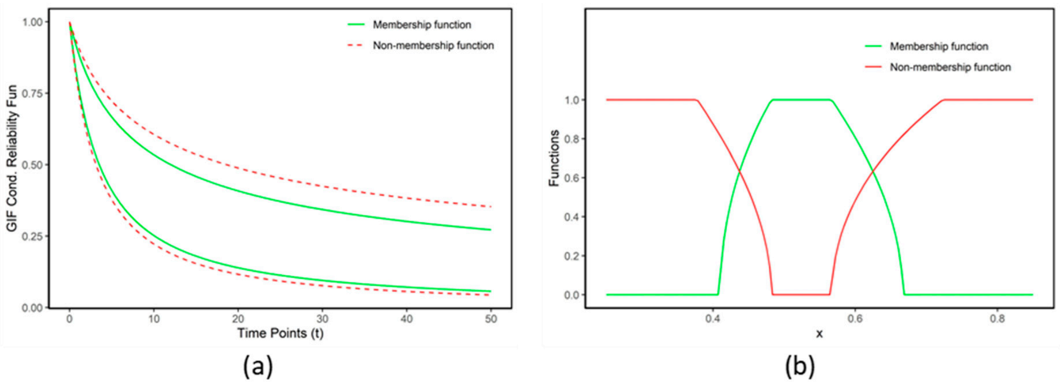

Figure 5. (**a**) The cut-sets of GenIFCR bands for $(\alpha = 0, \beta = 1)$. (**b**) Membership and non-membership curves of GenIFCR.

Finally, the cut-sets of generalized intuitionistic fuzzy hazard (GenIFH) bands based on the following expressions are given in Table 4.

$$h_\mu(t)[\alpha, 2] = \left[\frac{(0.5+0.2\alpha^2)\eta\, t^\eta}{1+t^\eta}, \frac{(1.1-0.2\alpha^2)\eta\, t^\eta}{1+t^\eta}\right],$$
$$h_\lambda(t)[\beta, 2] = \left[\frac{(0.7-0.3\beta^2)\eta\, t^\eta}{1+t^\eta}, \frac{(0.9+0.3\beta^2)\eta\, t^\eta}{1+t^\eta}\right]. \tag{38}$$

Table 4. The cut-sets of GenIFH bands.

(α,β)	$h_\mu(t)[\alpha,2]$	$h_\lambda(t)[\beta,2]$	$h(t)[\alpha,\beta,2]$
(0, 1)	[0.1667, 0.3667]	[0.1333, 0.4000]	[0.1667, 0.3667]
(0.3, 0.9)	[0.1727, 0.3607]	[0.1523, 0.3810]	[0.1727, 0.3607]
(0.5, 0.7)	[0.1833, 0.3500]	[0.1843, 0.3490]	[0.1843, 0.3490]
(0.7, 0.5)	[0.1993, 0.3340]	[0.2083, 0.3250]	[0.2083, 0.3250]
(0.9, 0.3)	[0.2207, 0.3127]	[0.2243, 0.3090]	[0.2243, 0.3090]
(1, 0)	[0.2333, 0.3000]	[0.2333, 0.3000]	[0.2333, 0.3000]

Analogously, the functions of membership and non-membership curves for GenIFH are given as

$$\mu_{h(2)}(x) = \begin{cases} \left(\frac{3x-0.5}{0.2}\right)^{0.5}, & x \in [0.1667,\ 0.2333], \\ 1, & x \in [0.2333,\ 0.3000], \\ \left(\frac{1.1-3x}{0.2}\right)^{0.5}, & x \in [0.3000,\ 0.3667], \\ 0, & \text{o.w}, \end{cases}$$
$$\lambda_{h(2)}(x) = \begin{cases} \left(\frac{0.7-3x}{0.3}\right)^{0.5}, & x \in [0.1333,\ 0.2333], \\ 0, & x \in [0.2333,\ 0.3000], \\ \left(\frac{3x-0.9}{0.3}\right)^{0.5}, & x \in [0.3000,\ 0.4000], \\ 1, & \text{o.w}, \end{cases} \quad (39)$$

In the same way, Figure 6a shows the decreasing GenIFH bands with time t for the case of least vagueness, which is attended for the greatest and lowest values of $\alpha = 0$ and $\beta = 1$, respectively. Also, the curves for membership and non-membership functions of hazard bands are graphically shown in Figure 6b.

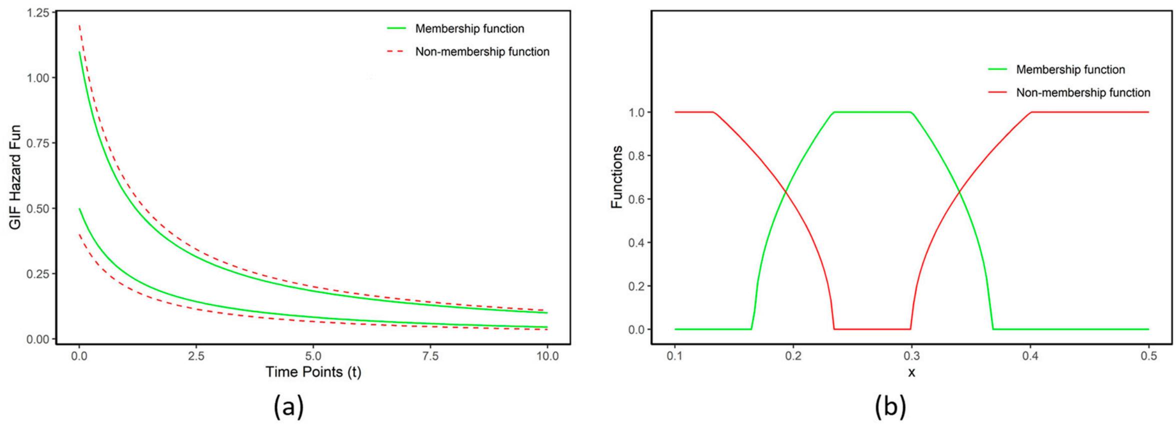

Figure 6. (**a**) The cut-sets of GenIFH bands $(\alpha = 0, \beta = 1)$. (**b**) Membership and non-membership curves of GenIFH.

6. Results and Discussion

This study employs a generalized intuitionistic fuzzy approach to examine the Burr XII lifetime distribution, introducing a novel methodological paradigm. The main goal is to evaluate uncertainty in many aspects of reliability measures in a comprehensive manner. The study achieves this goal by precisely quantifying the inherent fuzziness in one of the essential shape parameters of the Burr XII distribution, indicated as $\overline{\gamma} = (a_1' = 0.4,\ a = 0.5,\ b = 0.7,\ c = 0.9,\ d = 1.1,\ d_1' = 1.2)$. Concerning the fuzzy method, we concentrated on a particular case of the generalized intuitionistic fuzzy framework containing predetermined values for $\mu = 1,\ \lambda = 0$. In this study, the parameter ε is set to a value of 2, and the cut-set approach is used to precisely determine the bands of various reliability measures.

The cut-set bands of the generalized intuitionistic fuzzy probability (GenIFP) bands for various sets are presented in Table 1, which indicates that for $x = 2$, the most reliable band can be found at the highest cut-set of the membership function and the lowest cut-set of the non-membership function. Furthermore, the α-cut of membership and β-cut of non-membership function for generalized intuitionistic fuzzy reliability (GenIFR) bands at time $t = 2$ have been shown in Table 2. It indicates that most precise bands are attended at the greatest α and lowest values of β for fixed time. Since the reliability bands change over time, we have also included the GenIFR bands for $(\alpha = 0, \beta = 1)$ over time t in Figure 2a and a few more sets in Figure 3. Based on Figures 2a and 3, we conclude that the reliability bands lead to a great reduction in the level of uncertainty over time t. Additionally, the membership and non-membership curves for $t = 2$ and $(\alpha = 0, \beta = 1)$ have been graphically shown in Figure 2b. It is important to discuss the significance of parameter ε as it has a substantial impact on the uncertainty in both reliability and probability characteristics. Figure 4 shows that the best reliable bands are obtained at the maximum value of parameter $\varepsilon = 2$.

Another characteristic discussed here is the generalized intuitionistic fuzzy conditional reliability (GenIFCR) bands, which are presented in Table 3 for t = 2 and for varying t in Figure 5a. They exhibit the same trend as the GenIFCR bands. Eventually, we also studied the generalized intuitionistic fuzzy hazard (GenIFH) bands, evaluating them in Table 4 for a fixed time parameter t and displaying results in Figure 6a for varying t. Notably, Table 4 shows improved precision in hazard bands, particularly for the greatest cut-set value for the membership function and the lowest cut-set value for the non-membership function at a fixed time. Also, Figure 6a shows an obvious reduction in bandwidth over time.

In conjunction with the above outcomes, our study encompasses a thorough examination detailing the computation of membership and non-membership functions for each reliability measure. This comprehensive analysis is visually presented in Figures 2b, 5b and 6b, employing a graphical representation with the green line symbolizing the membership function and the red line depicting the non-membership function.

In summary, the findings from Tables 1–4 underscore the significance of optimizing the cut-sets for membership and non-membership, revealing improved precision in probability and diverse reliability measures. Furthermore, the observed trend across all reliability measures indicates a systematic decrease in the value of uncertainty or fuzziness corresponding to increasing time t.

7. Conclusions

This study assesses the fuzzy reliability of the Burr XII lifetime distribution using the notion of generalized intuitionistic fuzzy sets. In the Burr type XII distribution with two parameters, one of the shape parameters remains fixed while the other is treated as a generalized intuitionistic fuzzy number (GenIFN) so as to calculate probability and various reliability measures. The fuzziness or uncertainty of parameters can be assessed using bands formed by the various reliability measures, with each individual pair of cut-set values related to a distinct band width. The results demonstrate that optimal bandwidth precision is reached when the cut-set of membership functions is large, paired by a small cut-set of non-membership functions. In addition, all measures exhibit a declining trend as time progresses. Remarkably, this approach surpasses alternative fuzzy sets by effectively exposing greater levels of ambiguity.

The current study focuses on enhancing the field of system reliability engineering. The presented technique provides a way to evaluate the reliability of different systems, including those organized in series and parallel configurations. In addition, the combination of extended intuitionistic fuzzy approaches with the Hausdorff distance also has the potential to greatly improve models used for spatial and image data analysis for further research and practical use.

Acknowledging certain limitations inherent in applying the generalized intuitionistic fuzzy technique in reliability analysis is imperative. A notable constraint arises in the heightened complexity of modeling and computational processes. This increased complexity poses potential challenges in real-world applications, necessitating careful consideration. The identified difficulties underscore the significance of further research endeavors aimed at overcoming these challenges in the practical domain of reliability analysis.

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Abbreviations

The following abbreviations are used in this manuscript:

GenIFSs	Generalized Intuitionistic Fuzzy Sets
GenIFN	Generalized Intuitionistic Fuzzy Number
GenIF	Generalized Intuitionistic Fuzzy

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Atanassov, K.T.; Stoeva, S. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
3. Atanassov, K.T. Type-1 fuzzy sets and intuitionistic fuzzy sets. *Algorithms* **2017**, *10*, 106. [CrossRef]
4. Zeng, S.; Chen, S.-M.; Kuo, L.-W. Multiattribute decision making based on novel score function of intuitionistic fuzzy values and modified VIKOR method. *Inf. Sci.* **2019**, *488*, 76–92. [CrossRef]
5. Yager, R.R. Pythagorean fuzzy subsets. In Proceedings of the 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), Edmonton, AL, USA, 24–28 June 2013.
6. Yager, R.R. Generalized orthopair fuzzy sets. *IEEE Trans. Fuzzy Syst.* **2016**, *25*, 1222–1230. [CrossRef]
7. Yager, R.R. Properties and applications of pythagorean fuzzy sets. In *Imprecision and Uncertainty in Information Representation and Processing: New Tools Based on Intuitionistic Fuzzy Sets and Generalized Nets*; Springer: Berlin/Heidelberg, Germany, 2016; pp. 119–136.
8. Liao, H.; Xu, Z. Priorities of intuitionistic fuzzy preference relation based on multiplicative consistency. *IEEE Trans. Fuzzy Syst.* **2014**, *22*, 1669–1681. [CrossRef]
9. Liu, F.; Tan, X.; Yang, H.; Zhao, H. Decision making based on intuitionistic fuzzy preference relations with additive approximate consistency. *J. Intell. Fuzzy Syst.* **2020**, *39*, 4041–4058. [CrossRef]
10. Yang, W.; Jhang, S.T.; Fu, Z.W.; Xu, Z.S.; Ma, Z.M. A novel method to derive the intuitionistic fuzzy priority vectors from intuitionistic fuzzy preference relations. *Soft Comput.* **2021**, *25*, 147–159. [CrossRef]
11. Kumar, M.; Singh, S.; Kumar, D. System reliability analysis based on Pythagorean fuzzy set. *Int. J. Math. Oper. Res.* **2023**, *24*, 253–285. [CrossRef]
12. Panchal, D. Reliability analysis of turbine unit using Intuitionistic Fuzzy Lambda-Tau approach. *Rep. Mech. Eng.* **2023**, *4*, 47–61. [CrossRef]
13. Song, Y.; Wang, X.; Zhu, J.; Lei, L. Sensor dynamic reliability evaluation based on evidence theory and intuitionistic fuzzy sets. *Appl. Intell.* **2018**, *48*, 3950–3962. [CrossRef]
14. Wang, Z.; Huang, H.-Z.; Li, Y.; Pang, Y.; Xiao, N.-C. An approach to system reliability analysis with fuzzy random variables. *Mech. Mach. Theory* **2012**, *52*, 35–46. [CrossRef]
15. Fang, Y.; Kan, S.; Yang, M.; Xie, C.; Pei, X. Application of fuzzy multi-state Bayesian networks in redundant hydraulic system's reliability analysis. *Comput. Integr. Manuf. Syst.* **2015**, *21*, 1856–1864.
16. Levitin, G.; Xing, L. Reliability and performance of multi-state systems with propagated failures having selective effect. *Reliab. Eng. Syst. Saf.* **2010**, *95*, 655–661. [CrossRef]
17. Xing, L.; Dai, Y. A new decision-diagram-based method for efficient analysis on multistate systems. *IEEE Trans. Dependable Secur. Comput.* **2008**, *6*, 161–174. [CrossRef]
18. Kumar, A.; Ram, M.; Goyal, N.; Bisht, S.; Kumar, S.; Pant, R. Analysis of Fuzzy Reliability of the System Using Intuitionistic Fuzzy Set. In *In the Intelligent Communication, Control and Devices: Proceedings of ICICCD 2020*; Springer: Berlin/Heidelberg, Germany, 2021; pp. 371–378.
19. Akbari, M.G.; Hesamian, G. Time-dependent intuitionistic fuzzy system reliability analysis. *Soft Comput.* **2020**, *24*, 14441–14448. [CrossRef]
20. Bhandari, A.S.; Kumar, A.; Ram, M. Fuzzy reliability evaluation of complex systems using intuitionistic fuzzy sets. In *Advancements in Fuzzy Reliability Theory*; IGI Global: Hershey, PA, USA, 2021; pp. 166–180.
21. Chachra, A.; Kumar, A.; Ram, M.; Triantafyllou, I.S. Statistical Fuzzy Reliability Assessment of a Blended System. *Axioms* **2023**, *12*, 419. [CrossRef]
22. Faulin, J.; Juan, A.A.; Martorell, S.; Ramírez-Márquez, J.-E. *Simulation Methods for Reliability and Availability of Complex Systems*; Springer: Berlin/Heidelberg, Germany, 2010; Volume 315.
23. Glushkov, A. *Methods of a Chaos Theory*; Astroprint Odessa: Odesa, Ukraine, 2012.
24. Shalizi, C.R. Methods and techniques of complex systems science: An overview. *Complex Syst. Sci. Biomed.* **2006**, 33–114. [CrossRef]

25. Cho, S.Y. Fuzzy System Reliability Analysis Using q-rung Orthopair Fuzzy Sets. *J. Knowl. Inf. Technol. Syst. (JKITS)* **2020**, *15*, 925–932.
26. Güneri, B.; Deveci, M. Evaluation of supplier selection in the defense industry using q-rung orthopair fuzzy set based EDAS approach. *Expert Syst. Appl.* **2023**, *222*, 119846. [CrossRef]
27. Khan, M.J.; Kumam, P.; Shutaywi, M.; Kumam, W. Improved Knowledge Measures for q-Rung Orthopair Fuzzy Sets. *Int. J. Comput. Intell. Syst.* **2021**, *14*, 1700–1713. [CrossRef]
28. Mateen, F.; Munir, A.; Nadeem, M. Software Reliability Growth Model Selection by using VIKOR Method Based on q-Rung Orthopair Fuzzy Entropy and Distance Measures. *Res. Sq.* **2023**. *preprint*. [CrossRef]
29. Hakim, A.; Fithriani, I.; Novita, M. Properties of Burr distribution and its application to heavy-tailed survival time data. Journal of Physics: Conference Series. In Proceedings of the 2nd Basic and Applied Sciences Interdisciplinary Conference 2018 (2nd BASIC 2018), Depok, Indonesia, 3–4 August 2018; IOP Publishing: Bristol, UK, 2021; p. 012016.
30. Kumar, D. The Burr type XII distribution with some statistical properties. *J. Data Sci.* **2017**, *15*, 509–533. [CrossRef]
31. Zayed, M.A.; Hassan, A.S.; Almetwally, E.M.; Aboalkhair, A.M.; Al-Nefaie, A.H.; Almongy, H.M. A Compound Class of Unit Burr XII Model: Theory, Estimation, Fuzzy, and Application. *Sci. Program.* **2023**, *2023*, 4509889. [CrossRef]
32. Almuhayfith, F.E.; Darwish, J.A.; Alharbi, R.; Marin, M. Burr XII Distribution for Disease Data Analysis in the Presence of a Partially Observed Failure Mode. *Symmetry* **2022**, *14*, 1298. [CrossRef]
33. Hussein, A.A.; Al-Mosawi, R. Estimating the Parameters of Burr Type XII Distribution with Fuzzy Observations. *REVSTAT-Stat. J.* **2023**, *21*, 405–424.
34. Aiachi, H.; Chadli, A.; Talhi, H. BAYESIAN INFERENCE of THREE-PARAMETER BURR XII BASED on PROGRESSIVELY TYPE II CENSORED OBSERVATIONS. *J. Appl. Probab.* **2022**, *17*, 071–092.
35. Al-Shomrani, A.A. An improvement in maximum likelihood estimation of the Burr XII distribution parameters. *AIMS Math.* **2022**, *7*, 17444–17460. [CrossRef]
36. EL-Sagheer, R.M.; Mahmoud, M.A.; Hasaballah, H.M. Bayesian inference for the randomly-censored three-parameter Burr XII distribution. *Appl. Math.* **2020**, *14*, 261–271.
37. Hussain, W.J.; Akkar, A.A.; Rasheed, H.A. Comparison of Robust and Bayesian Methods for Estimating the Burr Type XII Distribution. *Int. J. Adv. Sci. Eng. Inf. Technol.* **2020**, *10*. [CrossRef]
38. Ismail, N.H.; Khalid, Z.M. Estimation of 2-and 3-parameter Burr type XII distribution using EM algorithm. *Malays. J. Fundam. Appl. Sci.* **2014**, *10*. [CrossRef]
39. Kalam, A.; Weihu, C.; Rahman, A.; Ahmad, M. ESTIMATION of FRECHET PARAMETERS with TIME-CENSORED DATA in ACCELERATED LIFE TESTING UTILISING the GEOMETRIC PROCESS. *Reliab. Theory Appl.* **2023**, *18*, 631–641. [CrossRef]
40. Jamkhaneh, E.B.; Nadarajah, S. A new generalized intuitionistic fuzzy set. *Hacet. J. Math. Stat.* **2015**, *44*, 1537–1551. [CrossRef]
41. Shabani, A.; Jamkhaneh, E.B. A new generalized intuitionistic fuzzy number. *J. Fuzzy Set Valued Anal.* **2014**, *24*, 1–10. [CrossRef]
42. Mondal, T.K.; Samanta, S. Generalized intuitionistic fuzzy sets. *J. Fuzzy Math.* **2002**, *10*, 839–862.
43. Ebrahimnejad, A.; Jamkhaneh, E.B. System reliability using generalized intuitionistic fuzzy Rayleigh lifetime distribution. *Appl. Appl. Math. Int. J. (AAM)* **2018**, *13*, 7.
44. Jamkhaneh, E.B. Reliability estimation under the fuzzy environments. *J. Math. Comput. Sci.* **2012**. [CrossRef]
45. Roohanizadeh, Z.; Baloui Jamkhaneh, E.; Deiri, E. The reliability analysis based on the generalized intuitionistic fuzzy two-parameter Pareto distribution. *Soft Comput.* **2023**, *27*, 3095–3113. [CrossRef]
46. Kalam, A.; Cheng, W.; Du, Y.; Zhao, X. Statistical Fuzzy Reliability Analysis: An Explanation with Generalized Intuitionistic Fuzzy Lomax Distribution. *Symmetry* **2023**, *15*, 2054. [CrossRef]