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Abstract: The construction of complex networks from electroencephalography (EEG) proves to be an effective method for representing emotion patterns in affection computing as it offers rich spatiotemporal EEG features associated with brain emotions. In this paper, we propose a novel method for constructing complex networks from EEG signals for emotion recognition, which begins with phase space reconstruction to obtain ordinal patterns and subsequently forms a graph network representation from the sequence of ordinal patterns based on the visibility graph method, named ComNet-PSR-VG. For the proposed ComNet-PSR-VG, the initial step involves mapping EEG signals into a series of ordinal partitions using phase space reconstruction, generating a sequence of ordinal patterns. These ordinal patterns are then quantified to form a symbolized new sequence. Subsequently, the resulting symbolized sequence of ordinal patterns is transformed into a graph network using the visibility graph method. Two types of network node measures, average node degree (AND) and node degree entropy (NDE), are extracted from the graph networks as the inputs of machine learning for EEG emotion recognition. To evaluate the effectiveness of the proposed construction method of complex networks based on the visibility graph of ordinal patterns, comparative experiments are conducted using two types of simulated signals (random and Lorenz signals). Subsequently, EEG emotion recognition is performed on the SEED EEG emotion dataset. The experimental results show that, with AND as the feature, our proposed method is 4.88% higher than the existing visibility graph method and 12.23% higher than the phase space reconstruction method. These findings indicate that our proposed novel method for constructing complex networks from EEG signals not only achieves effective emotional EEG pattern recognition but also exhibits the potential for extension to other EEG pattern learning tasks, suggesting broad adaptability and application potential for our method.

Keywords: emotion recognition; complex network; ordinal patterns

1. Introduction

The emotional dimensions of electroencephalography (EEG) have garnered increasing recognition, owing to its extensive applications in diagnosing mental illnesses and facilitating human–computer interaction [1,2]. By delving into the study of emotional patterns within EEG, we can enrich our comprehension of human behavior, refine psychological health treatment methodologies, and cultivate more intelligent and responsive systems within the realm of human–computer interaction. In recent years, the efficacy of complex networks in unraveling the spatiotemporal characteristics and dynamic shifts in emotional EEG has become evident. EEG signals, serving as physiological indicators of brain activity, contribute significantly to this exploration. Given the intricate structure and interconnections within the brain network, the adoption of complex networks for analyzing both brain networks and emotional EEG has garnered increasing attention. Research



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Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). grounded in complex networks offers a more holistic insight into the intricate topology and information transmission among distinct brain regions [3]. In [4], Lu et al. constructed a new complex network pattern based on the arrangement characteristics of time series and achieved excellent results in brain state recognition based on EEG signals in physiology and pathology. In [2], Yao et al. constructed a complex network of EEG signals using a viewable approach and extracted spatial network features, achieving high resolution in EEG emotion recognition. The transformation of EEG data into complex networks proves to be a valuable approach, providing a more effective representation of the complexity and dynamics inherent in brain activity. This transformation enhances our capacity to accurately capture the neural mechanisms associated with emotions. Consequently, this avenue of research holds the promise of advancing our understanding of emotional EEG, paving the way for innovative developments in neuroscience and human–computer interaction.

Complex network methods have the capability to unveil intricate interactions and connectivity patterns among various brain regions, a collaboration crucial in emotional processing. By scrutinizing connection patterns within complex networks, a deeper comprehension of the functions and interactions among different brain regions during emotional processing is attained. Unlike time-domain or frequency-domain methods applied in EEG signal analysis, complex networks can encapsulate both global and local features within the brain network, thus surpassing the constraints of localized time- or frequency-domain features [3]. The dynamic fluctuations within the brain network across different time points are observable through the construction of complex networks, providing a more profound insight into the spatiotemporal characteristics of brain activity during emotional processes.

To comprehensively analyze the connection density of nodes in complex networks from both local and global perspectives, effective measures, such as average node degree (AND) and node degree entropy (NDE), come into play. The AND serves as a valuable metric to offer overall insights into the connection density of nodes in a network, providing a descriptive overview of the network's general properties. Meanwhile, NDE plays a pivotal role in the analysis of complex networks, aiming to articulate the uncertainty and diversity inherent in the degree distribution among nodes. The degree of a node denotes the number of edges connected to it, and NDE takes into consideration the distribution of these degrees, shedding light on the quantity and relative frequency of nodes with varying degrees in the network. This metric offers crucial information about the degree distribution across nodes, allowing for a deeper understanding of how nodes interconnect and the prevalence of nodes with similar or distinct degrees. By capturing the uncertainty inherent in degree distribution, NDE becomes a powerful tool for unraveling the intricacies of network structure. It operates as a metric for gauging the complexity of the network with highly structured networks exhibiting higher node degree entropy. Additionally, NDE can be harnessed to scrutinize the correlation between node degrees, uncovering connections between nodes with specific degrees. This aspect proves instrumental in capturing features of degree correlation, providing valuable insights into the network's organization. In essence, NDE, by encapsulating the diversity and uncertainty present in degree distribution, contributes supplementary information for a more profound and nuanced analysis of complex networks.

The phase space reconstruction method involves deriving a set of multidimensional vectors from the original time series using embedding dimensions and delay time estimation techniques [5]. These vectors serve as nodes in the complex network, and the edges connecting these nodes are determined based on the similarity between vectors. However, this method faces instability issues during the embedding dimensions and delay time estimation process. Additionally, establishing the optimal threshold for edge relationship judgment proves challenging, resulting in diminished robustness in practical applications [4,6]. On the other hand, the visibility graph construction method regards data points in the original time series as nodes in the network with the visual relationships between these data points serving as edges [7,8]. In contrast to the phase space reconstruction method, the visibility graph construction method boasts fewer parameters and

enhanced algorithmic robustness [9–11]. However, it is important to note that the size of the network in this approach is directly proportional to the length of the time series. Consequently, when analyzing longer time series, the complexity of the network increases correspondingly, leading to heightened computational complexity in extracting subsequent features from the complex network [12–15].

In our research, we propose a pioneering method for constructing complex networks, which diverges from traditional approaches. The novelty of our method lies in the fusion of phase space reconstruction techniques and visibility graph methods, enabling the simultaneous depiction and analysis of complex network structures and dynamic behaviors from both temporal and spatial viewpoints. Phase space reconstruction delves into the internal relationships and dynamic behaviors of network nodes, while visibility graph construction highlights the overarching structure and connectivity patterns of nodes [16,17].

By amalgamating the phase space reconstruction and visibility graph methods, we harness the advantages of both approaches, thereby enhancing the accuracy and robustness of complex network construction. The specific implementation can be tailored and fine-tuned according to practical needs [18]. Through the integration of these two methods, we attain a more comprehensive comprehension of network properties and patterns. By concurrently leveraging the benefits of phase space reconstruction and visibility graph construction, we augment the efficiency and precision of our analyses. In summary, the main contributions of our work include the following:

- A novel method for constructing complex networks from EEG signals, named ComNet-PSR-VG, is introduced by exploiting both the phase space reconstruction method and the visibility graph method;
- (2) Employing the proposed ComNet-PSR-VG method to effectively identify EEG emotion states, obtaining outstanding classification outcomes of emotion recognition.

The remainder of the paper is as follows: The second part presents the proposed new method for constructing complex networks and the extracted network structure features; the third part presents the results of the data analysis and EEG emotion classification experiments; the fourth part compares our method with existing related research through experiments and results; and the last part is the conclusion of the article.

2. Materials and Methods

Our proposed method includes several key steps, as shown in Figure 1:

- Recording the corresponding emotional EEG signals generated by different emotional stimuli;
- Constructing complex networks for each channel of EEG signals using the proposed method and proposing network structure entropy features;
- Extracting entropy features of network structure;
- Inputting these features as feature sequences into the machine-learning model to
 obtain the corresponding classification results.



Figure 1. Framework of the proposed EEG emotion recognition.

2.1. Experimental Dataset

In our study, we utilized the SEED (SJTU Emotion EEG Dataset), an openly available dataset for thorough analysis. This dataset encompasses data from 15 Chinese subjects with a gender distribution of 7 males and 8 females and an average age of 23.27 years (standard deviation: 2.37). The emotional stimuli for the participants were derived from 15 Chinese film clips, each designed to elicit positive, neutral, or negative emotions, and each film lasted approximately 4 min. To execute our experiments, each participant engaged in 15 trials, resulting in a total of 45 trials (15 trials for each of the three emotional categories: positive, neutral, and negative). The experimental design comprised three distinct groups of experiments [19]. In each trial, the subjects were exposed to emotional stimuli through designated film clips, inducing the specified emotion (positive, neutral, or negative). This rigorous experimental setup aimed to comprehensively capture the varied responses to emotional stimuli across the different emotional categories.

2.2. Construction of Complex Networks from EEG Signals Based on Visibility Graph of Ordinal Patterns

The signal from each channel in EEG can be treated as a time series $\{x_i\}$, where i = 1, 2, ... N. Initially, the phase space reconstruction method is used to reconstruct this time series into a sequence [4] using embedding dimensions *d* and time delay τ . The resulting sequence can be written as follows:

$$v_j = (x_j, x_{j+\tau}, x_{j+2\tau}) \ j = 1, 2, 3, \dots, L$$
 (1)

where $L = N - (d - 1) * \tau$ and denotes the number of partitions v_i in the resulting sequence.

Subsequently, each partition v_j is mapped into an ordinal pattern $O^{(i)} = (\pi_0, \pi_1, \pi_2, ..., \pi_{d-1})$ where $\pi_i \in \{0, 1, 2, ..., d-1\}$ $(\pi_i \neq \pi_j \text{ if } i \neq j)$. Specifically, the indices of each element in the partition $v_i = (x_i, x_{i+\tau}, x_{i+2\tau}, ..., x_{i+(d-1)\tau})$ are rewritten to $v_i = (x_{i+\pi_0}, x_{i+\pi_1}, x_{i+\pi_2}, ..., x_{i+\pi_{d-1}})$, according to the ascending order of the values of elements in the partition v_i :

$$x_{i+\pi_0} \le x_{i+\pi_1} \le x_{i+\pi_2} \dots \le x_{i+\pi_{d-1}}, \forall x_{i+\pi_k} \in v_i \text{ and } \pi_k = \{0, 1, 2, \dots, d-1\}$$
(2)

For example, taking the $\{18, 9, 5, 11\}$ as a partition, it can be mapped to an ordinal pattern $\{2, 1, 3, 0\}$.

Finally, we introduce a metric for quantifying the ordinal patterns, denoted as the ordinal pattern number (OPN) [4]. Its formulation is articulated as follows:

$$OPN\left(O^{(i)}\right) = Inv\left(\pi_{0}\right) \times (d-1)! + Inv\left(\pi_{1}\right) \times (d-2)! + \dots + Inv\left(\pi_{d-2}\right) \times (1)! + 1 \tag{3}$$

where (·)! denotes the factorial function, and *Inv* (π_i) represents the inverse number of each element π_i in the ordinal pattern $O^{(i)} = (\pi_0, \pi_1, \pi_2, ..., \pi_{d-1})$. In accordance with Equation (3), the minimum value of the *OPN* is 1, which corresponds to the permutation $\pi = (0, 1, 2, ..., d - 1)$ in ascending order, the maximum value of the *OPN* is *d*!, and descending order is $\pi = (d - 1, ..., 2, 1, 0)$.

Following the aforementioned time series transformation and employing the phase space reconstruction method and ordinal pattern quantization, the time series with a data length of N is transformed into a symbol sequence with a length of L. Subsequently, utilizing the visibility graph method [4], the resulting symbol sequence is mapped into a graph network. To clearly illustrate the proposed method, which constructs a network, the basic process of constructing a complex network from a time series is shown in Figure 2a. Figure 2b presents the proposed method for time-series mapping to the OPN of network nodes.



Figure 2. (a) Construction of complex networks from time series based on the visibility graph of ordinal patterns; (b) The proposed mapping algorithm for time-series mapping to the OPN of network nodes.

2.3. Extracting Network Entropy Measures from Complex Networks

Network measures are commonly expressed through diverse network structural parameters, such as nodes and links, which typically represent network-related features and characterize the patterns of the network. As one of the classical network measures, the average node degree (AND) serves as a valuable tool for offering comprehensive insights into the connection density of nodes within the network. This network node measure serves as an effective descriptor of the overall properties of the network. It captures the average connection strength among neighboring nodes, facilitating an understanding of the distribution of node degrees and the characteristics of connections in the network. The calculation expression for AND is as follows:

$$k_{nn} = \frac{1}{N} \sum_{i=1}^{N} k_{nn}^{i}$$
 (4)

where k_{nn}^{i} indicates the degree of neighboring nodes for a node.

Network entropy, derived from information theory, is a measure of disorder used to quantify the information content encoded within a graph network. It provides a quantitative metric to assess network complexity. As one of the crucial network structure entropies, the strength of node degree entropy (NDE) lies in its comprehensive and unified depiction of the degree distribution within the network structure, determined through the consideration of neighbor degrees of nodes. The NDE proves highly effective in assessing node heterogeneity concerning neighbor degrees with its calculation expressed as follows:

$$H = -\sum_{i} p_i \log p_i \tag{5}$$

 p_i is the probability description of the node degree, which can be expressed in the following form:

$$p_i = \frac{d_i}{\sum\limits_{j=1}^N d_j} \tag{6}$$

 d_i is the number of neighbors in a node network.

2.4. Machine-Learning Model

The support vector machine (SVM) stands as a pivotal classification model in the realm of machine learning with the primary goal of delineating samples by identifying an optimal hyperplane. Its fundamental objective centers around maximizing intervals for effective segmentation. In our research, we leveraged individual channels of EEG signals as distinctive structural attributes within a network. The SVM served as our classifier, adept at distinguishing between positive and negative emotions. Harnessing kernel-based capabilities, the SVM exhibited prowess in achieving both linear and nonlinear classifications, thanks to diverse kernel functions with varying performance characteristics. Our study meticulously scrutinized multiple prevalent SVM kernels, ultimately identifying the radial basis function (RBF) as the most efficient performer. For our SVM classifier, we utilized the LIBSVM software package (https://www.csie.ntu.edu.tw/~cjlin/libsvm/), specifically implementing the RBF kernel. The configuration of SVM parameters involved values such as S, T, and C alongside default settings. S is the model setting type for SVM, T is the kernel function type, and C is the cost. Notably, T was set at 2, while S stood at 0. Determining the optimal C value entailed a meticulous one-step search within the parameter space $(10^{-3:2})$. Our methodological framework, which integrates complex network feature measures for emotive recognition via the SVM classifier, is comprehensively illustrated in Figure 3.



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Figure 3. Flowchart of machine-learning classification using complex network features.

2.5. Performance Evaluation

In our study, accuracy, sensitivity, and specificity serve as the performance metrics for evaluating the EEG emotion recognition task. Positive emotion is designated as positive instances, while negative emotion is designated as negative instances. The mathematical definitions of these evaluation metrics are expressed as follows:

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN}$$
(7)

$$Sensitivity = \frac{TP}{TP + FN}$$
(8)

$$Specificity = \frac{TN}{TN + FP}$$
(9)

where *TP* represents the number of the true positive test samples correctly classified as positive, *FN* represents the number of the true positive test samples incorrectly classified as negative, *TN* represents the number of the true negative test samples correctly classified as negative, and *FP* represents the number of the true negative test samples incorrectly classified as positive.

3. Results

In our experiment, we first evaluate the performance of the proposed complex network construction method using simulated signals. We employ numerically generated time series with well-defined properties to initiate our empirical exploration. Within our investigation, we delve into the analysis of numerically simulated chaotic signals, widely acknowledged as robust approximations of numerous real-world datasets. Furthermore, we evaluate the performance of the proposed method, which constructs a complex network using EEG emotion signals. We broaden the scope of our proposed approach for network construction to analyze EEG signals, thus shedding light on its prospective applications.

3.1. Performance Evaluation of the Proposed Complex Network Method Using the Simulated Signals

The purpose of the experiment is to use Lorenz signals and random signals as examples to verify the ability of our method to convert time series into network representations. Random time series are comprised of sequences of sequentially uncorrelated random variables. In our study, the random signals utilized consist of uniformly distributed pseudo-random numbers within the interval (0, 1). Figure 4a illustrates an example of the random time series used in our study, comprising 2000 samples (data points). To further underscore the robust applicability of the proposed method, which constructs a complex network for time series analysis, we extend our investigation to constructing networks for chaotic signals. In our experimentation, simulated chaotic signals are generated using a Lorenz system with the system function defined by Equation (10). This equation yields components x, y, and z, corresponding to the convection velocity, temperature difference, and temperature gradient components, respectively. Figure 4b portrays an example of the x component of the Lorenz system employed in our experiment, comprising 2000 samples.



Figure 4. Experimental results from random signals and Lorenz signals by the proposed network construction method. (a) An example of random time series; (b) An example of x component of Lorenz system by the proposed network construction method.

In the context of our proposed complex network methodology for constructing networks from random signals, the scalar time series undergo an initial reconstruction process into a sequence of ordinal partitions. This reconstruction is based on the phase space reconstruction method, utilizing different embedded dimensions (d = 6) with a fixed time lag (τ = 2). In accordance with the definition of the proposed method, which constructs a complex network, each ordinal partition is considered a network node, characterized by a specific set of ordinal patterns.

As shown in Figure 5a,b, the experimental results of the adjacency matrix of the unweighted network structure for the random signal and Lorenz signal x components of two thousand samples are presented, based on the proposed new method with embedded dimension d = 6 with time lag $\tau = 2$.



Figure 5. Experimental results for the adjacency matrix of the network construction from random signals and Lorenz signals by the proposed network construction method using embedded dimension d = 6 with time lag $\tau = 2$. (a) The result of the adjacency matrix for the random signal; (b) The result of the adjacency matrix for the Lorenz signal x components.

We established 10 sets of Lorenz signals and 10 sets of random signals, employing the proposed method to extract the NDE and AND network features from these respective signal sets. A comparative analysis of the feature results was conducted. Figure 6a shows the AND results for the 10 sets of Lorenz signals; the range of the AND values is from 330 to 390. Figure 6b shows the AND results for the 10 sets of random signals; the range of the AND values is from 5.35 to 5.55. From Figure 6a,b, it can be concluded that the AND value of the Lorenz signal is significantly higher than that of the random signal. In Figure 6c, a box plot is presented for the NDE results, illustrating a comparison between the Lorenz signals and random signals. The median NDE value of the Lorenz signal is 4.34, while the median NDE value of the random signal. The time series with different characteristics exhibit significant differences in their network parameters, which is the significance demonstrated by Figure 6.



Figure 6. Experimental results of extracting NDE and AND from Lorenz signals and random signals using the proposed method. (**a**) AND results based on Lorenz signals, (**b**) AND results based on random signals, (**c**) NDE results based on Lorenz signals and random signals.

3.2. Performance Evaluation of EEG Emotion Recognition Based on the Proposed Complex Network Construction Method

In Figure 7, the EEG data utilized in this study span a duration of 2 min, carefully selected from the midpoint of the 62-channel EEG signals (specifically, from 60 s to 180 s). The SEED dataset encompasses EEG signals from 15 subjects, each with 62 channels. For each channel, we embarked on constructing a complex network using three distinct methods. Subsequently, we extracted the network node degree entropy, employed it as the input for the machine-learning models, and garnered the ensuing classification results. Figure 7 and Table 1 elucidate the comparative outcomes of the three methods for classifying positive and negative emotions within the SEED dataset. Figure 7a contrasts the outcomes for positive and negative emotions based on the AND features, while Figure 7b compares the results based on the NDE features. Upon scrutinizing the classification results in Figure 7, it becomes evident that the proposed method's performance in classifying positive and negative emotions outshines significantly when compared to the outcomes of the other two conventional complex network construction methods.



Figure 7. The intra-individual comparison of classification results (both binary and triple) among the different methods is based on the SEED dataset. (**a**) Comparison of outcomes for positive and negative emotions based on the AND features; (**b**) Comparison of outcomes for positive and negative emotions based on the NDE features.

Feature	Method	Sensitivity (%)	Specificity (%)	Accuracy (%)
	OPVG	90.96 ± 4.06	91.24 ± 6.46	91.39 ± 4.69
AND	VG	84.69 ± 8.63	86.54 ± 9.04	86.51 ± 3.19
	PSR	79.3 ± 8.49	80.08 ± 8.40	79.16 ± 3.69
NIDE	OPVG	84.84 ± 6.97	85.99 ± 7.29	85.39 ± 7.09
NDE	VG	82.40 ± 7.98	83.36 ± 8.79	82.84 ± 8.35
	PSR	81.01 ± 7.23	82.40 ± 8.01	81.66 ± 7.57

Table 1. The performance of the ComNet-PSR-VG method in constructing both AND and NDE features to identify positive and negative emotions from EEG signals in the SEED dataset is detailed. The values are presented as means \pm standard deviations with positive emotions specified as positive instances and negative emotions as negative instances.

In this study, we conducted a comparative analysis of the impact of various data lengths on classification outcomes. Figure 8 presents our exploration using data spans of 30 s (from 60 to 90 s), 45 s (from 60 to 105 s), 60 s (from 60 to 120 s), 75 s (from 60 to 135 s), 90 s (from 60 to 150 s), 105 s (from 60 to 175 s), and 120 s (from 60 to 180 s) extracted from the SEED dataset. We employed our proposed methodology to construct complex networks for each of these seven data lengths. Subsequently, we derived the NDE feature from the constructed complex networks and inputted them into machine-learning models for classification. The outcomes depicted in Figure 8 reveal that, concerning the NDE

feature, the classification performance for the 2 min data surpasses that of other durations. Our experimental findings indicate that selecting longer-duration data yields improved classification outcomes compared to shorter durations. Notably, with a data duration of 45 s, the proportion of redundant information increases, resulting in a slight decline in classification accuracy. Nevertheless, the overarching trend illustrates that, as the duration of the data increases, classification accuracy tends to enhance, reaching its pinnacle and experiencing minimal variance with a 120 s data duration.



Figure 8. Comparison of classification results using the proposed method on different data durations.

4. Discussion

In our research, we propose an innovative approach to constructing complex networks for EEG analysis, specifically targeting emotion recognition. This method synergizes the features of phase space and visibility, demonstrating remarkable performance in emotion recognition based on EEG signals across two categories. Our proposed method differs from the existing approaches in several key aspects.

First, the selection of nodes for constructing complex networks diverges from the conventional methods. While existing approaches typically select ordinal numbers of time series as nodes, our method employs two parameters—dimension and delay—to map time series to phase space. Nodes in this phase space then serve as the foundation for constructing complex networks. Subsequently, these nodes are mapped into complex networks using visibility methods. The rationale behind the success of our method lies in the belief that the amalgamation of temporal and spatial features captures more physiological information than relying solely on temporal features. To elucidate the distinctions in representations of EEG signals in separate time-domain features and in combination with spatiotemporal features, we conducted a comprehensive analysis. Specifically, we performed time-domain feature analysis and spatiotemporal feature analysis on EEG emotional signals and EEG epilepsy signals separately. Subsequently, we compared the results obtained from these analyses. This meticulous approach provides insights into the efficacy of our proposed method, shedding light on its potential advantages in understanding and categorizing EEG signals related to emotions.

To demonstrate the superior performance of our proposed method in EEG emotion classification, we conducted a comprehensive comparison with recent studies that utilized the same SEED dataset. Our evaluation involved benchmarking against studies conducted by Zheng, Li, and Song.

In Zheng's research, the group sparse canonical correlation analysis (GSCCA) method was introduced to perform simultaneous electroencephalogram (EEG) channel selection and emotion recognition. Li's study utilized the graph regularized sparse linear regression (GRSLR) approach to address EEG emotion recognition problems, while Song's study employed dynamical graph convolutional neural networks (DGCNN) for EEG emotion recognition.

Upon analyzing the results, as depicted in Table 2, the individual EEG emotion classification accuracies for Zheng's study, Li's study, and Song's study were 82.96%, 87.39%, and 90.40%, respectively. Notably, our proposed ComNet-PSR-VG method achieved an

outstanding individual EEG emotion classification accuracy of 91.39%, signifying a significant enhancement in classification performance. These outcomes suggest that our method outperforms the benchmark studies in EEG emotion classification. The proposed ComNet-PSR-VG method effectively preserves crucial spatial structural information within the EEG, enabling more accurate and efficient classification of emotions. The experimental results underscore the method's robustness and its ability to achieve superior performance in the realm of EEG emotion recognition.

Title 1	Dataset	Methodology	Mean Accuracy	StdACC
Zheng's study [20]	SEED	GSCCA	82.96%	9.95%
Li's study [21]	SEED	GRSLR	87.39%	8.64%
Song's study [22]	SEED	DBN-CRF	90.40%	8.49
Our work	SEED	NEM	91.39%	4.69%

Table 2. The results of classification accuracy from Zheng's study, Li's study, and Song's study.

In the realm of EEG emotion recognition, the temporal and spatial characteristics of features harbor abundant information, enabling a more comprehensive depiction of brain activity patterns and subsequently enhancing the precision of emotion recognition [23,24]. Tao's investigation [25] employs attention-based convolutional recurrent neural networks (ACRNN) to dynamically assign weights to different channels, integrating extended self-attention into the RNN. This methodology yields features that retain rich information across channels and time, demonstrating significant superiority over traditional emotion recognition methods. In Wang's study [26], a hybrid spatial–temporal feature fusion neural network (STFFNN) is introduced, amalgamating extracted features through convolutional neural networks (CNN) for spatial learning and utilizing Bi LSTM for network storage by merging temporal and spatial features. In our study, we also extract features preserving rich spatial and temporal information. However, our approach involves constructing a new spatial network for EEG signals within the framework of complex networks to enhance the extraction of EEG information.

Emotion recognition based on EEG signals holds promising applications, including auditory attention research and clinical psychiatric investigations. Despite these prospects, there are inherent limitations in the current research. This article presents a novel complex network achieved through the fusion of phase space reconstruction and visibility graph, thereby retaining the intricate temporal and spatial features of EEG signals. The absence of a standardized criterion for selecting spatial dimensions and time-delay parameters in phase space construction necessitates a discussion tailored to different signals and research contexts. Moreover, emotional stimulation introduces a certain impact on the selection of EEG patterns and features. In Chen's study [27], a discernible relationship between emotion and cognition was identified in specific regions during emotional interference, encompassing the bilateral dorsal anterior cingulate cortex, anterior insula, left inferior frontal gyrus, and superior parietal lobule, which exhibit sustained effects in these areas. Research affirms the nervous system's involvement in various interference processing types with the regulation of emotional and cognitive interference relying on interactions within extensive distributed brain networks. In Di Plinio's investigation [28], the pivotal role of the default mode network (DMN) region and executive region in emotional interference processes was demonstrated. Negative emotional interference prompts activity regulation in diverse regions, such as the frontal and parietal lobes, correlating with the regulation of functional connections between these task-activation regions and DMN regions. Both studies highlight that emotional interference triggers engagement in emotional processing activities in specific brain regions, influencing characteristic responses within the brain

network. Consequently, subsequent EEG emotion classification research should factor in the impact of emotional interference and opt for suitable classification modes and features.

5. Conclusions

In this paper, we present a novel approach to construct complex networks for EEG emotion recognition by synergizing the phase space reconstruction and visibility graph methods. The main innovation in our proposed method lies in the seamless integration of the phase space reconstruction and visibility graph methods. From the perspective of the visibility graph of ordinal patterns, we proposed a new construction method of complex networks from EEG signals, ComNet-PSR-VG. With the help of the phase space reconstruction method, EEG signals are mapped to a series of ordered partitions and symbolized to obtain a sequence of ordinal patterns. Subsequently, the generated symbolic sequence of ordinal patterns is transformed into a graph network using the visibility graph method. To validate the effectiveness and versatility, we constructed the experiment on random signals, Lorenz signals, and the SEED emotion dataset by the proposed method. Two types of network node measures, AND and NDE, are extracted from the resulting graph networks. These extracted network features are then utilized as the input features for emotion classification, employing SVM as the pattern classifier to discern positive and negative emotions. The experimental results demonstrated outstanding classification performance, reinforcing the effectiveness and universality of our method. Furthermore, we compared our experimental results with existing research methods, showcasing the superior performance of our proposed entropy measure in EEG emotion recognition. The outstanding generalization observed in our proposed method suggests its significant practical potential in the field of EEG emotion recognition. Overall, our method stands out as a promising and effective approach for EEG emotion recognition, paving the way for advancements in the broader domain of EEG pattern-learning research.

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References

- Yong, Z.; Donner, R.V.; Marwan, N.; Donges, J.F.; Kurths, J. Complex network approaches to nonlinear time series analysis. *Phys. Rep.* 2018, 787, 1–97. [CrossRef]
- Yao, L.; Wang, M.; Lu, Y.; Li, H.; Zhang, X. EEG-Based Emotion Recognition by Exploiting Fused Network Entropy Measures of Complex Networks across Subjects. *Entropy* 2021, 23, 984. [CrossRef] [PubMed]
- McCullough, M.; Small, M.; Iu, H.H.C.; Stemler, T. Multiscale ordinal network analysis of human cardiac dynamics. *Philos. Trans.* R. Soc. A Math. Phys. Eng. Sci. 2017, 375, 20160292. [CrossRef]
- 4. Lu, Y.; Yao, L.; Li, H.; Tasleem, K.; Zhang, Z.; Gao, P.; Wang, M. A new network representation for time series analysis from the perspective of combinatorial property of ordinal patterns. *Heliyon* **2023**, *9*, e22455. [CrossRef] [PubMed]
- 5. Lacasa, L.; Luque, B.; Ballesteros, F.; Luque, J.; Nuño, J.C. From time series to complex networks: The visibility graph. *Proc. Natl. Acad. Sci. USA* **2008**, *105*, 4972–4975. [CrossRef]
- 6. Pessa, A.A.B.; Ribeiro, H.V. Characterizing stochastic time series with ordinal networks. Phys. Rev. E 2019, 100, 042304. [CrossRef]

- Zheng, W.-L.; Zhu, J.-Y.; Lu, B.-L. Identifying Stable Patterns over Time for Emotion Recognition from EEG. *IEEE Trans. Affect. Comput.* 2019, 10, 417–429. [CrossRef]
- 8. Pham, T.D. Quantification analysis of fuzzy recurrence plots. *Europhys. Lett.* 2022, 137, 62002. [CrossRef]
- 9. Gao, Z.; Jin, N. Complex network from time series based on phase space reconstruction. Chaos 2009, 19, 033137. [CrossRef]
- Liu, X.; Fu, Z. A Novel Recognition Strategy for Epilepsy EEG Signals Based on Conditional Entropy of Ordinal Patterns. *Entropy* 2020, 22, 1092. [CrossRef]
- 11. Himmel, A.-S.; Hoffmann, C.; Kunz, P.; Froese, V.; Sorge, M. Computational complexity aspects of point visibility graphs. *Discret. Appl. Math.* **2018**, 254, 283–290. [CrossRef]
- 12. McCullough, M.; Small, M.; Stemler, T.; Iu, H.H.-C. Time lagged ordinal partition networks for capturing dynamics of continuous dynamical systems. *Chaos* 2015, *25*, 053101. [CrossRef]
- 13. Kulp, C.W.; Chobot, J.M.; Freitas, H.R.; Sprechini, G.D. Using ordinal partition transition networks to analyze ECG data. *Chaos* **2016**, *26*, 073114. [CrossRef] [PubMed]
- 14. Donner, R.V.; Small, M.; Donges, J.F.; Marwan, N.; Zou, Y.; Xiang, R.; Kurths, J. Recurrence-Based Time Series Analysis by Means of Complex Network Methods. *Int. J. Bifurc. Chaos* **2011**, *21*, 1019–1046. [CrossRef]
- 15. Zhang, J.; Sun, J.; Luo, X.; Zhang, K.; Nakamura, T.; Small, M. Characterizing pseudoperiodic time series through the complex network approach. *Phys. D Nonlinear Phenom.* 2008, 237, 2856–2865. [CrossRef]
- 16. Marques, J.A.L.; Cortez, P.C.; Madeiro, J.P.V.; De Albuquerque, V.H.C.; Fong, S.J.; Schlindwein, F.S. Nonlinear characterization and complexity analysis of cardiotocographic examinations using entropy measures. *J. Supercomput.* **2018**, *76*, 1305–1320. [CrossRef]
- 17. Yang, Y.; Yang, H. Complex network-based time series analysis. Phys. A Stat. Mech. Its Appl. 2008, 387, 1381–1386. [CrossRef]
- 18. Knuth, D.E. *The Art of Computer Programming*; Pearson Education: London, UK, 1981.
- Zheng, W.-L.; Lu, B.-L. Investigating Critical Frequency Bands and Channels for EEG-Based Emotion Recognition with Deep Neural Networks. *IEEE Trans. Auton. Ment. Dev.* 2015, 7, 162–175. [CrossRef]
- Zheng, W. Multichannel EEG-based emotion recognition via group sparse canonical correlation analysis. *IEEE Trans. Cogn. Dev.* Syst. 2017, 9, 281–290. [CrossRef]
- Li, Y.; Zheng, W.; Cui, Z.; Zong, Y.; Ge, S. EEG emotion recognition based on graph regularized sparse linear regression. *Neural Process. Lett.* 2019, 49, 555–571. [CrossRef]
- Song, T.; Zheng, W.; Song, P.; Cui, Z. EEG emotion recognition using dynamical graph convolutional neural networks. *IEEE Trans. Affect. Comput.* 2018, 11, 532–541. [CrossRef]
- 23. Gao, Z.; Li, R.; Ma, C.; Rui, L.; Sun, X. Core-Brain-Network-Based Multilayer Convolutional Neural Network for Emotion Recognition. *IEEE Trans. Instrum. Meas.* 2021, 70, 2510209. [CrossRef]
- 24. Jafari, M.; Shoeibi, A.; Khodatars, M.; Bagherzadeh, S.; Shalbaf, A.; García, D.L.; Gorriz, J.M.; Acharya, U.R. Emotion recognition in EEG signals using deep learning methods: A review. *Comput. Biol. Med.* **2023**, *165*, 107450. [CrossRef]
- Tao, W.; Li, C.; Song, R.; Cheng, J.; Liu, Y.; Wan, F.; Chen, X. EEG-Based Emotion Recognition via Channel-Wise Attention and Self Attention. *IEEE Trans. Affect. Comput.* 2023, 14, 382–393. [CrossRef]
- Wang, Z.; Wang, Y.; Zhang, J.; Hu, C.; Yin, Z.; Song, Y. Spatial-Temporal Feature Fusion Neural Network for EEG-Based Emotion Recognition. *IEEE Trans. Instrum. Meas.* 2022, 71, 2507212. [CrossRef]
- Chen, T.; Becker, B.; Camilleri, J.; Wang, L.; Yu, S.; Eickhoff, S.B.; Feng, C. A domain-general brain network underlying emotional and cognitive interference processing: Evidence from coordinate-based and functional connectivity meta-analyses. *Brain Struct. Funct.* 2018, 223, 3813–3840. [CrossRef] [PubMed]
- Plinio, S.D.; Ferri, F.; Marzetti, L.; Romani, G.L.; Northoff, G.; Pizzella, V. Functional connections between activated and deactivated brain regions mediate emotional interference during externally directed cognition. *Hum. Brain Mapp.* 2018, 39, 3597–3610. [CrossRef] [PubMed]

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