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Article

On Line Diagrams Plus Modality

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Abstract: In this paper, we produce an extension of Englebretsen's line diagrams in order to represent modal syllogistic, i.e., we add some diagrammatic objects and rules to his system in order to reason about modal syllogistics in a diagrammatic, linear fashion.

Keywords: logical diagrams; modal logic; term logic

1. Introduction

If we assume, albeit provisionally, that reasoning is a process that permits the production of some information (i.e., a conclusion) given prior information (i.e., a set of premises)—following certain rules, of course—and that said information can be represented through diagrams, it can be said that diagrammatic inference is the relation that defines our general intuitions about the informal notion of visual inference, and so classical structural norms such as reflexivity, monotonicity, and cut should follow ex hypothesi [1]. With this assumption in mind, we could say a logical diagram—a deductive logical diagram, we should add—is a diagram within a system of diagrams that is sound and complete with respect to a class of valid inferences given a deductive base [2].

Granted this notion of a logical diagram, in this paper, we offer a simple extension of Englebretsen's logical diagrams in order to represent modal syllogistic, i.e., we add some diagrammatic objects and rules to his system in order to reason about modal syllogistic in a diagrammatic, linear fashion. To reach this goal, we briefly explain some core features of (assertoric and modal) syllogistic and Englebretsen's diagrammatic system, and then we develop our proposal.

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2. Preliminaries

2.1. Assertoric Syllogistic

Assertoric syllogistic ($\mathcal{SYLL}$), the system at the core of traditional, Aristotelian logic, is a logic that makes use of categorical statements. A categorical statement is a statement composed of two terms, a quantity, and a quality. The subject and the predicate are called terms: the term-schema S stands for the subject term and the term-schema P stands for the predicate. The quantity may be either universal (*All*) or particular (*Some*), and the quality may be either affirmative (*is*) or negative (*is not*). Thus, formally, we say a categorical statement is a statement of the form:

$$\langle Quantity \rangle\ \langle \mathsf{S} \rangle\ \langle Quality \rangle\ \langle \mathsf{P} \rangle$$

where $Quantity \in \{All,\ Some\}$, $Quality \in \{is,\ is\ not\}$, and S and P are term-schemes.

The combination of these components produce the four categorical statements: the universal affirmative, "All S is P," represented by the label a and thus shortened as SaP; the universal negative, "All S is not P" (SeP); the particular affirmative, "Some S is P" (SiP); and the particular negative, "Some S is not P" (SoP).

Three categorical statements define a mood, that is, a sequence of categorical statements ordered in such a way that the first two are regarded as premises (major and minor, in that order) and the last one as a conclusion. A categorical syllogism, then, is a mood with three terms one of which appears in both premises but not in the conclusion (known as the middle term and usually represented with the term-schema M), and two other terms that appear exactly once in each premise and together in the conclusion. The middle term works as a link between these remaining terms and, according to its position, four figures or arrangements can be set up in order to encode the valid syllogistic moods (Table 1) (for the sake of brevity, but without loss of generality, we omit the syllogisms that require existential import).

Table 1. Valid syllogistic moods.

Figure 1	Figure 2	Figure 3	Figure 4
1. MaP 2. SaM ⊢ SaP	1. PeM 2. SaM ⊢ SeP	1. MiP 2. MaS ⊢ SiP	1. PaM 2. MeS ⊢ SeP
1. MeP 2. SaM ⊢ SeP	1. PaM 2. SeM ⊢ SeP	1. MaP 2. MiS ⊢ SiP	1. PiM 2. MaS ⊢ SiP
1. MaP 2. SiM ⊢ SiP	1. PaM 2. SoM ⊢ SoP	1. MoP 2. MaS ⊢ SoP	1. PeM 2. MiS ⊢ SoP
1. MeP 2. SiM ⊢ SoP	1. PeM 2. SiM ⊢ SoP	1. MeP 2. MiS ⊢ SoP	

2.2. Modal Syllogistic

Modal syllogistic ($\mathcal{SYLL}^{\mu}$), a system around the core of traditional, Aristotelian logic, is a logic that results from the addition of the modal operators of necessity (□) and possibility (◇), and their respective rules (again, for the sake of brevity, we do not need to show these rules in this moment, but we will refer to them in the proof below), to assertoric syllogistic. Since its proposal in *Pr. An.* I, 3, 8–22, it has received lots of attention, and even when this attention has not been always favorable [3–8], recent studies have recovered some of its features [9–12].

Given the addition of these operators, in modal syllogistic, we obtain three kinds of statements: categorical statements *simpliciter*, *de re* modal statements of the form

$$\langle \mathit{Quantity} \rangle \; \langle \mathsf{S} \rangle \; \langle \mathit{Quality} \rangle \; \langle \mathit{Modality} \rangle \; \langle \mathsf{P} \rangle$$

and *de dicto* modal statements of the form

$$\langle \mathit{Modality} \rangle \; (\langle \mathit{Quantity} \rangle \; \langle \mathsf{S} \rangle \; \langle \mathit{Quality} \rangle \; \langle \mathsf{P} \rangle)$$

where $\mathit{Modality} \in \{\diamond, \Box\}$, and the rest of components are defined as above.

With these statements, and in the interest of time, consider, as a representative example, the following *de re* (Table 2) and *de dicto* (Table 3) valid syllogistic moods.

Table 2. Some valid *de re* syllogisms.

1. Ma□P 2. Sa□M ⊢ Sa□P	1. Ma□P 2. Sa□M ⊢ SaP	1. Ma□P 2. Sa□M ⊢ Sa◇P	1. Ma□P 2. SaM ⊢ Sa□P
1. Ma□P 2. SaM ⊢ SaP	1. Ma□P 2. SaM ⊢ Sa◇P	1. Ma◇P 2. Sa□M ⊢ Sa◇P	1. MaP 2. Sa□M ⊢ SaP
1. MaP 2. Sa□M ⊢ Sa◇P	1. MaP 2. SaM ⊢ SaP	1. MaP 2. SaM ⊢ Sa◇P	1. Ma◇P 2. SaM ⊢ Sa◇P

Table 3. Some valid *de dicto* syllogisms.

1. $\Box(\mathsf{MaP})$ 2. $\Box(\mathsf{SaM})$ $\vdash \Box(\mathsf{SaP})$	1. $\Box(\mathsf{MaP})$ 2. $\Box(\mathsf{SaM})$ $\vdash \mathsf{SaP}$	1. $\Box(\mathsf{MaP})$ 2. $\Box(\mathsf{SaM})$ $\vdash \diamond(\mathsf{SaP})$
1. $\Box(\mathsf{MaP})$ 2. SaM $\vdash \mathsf{SaP}$	1. $\Box(\mathsf{MaP})$ 2. SaM $\vdash \diamond(\mathsf{SaP})$	1. MaP 2. $\Box(\mathsf{SaM})$ $\vdash \mathsf{SaP}$
1. MaP 2. $\Box(\mathsf{SaM})$ $\vdash \diamond(\mathsf{SaP})$	1. MaP 2. SaM $\vdash \mathsf{SaP}$	1. MaP 2. SaM $\vdash \diamond(\mathsf{SaP})$

2.3. Englebretsen's Line Diagrams

Englebretsen [13,14] has developed a quite powerful system of line diagrams, let us call it $\mathcal{ENGL}$, with which we can reason about syllogisms (in all fairness, it must be mentioned that $\mathcal{ENGL}$ can be used not just to reason about syllogistic, but also about singular, relational, and compound statements —Leibniz's *desideratum*—; however, for our current purposes, this presentation is enough). The signature for this system requires (labeled) solid lines and (labeled) dots (Figure 1).

Figure 1. Signature of $\mathcal{ENGL}$.

In order to reason about syllogisms, $\mathcal{ENGL}$ provides a basic syntax for the categorical statements: straight lines stand for terms while the relations between the lines stand for the relations between terms, as in Figure 2.

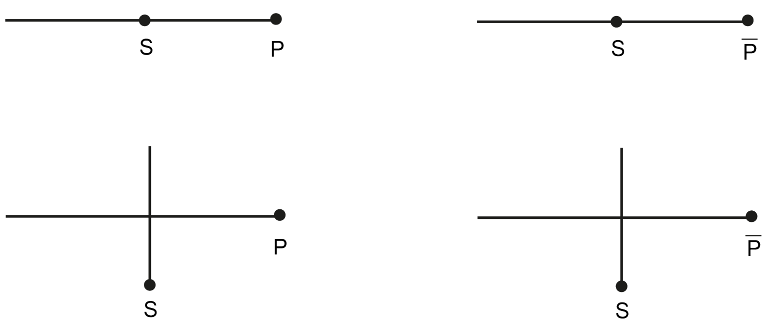


Figure 2. Categorical statements. Clockwise: SaP, SeP, SoP, SiP.

With the help of these basic diagrams, we can develop syllogistic inferences quite simply and elegantly through a visual representation of the *dictum de omni et nullo* principle (DON for short). This principle states a relation between terms such that everything that is affirmed (or denied) of the whole can be affirmed (or denied) of the parts. Visually, we can accommodate this principle by erasing the middle term, M, in the following diagrams, so that the conclusion gets drawn by drawing down the premises (Figure 3).

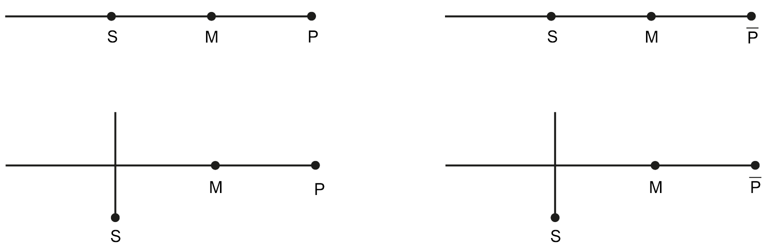


Figure 3. Syllogisms of the first figure. Clockwise: $\mathsf{MaPSaM} \vdash \mathsf{SaP}$, $\mathsf{MePSaM} \vdash \mathsf{SeP}$, $\mathsf{MePSiM} \vdash \mathsf{SoP}$, $\mathsf{MaPSiM} \vdash \mathsf{SiP}$.

3. On Line Diagrams Plus Modality

Taking $\mathcal{ENGL}$ as a basic system, we will add two kinds of objects (the square-ish and the diamond-ish diagrams) and some rules in order to produce $\mathcal{ENGL}^{\mu}$, an extension of Englebretsen's line diagrams, in order to represent modal syllogistic, $\mathcal{SYLL}^{\mu}$. Our intuition can be better appreciated with the aid of the following diagram, where the solid lines represent the preliminaries, and the dashed lines represent our goals:

$$\begin{tikzcd}
\mathcal{SYLL} \arrow[r, "\text{extends to}"] \arrow[d, "\text{modeled by}"'] & \mathcal{SYLL}^{\mu} \arrow[d, dashed, "\text{modeled by}"] \\
\mathcal{ENGL} \arrow[r, dashed, "\text{extends to}"] & \mathcal{ENGL}^{\mu}
\end{tikzcd}$$

3.1. Syntax and Rules

So, first, the syntax for this new system is given by (labeled) solid lines, (labeled) dots (an in $\mathcal{ENGL}$), and by the addition of the square-ish and diamond-ish diagrams. In order to observe these new objects (namely, the square-ish and diamond-ish diagrams), consider the rules for the system (Figures 4–6). So, in Figure 4, we observe the *de re* rules, from left to right: the elimination of a *de re* square, and the introduction of a *de re* diamond. In Figure 5, we have the *de dicto* rules, the elimination of a *de dicto* square, and the introduction of a *de dicto* diamond. Finally, in Figure 6 we have some jointing rules, the joint *de dicto* rule, and the joint *de dicto/de re* rule. But some clarifications may be in order here. First, in Figures 4–6, we use a line between the diagrams of $\mathcal{ENGL}^{\mu}$ only as a device to explain how the initial diagram results in a new diagram after applying the rule. Second, it can be claimed that the square and the diamond are just syntactical marks, not diagrams, when used in the *de re* sense, and that the *de dicto* squares and diamonds are not really squares or diamonds. These are fair observations: it is true that when used in the *de re* sense, the square and the diamond can be seen just as syntactical marks, and it is also true that the *de dicto* squares and diamonds are not really squares or diamonds; that is why we call them square-ish and diamond-ish diagrams, and we have worked with them in such fashion out of convenience.

Figure 4. *De re* rules. Elimination of a *de re* square, and introduction of a *de re* diamond.

Figure 5. *De dicto* rules. Elimination of a *de dicto* square, and introduction of a *de dicto* diamond.

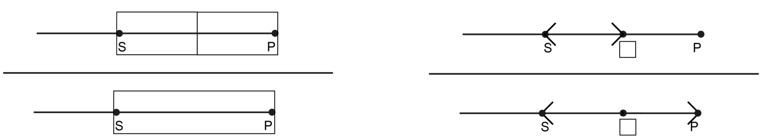


Figure 6. Jointing rules. Joint *de dicto* square, and joint *de dicto/de re*.

As should be expected, the idea is that a diagram follows validly from some other diagram if and only if it follows from an application of DON (as in $\mathcal{ENGL}$) or the previous rules, which we can summarily deploy like this:

1. Elimination of a *de re* square: $\mathsf{S}\boxed{\mathsf{P}} \vdash \mathsf{SP}$.
2. Introduction of a *de re* diamond: $\mathsf{SP} \vdash \mathsf{S}\langle\mathsf{P}\rangle$.

3. Elimination of a *de dicto* square: $\boxed{\mathsf{SP}} \vdash \mathsf{SP}$.
4. Introduction of a *de dicto* diamond: $\mathsf{SP} \vdash \langle \mathsf{SP} \rangle$.
5. Joint *de dicto* square: $\boxed{\text{de dicto}}\boxed{\text{de dicto}} \vdash \boxed{\text{de dicto de dicto}}$.
6. Joint de *dicto/de re*: $\langle \text{de dicto} \rangle\ \boxed{\text{de dicto/de re}} \vdash \langle \text{de dicto} \rangle$.

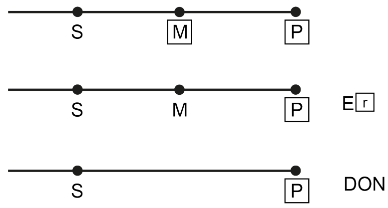
And now, in order to show how these diagrams and rules work, let us consider some examples.

3.2. Examples

Example 1. *First, let us consider a valid de re syllogism and its corresponding diagram.*

1. $\mathsf{Ma}\Box\mathsf{P}$
2. $\mathsf{Sa}\Box\mathsf{M}$

$\vdash \mathsf{Sa}\Box\mathsf{P}$

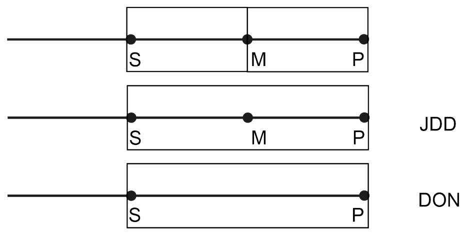


In this example, the first diagram represents the arrangement of the premises. The second diagram is the result of applying an elimination of a *de re* square to the middle term M, and finally, the last diagram is the result of applying DON to the previous diagram, thus erasing the middle term M, and hence obtaining the diagram of the conclusion.

Example 2. *Now, let us consider a valid de dicto syllogism.*

1. $\Box(\mathsf{MaP})$
2. $\Box(\mathsf{SaM})$

$\vdash \Box(\mathsf{SaP})$

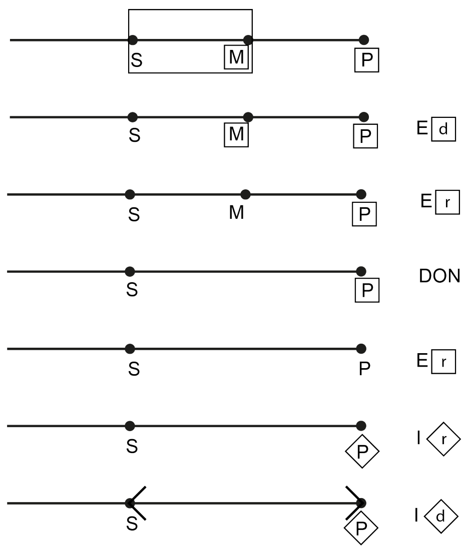


In this example, the first diagram represents the arrangement of the premises. The second diagram is the result of applying a joint *de dicto* rule to the first diagram, and finally, the last diagram is the result of applying DON to the previous diagram, thus erasing the middle term M, and hence obtaining the diagram of the conclusion.

Example 3. *Now consider a valid syllogism that combines de dicto and de re modalities.*

1. $\mathsf{Ma}\Box\mathsf{P}$
2. $\Box(\mathsf{Sa}\Box\mathsf{M})$

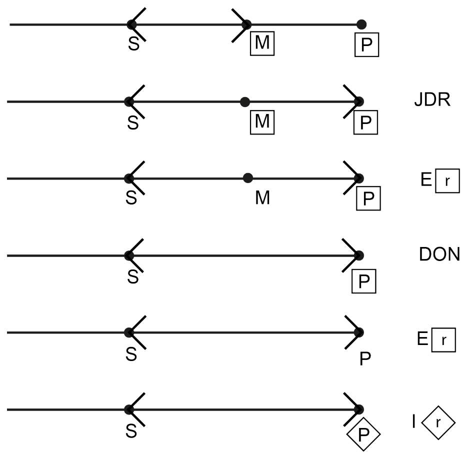
$\vdash \Diamond(\mathsf{Sa}\Diamond\mathsf{P})$



This sequence is a little bit longer. Again, the first diagram represents the premises. The second diagram is the result of applying an elimination of a *de dicto* square. The third line is the result of using an elimination of a *de re* square to the middle term M. The fourth diagram is the result of applying DON to the previous diagram, thus erasing the middle term M. The fifth line is an elimination of a *de re* square to the extreme term P. The sixth line is an introduction of a *de re* diamond to the extreme term P. And last, the final line is an introduction of a *de dicto* diamond, thus obtaining the diagram of the conclusion.

Example 4. *Another valid, combined syllogism.*

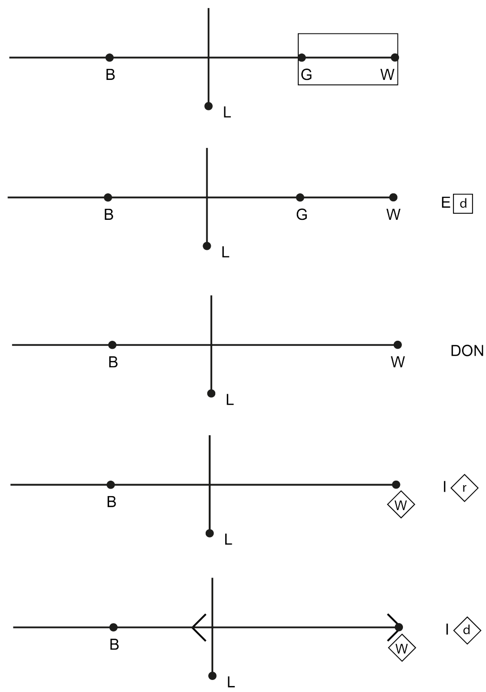
$$\begin{array}{r} 1.\ \mathsf{Ma}\Box\mathsf{P} \\ 2.\ \Diamond(\mathsf{Sa}\Box\mathsf{M}) \\ \vdash \Diamond(\mathsf{Sa}\Diamond\mathsf{P}) \end{array}$$



Once again, the first diagram represents the premises. The second diagram is the result of applying a joint *de dicto/de re*. The third line results from using an elimination of a *de re* square to the middle term M. The fourth diagram is the result of applying DON to the previous diagram. The fifth line is an elimination of a *de re* square to the extreme term P. The final line is an introduction of a *de re* diamond to the extreme term P.

Example 5. *Finally, let us consider a valid, relational, de dicto and de re syllogism (in plain English, this syllogism could say, for example, that since (1) all brides love some groom, and (2) necessarily every groom is wonderful, it follows that all brides possibly love some possibly wonderful people).*

$$\begin{array}{r} 1.\ \mathsf{Ba(LiG)} \\ 2.\ \Box\mathsf{(GaW)} \\ \vdash \mathsf{Ba}\Diamond\mathsf{(Li}\Diamond\mathsf{W)} \end{array}$$



As expected, the first diagram represents the premises. The second diagram is the result of applying an elimination of a *de dicto* square. The third line is the result of applying DON to the previous diagram. The fourth line is an introduction of a *de re* diamond to the extreme term W. Finally, the final line is an introduction of a *de dicto* diamond.

3.3. A Formal Result

Up to this point, we have seen that $\mathcal{ENGL}$ can be extended so as to obtain $\mathcal{ENGL}^{\mu}$. Now, we would like to offer some evidence to the effect that $\mathcal{SYLL}^{\mu}$ can be modeled by $\mathcal{ENGL}^{\mu}$. So, consider the next:

Proposition 1. *A syllogism is valid in* $\mathcal{SYLL}^{\mu}$ *iff its diagram is valid in* $\mathcal{ENGL}^{\mu}$.

Proof. From left to right, we proceed by construction. So, suppose $\mathfrak{S}$ is an arbitrary syllogism that is valid in $\mathcal{SYLL}^{\mu}$; then, we have to show that its corresponding diagram in $\mathcal{ENGL}^{\mu}$ follows from DON or an application of rules 1 to 6. Now, if $\mathfrak{S}$ is valid in $\mathcal{SYLL}^{\mu}$, then, according to [9], (a) the middle term of $\mathfrak{S}$ is distributed (i.e., it is universal or negative) in at least one premise, (b) every term distributed in the conclusion of $\mathfrak{S}$ is distributed in the premises of $\mathfrak{S}$, (c) the number of particular (resp. negative) premises of $\mathfrak{S}$ is equal to the number of particular (resp. negative) conclusions of $\mathfrak{S}$, (d) the conclusion of $\mathfrak{S}$ is not stronger than any premise of $\mathfrak{S}$ (according to [9], there is a transitivity or "strength" of modal operators in such a way that $\Box\mathsf{T}$ implies $\mathsf{T}\Box$, $\mathsf{T}\Box$ implies T, T implies $\mathsf{T}\Diamond$, and $\mathsf{T}\Diamond$ implies $\Diamond\mathsf{T}$. So, a first statement (or term) is stronger than a second statement (or term) if and only if the first implies the second but not the other way around. The intuition is that a necessary condition for the validity of any inference is that the conclusion cannot exceed any premise in strength: the scholastics called this the *peiorem* rule, namely, *peiorem semper sequiter conclusio partem*), and (e) the number of *de dicto* $\Diamond$ premises of $\mathfrak{S}$ is not greater than the number of *de dicto* $\Diamond$ conclusions of $\mathfrak{S}$.

Given conditions (a) to (d), $\mathfrak{S}$ must be of one of the following *de re* forms (Table 4):

Table 4. Valid *de re* syllogisms.

1. $\mathsf{M}\langle a,a,e,e\rangle\mathsf{P}$ 2. $\mathsf{S}\langle a,i,a,i\rangle\mathsf{M}$ $\vdash \mathsf{S}\langle a,i,e,o\rangle\{\mathsf{P},\Diamond\mathsf{P}\}$	1. $\mathsf{M}\langle a,a,e,e\rangle\Box\mathsf{P}$ 2. $\mathsf{S}\langle a,i,a,i\rangle\Box\mathsf{M}$ $\vdash \mathsf{S}\langle a,i,e,o\rangle\{\Box\mathsf{P},\mathsf{P},\Diamond\mathsf{P}\}$	1. $\mathsf{M}\langle a,a,e,e\rangle\Box\mathsf{P}$ 2. $\mathsf{S}\langle a,i,a,i\rangle\mathsf{M}$ $\vdash \mathsf{S}\langle a,i,e,o\rangle\{\Box\mathsf{P},\mathsf{P},\Diamond\mathsf{P}\}$
1. $\mathsf{M}\langle a,a,e,e\rangle\mathsf{P}$ 2. $\mathsf{S}\langle a,i,a,i\rangle\Box\mathsf{M}$ $\vdash \mathsf{S}\langle a,i,e,o\rangle\{\mathsf{P},\Diamond\mathsf{P}\}$	1. $\mathsf{M}\langle a,a,e,e\rangle\Diamond\mathsf{P}$ 2. $\mathsf{S}\langle a,i,a,i\rangle\Box\mathsf{M}$ $\vdash \mathsf{S}\langle a,i,e,o\rangle\Diamond\mathsf{P}$	1. $\mathsf{M}\langle a,a,e,e\rangle\Diamond\mathsf{P}$ 2. $\mathsf{S}\langle a,i,a,i\rangle\mathsf{M}$ $\vdash \mathsf{S}\langle a,i,e,o\rangle\Diamond\mathsf{P}$

Now, let us model these syllogisms in $\mathcal{ENGL}^{\mu}$ in order to obtain their respective diagrams (Figure 7).

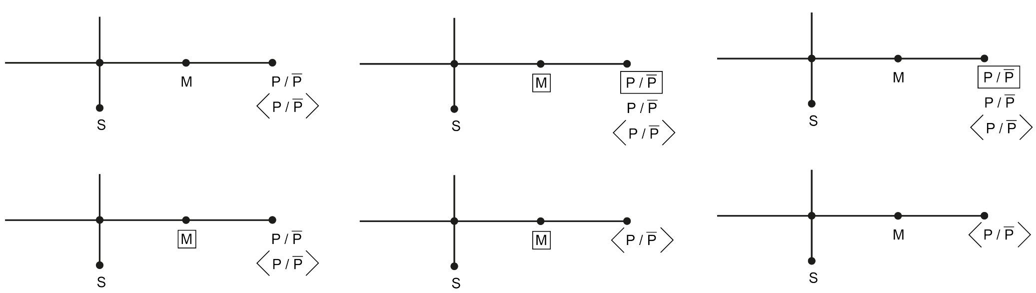


Figure 7. Valid *de re* syllogisms in $\mathcal{ENGL}^{\mu}$.

Notice that if conditions (a), (b), and (c) hold, the diagram of $\mathfrak{S}$ in $\mathcal{ENGL}^{\mu}$, $\mathfrak{D}$, looks exactly like the diagram of a valid syllogism in $\mathcal{ENGL}$. Additionally, if condition (d) holds, then the order of the new *de re* rules also holds. Indeed, if condition (d) is met, then $\mathfrak{D}$ follows from ordered applications of rules 1, 2, 3, and 4. Condition (e) is trivial in this case because there are zero *de dicto* $\diamond$ statements.

And that does it for the *de re* syllogisms; now, given conditions (a) to (e), $\mathfrak{S}$ must be of one of the following *de dicto* forms (Table 5):

Table 5. Valid *de dicto* syllogisms.

1. $\Box(\mathsf{M}\langle a,a,e,e\rangle \mathsf{P})$ 2. $\Box(\mathsf{S}\langle a,i,a,i\rangle \mathsf{M})$ $\vdash \{\Box(\mathsf{S}\langle a,i,e,o\rangle \mathsf{P}), \mathsf{S}\langle a,i,e,o\rangle \mathsf{P}, \diamond(\mathsf{S}\langle a,i,e,o\rangle \mathsf{P})\}$	1. $\Box(\mathsf{M}\langle a,a,e,e\rangle \mathsf{P})$ 2. $\mathsf{S}\langle a,i,a,i\rangle \mathsf{M}$ $\vdash \{\mathsf{S}\langle a,i,e,o\rangle \mathsf{P}, \diamond(\mathsf{S}\langle a,i,e,o\rangle \mathsf{P})\}$
1. $\mathsf{M}\langle a,a,e,e\rangle \mathsf{P}$ 2. $\Box(\mathsf{S}\langle a,i,a,i\rangle \mathsf{M})$ $\vdash \{\mathsf{S}\langle a,i,e,o\rangle \mathsf{P}, \diamond(\mathsf{S}\langle a,i,e,o\rangle \mathsf{P})\}$	1. $\mathsf{M}\langle a,a,e,e\rangle \mathsf{P}$ 2. $\mathsf{S}\langle a,i,a,i\rangle \mathsf{M}$ $\vdash \{\mathsf{S}\langle a,i,e,o\rangle \mathsf{P}, \diamond(\mathsf{S}\langle a,i,e,o\rangle \mathsf{P})\}$

Likewise, let us model these syllogisms in $\mathcal{ENGL}^{\mu}$ in order to obtain their respective diagrams (Figure 8).

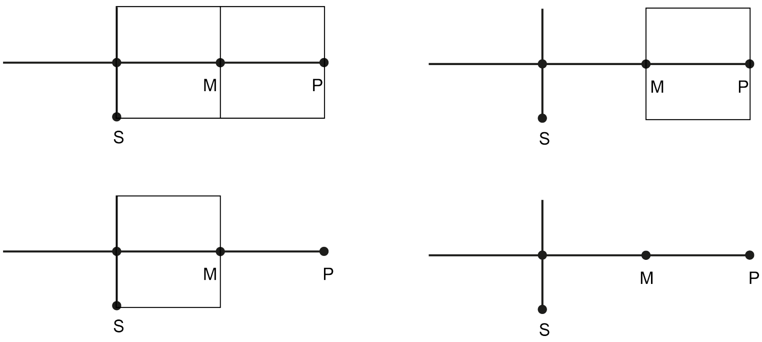


Figure 8. *De dicto* syllogisms.

Observe, then, that if conditions (a), (b), and (c) hold, the diagram of $\mathfrak{S}$ in $\mathcal{ENGL}^{\mu}$, $\mathfrak{D}$, looks exactly like the diagram of a valid syllogism in $\mathcal{ENGL}$. If condition (d) holds, then $\mathfrak{D}$ follows from ordered applications of rules 1, 2, 3, and 4. And if condition (e) is met, then the number of a *de dicto* $\diamond$ diagrams of $\mathfrak{D}$ is not greater than the number of *de dicto* $\diamond$ conclusions.

Finally, if $\mathfrak{S}$ is a combination of valid *de dicto* and *de re* forms, then $\mathfrak{S}$ can be reduced to a *de dicto* form, since *de re* syllogisms can be reduced to assertoric syllogisms by collapsing the modal *de re* terms into simple, assertoric terms.

From right to left, we proceed by *reductio*. Thus, suppose an arbitrary diagram $\mathfrak{D}$ is valid in $\mathcal{ENGL}^{\mu}$ but its corresponding syllogism $\mathfrak{S}$ is invalid in $\mathcal{SYLL}^{\mu}$. If $\mathfrak{S}$ is invalid in $\mathcal{SYLL}^{\mu}$, then $\mathfrak{S}$ does not comply with at least one of the conditions (a) to (e). Now, if $\mathfrak{D}$ is valid in $\mathcal{ENGL}^{\mu}$, it complies with DON or the modal rules 1 to 6. If $\mathfrak{D}$ complies with DON, then conditions (a) to (c) of $\mathcal{SYLL}^{\mu}$ must hold for $\mathfrak{S}$. Indeed, if $\mathfrak{D}$ complies with DON, then the diagram of the middle term M is between the diagrams of S and P, which implies that M is distributed (which is condition (a)); the diagram of S and P is in the required position of the conclusion, so that every term distributed in the conclusion of $\mathfrak{S}$ is distributed in the premises of $\mathfrak{S}$ (which is condition (b)), and the number of intersections (resp. negated terms) in the diagram of the premises is equal to the number of intersections (resp. negated terms) in the diagram of the conclusion (which is condition (c)). And now, if rules 1 to 4 hold for $\mathfrak{D}$, then $\mathfrak{S}$ must follow the order of strength of the modal operators, so that the conclusion of $\mathfrak{S}$ is not stronger than any premise of $\mathfrak{S}$ (which is condition (d)); finally, if $\mathfrak{D}$ follows from rules 5 or 6, then the number of *de dicto* $\diamond$ premises of $\mathfrak{S}$ is not greater than the number of *de dicto* $\diamond$ conclusions of $\mathfrak{S}$ (which is condition (e)). But then, if $\mathfrak{D}$ complies with DON or rules 1 to 6, $\mathfrak{S}$ follows conditions (a) to (e), and then $\mathfrak{S}$ is valid in $\mathcal{SYLL}^{\mu}$, which contradicts our initial assumption. $\Box$

4. Final Remarks

In this paper, we have offered a simple extension of Englebretsen's line diagrams in order to represent modal syllogistics. We can wrap this up with the help of the next diagram:

$$\begin{tikzcd}
\mathcal{SYLL} \arrow[r, "\text{extends to}"] \arrow[d, "\text{modeled by}"'] & \mathcal{SYLL}^{\mu} \arrow[d, "\text{modeled by}"] \\
\mathcal{ENGL} \arrow[r, "\text{extends to}"] & \mathcal{ENGL}^{\mu}
\end{tikzcd}$$

This exercise, though humble, shows the power of Englebretsen's diagrams in the sense that they are so basic that they allow us to add different objects and rules in order to increase both their deductive and expressive powers (cf. [15]), and opens a non-zero-sum discussion between systems that represent modal logic using diagrams, for instance, between Peircean existential graphs and Englebretsen diagrams in terms of their logical parsimony in the sense that whatever the results of such a discussion, we will all win. Let us win!

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Abbreviations

The following abbreviations are used in this manuscript:

$\mathcal{ENGL}$	Englebretsen's line diagrams
$\mathcal{ENGL}^{\mu}$	Englebretsen's line diagrams plus modality
DON	*Dictum de omni et nullo*
Pr. An.	Aristotle's *Prior Analytics*
$\mathcal{SYLL}$	Assertoric syllogistic
$\mathcal{SYLL}^{\mu}$	Modal syllogistic

References

1. Castro-Manzano, J.M. Remarks on the Idea of Non-monotonic (Diagrammatic) Inference. *Rev. Filos. Open Insight* **2017**, *8*, 243–263. [CrossRef]
2. Castro-Manzano, J.M. When Do We Say That a Diagram is a Logic Diagram? A Comparative Study. *Cogency J. Reason. Argum.* **2021**, *13*, 71–103. [CrossRef]
3. Becker, A. *Die Aristotelische Theorie der Möglichkeitsschlüsse*; Wissenschaftliche Buchgesellschaft: Berlin, Germany, 1968.
4. Bocheński, J. *Formale Logik*; Karl Alber: München, Germany, 1962.
5. Łukasiewicz, J. *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*; Clarendon Press: Oxford, UK, 1957.
6. Kneale, W.; William Kneale, M.; Kneale, W.; Kneale, M.; Conte, A.; Press, O.U. *The Development of Logic*; Clarendon Press: Oxford, UK, 1962.
7. Hintikka, J.; Hintikka, K. *Time & Necessity: Studies in Aristotle's Theory of Modality*; Clarendon Press: Oxford, UK, 1973.
8. Striker, G. Assertoric vs. Modal Syllogistic. *Anc. Philos.* **1994**, *14*, 39–51. [CrossRef]
9. Englebretsen, G. Preliminary notes on a new modal syllogistic. *Notre Dame J. Form. Log.* **1988**, *29*, 381–395. [CrossRef]
10. Rini, A.A. Is There a Modal Syllogistic? *Notre Dame J. Form. Log.* **1998**, *39*, 554–572. [CrossRef]
11. Thom, P. *The Logic of Essentialism: An Interpretation of Aristotle's Modal Syllogistic*; The New Synthese Historical Library: Dordrecht, The Netherlands, 2012.
12. Malink, M. *Aristotle's Modal Syllogistic*; Harvard University Press: Cambridge, MA, USA, 2013.
13. Englebretsen, G. Linear diagrams for syllogisms (with relationals). *Notre Dame J. Form. Log.* **1991**, *33*, 37–69. [CrossRef]

14. Englebretsen, G. *Something to Reckon with: The Logic of Terms*; Canadian Electronic Library: Books Collection; University of Ottawa Press: Ottawa, ON, Canada, 1996.
15. Englebretsen, G.; Castro-Manzano, J.M.; Pacheco-Montes, J.R. *Figuring It Out: Logic Diagrams*; Philosophical Analysis; De Gruyter: Berlin, Germany, 2019.