physical sciences forum

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Proceeding Paper

New Black Hole Solutions in $\mathcal{N}=2$ and $\mathcal{N}=8$ Gauged Supergravity †

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† Presented at the 1st Electronic Conference on Universe, 22–28 February 2021; Available online: https://ecu2021.sciforum.net/.

Abstract: We present a $\mathcal{N}=2$ supergravity model that interpolates between all the single dilaton truncations of the gauged $\mathrm{SO}(8)$, $\mathcal{N}=8$ supergravity. We provide new explicit non-extremal charged black hole solutions and their supersymmetric limits, exploiting the non-trivial transformation of the Fayet–Iliopoulos parameters under electromagnetic duality to connect the electric and the magnetic configurations. We also provide the asymptotic charges, thermodynamics and boundary conditions of these black hole configurations. We then construct a new supersymmetric truncation of the maximal supergravity, with this new sector featuring non-extremal and supersymmetric black holes.

Keywords: supergravity; black hole solutions; black hole thermodynamics; BPS black holes

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Citation: Gallerati, A. New Black Hole Solutions in $\mathcal{N}=2$ and $\mathcal{N}=8$ Gauged Supergravity. *Phys. Sci. Forum* **2021**, *2*, 50. https://doi.org/10.3390/ECU2021-09297

Academic Editor: Gonzalo Olmo

Published: 22 February 2021

1. Introduction

Anti-de Sitter (AdS) black hole configurations are an amazing and stimulating field of research due to the role they play in high energy theory as well as in the phenomenology of the AdS/CFT conjecture [1]. In particular, classical AdS black hole solutions can reveal specific features about the dual, strongly coupled gauge theory, therefore providing a possible description of many condensed matter phenomena.

The thermodynamic properties of AdS black holes were first analyzed in [2] and subsequently extended to various other AdS black hole configurations [3–6]. These studies described how black holes have specific phase structures, giving rise to critical phenomena analogous to other common thermodynamic systems. Of particular interest are black hole solutions preserving a certain amount of supersymmetry, since they allow mapping of a weak (string) coupling description of the system thermodynamics to the strong-coupling regime, where a formulation in terms of a black hole configuration is valid [7]. Moreover, these solutions can be exploited to study the BPS attractor flows in AdS spacetime [8–21].

In the following, we will discuss the new exact charged hairy black hole solutions in gauged $\mathcal{N}=2$, $D=4$ supergravity of [22] (these new solutions generalize the uncharged configurations of [23,24]), interpolating between four single dilaton truncations of the maximal $\mathrm{SO}(8)$, $\mathcal{N}=8$ supergravity. In particular, we will provide the explicit expressions for two new and different families of non-extremal black hole solutions, analyzing the duality relation between them. We will then investigate the thermodynamic properties of our new solutions, together with the analysis of the related boundary conditions. We will also study which conditions of the parameters give rise to BPS configuration ones. Finally, we will characterize certain models within the general class under consideration as consistent truncations of the maximal $\mathcal{N}=8$, $D=4$ gauged supergravity.

2. Results

Let us consider an extended $\mathcal{N}=2$ supergravity theory in $D=4$, coupled to n_{v} vector multiplets and no hypermultiplets, in the presence of Fayet–Iliopoulos (FI) terms.

The model describes $n_v + 1$ vector fields A^Λ_μ, $(\Lambda = 0, \ldots, n_v)$ and $n_s = n_v$ complex scalar fields z^i $(i = 1, \ldots, n_s)$. The bosonic gauged Lagrangian has the explicit form

$$\frac{1}{\sqrt{-g}}\,\mathcal{L}_{\text{BOS}} \;=\; -\frac{R}{2} + g_{i\bar{\jmath}}\,\partial_\mu z^i\,\partial^\mu \bar{z}^{\bar{\jmath}} + \frac{1}{4}\,\mathcal{I}_{\Lambda\Sigma}(z,\bar{z})\,F^\Lambda_{\mu\nu}\,F^{\Sigma\,\mu\nu} + \frac{1}{8\,\sqrt{-g}}\,\mathcal{R}_{\Lambda\Sigma}(z,\bar{z})\,\varepsilon^{\mu\nu\rho\sigma}\,F^\Lambda_{\mu\nu}\,F^\Sigma_{\rho\sigma} - V(z,\bar{z})\,, \tag{1}$$

where $g = \det(g_{\mu\nu})$ and with the $n_v + 1$ vector field strengths:

$$F^\Lambda_{\mu\nu} \;=\; \partial_\mu A^\Lambda_\nu - \partial_\nu A^\Lambda_\mu\,. \tag{2}$$

The n_s complex scalars z^i couple to the vector fields in a non-minimal way through the real symmetric matrices $\mathcal{I}_{\Lambda\Sigma}(z,\bar{z})$, $\mathcal{R}_{\Lambda\Sigma}(z,\bar{z})$ and span a special Kähler manifold $\mathscr{M}_{\text{SK}}$, while the scalar potential $V(z,\bar{z})$ originates from electric–magnetic FI terms.

2.1. The Model

Let us now focus on a $\mathcal{N} = 2$ theory with no hypermultiplets and a single vector multiplet $(n_v = 1)$, with a complex scalar field z. The geometry of the special Kähler manifold is characterized by a prepotential of the form:

$$\mathcal{F}(\mathcal{X}^\Lambda) \;=\; -\frac{i}{4}\,(\mathcal{X}^0)^n\,(\mathcal{X}^1)^{2-n}\,, \tag{3}$$

with $\mathcal{X}^\Lambda(z)$ being components of a holomorphic section of the symplectic bundle over the manifold and the coordinate z being identified with the ratio $\mathcal{X}^1/\mathcal{X}^0$, in a local patch in which $\mathcal{X}^0 \neq 0$. For special values of n, the model is a consistent truncation of the STU model (the STU model [25–27] is a $\mathcal{N} = 2$ supergravity coupled to $n_v = 3$ vector multiplets and characterized, in a suitable symplectic frame, by the prepotential $\mathcal{F}_{\text{STU}}(\mathcal{X}^\Lambda) = -\frac{i}{4}\sqrt{\mathcal{X}^0\,\mathcal{X}^1\,\mathcal{X}^2\,\mathcal{X}^3}$, together with symmetric scalar manifold of the form $\mathscr{M}_{\text{STU}} = (\text{SL}(2,\mathbb{R})/\text{SO}(2))^3$ spanned by the three complex scalars $z^i = \mathcal{X}^i/\mathcal{X}^0$ $(i = 1,2,3)$; this model is in turn a consistent truncation of the maximal $\mathcal{N} = 8$ theory in four dimensions with SO(8) gauge group [28–30]).

If we set $\mathcal{X}^0 = 1$, the holomorphic section Ω^M of the model reads:

$$\Omega^M = \begin{pmatrix} 1 \\ z \\ -\frac{i}{4}\,n\,z^{2-n} \\ -\frac{i}{4}\,(2-n)\,z^{1-n} \end{pmatrix}, \tag{4}$$

and the Kähler potential $\mathcal{K}$ has the expression

$$e^{-\mathcal{K}} \;=\; \frac{1}{4}\,z^{1-n}\,\big(n\,z - (n-2)\,\bar{z}\big) + \text{c.c.} \tag{5}$$

The theory is deformed with the introduction of abelian electric–magnetic FI terms defined by a constant symplectic vector $\theta_M = (\theta_1, \theta_2, \theta_3, \theta_4)$, encoding the gauge parameters of the model. The scalar potential $V(z,\bar{z})$ can then be obtained from:

$$V \;=\; \Big(g^{i\bar{\jmath}}\,\mathcal{U}^M_i\,\bar{\mathcal{U}}^N_{\bar{\jmath}} - 3\,\mathcal{V}^M\,\bar{\mathcal{V}}^N\Big)\theta_M\,\theta_N \;=\; -\frac{1}{2}\,\theta_M\,\mathcal{M}^{MN}\,\theta_N - 4\,\mathcal{V}^M\,\bar{\mathcal{V}}^N\theta_M\,\theta_N\,, \tag{6}$$

where $\mathcal{V}^M = e^{\mathcal{K}/2}\,\Omega^M$, $\mathcal{U}^M_i = \mathcal{D}_i\,\mathcal{V}^M$ and $\mathcal{M}(\phi)$ are the symplectic, symmetric and negative definite matrices encoding the non-minimal couplings of the scalars to the vector fields of the theory.

Writing $z = e^{\lambda\phi} + i\chi$, the truncation to the dilaton field ϕ (i.e., $\chi = 0$) is consistently provided:

$$(2-n)\,\theta_1\,\theta_3 - n\,\theta_2\,\theta_4 = 0\,, \tag{7}$$

and the metric restricted to the dilaton reads:

$$ds^2 = 2\,g_{z\bar z}\,dz\,d\bar z\,\Big|_{\substack{\chi=0\\ d\chi=0}} = \frac{1}{2}\lambda^2\,n\,(2-n)\,d\phi^2\,, \tag{8}$$

and is positive provided $0 < n < 2$. Choosing $\lambda = \sqrt{\frac{2}{n\,(2-n)}}$ the kinetic term for ϕ is then canonically normalized. As a function of the dilaton only, the scalar potential has the following explicit form:

$$\begin{aligned} V(\phi) = &-2\,e^{\lambda\,\phi\,(n-2)}\left(\frac{2\,n-1}{n}\,\theta_1^2 + 4\,\theta_1\,\theta_2\,e^{\lambda\,\phi} + \frac{2\,n-3}{n-2}\,\theta_2^2\,e^{2\,\lambda\,\phi}\right) - \\ &-\frac{1}{8}\,e^{-\lambda\,\phi\,(n-2)}\left((2\,n-1)\,n\,\theta_3^2 - 4\,\theta_3\,\theta_4\,n\,(n-2)\,e^{-\lambda\,\phi} + (n-2)\,(2\,n-3)\,\theta_4^2\,e^{-2\,\lambda\,\phi}\right). \end{aligned} \tag{9}$$

2.1.1. Redefinitions

Let us now make the shift

$$\phi \to \phi - \frac{2\,\nu}{\lambda\,(\nu+1)}\,\log(\theta_2\,\xi)\,, \tag{10}$$

and redefine the FI terms as:

$$\theta_1 = \frac{\nu+1}{\nu-1}\,\theta_2^{-\frac{\nu-1}{\nu+1}}\,\xi^{-\frac{2\nu}{\nu+1}}\,, \qquad \theta_3 = 2\,\alpha(\xi\,\theta_2)^{\frac{\nu-1}{\nu+1}}\,s\,, \qquad \theta_4 = \frac{2\,\alpha}{\theta_2\,\xi\,s}\,, \tag{11}$$

having defined the quantity $\nu = (n-1)^{-1}$ and having also introduced the parameters α, s and

$$\xi = \frac{2\,L\,\nu}{\nu-1}\,\frac{1}{\sqrt{1-\alpha^2\,L^2}}\,, \tag{12}$$

expressed in terms of the AdS radius L. Let us recall that the truncation to the dilaton is consistent provided Equation (7) is satisfied and, in light of the new parametrization (11), this condition requires

$$(s^2-1)\,(\nu^2-1)\,\alpha\,\sqrt{1-L^2\,\alpha^2} = 0\,, \tag{13}$$

which is solved, excluding values $n = 0$ and $n = 2$, either for pure electric FI terms ($\alpha = 0$) or for $s = \pm 1$. Since we are interested in dyonic FI terms, we shall restrict ourselves to the latter case.

After the shift (10), the scalar field z is expressed as

$$z = (\theta_2\,\xi)^{-\frac{2\nu}{\nu+1}}\,e^{\lambda\,\phi}\,, \tag{14}$$

and the same redefinition for the potential (with $s = \pm 1$) yields

$$\begin{aligned} V(\phi) = &-\frac{\alpha^2}{\nu^2}\left(\frac{(\nu-1)(\nu-2)}{2}\,e^{-\phi\,\ell\,(\nu+1)} + 2(\nu^2-1)\,e^{-\phi\,\ell} + \frac{(\nu+1)(\nu+2)}{2}\,e^{\phi\,\ell\,(\nu-1)}\right) + \\ &+\frac{\alpha^2-L^{-2}}{\nu^2}\left(\frac{(\nu-1)(\nu-2)}{2}\,e^{\phi\,\ell\,(\nu+1)} + 2(\nu^2-1)\,e^{\phi\,\ell} + \frac{(\nu+1)(\nu+2)}{2}\,e^{-\phi\,\ell\,(\nu-1)}\right), \end{aligned} \tag{15}$$

where $\ell = \dfrac{\lambda}{\nu}$ and having disposed of θ_2 by the above redefinitions.

After the restriction to the dilaton truncation, the matrix $\mathcal{R}_{\Lambda\Sigma}$ vanishes and the action has the form

$$\mathscr{S} \;=\; \frac{1}{8\pi G}\int d^4x\,\sqrt{-g}\left(-\frac{R}{2}+\frac{\partial_\mu\phi\,\partial^\mu\phi}{2}+\frac{1}{4}\,\mathcal{I}_{\Lambda\Sigma}(\phi)\,F^{\Lambda}_{\mu\nu}\,F^{\Sigma\,\mu\nu}-V(\phi)\right), \tag{16}$$

If we define the canonically normalized gauge fields

$$\bar{F}^1 = \frac{1}{2}\sqrt{\frac{1+\nu}{\nu}}\,(\theta_2\,\xi)^{\frac{1-\nu}{1+\nu}}\,F^1\,, \qquad \bar{F}^2 = \frac{1}{2}\sqrt{\frac{-1+\nu}{\nu}}\,(\theta_2\,\xi)\,F^2\,, \tag{17}$$

the action can be expressed as

$$\mathscr{S} \;=\; \frac{1}{8\pi G}\int d^4x\,\sqrt{-g}\left(-\frac{R}{2}+\frac{\partial_\mu\phi\,\partial^\mu\phi}{2}-\frac{1}{4}\,e^{(-1+\nu)\,\ell\,\phi}\left(\bar{F}^1\right)^2-\frac{1}{4}\,e^{-(1+\nu)\,\ell\,\phi}\left(\bar{F}^2\right)^2-V(\phi)\right). \tag{18}$$

2.2. Hairy Black Hole Solutions

Now, we construct two distinct families of solutions, which we refer to as electric and magnetic, respectively.

2.2.1. Family 1—Electric Solutions

A first family of solutions is given by

$$\phi = -\ell^{-1}\ln(x)\,, \qquad \bar{F}^1_{tx} = Q_1\,x^{-1+\nu}, \qquad \bar{F}^2_{tx} = Q_2\,x^{-1-\nu}, \qquad \Upsilon(x) = \frac{x^{\nu-1}\,\nu^2 L^2}{\eta^2\,(x^\nu-1)^2}\,, \tag{19a}$$

$$\begin{aligned} f(x) \;=\; &\frac{x^{2-\nu}\,\eta^2\,(x^\nu-1)^2}{\nu^2}\,\frac{k}{L^2}+\alpha^2 L^2\left(-1+\frac{x^2}{\nu^2}\,\big((\nu+2)\,x^{-\nu}-(\nu-2)\,x^\nu+\nu^2-4\big)\right)+\\ &+1+\frac{x^{2-\nu}\,\eta^2\,(x^\nu-1)^3}{\nu^3 L^2}\left(\frac{Q_1^2}{(\nu+1)}-\frac{Q_2^2}{(\nu-1)}\,x^{-\nu}\right), \end{aligned} \tag{19b}$$

$$ds^2 \;=\; \Upsilon(x)\left(f(x)\,dt^2-\frac{\eta^2}{f(x)}\,dx^2-L^2\,d\Sigma_k\right), \tag{19c}$$

where $d\Sigma_k^2 = d\theta^2+\frac{\sin^2(\sqrt{k}\,\theta)}{k}\,d\varphi^2$ is the metric on the $2D$-surfaces $\Sigma_k = \{\mathbb{S}^2, \mathbb{H}^2, \mathbb{R}^2\}$ (sphere, hyperboloid and flat space) with constant scalar curvature $R = 2\,k$.

Boundary Conditions, Mass and Thermodynamics for the Electric Solutions

To make contact with AdS canonical coordinates, we consider the following fall-off for the metric function:

$$\Upsilon(x) \;=\; \frac{r^2}{L^2}+O\left(r^{-2}\right). \tag{20}$$

The change in coordinates that provides the above asymptotic behavior is given by

$$x \;=\; 1\pm\left(\frac{L^2}{\eta\,r}+L^6\,\frac{1-\nu^2}{24\,(\eta\,r)^3}\right)+L^8\,\frac{\nu^2-1}{24\,(\eta\,r)^4}\,, \tag{21}$$

where the $+$ $(-)$ sign holds for $x>1$ $(x<1)$. The corresponding asymptotic behavior of the scalar field is

$$\phi \;=\; L^2\,\frac{\phi_0}{r}+L^4\,\frac{\phi_1}{r^2}+O\left(r^{-3}\right) \;=\; \mp L^2\,\frac{1}{\ell\,\eta\,r}+L^4\,\frac{1}{2\,\ell\,\eta^2\,r^2}+O\left(r^{-3}\right), \tag{22}$$

with normalized ϕ_0 and ϕ_1, to match their conformal and engineering dimension. The coefficients of the leading and subleading terms in the scalar asymptotic expansion read

$$\phi_0 = \mp\frac{1}{\ell\,\eta}\,, \qquad \phi_1 = \frac{\ell}{2}\,\phi_0^2\,, \tag{23}$$

and correspond to AdS invariant boundary conditions, the boundary conformal symmetry therefore being preserved. The asymptotic expansion of the spacetime (19) reads

$$g_{tt} \;=\; \frac{r^2}{L^2} + k - \frac{\mu_{\rm E}\,L^4}{r} + O\Big(r^{-2}\Big)\,, \qquad g_{rr} \;=\; -\frac{L^2}{r^2} - L^6\,\frac{k\,L^{-2} + \frac{1}{2}\,\phi_0^2}{r^4} + O\Big(r^{-5}\Big)\,, \tag{24}$$

where

$$\mu_{\rm E} = \pm\left(\frac{\nu^2-4}{3\,\eta^3}\,\alpha^2\,L^2 - \frac{k}{\eta\,L^2} + \frac{Q_2^2}{\eta\,(\nu-1)\,L^2} - \frac{Q_1^2}{\eta\,(\nu+1)\,L^2}\right). \tag{25}$$

The black hole mass can be read off from the above expansion [31–33] and reads

$$M_{\rm E} \;=\; L^4\,\frac{\mu_{\rm E}\,\sigma_k}{8\pi G}\,, \tag{26}$$

where $\sigma_k = \int d\Sigma_k$, while the temperature and the entropy are given by

$$T \;=\; \frac{|f(x)'|}{4\pi\,\eta}\bigg|_{x=x_+}\,, \qquad S \;=\; \frac{L^2\,\Upsilon(x_+)\,\sigma_k}{4\,G}\,, \tag{27}$$

where $f(x_+) = 0$. The physical charges and electric potentials are

$$q_1 = \frac{L^2\,Q_1\,\sigma_k}{8\pi G\,\eta}\,, \qquad \Phi_1^{\rm E} = Q_1\,\frac{x_+^\nu - 1}{\nu}\,; \qquad q_2 = \frac{L^2\,Q_2\,\sigma_k}{8\pi G\,\eta}\,, \qquad \Phi_2^{\rm E} = Q_2\,\frac{1-x_+^{-\nu}}{\nu}\,, \tag{28}$$

and it is possible to verify that these quantities satisfy the first law of thermodynamics:

$$dM_{\rm E} \;=\; T\,dS + \Phi_1^{\rm E}\,dq_1 + \Phi_2^{\rm E}\,dq_2\,. \tag{29}$$

2.2.2. Family 2—Magnetic Solutions

A second family of solutions is given by

$$\phi = \ell^{-1}\ln(x)\,, \qquad \bar{F}^1_{\theta\varphi} = P_1\,\frac{\sin(\sqrt{k}\,\theta)}{\sqrt{k}}\,, \qquad \bar{F}^2_{\theta\varphi} = P_2\,\frac{\sin(\sqrt{k}\,\theta)}{\sqrt{k}}\,, \qquad \Upsilon(x) = \frac{x^{\nu-1}\,\nu^2\,L^2}{\eta^2\,(x^\nu-1)^2}\,, \tag{30a}$$

$$\begin{aligned} f(x) \;=\; & \frac{x^{2-\nu}\,\eta^2\,(x^\nu-1)^2}{\nu^2}\,\frac{k}{L^2} + \Big(1-\alpha^2L^2\Big)\left(-1 + \frac{x^2}{\nu^2}\,\Big(\,(\nu+2)\,x^{-\nu} - (\nu-2)\,x^\nu + \nu^2 - 4\,\Big)\right) + \\ & + 1 + \frac{x^{2-\nu}\,\eta^4\,(x^\nu-1)^3}{\nu^3\,L^6}\left(\frac{P_1^2}{(\nu+1)} - \frac{P_2^2}{(\nu-1)}\,x^{-\nu}\right), \end{aligned} \tag{30b}$$

$$ds^2 \;=\; \Upsilon(x)\left(f(x)\,dt^2 - \frac{\eta^2}{f(x)}\,dx^2 - L^2\,d\Sigma_k\right). \tag{30c}$$

The electric and magnetic solutions are related to each other by means of electromagnetic duality

$$\phi \;\to\; -\phi\,, \qquad \alpha^2 \;\to\; L^{-2} - \alpha^2\,, \tag{31}$$

and corresponding transformation of the electromagnetic fields. In each family, the asymptotic region is located at the pole of order 2 of the conformal factor $\Upsilon(x)$, namely $x = 1$. The geometry and scalar field are singular at $x = 0$ and $x = \infty$ and, therefore, the configuration contains two disjoint geometries given by $x \in (1, \infty)$ or $x \in (0, 1)$.

Boundary Conditions, Mass and Thermodynamics for the Magnetic Solutions

The metric of the magnetic family can be obtained from the electric solutions by means of the transformation

$$Q_i \;\to\; \frac{\eta}{L^2}\,P_i\,, \qquad\qquad \alpha^2 \;\to\; L^{-2}-\alpha^2\,, \tag{32}$$

therefore, all the discussions on mass and thermodynamics carry on from the electric to the magnetic case. The main difference is that the scalar boundary leading and subleading terms now satisfy

$$\phi_1 = -\frac{\ell}{2}\,\phi_0^2\,, \tag{33}$$

which correspond again to AdS invariant boundary conditions. We now define

$$\mu_{\mathrm{M}} = \pm\left(\frac{\nu^2-4}{3\,\eta^3}\,(1-\alpha^2\,L^2) - \frac{k}{\eta\,L^2} + \frac{\eta\,P_2^2}{(\nu-1)\,L^6} - \frac{\eta\,P_1^2}{(\nu+1)\,L^6}\right), \tag{34}$$

where the $+$ $(-)$ corresponds to $x>1$ $(x<1)$. The black hole mass is

$$M_{\mathrm{M}} \;=\; L^4\,\frac{\mu_{\mathrm{M}}\,\sigma_k}{8\pi G}\,, \tag{35}$$

while the magnetic charges and potentials are

$$p_1 = \frac{P_1\;\sigma_k}{8\pi G}\,, \qquad \Phi_1^{\mathrm{M}} = \frac{P_1\,\eta}{L^2}\,\frac{x_+^{\nu}-1}{\nu}\,; \qquad\qquad p_2 = \frac{P_2\,\sigma_k}{8\pi G}\,, \qquad \Phi_2^{\mathrm{M}} = \frac{P_2\,\eta}{L^2}\,\frac{1-x_+^{-\nu}}{\nu}\,. \tag{36}$$

It is possible to verify that these quantities satisfy the first law of thermodynamics:

$$dM_{\mathrm{M}} \;=\; T\,dS + \Phi_1^{\mathrm{M}}\,dp_1 + \Phi_2^{\mathrm{M}}\,dp_2\,. \tag{37}$$

3. Discussion

3.1. Duality Relation between the Two Families of Solutions

The two families of solutions are related by a non-perturbative electric–magnetic duality symmetry. The latter is a global symmetry of the ungauged theory and is extended to the gauged one if the constant tensor θ_M is made to transform under it as well. In general, a transformation of θ_M would imply a change in the theory and the duality would be interpreted as an equivalence between different models. Such a transformation can be absorbed in a redefinition of a single parameter α in θ_M: the two solutions related by duality satisfy the field equations of the same model with two (dual) values of the α parameter.

Consider a generic $\mathcal{N}=2$ theory described by the Lagrangian (1), where the scalar manifold (which is of a special Kähler type in the absence of hypermultiplets) is now described in terms of real scalars $\phi(x)\equiv\phi^s(x)$ and where $\mathscr{G}_{rs}(\phi)$ is the metric on the scalar manifold. The scalar potential $V(\theta,\phi)$ is given by Equation (6), explicitly depending on both the scalar fields and the FI θ_M. To describe the duality, it is useful to define the magnetic field strengths as:

$$G_{\Lambda\,\mu\nu} \;=-\,\epsilon_{\mu\nu\rho\sigma}\frac{\delta\mathscr{L}_{\mathrm{BOS}}}{\delta F^{\Lambda}_{\rho\sigma}} \;=\; \mathcal{R}_{\Lambda\Sigma}\,F^{\Sigma}_{\mu\nu} - \mathcal{I}_{\Lambda\Sigma}\,{}^{*}F^{\Sigma}_{\mu\nu}\,, \tag{38}$$

and to introduce the symplectic field strength vector:

$$\mathbb{F}^M = \begin{pmatrix} F^{\Lambda}_{\mu\nu} \\ G_{\Lambda\,\mu\nu}\end{pmatrix}, \tag{39}$$

together with the $2(n_{\mathrm{v}}+1)\times 2(n_{\mathrm{v}}+1)$ matrices

$$\mathcal{M}_{MN}(\phi)\;\equiv\;\begin{pmatrix} \mathcal{I}_{\Lambda\Sigma}+(\mathcal{R}\,\mathcal{I}^{-1}\,\mathcal{R})_{\Lambda\Sigma} & -(\mathcal{R}\,\mathcal{I}^{-1})_{\Lambda}{}^{\Gamma} \\ -(\mathcal{I}^{-1}\,\mathcal{R})^{\Delta}{}_{\Sigma} & (\mathcal{I}^{-1})^{\Delta\Gamma}\end{pmatrix}, \qquad \mathbb{C}\;\equiv\;\begin{pmatrix} \mathbf{0} & \mathbb{1} \\ -\mathbb{1} & \mathbf{0}\end{pmatrix}. \tag{40}$$

A feature of special Kähler manifolds is the existence of a flat symplectic bundle structure on it, within which the matrix $\mathcal{M}_{MN}$ can be regarded as a metric on the symplectic fiber. As a consequence of this, with each isometry $\mathbf{g}$ in the isometry group G of the scalar manifold, there corresponds a constant $2(n_{\rm v}+1)\times 2(n_{\rm v}+1)$ symplectic matrix $\mathscr{R}[\mathbf{g}]=\mathscr{R}[\mathbf{g}]^M{}_N$ such that, if $\phi'=\phi'^s=\phi'(\phi)$ describes the (non-linear) action of $\mathbf{g}$ on the scalar fields, we have that $\mathcal{M}_{MN}(\phi)$ transforms as a fiber metric [34,35]:

$$\mathcal{M}(\phi')\;=\;(\mathscr{R}[\mathbf{g}]^{-1})^T\mathcal{M}(\phi)\,\mathscr{R}[\mathbf{g}]^{-1}\,,\tag{41}$$

where we have suppressed the symplectic indices. If, for any isometry $\mathbf{g}$, we transform the fields $\mathbb{F}^M_{\mu\nu}$ and the constant vector θ_M correspondingly,

$$\mathbb{F}^M_{\mu\nu}\;\rightarrow\;\mathbb{F}'^M_{\mu\nu}=\mathscr{R}[\mathbf{g}]^M{}_N\,\mathbb{F}^N_{\mu\nu}\,,\qquad\quad \theta_M\;\rightarrow\;\theta'_M=(\mathscr{R}[\mathbf{g}]^{-1})^N{}_M\,\theta_N\,,\tag{42}$$

we find that the equations of motion are formally left invariant. Aside from this duality equivalence, one can just redefine the field strengths and the FI constants without altering the physics of the model:

$$\mathbb{F}_{\mu\nu}\;\rightarrow\begin{pmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{0} & (\mathbf{A}^{-1})^T\end{pmatrix}\mathbb{F}_{\mu\nu}\,,\qquad\quad \theta\;\rightarrow\begin{pmatrix}(\mathbf{A}^{-1})^T & \mathbf{0}\\ \mathbf{0} & \mathbf{A}\end{pmatrix}\theta\,,\tag{43}$$

where $\mathbf{A}=(A^\Lambda{}_\Sigma)$ is a generic, real, invertible matrix, not related to the isometries of the scalar manifold. This just amounts to choosing a different basis in the symplectic fiber.

The above mechanism also works for our 1-scalar truncated model (with trivial metric $\mathscr{G}(\phi)=1$), provided the isometry does not switch on the truncated axion χ (imaginary part of z). The absence of the axion also implies $\mathcal{R}_{\Lambda\Sigma}=0$. The isometries of the scalar field are

$$\phi\;\rightarrow\;\phi'=\phi+\beta\,,\tag{44a}$$

$$\phi\;\rightarrow\;\phi'=-\phi\,,\tag{44b}$$

with β being a constant. The first of the above isometries was used in Section 2.1.1 to reabsorb the θ_2 dependence of the FI terms, making them basically only dependent on α. This was then followed by a corresponding redefinition of the field strengths and of the FI parameters of the kind (43). Let us now consider the duality transformation associated with the isometry (44b). Using the property $\mathcal{I}_{\Lambda\Sigma}(-\phi)=(\mathcal{I}^{-1})^{\Lambda\Sigma}(\phi)$, it is easily verified that the associated duality transformation $\mathscr{R}[\mathbf{g}]$ is simply given by $\mathbb{C}$. Indeed, we have:

$$\mathcal{M}(\phi')\;=\;\mathcal{M}(-\phi)\;=\;(\mathbb{C}^{-1})^T\mathcal{M}(\phi)\,\mathbb{C}^{-1}\,.\tag{45}$$

By the same token, we can compute the duality transformed field strengths:

$$\begin{pmatrix}\bar{F}'^\Lambda_{\mu\nu}\\ \bar{G}'_{\Lambda\,\mu\nu}\end{pmatrix}\;=\;\mathbb{C}\begin{pmatrix}\bar{F}^\Lambda_{\mu\nu}\\ \bar{G}_{\Lambda\,\mu\nu}\end{pmatrix}\;=\;\begin{pmatrix}\bar{G}_{\Lambda\,\mu\nu}\\ -\bar{F}^\Lambda_{\mu\nu}\end{pmatrix}.\tag{46}$$

After the action of the shift symmetry (44a) and the abovementioned redefinition, the resulting FI parameter vector, to be denoted by $\bar{\theta}_M$, has the form:

$$\bar{\theta}_M\;=\;\left(\sqrt{\frac{\nu+1}{\nu}}\sqrt{L^{-2}-\alpha^2}\,,\;\sqrt{\frac{\nu-1}{\nu}}\sqrt{L^{-2}-\alpha^2}\,,\;\alpha\,s\,\sqrt{\frac{\nu+1}{\nu}}\,,\;\frac{\alpha}{s}\sqrt{\frac{\nu-1}{\nu}}\right).\tag{47}$$

We denote by $\bar{\theta}'$ the transformed of $\bar{\theta}$ by $\mathbb{C}$, that is, $\bar{\theta}'\;=\;\mathbb{C}\,\bar{\theta}$. One can then verify that

$$V(\bar{\theta}',\phi')\;=\;V(\bar{\theta},\phi)\,.\tag{48}$$

Since $\bar{\theta}$ only depends on α, we will simply denote the potential by $V(\alpha, \phi)$. This action, apart from signs, basically amounts, in $\bar{\theta}$, to changing $\alpha^2 \to L^{-2} - \alpha^2$. In particular, we find:

$$V(\alpha, \phi) \;=\; V(\alpha', \phi')\,, \tag{49}$$

where $\alpha'^2 = L^{-2} - \alpha^2$. It is possible to verify that, if the electric solutions

$$\phi(x)\,,\;\; \bar{F}^{\Lambda}_{\mu\nu}(x)\,,\;\; g_{\mu\nu}(x)\,, \tag{50}$$

solve the field equations with FI terms defined by the parameter α, then the magnetic configuration

$$\phi'(x) = -\phi(x)\,,\quad \bar{G}'_{\Lambda\,\mu\nu}(x) = -\bar{F}^{\Lambda}_{\mu\nu}(x)\,,\quad g'_{\mu\nu}(x) = g_{\mu\nu}(x)\,, \tag{51}$$

is a solution with parameter α'. The identification $\bar{G}'_{\Lambda\,\mu\nu}(x) = -\bar{F}^{\Lambda}_{\mu\nu}(x)$ means that the magnetic field strengths (apart from a sign) will have the same form as the original electric ones, so that we can relate its magnetic charge parameters P'_i with the original electric ones: $P'_i = -Q_i$.

3.2. Supersymmetric Solutions

Now, we want to study supersymmetric configurations for our model, imposing the vanishing of the SUSY variations [22,36]. First, it is useful to make a change of coordinates that puts metric (19c) in the standard form:

$$ds^2 \;=\; e^{2\,U(r)}\,dt^2 - e^{-2\,U(r)}\left(dr^2 + e^{2\,\Psi(r)}\,d\Sigma_k^2\right)\,. \tag{52}$$

This can be achieved through the change in coordinate

$$x(r) \;=\; \left(1 + \frac{L^2\,\nu}{\eta\,(r-c)}\right)^{\frac{1}{\nu}}\,, \tag{53}$$

with c being a constant.

3.2.1. Family 1

The scalar field z in the new parametrization has the form

$$z \;=\; (\theta_2\,\xi)^{-\frac{2\nu}{1+\nu}}\left(1 + \frac{L^2\,\nu}{\eta\,(r-c)}\right)^{-1}\,, \tag{54}$$

while the electric–magnetic charges explicitly read

$$\Gamma^M \;=\; \begin{pmatrix} m^{\Lambda} \\ e_{\Lambda} \end{pmatrix} \;=\; \begin{pmatrix} 0 \\ 0 \\ \frac{L^2}{2\,\eta}\,Q_1\,\sqrt{\frac{1+\nu}{\nu}}\,(\theta_2\,\xi)^{\frac{1-\nu}{1+\nu}} \\ \frac{L^2}{2\,\eta}\,Q_2\,\sqrt{\frac{-1+\nu}{\nu}}\,\theta_2\,\xi \end{pmatrix}\,. \tag{55}$$

The solution is supersymmetric if

$$\begin{aligned} Q_1 &= -\,Q_2\,\sqrt{\frac{-1+\nu}{1+\nu}} + \frac{k\,\eta}{\alpha\,L^2}\,\sqrt{\frac{\nu}{1+\nu}}\,, \\ Q_2 &= \left(\frac{k\,\eta}{2\,\alpha\,L^2} + \frac{\alpha\,L^2\,(1+\nu)}{2\,\eta}\right)\sqrt{\frac{-1+\nu}{\nu}}\,. \end{aligned} \tag{56}$$

3.2.2. Family 2

The scalar field z reads

$$z \,=\, (\theta_2\,\xi)^{-\frac{2\nu}{1+\nu}}\left(1+\frac{L^2\,\nu}{\eta\,(r-c)}\right), \tag{57}$$

while the electric–magnetic charges have the form

$$\Gamma^M \,=\, \begin{pmatrix} m^\Lambda \\ e_\Lambda \end{pmatrix} \,=\, \begin{pmatrix} 2\,P_1\,\sqrt{\frac{\nu}{1+\nu}}\,(\theta_2\,\xi)^{\frac{-1+\nu}{1+\nu}} \\ 2\,P_2\,\sqrt{\frac{\nu}{-1+\nu}}\,(\theta_2\,\xi)^{-1} \\ 0 \\ 0 \end{pmatrix}. \tag{58}$$

The solution is supersymmetric if

$$\begin{aligned} P_1 &= -\,P_2\,\sqrt{\frac{-1+\nu}{1+\nu}}+\frac{k\,L}{\sqrt{1-\alpha^2\,L^2}}\,\sqrt{\frac{\nu}{1+\nu}}\,, \\ P_2 &= \left(\frac{k\,L}{2\,\sqrt{1-\alpha^2\,L^2}}+\frac{L^3\,(1+\nu)}{2\,\eta^2}\,\sqrt{1-\alpha^2\,L^2}\right)\sqrt{\frac{-1+\nu}{\nu}}\,. \end{aligned} \tag{59}$$

The supersymmetric magnetic condition can be obtained from the supersymmetric electric condition by means of the duality transformation

$$Q_i \rightarrow \frac{\eta}{L^2}\,P_i\,, \qquad \alpha^2 \,\rightarrow\, L^{-2}-\alpha^2\,. \tag{60}$$

3.3. BPS Black Holes of Finite Area

3.3.1. Family 1: BPS Electric Black Holes

We found that the electric family has BPS black holes of finite area only when $\alpha^2 = L^{-2}$. In this case, the lapse function has a double zero, as expected:

$$f(x_+) = 0 \quad\Longrightarrow\quad x_\pm^\nu = \frac{\nu^2-1-k\,\eta^2\,L^{-2}}{\nu-1-k\,\eta^2\,L^{-2}} \pm \nu\,\frac{\sqrt{\nu^2-1-2\,k\,\eta^2\,L^{-2}}}{\nu-1-k\,\eta^2\,L^{-2}}\,. \tag{61}$$

It is possible to verify that $x_\pm(\nu) = x_\pm(-\nu)$, namely, $x_\pm$ is an even function of ν. Hence, we can restrict our analysis to the $\nu > 1$ interval.

The asymptotic region of the spacetime is located at $x = 1$ and, therefore, the region of the spacetime at $x > 1$ is disconnected from the region at $x < 1$, therefore representing different spacetimes. The location of the horizon can be characterized as follows:

- $k = 0$: in the flat case, the location of the horizon is very simple (see (61) above) and it follows that $x_+^\nu > 0$ and $x_-^\nu < 0$, so we conclude that only x_+ exists;
- $k = -1$: in the hyperbolic case, $x_+ > 1$ always exists while the solution $0 < x_- < 1$ exists provided $\eta^2\,L^{-2} > \nu+1$;
- $k = +1$: for a spherical black hole, only x_+ exists, provided $\nu-1 \,>\, \eta^2\,L^{-2} \,>\, 0$.

3.3.2. Family 2: BPS Magnetic Black Holes

Again, we shall consider only the case $\nu > 1$. This family has BPS black holes of finite area only when $\alpha^2 = 0$, namely when the gauging is purely electric. In this case, the metric of the magnetic solution exactly coincides with the metric of the electric solutions. Hence, the analysis of the location of the horizons is exactly the same. The extremality of the magnetic solutions is the same as for the electric solutions when the electric charges and potentials are interchanged by their magnetic counterparts.

3.4. $\mathcal{N} = 8$ Truncations

3.4.1. Uncharged Case

The infinitely many theories we have described in the previous sections contain all the possible one-dilaton consistent truncations of the ω-deformed SO(8) gauged maximal supergravities. Let us briefly discuss this result.

In the dyonic models with gauge group SO(8), the 28 generators are gauged by the 28 vector fields in a symplectic frame which is related to the one of [37,38] by an SO(2) transformation parameterized by an angle ω. The physically independent values of ω lie within the interval $0 \le \omega \le \pi/8$, $\omega = 0$ corresponding to the original theory by de Wit and Nicolai. The 70 scalar fields parameterize the coset $E_{7(7)}/SU(8)$ and, upon truncation to gravity and scalar field sector (note 1: we choose a parametrization covariant under SU(8), the scalars splitting into the representations $\mathbf{35}_c$ and $\mathbf{35}_v$ of the gauge group SO(8); the former can be truncated out, while the latter scalars span the submanifold $SL(8,\mathbb{R})/SO(8)$; the local SO(8) transformations can be used to diagonalize the coset representative and thus to further truncate the theory to the seven scalars, parameterizing the non-compact Cartan subalgebra of $SL(8,\mathbb{R})$), we are led to consider the following action [39,40]:

$$I(g_{\mu\nu},\vec{\phi}\,) = \int_{\mathcal{M}} d^4x\,\sqrt{-g}\left[-\frac{R}{2}+\frac{1}{2}\,(\partial\vec{\phi}\,)^2 - V(\vec{\phi}\,)\right], \tag{62}$$

where $\vec{\phi} = (\phi_i)$, with $i = 1,\dots,8$ and $\sum_{i=1}^{8}\phi_i = 0$. The potential is given by

$$V(\vec{\phi}\,) = -\frac{g^2}{32}\left[\cos^2(\omega)\left(\left(\sum_{i=1}^{8}X_i\right)^2 - 2\sum_{i=1}^{8}X_i^2\right) + \sin^2(\omega)\left(\left(\sum_{i=1}^{8}X_i^{-1}\right)^2 - 2\sum_{i=1}^{8}X_i^{-2}\right)\right], \tag{63}$$

where

$$X_i = e^{2\phi_i}\,, \qquad \prod_{i=1}^{8}X_i = 1\,. \tag{64}$$

Let us consider now a single scalar field reduction preserving $SO(p)\times SO(8-p)$. This is effected through the following identification:

$$\phi_1 = \cdots = \phi_p = \frac{1}{2\sqrt{2}}\,\sigma\,\phi\,, \qquad \phi_{p+1} = \cdots = \phi_8 = -\frac{1}{2\sqrt{2}}\,\frac{\phi}{\sigma}\,, \tag{65}$$

where we have defined:

$$\sigma = \sqrt{\frac{8-p}{p}} = \sqrt{\frac{\nu-1}{\nu+1}}\,, \qquad p = \frac{4\,(\nu+1)}{\nu}\,. \tag{66}$$

With the above choice we have:

$$X_1 = \cdots = X_p = X := e^{\frac{1}{\sqrt{2}}\sigma\phi}\,, \qquad X_{p+1} = \cdots = X_8 = Y := e^{-\frac{1}{\sqrt{2}}\frac{\phi}{\sigma}}\,, \tag{67}$$

and the previous action (62) reduces to the one we are studying here. The action is invariant under $\sigma \to 1/\sigma$, $\phi \to -\phi$ and $p \to 8-p$. This action, consistent truncation of the ω-rotated SO(8)-gauged maximal supergravity, coincides, in the absence of vector fields, with the action (16) upon the following identification:

$$g = \frac{\sqrt{2}}{L}\,, \qquad \cos(\omega) = L\,\alpha\,, \qquad \sin(\omega) = \sqrt{1-L^2\,\alpha^2}\,. \tag{68}$$

Changing ϕ into $-\phi$ in (67), the above identifications change correspondingly:

$$g = \frac{\sqrt{2}}{L}\,, \qquad \sin(\omega) = L\,\alpha\,, \qquad \cos(\omega) = \sqrt{1-L^2\,\alpha^2}\,. \tag{69}$$

From the above relation between p and ν, we conclude that the single scalar field models considered in this work, if all vector fields are set to zero, coincide with truncations of the ω-deformed SO(8) gauged maximal supergravity to the singlet sector with respect to the following subgroups of the SO(8) gauge group:

$$\begin{aligned} \nu = \tfrac{4}{3} &\quad\rightarrow\quad \mathrm{SO}(7)\ , \\ \nu = 2 &\quad\rightarrow\quad \mathrm{SO}(6)\times\mathrm{SO}(2)\ , \\ \nu = 4 &\quad\rightarrow\quad \mathrm{SO}(5)\times\mathrm{SO}(3)\ , \\ \nu = \infty &\quad\rightarrow\quad \mathrm{SO}(4)\times\mathrm{SO}(4)\ . \end{aligned} \tag{70}$$

The values $\nu = \infty$ or $\nu = \pm 2$ correspond to models which can be embedded in the STU truncation of the SO(8) gauged $\mathcal{N} = 8$ supergravity. Therefore, the black hole solutions discussed in this work, in the absence of electric and magnetic charges, can all be embedded in the maximal supergravity. Since the SO(8) gauged maximal supergravity can be uplifted to $D = 11$ supergravity only for $\omega = 0$ [41], only for $L\,\alpha = 0$ or equivalently for $L\,\alpha = \pm 1$ our solutions can be embedded in the eleven-dimensional theory through maximal supergravity, by means of the formulas presented in [39,40].

3.4.2. Charged Case

The $\nu = 4$ case

Let us focus first on the $\omega = 0$ case. The solutions describe gravity coupled to one scalar field and two vector fields. When identified with fields in the maximally supersymmetric model, the scalar and the two vectors should not excite the other fields in the model, such as the scalars in the $\mathbf{35}_c$ of SO(8) (see note 1). This condition, in turn, requires, in our solution, $F^\Lambda \wedge F^\Sigma = 0$ (only in this subsection do we denote by Λ, Σ the indices labeling the 28 vectors of the maximal theory, by $\lambda, \sigma = 1, 2$ those of the two vectors surviving the truncation, by M, N the symplectic indices of the 56 electric and magnetic charges, by $m, n = 1, \ldots, 4$ the symplectic index labeling the two electric and two magnetic charges in the truncated model; the corresponding indices in the new, ω-rotated symplectic frame are distinguished from those in the original frame by a hat). Of the remaining scalar fields in the $\mathbf{35}_v$ of SO(8), 28 are gauged away so that we are left with the seven independent scalar fields $\vec{\phi} = \phi_i$, with $i = 1, \ldots, 8$ and $\sum_{i=1}^{8} \phi_i = 0$, parameterizing the Cartan subalgebra of $\mathfrak{e}_{7(7)}$ (Lie algebra of $E_{7(7)}$).

Next, we want to further truncate the theory to the scalar field ϕ, which is singlet with respect to the subgroup SO(5) × SO(3) of SO(8). When the solution is charged, the two vector fields involved in it should be identified with two of the 28 vectors on the maximal model which do not source the six scalar fields among $\vec{\phi}$ that we wish to set to zero. To this end, let us write $\vec{\phi}$ as follows:

$$\begin{aligned} &\phi_1 = -\frac{1}{2}\sqrt{\frac{3}{10}}\,\phi + \varphi_1\,, \qquad \phi_2 = -\frac{1}{2}\sqrt{\frac{3}{10}}\,\phi + \varphi_2\,, \qquad \phi_3 = -\frac{1}{2}\sqrt{\frac{3}{10}}\,\phi + \varphi_3\,, \\ &\phi_4 = -\frac{1}{2}\sqrt{\frac{3}{10}}\,\phi + \varphi_4\,, \qquad \phi_5 = -\frac{1}{2}\sqrt{\frac{3}{10}}\,\phi - \sum_{k=1}^{4}\varphi_k\,, \qquad \phi_6 = \frac{1}{2}\sqrt{\frac{5}{6}}\phi + \varphi_5\,, \\ &\phi_7 = \frac{1}{2}\sqrt{\frac{5}{6}}\phi + \varphi_6\,, \qquad \phi_8 = \frac{1}{2}\sqrt{\frac{5}{6}}\phi - \varphi_5 - \varphi_6\,. \end{aligned} \tag{71}$$

Writing the SO(8) generators as $T_{IJ} = -T_{JI}$ $(I, J = 1, \ldots, 8)$, the equations for the scalars φ_ℓ $(\ell = 1, \ldots 6)$ are satisfied when $\varphi_\ell \equiv 0$, and the scalar ϕ enters the kinetic terms of the vector fields as in (18) with $\nu = 4$ if J_1 and J_2 are chosen as follows:

$$J_1 = \sqrt{\frac{2}{5}}\left(T_{12} + \frac{\varepsilon_1}{\sqrt{2}}\,T_{34} + \frac{\varepsilon_2}{\sqrt{2}}\,T_{35} + \frac{\varepsilon_3}{\sqrt{2}}\,T_{45}\right), \qquad J_2 = \frac{1}{\sqrt{3}}\left(T_{67} + \varepsilon_4\,J_{68} + \varepsilon_5\,J_{78}\right), \tag{72}$$

where $\varepsilon_\ell^2 = 1$. This identifies the two vector fields A^1_μ and A^2_μ out of A^{IJ}_μ:

$$\frac{1}{2}\, A^{IJ}_\mu\, T_{IJ} \;=\; A^1_\mu\, J_1 + A^2_\mu\, J_2\,, \tag{73}$$

so that the two field strengths $\bar{F}^1_{\mu\nu}$, $\bar{F}^2_{\mu\nu}$ in (18) are identified with the $F^{IJ}_{\mu\nu}$ of the maximal theory as follows:

$$\begin{aligned} &F^{12}_{\mu\nu} = \sqrt{\frac{2}{5}}\,\bar{F}^1_{\mu\nu}\,, \quad F^{34}_{\mu\nu} = \frac{\varepsilon_1}{\sqrt{5}}\,\bar{F}^1_{\mu\nu}\,, \quad F^{35}_{\mu\nu} = \frac{\varepsilon_2}{\sqrt{5}}\,\bar{F}^1_{\mu\nu}\,, \quad F^{45}_{\mu\nu} = \frac{\varepsilon_3}{\sqrt{5}}\,\bar{F}^1_{\mu\nu}\,, \\ &F^{67}_{\mu\nu} = \frac{1}{\sqrt{3}}\,\bar{F}^2_{\mu\nu}\,, \quad F^{68}_{\mu\nu} = \frac{\varepsilon_4}{\sqrt{3}}\,\bar{F}^2_{\mu\nu}\,, \quad F^{78}_{\mu\nu} = \frac{\varepsilon_5}{\sqrt{3}}\,\bar{F}^2_{\mu\nu}\,. \end{aligned} \tag{74}$$

When $\omega \neq 0$, the same generators T_{IJ} are gauged by linear combinations of A^{IJ}_μ and $A_{IJ\,\mu}$ of the form

$$\hat{A}^{IJ}_\mu \;=\; \cos(\omega)\, A^{IJ}_\mu - \sin(\omega)\, A_{IJ\,\mu}\,, \tag{75}$$

which means that the gauging of SO(8) is performed in a different symplectic frame in which the electric vector fields are $\hat{A}^{IJ}_\mu$. Let us denote by $\hat{A}^{\hat{M}}_\mu = (\hat{A}^{\hat{\Lambda}}_\mu,\, \hat{A}_{\hat{\Lambda}\,\mu})$, where $\hat{A}^{\hat{\Lambda}}_\mu = \hat{A}^{IJ}_\mu$, the vector fields and their magnetic duals in the new symplectic frame, and by $A^M_\mu = (A^\Lambda_\mu,\, A_{\Lambda\,\mu})$ the same vectors in the old frame. Let $\mathbb{F}^{\hat{M}}_{\mu\nu} = \partial_\mu \hat{A}^{\hat{M}}_\nu - \partial_\nu \hat{A}^{\hat{M}}_\mu$ and $\mathbb{F}^M_{\mu\nu} = \partial_\mu A^M_\nu - \partial_\nu A^M_\mu$ be the corresponding field strengths. We have the following relation:

$$\mathbb{F}^{\hat{M}}_{\mu\nu} \;=\; E_M{}^{\hat{M}}\,\mathbb{F}^M_{\mu\nu}\,, \qquad\qquad E_M{}^{\hat{M}} = \begin{pmatrix} \cos(\omega)\,\mathbb{1}_{28\times 28} & \sin(\omega)\,\mathbb{1}_{28\times 28} \\ -\sin(\omega)\,\mathbb{1}_{28\times 28} & \cos(\omega)\,\mathbb{1}_{28\times 28} \end{pmatrix}. \tag{76}$$

In the new frame, the parameter ω will also enter the kinetic matrices $\mathcal{I}_{\hat{\Lambda}\hat{\Sigma}}(\phi,\omega)$, $\mathcal{R}_{\hat{\Lambda}\hat{\Sigma}}(\phi,\omega)$, that is, the components of the new symplectic matrix $\mathcal{M}_{\hat{M}\hat{N}}(\phi,\omega)$, which is expressed in terms of the ω-independent $\mathcal{M}_{MN}(\phi)$ in the original frame through the relation:

$$\mathcal{M}_{\hat{M}\hat{N}}(\phi,\omega) \;=\; E^{-1}(\omega)_{\hat{M}}{}^M\, E^{-1}(\omega)_{\hat{N}}{}^N\, \mathcal{M}_{MN}(\phi)\,. \tag{77}$$

Upon truncating scalar and vector fields as described above, the two vectors will only enter the bosonic action through the corresponding field strengths. Written in the new symplectic frame, the kinetic terms of $\hat{A}^{\hat{\lambda}}_\mu = (\hat{A}^1_\mu,\, \hat{A}^2_\mu)$ will depend on the ω parameter through the restrictions $\mathcal{I}_{\hat{\lambda}\hat{\sigma}}(\phi,\omega)$, $\mathcal{R}_{\hat{\lambda}\hat{\sigma}}(\phi,\omega)$ of $\mathcal{I}_{\hat{\Lambda}\hat{\Sigma}}(\phi,\omega)$, $\mathcal{R}_{\hat{\Lambda}\hat{\Sigma}}(\phi,\omega)$ to the two vectors. This dependence can, however, be undone at the level of the bosonic field equations and Bianchi identities since the latter depend on $\mathbb{F}^{\hat{m}}_{\mu\nu} = (F^{\hat{\lambda}}_{\mu\nu},\, G_{\hat{\lambda}\mu\nu})$ only in symplectic-invariant contractions with the matrix $\mathcal{M}_{\hat{m}\hat{n}}(\phi,\omega)$ and its derivatives. This means that the dependence on ω of the terms involving the vector field strengths can be disposed of through a redefinition of the latter, which amounts to writing them in terms of the field strengths $\mathbb{F}^m_{\mu\nu}$ in the original frame (consisting of $\bar{F}^1_{\mu\nu}$, $\bar{F}^2_{\mu\nu}$ and their magnetic duals) through the matrix E. Upon this redefinition, the bosonic field equations of the truncated model coincide with those obtained from the action (18), with $\nu = 4$, provided we identify:

$$g = \frac{\sqrt{2}}{L}\,, \qquad \sin(\omega) = L\,\alpha\,, \qquad \cos(\omega) = \sqrt{1 - L^2\alpha^2}\,. \tag{78}$$

The embedding of the $\nu = 4/3$ model in the maximal theory is more subtle and will be dealt with in the future. In the remaining cases $\nu = \infty$ or $\nu = \pm 2$, our solutions can be extended to charged solutions within $\mathcal{N} = 8$ SO(8) gauged supergravity, within the bosonic part of the STU truncation of it.

Acknowledgments: I would like to thank M. Trigiante, A. Anabalon and D. Astefanesei for discussions.

Conflicts of Interest: The author declares no conflict of interest.

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