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A Green Supply Chain with Sales Effort under a Cost-Sharing Contract

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Abstract: Due to social and psychological factors, the COVID-19 pandemic has impacted international trade, dampened consumption globally, and resulted in conservative investment and spending. To stimulate economic recovery while promoting the establishment of a positive consumption awareness among people, enterprises endeavor to enhance competitiveness and expand market share through various means, such as advertising and discounting. With more attention paid to environmentally friendly products, there are greater challenges encountered by green supply chain management. In this study, a green supply chain network problem is analyzed, involving a manufacturer and a retailer. In addition to the construction of centralized and decentralized decision models, two cost-sharing contracts are adopted to reduce promotion costs for the retailer and improve the level of greenery in products. With the help of game theory, equilibrium decisions can be made by solving the models. According to the results of numerical experiment, cost-sharing contracts can effectively improve the level of greenery in products and the profitability of the supply chain, despite the inability to achieve a win-win situation.

Keywords: green supply chain; sales effort; green level; cost-sharing contract; game model

MSC: 91B24



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1. Introduction

Because of the COVID-19 pandemic, the global economy has taken a hard hit. With many enterprises forced into closure or reduced production, there is a sharp decline in production capacity. International trade is an important part of the global economy, but its recovery is severely affected by the pandemic, as it has caused disruptions to international supply chains, logistical difficulties, and other issues. Furthermore, the economic growth in various countries is hindered by the weak demand of consumers in the global market and a large amount of unsold goods. To stimulate economic recovery and enhance the confidence of investors, the government has taken a series of measures, such as promoting industrial upgrading and structural adjustment, encouraging people to develop a positive awareness of consumption and investment, and so on. Considering a highly competitive external environment, enterprises can not survive market competition without enhancing their own market competitiveness. To stimulate market demand, advertising, promotion, personnel promotion, and sales promotion are commonly practiced.

IKEA is a typical example. As a globally renowned home furnishing brand, IKEA has consistently emphasized the significance of environmental protection. Under its green marketing strategy, IKEA promotes both environmentally friendly products and environmental concepts in different ways, such as vigorously promoting environmental concepts in advertising, encouraging consumers to purchase environmentally friendly products, and proactively engaging in various environmental activities. These measures allow IKEA to create an environmental brand image, attract a large number of consumers with strong environmental awareness, and gain significant benefits from the market. Coca Cola is

another example. Coca Cola, as one of the world's largest beverage companies, has consistently committed itself to promoting the concepts in relation to environmental protection. Under its green marketing strategy, Coca Cola not only introduces environmentally friendly packaging, but also engages in various environmental public welfare activities proactively, such as organizing environmental volunteer activities and conducting environmental publicity. Through these measures, Coca Cola has successfully established an environmental image, earning recognition and support from its consumers. Finally, Nike is exemplified. As a world-renowned sports brand, Nike has always attached much significance to the concepts of environmental protection. Under its green marketing strategy, Nike launches environmental sports equipment, and takes participation in various environmental public welfare activities proactively, such as organizing environmental campaigns and conducting environmental publicity. These measures enable Nike to establish an environmental image, attract many consumers with strong environmental awareness, and gain significant market benefits. In summary, the importance and necessity of green marketing are fully demonstrated by the above successful green marketing cases. With the promotion of environmentally friendly products and services, the establishment of an environmental image, and the active participation in environmental public welfare activities, enterprises can gain a good social reputation, earn recognition and support from consumers, and receive market benefits sustainably. Therefore, all enterprises are supposed to attach significance to green marketing, promote environmentally friendly products and services vigorously, build an environmental image, and contribute to the promotion of green consumption. The above cases are expected to inspire everyone and promote the widespread practice of green marketing in various industries.

Inspired by the above examples, we aim to reveal how sales efforts influence the pricing decisions for green products, and how appropriate contract agreements can be chosen to improve product greening or marketing efforts, for the maximum supply chain profits. In this study, a green supply chain network with a single manufacturer and a single retailer is studied. The former produces green products and invest in greening improvements, for sale to the retailer. By influencing green level and sales scale, customer green sensitivity indirectly affects the profits. Considering sales effort, the game models with customer green sensitivity is constructed. In addition, two cost-sharing contracts are applied in this study to green supply chains for discussion about whether the proposed contracts contribute to channel coordination in the new environment by relaxing assumptions. The main research objective of this paper is to answer the following questions:

1. What is the impact of sales effort on the decision making and profitability of supply chain participants?
2. Can the proposed cost-sharing contracts coordinate the entire supply chain and achieve an increase in supply chain profits?
3. Which cost-sharing contract would allow the manufacturer or the retailer to gain more profits?

To answer these questions, the existing literature is extended by constructing a green product supply chain that consists of a manufacturer who determines the level of greenery in products and a retailer who determines product marketing efforts. Game models are established under centralized contract, decentralized contract, cost-sharing contract, and cost-sharing negotiation, respectively. In the cost-sharing model, there are two scenarios under consideration. One is that the retailer actively shares a certain proportion of green investment cost with the manufacturer, and the other is that the manufacturer actively shares a certain proportion of promotional cost with the retailer. In the cost negotiation model, these two situations are considered as well. Finally, the total profit of the supply chain is compared under these contract models.

The main contributions of this paper are as follows. Firstly, an innovation is achieved in integrating green promotions into green product supply chains under cost-sharing contracts, which is inadequately explored in the existing literature. Secondly, different sharing contract strategies are analyzed to reveal how these contracts support the supply

chain in improving the green level, and whether this improvement is beneficial to the enterprise. It can be found out that the green level of the product is improved when salespeople endeavor to sell, with higher returns generated for the enterprise. Thirdly, it is discovered that the negotiation of greening cost is beneficial to the manufacturer compared with the game model under the context of centralized decision making, when market demand is determined by the sales effort and green level. In addition, it has also been found that compared to other models, the green cost negotiation model reduces the profits for sellers, which provides an important guidance for practice. Finally, it is revealed in the study that the sales efforts of retailers can increase the profits generated for the entire supply chain to some extent. These important findings supplement the existing literature on green product supply chains, providing a theoretical guidance for enterprises on how to choose contract strategies.

The study is structured as follows. In Section 2, a brief review of the relevant literature is conducted. In Section 3, the problem and assumptions are introduced. In Section 4, game models are proposed and analyzed to determine the optimal strategies for each model. In Section 5, the optimal strategies between models are compared. In Section 6, numerical experiments are conducted to analyze the influence of parameters on decision variables. Furthermore, the impact of cost-sharing contracts on the problem is analyzed. In Section 7, a practical case is analyzed and insights in management are obtained. In Section 8, the research results are summarized and the prospects for future research are indicated.

2. Literature Review

This study revolves around the impact of marketing efforts and contract mechanisms on pricing strategies in the context of green product supply chains. Therefore, a brief review of the literature will be conducted in this section from these perspectives.

2.1. Sales Effort

In the current market environment with fierce competition, advertising promotions, price reductions, and other sales efforts are crucial to enterprises in terms of improving competitiveness and expanding market share. Implementing appropriate promotional strategies enables enterprises to attract more customers, increase sales, and improve profitability. As e-commerce develops, promotion has played an increasingly evident role in the supply chain. Cai et al. [1] introduced a cost-sharing contract to conduct a difference analysis between the sales efforts of the supplier and the retailer. Datta et al. [2] proposed a dynamic system with sales effort, indicating that sales effort can increase revenue, attract customers, and reinforce a competitive advantage. As a prevalent and serious problem, the free riding effect lowers the expected level of sales effort for brick and mortar retailers. Considering the coordinated sales effort of the retailer, Sun and Liu [3] formulated a price-match and bi-directional compensation contract for a two-channel supply chain. When consumers choose offline channels, the overall profit of the supply chain is reduced by the free riding coefficient whether under decentralized or centralized decision making [4]. Tian et al. [5] analyzed a dynamic game model of a multi-channel supply chain consisting of a manufacturer and two retailers, revealing that system stability can be reduced by the significant adjustment to order quantity, channel preferences, and sales efforts. Wu et al. [6] applied a game model to explore the impact of sales effort on the profits, discovering that if this manufacturer assists two retailers, it leads to a scenario where the weaker retailer invests more effort while the dominant retailer obtains the opposite result. Duan et al. [7] established a game theory model to study the impact of sales manager's efforts on the supply chain, comparing the equilibrium decisions and profits. Yang et al. [8] developed the models fit for an agricultural supply chain with sales effort, and compared the optimal decisions under different option contracts.

2.2. Contract Theory

In definition, a supply contract is an agreement reached between different members of a supply chain, which dictates the rights and obligations of each party to work on the coordinated operation of the whole supply chain. Contracts can be used to promote coordination and cooperation among members of the supply chain by establishing trust, determining responsibility, improving efficiency, and optimizing costs. Thus, the efficiency and competitiveness of the entire supply chain are enhanced. Bai et al. [9] explored a sustainable supply chain system with time-varying demand, proposing two contracts to coordinate this system. Xie et al. [10] investigated a dual channel closed-loop supply chain to present a revenue-sharing-cost-sharing contract for the improved profits of supply chain members in various channels. Li et al. [11] investigated the impact of sharing contracts on emission reduction efforts and corporate profitability, developing a game model to test the equilibrium decisions of channel members. He et al. [12] established three differential game models with cost-sharing decisions by considering a carbon reduction supply chain problem to explore the optimal decision. Zhao and Zhang [13] constructed three two-level photovoltaic supply chains to effectively shorten the delivery cycle under revenue sharing and cost-sharing contracts. Xu et al. [14] developed a low-carbon supply chain model for manufacturers and distributors, formulated two cost-sharing contracts to improve the reputation of low-carbon products, and increased the low-carbon efforts of supply chain members.

2.3. Promotion Efforts Mixed with the Contract Strategy

When it comes to the pricing problem with green supply chain, sales efforts are often treated as decision variables, and new contracts are introduced as incentive mechanisms to achieve a win-win situation for all members [15–17]. Yang et al. [18] proposed a green product supply chain in which the retailer can regularly conduct promotional activities. Furthermore, the manufacturer can invest in improving the green level of their products. Finally, a cost-sharing contract was proposed to mitigate this negative impact. Mondal and Giri [19] examined the impact of marketing effort and old product recycling rate on a two-period closed-loop supply chain decision, demonstrating that encouraging green innovation or marketing effort can enhance the performance of the supply chain. Gu et al. [20] explored a dual-channel supply chain model that considers offline on sale services, demonstrating that the optimal wholesale price and retail price are positively correlated with the quality of service available for sale. Through the construction of several supply chain models involving multiple manufacturers and one retailer, Zhao et al. [21] illustrated that an increase in demand sensitivity can increase the profits generated for all members. Saha et al. [22] developed a supply chain model that involves green practices and advertising to demonstrate that implementing a proposed shared contract policy is more beneficial than in the decentralized situation to supply chain participants.

3. Problem Description

Despite a reference made to the existing literature, this study differs in the following aspects. Our consideration is given to the manufacturer who invests in developing green technologies to produce green products and the retailer who invests in marketing to promote green products. Customers are sensitive to the price, green level, and promotional advertising of green products. Furthermore, they are influenced by these factors when choosing products. In the proposed supply chain, the manufacturer and the retailer sign contracts and sell green products to promote the development of green products. The manufacturer shares the promotional cost with the retailer, while the retailers shares the greening cost with the manufacturer. This is a situation that has not been explored in the existing literature. Secondly, several strategies are comparatively studied to draw interesting conclusions for each supply chain member to make optimal decisions, with some interesting results obtained that have not been presented in the existing literature.

Next Table 1 is an introduction to the symbols to be used in the paper.

Table 1. Notations.

Notations	Definitions
a	basic market demand
b	sensitivity to price
c	production cost
D	consumer demand
e	sales effort of the retailer
f	sales effort cost coefficient
m	the retailer's margins
w	wholesale price of the manufacturer
r	sensitivity of marketing efforts
p	selling price of the retailer
I	the coefficient of green investment
α	consumer sensitivity to green improvement
θ	the level of green innovation
π_m	the manufacturer's profit
π_r	the retailer's profit
π_{sc}	the total supply chain's profit

Below are some assumptions made for the model:

1. The manufacturer produces green products and sells them to the retailer. In order to improve the green level of products, the manufacturer need to invest in research and development. When the green investment parameter is assumed to be I , the R&D cost of green products is $I\theta^2$. In addition, green improvement does not affect marginal production cost for the manufacturer. Sensitive to the green level of the product, consumers are more inclined to purchase green products [23].
2. Similar to [24–27], the demand function is expressed as $D = a - bp + \alpha\theta + re$, where the b , α , and r are positive constants and D is invariably positive. Furthermore, a represents the basic market demand, b indicates the sensitivity to price, α denotes the sensitivity to green improvement, and r refers to the sensitivity of marketing efforts.
3. Similar to [28], a quadratic function widely used in management is applied to describe the marketing effort cost of the retailers $\frac{f}{2}e^2$.
4. The information between supply chain members is symmetrical.

Under the above assumptions, profit functions are proposed for the manufacturer and the retailer:

$$\pi_M = (w - c)(a - bp + \alpha\theta + re) - I\theta^2, \quad (1)$$

$$\pi_R = (p - w)(a - bp + \alpha\theta + re) - \frac{f}{2}e^2. \quad (2)$$

4. Modeling and Analysis

In this section, game models are constructed for centralized decision making, decentralized decision making, and cost-sharing contracts, respectively. Figure 1 shows the process of solving the model.

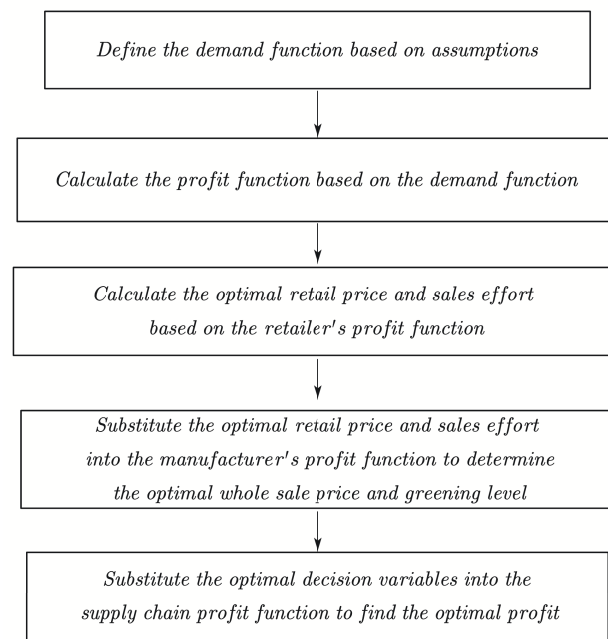


Figure 1. Flowchart.

4.1. Centralized Decision-Making Model

Firstly, a game model is established under centralized decision making. In this case, the manufacturer and the retailer are treated as a collaborative whole. Both of them make decisions with the aim to maximize the overall profits of the supply chain. Despite the clear advantages created by a centralized decision model in some cases, it requires a large amount of data and information support. Besides, it is necessary to establish a stable and reliable supply chain partnership.

The profit function of the supply chain is:

$$\pi_{SC} = (p - c)(a - bp + \alpha\theta + re) - I\theta^2 - \frac{f}{2}e^2. \quad (3)$$

The inverse derivation method can be used to determine first and second partial derivatives for p , θ , and e in Equation (3):

$$\frac{\partial \pi_{SC}}{\partial p} = a - 2bp + \alpha\theta + re + bc, \quad (4)$$

$$\frac{\partial \pi_{SC}}{\partial \theta} = p\alpha - c\alpha - 2I\theta, \quad (5)$$

$$\frac{\partial \pi_{SC}}{\partial e} = pr - cr - fe. \quad (6)$$

$$\frac{\partial^2 \pi_{SC}}{\partial p^2} = -2b, \frac{\partial^2 \pi_{SC}}{\partial \theta^2} = -2I, \frac{\partial^2 \pi_{SC}}{\partial e^2} = -f, \frac{\partial^2 \pi_{SC}}{\partial p \partial \theta} = \alpha, \frac{\partial^2 \pi_{SC}}{\partial p \partial e} = r, \frac{\partial^2 \pi_{SC}}{\partial \theta \partial e} = 0, \quad (7)$$

Then, the Hessian matrix of π_{SC} can be obtained:

$$H_1 = \begin{bmatrix} \frac{\partial^2 \pi_{SC}}{\partial p^2} & \frac{\partial^2 \pi_{SC}}{\partial p \partial \theta} & \frac{\partial^2 \pi_{SC}}{\partial p \partial e} \\ \frac{\partial^2 \pi_{SC}}{\partial \theta \partial p} & \frac{\partial^2 \pi_{SC}}{\partial \theta^2} & \frac{\partial^2 \pi_{SC}}{\partial \theta \partial e} \\ \frac{\partial^2 \pi_{SC}}{\partial e \partial p} & \frac{\partial^2 \pi_{SC}}{\partial e \partial \theta} & \frac{\partial^2 \pi_{SC}}{\partial e^2} \end{bmatrix} = \begin{bmatrix} -2b & \alpha & r \\ \alpha & -2I & 0 \\ r & 0 & -f \end{bmatrix} \quad (8)$$

When the conditions $4bI - \alpha^2 > 0$ and $2r^2I + \alpha^2f - 4bfI < 0$ are met, H_1 is negative definite, indicating that π_{SC} is concave with respect to p , e and θ . Therefore, by making the

first derivative equal to 0 and combining them, the optimal solution can be obtained for retail price, sales effort, and green level.

Proposition 1. Under the constraint that $4bI - \alpha^2 > 0$ and $2r^2I + \alpha^2f - 4bfI < 0$, the optimal retail price, sales effort, and green level are:

$$p^* = \frac{2fIa - fca^2 - 2Icr^2 + 2fIbc}{4fbI - f\alpha^2 - 2Ir^2}, \quad (9)$$

$$\theta^* = \frac{f\alpha(a - bc)}{4fbI - f\alpha^2 - 2Ir^2}, \quad (10)$$

$$e^* = \frac{2Ir(a - bc)}{4fbI - f\alpha^2 - 2Ir^2}. \quad (11)$$

Proof. When the derivatives of π_{SC} are equal to 0, they are combined to obtain the Equations (9)–(11). \square

Finally, p^* , θ^* , and e^* are substituted into (3) to determine the optimal profit of the supply:

$$\pi_{SC}^* = \frac{fI(a - bc)^2}{4Ibf - f\alpha^2 - 2Ir^2}.$$

4.2. Decentralized Decision-Making Model

Referred to as the delegation of decision-making power by enterprises to various departments or individuals, decentralized decision making allows them to make independent decisions and better respond to market changes. The most significant advantage of decentralized decision making is to improve the flexibility and adaptability of the enterprise. At the same time, each department or individual is allowed to make decisions based on their own practicalities, which gives full play to the internal self-regulation mechanism of the enterprise. In addition, decentralized decision making can promote the innovation and development of the enterprise by stimulating the enthusiasm and creativity of each member. As a leader, the manufacturer first determines wholesale price w and green level θ based on the retailer's response function for the maximum profit. On this basis, the retailer sets the retail price p and sales effort e .

The profit function of the retailer can be changed into:

$$\pi_R(m) = m(a - b(m + w) + \alpha\theta + re) - \frac{f}{2}e^2. \quad (12)$$

Through the inverse derivation method, partial derivatives can be taken for e and m in Equation (12):

$$\frac{\partial \pi_R}{\partial e} = mr - fe, \quad (13)$$

$$\frac{\partial \pi_R}{\partial m} = a - bw - 2mb + \alpha\theta + re, \quad (14)$$

$$\frac{\partial^2 \pi_R}{\partial e^2} = -f, \quad \frac{\partial^2 \pi_R}{\partial m^2} = -2b. \quad (15)$$

Thus, π_R is concave in m and e . By making (13) and (14) equal to 0 and combining them, it can be obtained that:

$$m = \frac{fa - fbw + f\alpha\theta}{2fb - r^2}, \quad (16)$$

$$e = \frac{ra - rbw + r\alpha\theta}{2fb - r^2}. \quad (17)$$

The profit function of the manufacturer can be changed into:

$$\pi_M(m) = (w - c)(a - b(m + w) + \alpha\theta + re) - I\theta^2. \quad (18)$$

By taking partial derivatives for w and θ in (18), it can be obtained that:

$$\frac{\partial \pi_M}{\partial w} = a - b(w + m) + \alpha\theta + re - (w - c)b, \quad (19)$$

$$\frac{\partial \pi_M}{\partial \theta} = (w - c)\alpha - 2I\theta, \quad (20)$$

$$\frac{\partial^2 \pi_M}{\partial w^2} = -2b, \frac{\partial^2 \pi_M}{\partial w \partial \theta} = \alpha, \frac{\partial^2 \pi_M}{\partial \theta^2} = -2I, \quad (21)$$

The determinant is $4bI - \alpha^2$. When $4bI - \alpha^2 > 0$, the Hessian matrix is negative definite. Thus, the profit function of the manufacturer π_M is concave in m and e .

By substituting (16) and (17) into (18) and taking the partial derivatives with respect to w and θ , it can be obtained that:

$$w = \frac{\alpha\theta + bc + a}{2b}, \quad (22)$$

$$\theta = \frac{(w - c)b\alpha f}{2I(2fb - r^2)}. \quad (23)$$

By combining (22) and (23), the optimal wholesale price and green level can be determined:

$$w^* = \frac{4Ib^2cf - 2Ibcr^2 - \alpha^2bcf + 4Iabf - 2Iar^2}{(8Ibf - 4Ir^2 - \alpha^2f)b}, \quad (24)$$

$$\theta^* = \frac{\alpha f(a - bc)}{8Ibf - 4Ir^2 - \alpha^2f}. \quad (25)$$

By substituting (24) and (25) into (16) and (17), respectively, the optimal margin and sales effort can be obtained:

$$m^* = \frac{2(a - bc)If}{8Ibf - 4Ir^2 - \alpha^2f}, \quad (26)$$

$$e^* = \frac{2(a - bc)Ir}{8Ibf - 4Ir^2 - \alpha^2f}. \quad (27)$$

Then it can be obtained that:

$$p^* = m^* + w^* = \frac{2Ib^2cf - 2Ibcr^2 - \alpha^2bcf + 6Iabf - 2Iar^2}{(8Ibf - 4Ir^2 - \alpha^2f)b}. \quad (28)$$

Finally, w^* , p^* , θ^* , and e^* are substituted into (3), (12), and (18) to obtain the maximum profits for the manufacturers, retailer, and supply chain as follows:

$$\pi_M^* = \frac{fI(a - bc)^2}{8Ibf - 4Ir^2 - \alpha^2f}, \quad (29)$$

$$\pi_R^* = \frac{2(2fb - r^2)(a - bc)^2I^2f}{(8Ibf - 4Ir^2 - \alpha^2f)^2}, \quad (30)$$

$$\pi_{SC}^* = \frac{(a - bc)^2(-\alpha^2f + 12Ibf - 6Ir^2)fI}{(8Ibf - 4Ir^2 - \alpha^2f)^2}. \quad (31)$$

4.3. Cost-Sharing Game Model

4.3.1. Promotion Cost-Sharing Model

In a market economy, implementing sales effort is effective in improving the market demand for products and increasing their market sales. Although the sales effort of supply chain members is beneficial to the entire supply chain, it still leads to insufficient motivation for their sales effort if the costs of promotions are borne solely by retailers. In addition, the promotional behavior of enterprise members can cause positive externalities and a dual marginal problem. For upstream and downstream companies, they can adopt a method of sharing promotional costs to incentivize member companies to implement the optimal promotional actions of the system, such as making the retailers closer to the market implement promotional efforts and manufacturers share part of the promotional costs. In this section, the cost-sharing mechanism is applied to eliminate the double marginal problem and prompt promotional enterprises into choosing the optimal sales effort of the system, and thus, achieve channel coordination. The following assumptions are made:

1. The manufacturer shares a certain proportion of sales costs. ϕ is used to represent the percentage shared by the retailer ($0 < \phi < 1$).
2. The manufacturer decides the wholesale price w and green innovation θ based on ϕ and the retailer's response function.
3. The retailer decides the retail price p of products according to the wholesale price w , level of green innovation θ , and sharing ratio ϕ .

Under these assumptions, the profit functions are obtained:

$$\pi_M = (w - c)(a - b(w + m) + \alpha\theta + re) - I\theta^2 - (1 - \phi)\frac{f}{2}e^2, \quad (32)$$

$$\pi_R = m(a - b(w + m) + \alpha\theta + re) - \phi\frac{f}{2}e^2. \quad (33)$$

Firstly, let us solve the profit function (33) of the retailer and take the partial derivatives of m and e to obtain:

$$\frac{\partial \pi_R}{\partial e} = mr - \phi fe, \quad (34)$$

$$\frac{\partial \pi_R}{\partial m} = a - bw - 2mb + \alpha\theta + re, \quad (35)$$

$$\frac{\partial^2 \pi_R}{\partial e^2} = -\phi f, \quad \frac{\partial^2 \pi_R}{\partial m^2} = -2b. \quad (36)$$

The determinant is $2\phi fb - r^2$. When $2\phi fb - r^2 > 0$, the Hessian matrix is negative definite. Therefore, π_R is concave in m and e .

By making (34) and (35) equal to 0 and combining them, it can be obtained that:

$$m = \frac{fa - fbw + f\alpha\theta}{2\phi fb - r^2}\phi, \quad (37)$$

$$e = \frac{ra - rbw + r\alpha\theta}{2\phi fb - r^2}. \quad (38)$$

By substituting (37) and (38) into (32) and taking the partial derivatives with respect to w and θ , it can be obtained that:

$$w = \frac{2\alpha bf\phi^2\theta + 2b^2cf\phi^2 + 2abf\phi^2 - 2\alpha\phi r^2\theta - bc\phi r^2 - 2a\phi r^2 + \alpha r^2\theta + ar^2}{b(4bf\phi^2 - 3r^2\phi + r^2)}, \quad (39)$$

$$\theta = \frac{\alpha f(-2b^2cf\phi^2 + 2b^2f\phi^2w + r^2\phi bc - 2b\phi r^2w + r^2\phi a + br^2w - ar^2)}{8Ib^2f^2\phi^2 - 8Ib\phi r^2 - \alpha^2f\phi r^2 + 2Ir^4 + \alpha^2fr^2}. \quad (40)$$

Additionally:

$$\frac{\partial^2 \pi_M}{\partial w^2} = -2b \left(1 - \frac{b\phi f}{2bf\phi - r^2} \right) - \frac{2br^2}{2bf\phi - r^2} - \frac{(1-\phi)f b^2 r^2}{(2bf\phi - r^2)^2} < 0,$$

$$\frac{\partial^2 \pi_M}{\partial \theta^2} = -2I - \frac{(1-\phi)f \alpha^2 r^2}{(2bf\phi - r^2)^2} < 0,$$

$$\frac{\partial^2 \pi_R}{\partial w \partial \theta} = -\frac{b\phi f \alpha}{2bf\phi - r^2} + \alpha + \frac{r^2 \alpha}{2bf\phi - r^2} + \frac{(1-\phi)f b \alpha r^2}{(2bf\phi - r^2)^2}.$$

It can be known that the value of the determinant is $\frac{8 \left(f \left(bI - \frac{\alpha^2}{8} \right) \phi^2 - \frac{3 \left(\phi - \frac{1}{3} \right) I r^2}{4} \right) f b^2}{(2bf\phi - r^2)^2}$.

For $f \left(bI - \frac{\alpha^2}{8} \right) \phi^2 - \frac{3 \left(\phi - \frac{1}{3} \right) I r^2}{4} < 0$, the Hessian matrix is negative definite.

By combining (39) and (40), the optimal wholesale price and green level can be determined:

$$w^* = \frac{\alpha^2 b c f \phi^2 - 4b^2 c f I \phi^2 - 4ab f I \phi^2 + 2bc I \phi r^2 + 4a I \phi r^2 - 2a I r^2}{b(\alpha^2 f \phi^2 - 8b f I \phi^2 + 6I \phi r^2 - 2I r^2)}, \quad (41)$$

$$\theta^* = \frac{f \phi^2 \alpha (cb - a)}{\alpha^2 f \phi^2 - 8b f I \phi^2 + 6I \phi r^2 - 2I r^2}. \quad (42)$$

By substituting (41) and (42) into (37) and (38), respectively, the optimal margin and sales effort can be obtained:

$$m^* = \frac{2(cb - a) f I \phi^2}{\alpha^2 f \phi^2 - 8b f I \phi^2 + 6I \phi r^2 - 2I r^2}, \quad (43)$$

$$e^* = \frac{2(cb - a) \phi I r}{\alpha^2 f \phi^2 - 8b f I \phi^2 + 6I \phi r^2 - 2I r^2}. \quad (44)$$

Then, the optimal retail price can be obtained:

$$p^* = m^* + w^* = \frac{6fb \left(\left(\frac{cb}{3} + a \right) I - \frac{c\alpha^2}{6} \right) \phi^2 - 4r^2 I \left(\frac{cb}{2} + a \right) \phi + 2a I r^2}{8 \left(bI - \frac{\alpha^2}{8} \right) f b \phi^2 - 6b I r^2 \phi + 2b I r^2}. \quad (45)$$

Next, w^* , m^* , θ^* , and e^* are inputted into (33), so as to obtain the partial derivatives with respect to ϕ :

$$\frac{\partial \pi_R}{\partial \phi} = \frac{2(\alpha^2 f \phi^2 + 16b f I \phi^2 - 16b f I \phi - 6I \phi r^2 + 6I r^2)(a - cb)^2 r^2 \phi^2 I^2 f}{(\alpha^2 f \phi^2 - 8b f I \phi^2 + 6I \phi r^2 - 2I r^2)^3}. \quad (46)$$

$$\begin{aligned} \frac{\partial \pi_R^2}{\partial \phi^2} = & \frac{1}{(-\alpha^2 f \phi^2 + 8b f I \phi^2 - 6I \phi r^2 + 2I r^2)^4} \left(512r^2(a - cb)^2 f I^2 \right. \\ & \left(\left(bI - \frac{\alpha^2}{8} \right) f^2 \left(bI + \frac{\alpha^2}{16} \right) \phi^4 - \frac{3 \left(\left(b^2 f + \frac{1}{8} r^2 b \right) I - \frac{(fb + \frac{r^2}{2}) \alpha^2}{8} \right) f I \phi^3}{2} \right. \\ & \left. \left. + \frac{\left(bI - \frac{\alpha^2}{2} \right) r^2 f I \phi^2}{4} + \frac{3b f I^2 \phi r^2}{8} - \frac{3I^2 r^4}{32} \right) \phi \right) \end{aligned} \quad (47)$$

Therefore, when:

$$\begin{aligned} & \left(bI - \frac{\alpha^2}{8}\right)f^2\left(bI + \frac{\alpha^2}{16}\right)\phi^4 - \frac{3\left(\left(b^2f + \frac{1}{8}r^2b\right)I - \frac{\left(fb + \frac{r^2}{2}\right)\alpha^2}{8}\right)fI\phi^3}{2} + \frac{\left(bI - \frac{\alpha^2}{2}\right)r^2fI\phi^2}{4} \\ & + \frac{3bfI^2\phi r^2}{8} - \frac{3I^2r^4}{32} < 0, \end{aligned}$$

π_R is concave in ϕ . Then, by making the first-order derivative of π_R equal to 0, the optimal sharing ratio is obtained:

$$\phi^* = \begin{cases} \frac{8bfI + 3Ir^2 + \sqrt{-6\alpha^2fIr^2 + 64b^2f^2I^2 - 48bfI^2r^2 + 9I^2r^4}}{f(\alpha^2 + 16bI)} \\ \frac{8bfI + 3Ir^2 - \sqrt{-6\alpha^2fIr^2 + 64b^2f^2I^2 - 48bfI^2r^2 + 9I^2r^4}}{f(\alpha^2 + 16bI)}. \end{cases} \quad (48)$$

As revealed by numerical experiments, the solution below is close to 0. Therefore, the values are usually taken from the one above.

Finally, the optimal solution is substituted into w^* , m^* , θ^* , e^* , π_M , π_R , and π_{SC} , through which their optimal solutions are obtained.

4.3.2. Cost-Sharing Model for Greening Improvement

In addition, the manufacturer has invested in the cost of green improvement. Furthermore, the effectiveness of the cost-sharing contract is explored by studying a model under the following scenario. The following assumptions are made:

1. The retailer offers to share a certain proportion of green improvement. ϕ is used to represent the percentage shared by the retailer ($0 < \phi < 1$).
2. The manufacturer decides the wholesale price w and green innovation θ based on ϕ and the retailer's response function.
3. The retailer decides the retail price p of products according to the wholesale price w , level of green innovation θ , and sharing ratio ϕ .

Under the above assumptions, the profit functions are obtained:

$$\pi_M = (w - c)(a - b(w + m) + \alpha\theta + re) - (1 - \phi)I\theta^2, \quad (49)$$

$$\pi_R = m(a - b(w + m) + \alpha\theta + re) - \phi I\theta^2 - \frac{f}{2}e^2. \quad (50)$$

There is a need to study the role of this cost-sharing contract and solve the situation where the manufacturer accepts the retailer's proposal. Firstly, by solving the profit function (49) of the retailer and taking the partial derivatives of m and e , it is obtained that:

$$\frac{\partial \pi_R}{\partial e} = mr - fe, \quad (51)$$

$$\frac{\partial \pi_R}{\partial m} = a - bw - 2mb + \alpha\theta + re, \quad (52)$$

$$\frac{\partial^2 \pi_R}{\partial e^2} = -f, \quad \frac{\partial^2 \pi_R}{\partial m^2} = -2b. \quad (53)$$

The determinant is $2fb - r^2$. When $2fb - r^2 > 0$, the Hessian matrix is negative definite. Therefore, π_R is concave in m and e .

By making (51) and (52) equal to 0 and combining them, it can be obtained that:

$$m = \frac{fa - fbw + f\alpha\theta}{2fb - r^2}, \quad (54)$$

$$e = \frac{ra - rbw + r\alpha\theta}{2fb - r^2}. \quad (55)$$

By substituting (54) and (55) into (49) and taking the partial derivatives with respect to w and θ , it can be obtained that:

$$w = \frac{\alpha\theta + cb + a}{2b}, \quad (56)$$

$$\theta = \frac{(c - w)bf\alpha}{2(2bf - r^2)(\phi - 1)I}. \quad (57)$$

Additionally:

$$\frac{\partial^2 \pi_M}{\partial w^2} = -2b \left(1 - \frac{bf}{2bf - r^2} \right) - \frac{2br^2}{2bf - r^2} < 0,$$

$$\frac{\partial^2 \pi_M}{\partial \theta^2} = -2(1 - \phi)I < 0,$$

$$\frac{\partial^2 \pi_R}{\partial w \partial \theta} = -\frac{bf\alpha}{2bf - r^2} + \alpha + \frac{r^2\alpha}{2bf - r^2}.$$

It can be known that the value of the determinant is $-\frac{8b^2f \left(((\phi - 1)Ib + \frac{a^2}{8})f - \frac{Ir^2(\phi - 1)}{2} \right)}{(2bf - r^2)^2}$.

For $(1 - \phi)(bf - \frac{r^2}{2})I - \frac{f\alpha^2}{8} > 0$, the Hessian matrix is negative definite.

By combining (56) and (57), the optimal wholesale price and green level can be obtained:

$$w^* = \frac{4b^2cfI\phi - 2bcI\phi r^2 + 4abfI\phi - 2aI\phi r^2 + \alpha^2bcf - 4b^2cfI + 2bcIr^2 - 4abfI + 2aIr^2}{(8fIb\phi - 4Ir^2\phi + f\alpha^2 - 8fIb + 4Ir^2)b}, \quad (58)$$

$$\theta^* = \frac{f\alpha(cb - a)}{8fIb\phi - 4Ir^2\phi + f\alpha^2 - 8fIb + 4Ir^2}. \quad (59)$$

By substituting (58) and (59) into (54) and (55), respectively, the optimal margin and sales effort can be obtained:

$$m^* = \frac{2(\phi - 1)(-cb + a)If}{8 \left(bf - \frac{r^2}{2} \right) (\phi - 1)I + f\alpha^2}, \quad (60)$$

$$e^* = \frac{2(\phi - 1)(-cb + a)Ir}{8 \left(bf - \frac{r^2}{2} \right) (\phi - 1)I + f\alpha^2}. \quad (61)$$

Then, the optimal retail price can be obtained:

$$p^* = m^* + w^* = \frac{6 \left(\frac{b^2cf}{3} + \left(-\frac{cr^2}{3} + af \right) b - \frac{ar^2}{3} \right) (\phi - 1)I + \alpha^2bcf}{8 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1)I + \frac{f\alpha^2}{8} \right) b}. \quad (62)$$

Next, w^* , m^* , θ^* , and e^* are inputted into (50), so as to obtain the partial derivatives with respect to ϕ :

$$\frac{\partial \pi_R}{\partial \phi} = \frac{I\alpha^2 f^2 \left(\phi \left(bf - \frac{r^2}{2} \right) I - \frac{f\alpha^2}{16} \right) (-cb + a)^2}{32 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1) I + \frac{f\alpha^2}{8} \right)^3}. \quad (63)$$

$$\frac{\partial \pi_R^2}{\partial \phi^2} = - \frac{I^2 \alpha^2 f^2 \left(\left(\phi + \frac{1}{2} \right) \left(bf - \frac{r^2}{2} \right) I - \frac{5f\alpha^2}{32} \right) \left(bf - \frac{r^2}{2} \right) (-cb + a)^2}{16 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1) I + \frac{f\alpha^2}{8} \right)^4} \quad (64)$$

Therefore, when:

$$\frac{5f\alpha^2}{32} - \left(\phi + \frac{1}{2} \right) \left(bf - \frac{r^2}{2} \right) I < 0,$$

π_R is concave in ϕ . Then, by making the first-order derivative of π_R equal to 0, the optimal sharing ratio is obtained:

$$\phi^* = \frac{f\alpha^2}{8(2bf - r^2)I} \quad (65)$$

Finally, w^* , p^* , θ^* , and e^* are substituted into (49) and (50) to obtain the maximum profits for the manufacturer, retailer, and supply chain:

$$\begin{aligned} \pi_M^* &= \frac{(a - cb)^2 (\alpha^2 f - 16bfI + 8I r^2) f}{4(3\alpha^2 f - 16bfI + 8I r^2)(2bf - r^2)}, \\ \pi_R^* &= \frac{f(a - cb)^2 (\alpha^2 f + 16bfI - 8I r^2)}{8(-3\alpha^2 f + 16bfI - 8I r^2)(2bf - r^2)}, \\ \pi_{SC}^* &= \frac{f(a - cb)^2 (\alpha^2 f - 48bfI + 24I r^2)}{8(3\alpha^2 f - 16bfI + 8I r^2)(2bf - r^2)}. \end{aligned}$$

4.4. Bargaining Cost-Sharing Game Model

To explore the role of bargaining in the cost-sharing contract, this mechanism is introduced into the game model. The Nash bargaining game model is applicable in various settings. Through preset expected values and strategic choices, both parties can conduct multiple rounds of negotiation to reach an agreement. By analyzing the model, the interrelationships between various strategies and variables in the bargaining process can be better understood. By assuming that a Nash bargaining process is followed by all members in the supply, the objective function is obtained as $\max_{\phi} \pi_B = \pi_M \pi_R$. When π_B reaches its maximum value, the corresponding ϕ is taken as the optimal cost-sharing rate.

4.4.1. Bargaining Model for Sharing Promotion Costs

Firstly, the model of the manufacturer sharing promotional costs is solved. The profit functions are (32) and (33). With w^* , m^* , θ^* , and e^* inputted into (32) and (33), the profit functions are obtained:

$$\pi_M = \frac{\phi^2 I f (-bc + a)^2}{(-\alpha^2 f + 8bfI)\phi^2 - 6I\phi r^2 + 2I r^2}, \quad (66)$$

$$\pi_R = \frac{2(2bf\phi - r^2)(-bc + a)^2 I^2 f \phi^3}{(\alpha^2 f \phi^2 - 8Ibf\phi^2 + 6I\phi r^2 - 2I r^2)^2}. \quad (67)$$

Then, the objective function is obtained:

$$\pi_B = \frac{2\phi^5 f^2 (-bc + a)^4 I^3 (r^2 - 2bf\phi)}{(\alpha^2 f\phi^2 - 8Ibf\phi^2 + 6I\phi r^2 - 2I r^2)^3}. \quad (68)$$

By taking the first-order and second-order partial derivatives of (68) with respect to ϕ , it is obtained that:

$$\frac{\partial \pi_B}{\partial \phi} = -\frac{2(-bc + a)^4 r^2 f^2 \phi^4 I^3 (\alpha^2 f\phi^2 + 28Ibf\phi^2 - 24bfI\phi - 12I\phi r^2 + 10I r^2)}{(\alpha^2 f\phi^2 - 8Ibf\phi^2 + 6I\phi r^2 - 2I r^2)^4}, \quad (69)$$

$$\begin{aligned} \frac{\partial \pi_B^2}{\partial \phi^2} = & \frac{1}{(-\alpha^2 f\phi^2 + 8Ibf\phi^2 - 6I\phi r^2 + 2I r^2)^5} \left(896 \left(\left(b^2 f^2 \phi^4 + \left(-\frac{9}{7} b^2 f^2 + \frac{3}{28} r^2 bf \right) \phi^3 \right. \right. \right. \\ & + \left(-\frac{5}{14} r^2 bf - \frac{9}{56} r^4 \right) \phi^2 + \left(\frac{15}{28} r^2 bf + \frac{15}{56} r^4 \right) \phi - \frac{5r^4}{28} \Big) I^2 \\ & \left. - \frac{5\phi^2 f \left(bf\phi^2 + \left(-\frac{9bf}{5} - \frac{6r^2}{5} \right) \phi + \frac{13r^2}{10} \right) \alpha^2 I}{56} - \frac{f^2 \phi^4 \alpha^4}{224} \right) r^2 \phi^3 f^2 (-bc + a)^4 I^3 \Big). \end{aligned} \quad (70)$$

Therefore, when:

$$\begin{aligned} & \left(b^2 f^2 \phi^4 + \left(-\frac{9}{7} b^2 f^2 + \frac{3}{28} r^2 bf \right) \phi^3 \right. \\ & + \left(-\frac{5}{14} r^2 bf - \frac{9}{56} r^4 \right) \phi^2 + \left(\frac{15}{28} r^2 bf + \frac{15}{56} r^4 \right) \phi - \frac{5r^4}{28} \Big) I^2 \\ & \left. - \frac{5\phi^2 f \left(bf\phi^2 + \left(-\frac{9bf}{5} - \frac{6r^2}{5} \right) \phi + \frac{13r^2}{10} \right) \alpha^2 I}{56} - \frac{f^2 \phi^4 \alpha^4}{224} < 0, \end{aligned} \quad (71)$$

π_R is concave in ϕ . Then, by making the first-order derivative of π_B equal to 0, the optimal sharing ratio is obtained:

$$\phi^* = \begin{cases} \frac{12bfI + 6I r^2 + \sqrt{-10\alpha^2 fI r^2 + 144b^2 f^2 I^2 - 136bf I^2 r^2 + 36I^2 r^4}}{f(\alpha^2 + 28Ib)} \\ \frac{12bfI + 6I r^2 - \sqrt{-10\alpha^2 fI r^2 + 144b^2 f^2 I^2 - 136bf I^2 r^2 + 36I^2 r^4}}{f(\alpha^2 + 28Ib)} \end{cases} \quad (72)$$

As revealed by numerical experiments, the solution below is close to 0, so that the values are usually taken from the one above.

Finally, the optimal solution is substituted into w^* , m^* , θ^* , e^* , π_M , π_R , and π_B to obtain their optimal solutions.

4.4.2. Bargaining Model for Cost Sharing of Greening

Next, the model for the retailer to share the cost of green improvement is solved.

With w^* , m^* , θ^* , and e^* inputted into (49) and (50), the profit functions are obtained:

$$\pi_M = \frac{(-cb + a)^2 (\phi - 1) f I}{8(bf - \frac{r^2}{2})(\phi - 1) I + f\alpha^2}, \quad (73)$$

$$\pi_R = \frac{\left((\phi - 1)^2 \left(bf - \frac{r^2}{2} \right) I - \frac{\phi f \alpha^2}{4} \right) I f (-cb + a)^2}{16 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1) I + \frac{f \alpha^2}{8} \right)^2}. \quad (74)$$

Then, the objective function is obtained:

$$\pi_B = \frac{I^2 f^2 (\phi - 1) \left((\phi - 1)^2 \left(bf - \frac{r^2}{2} \right) I - \frac{\phi f \alpha^2}{4} \right) (-cb + a)^4}{128 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1) I + \frac{f \alpha^2}{8} \right)^3}. \quad (75)$$

By taking the first-order and second-order partial derivatives of (75) with respect to ϕ , it is obtained that:

$$\frac{\partial \pi_B}{\partial \phi} = \frac{5I^2 f^3 \alpha^2 (-cb + a)^4 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1) \left(\phi - \frac{1}{5} \right) I - \frac{f \alpha^2 (\phi - \frac{1}{2})}{10} \right)}{1024 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1) I + \frac{f \alpha^2}{8} \right)^4}, \quad (76)$$

$$\frac{\partial^2 \pi_B}{\partial \phi^2} = - \frac{5I^2 f^3 \left(\left(bf - \frac{r^2}{2} \right)^2 \left(\phi + \frac{1}{5} \right) (\phi - 1) I^2 - \frac{11 \left(bf - \frac{r^2}{2} \right) f (\phi - \frac{5}{11}) \alpha^2 I}{40} + \frac{\alpha^4 f^2}{160} \right) \alpha^2 (-cb + a)^4}{512 \left(\left(bf - \frac{r^2}{2} \right) (\phi - 1) I + \frac{f \alpha^2}{8} \right)^5} \quad (77)$$

Therefore, when:

$$\frac{11 \left(bf - \frac{r^2}{2} \right) f (\phi - \frac{5}{11}) \alpha^2 I}{40} - \left(bf - \frac{r^2}{2} \right)^2 \left(\phi + \frac{1}{5} \right) (\phi - 1) I^2 - \frac{\alpha^4 f^2}{160} < 0$$

π_R is concave in ϕ . Then, by making the first-order derivative of π_B equal to 0, the optimal sharing ratio is obtained:

$$\phi^* = \begin{cases} \frac{f \alpha^2 + 12bfI - 6I r^2 + \sqrt{\alpha^4 f^2 + 4f^2 \alpha^2 Ib - 2f \alpha^2 I r^2 + 64I^2 b^2 f^2 - 64I^2 bf r^2 + 16I^2 r^4}}{10(2bf - r^2)I} \\ \frac{f \alpha^2 + 12bfI - 6I r^2 - \sqrt{\alpha^4 f^2 + 4f^2 \alpha^2 Ib - 2f \alpha^2 I r^2 + 64I^2 b^2 f^2 - 64I^2 bf r^2 + 16I^2 r^4}}{10(2bf - r^2)I} \end{cases} \quad (78)$$

As revealed by numerical experiments, the above solution exceeds 1, so that it is usually taken from the value below.

Finally, the optimal solution is substituted into w^* , m^* , θ^* , e^* , π_M , π_R , and π_B to obtain their optimal solutions.

5. Comparison between Models

In this section, the impact of the proposed cost-sharing contract and parameter settings on the models is explored. Based on the previous models, a comparative analysis is conducted on the relationship between the optimal decision variables in the models, with subscripts used to represent the optimal values in different models.

Proposition 2. *In the cost-sharing model for greening improvement, the cost-sharing ratio ϕ is directly proportional to the square of the sensitivity of green improvement α^2 . Meanwhile, it is inversely proportional to the green investment coefficient I .*

Proof. By taking partial derivatives of α and I for Formula (65), it is obtained that:

$$\frac{\partial \phi^*}{\partial \alpha} = \frac{f \alpha}{4(2bf - r^2)I} > 0, \quad (79)$$

$$\frac{\partial \phi^*}{\partial I} = - \frac{f \alpha^2}{8(2bf - r^2)I^2} < 0. \quad (80)$$

□

According to Formula (79), when the sensitivity of green improvement increases, the retailer is willing to bear more improvement costs for more profits. According to Formula (80), when the green improvement coefficient is excessively high, the retailer lacks willingness to bear more improvement costs for self-interests. This implies that the cost-sharing contract is influenced by the customer's sensitivity to green improvement and the coefficient of green improvement.

Proposition 3. *The sales effort in the centralized model e_c^* is greater than in the decentralized model e_d^* .*

Proof. The optimal sales effort under two models are:

$$e_c^* = \frac{2Ir(a-bc)}{4fbI - 2Ir^2 - \alpha^2 f'} \quad (81)$$

$$e_d^* = \frac{2Ir(a-bc)}{8fbI - 4Ir^2 - \alpha^2 f}. \quad (82)$$

□

According to Proposition 1 that θ^* exceeds 0, it can be concluded that $a - bc > 0$. Due to $8fbI - 4Ir^2 > 4fbI - 2Ir^2$, $e_c^* > e_d^*$, which indicates that the willingness of the retailer to make sales effort diminishes under the context of decentralized decision making, which is possibly attributed to the higher profits under decentralized decision making than under centralized decision making.

Proposition 4. *The optimal green level meets the following condition: $\theta_c^* > \theta_g^* > \theta_d^*$, where θ_g^* represents the green level in the green cost-sharing model.*

Proof. The optimal green levels for these models are:

$$\theta_c^* = \frac{f\alpha(a-bc)}{4fbI - f\alpha^2 - 2Ir^2}, \quad (83)$$

$$\theta_d^* = \frac{f\alpha(a-bc)}{8fbI - f\alpha^2 - 4Ir^2}. \quad (84)$$

$$\theta_g^* = \frac{2f\alpha(a-bc)}{16fbI - 3f\alpha^2 - 8Ir^2}. \quad (85)$$

□

Similar to Proposition 3, this conclusion can be drawn directly from (83) and (84): $\theta_c^* > \theta_d^*$. In addition, there are some conclusions that can also be drawn: $4fbI - f\alpha^2 - 2Ir^2 > 0$, $8fbI - f\alpha^2 - 4Ir^2 > 0$ and $16fbI - 3f\alpha^2 - 8Ir^2 > 0$.

Due to $\theta_c^* - \theta_g^* = \frac{f\alpha(8fbI - f\alpha^2 - 4Ir^2)(a-bc)}{(16fbI - 3f\alpha^2 - 8Ir^2)(4fbI - f\alpha^2 - 2Ir^2)} > 0$, we have $\theta_c^* > \theta_g^*$.

According to $\theta_d^* - \theta_g^* = \frac{(a-bc)\alpha^3 f^2}{(3\alpha^2 f - 16fbI + 4Ir^2)(8fbI - f\alpha^2 - 4Ir^2)} < 0$, we have $\theta_d^* < \theta_g^*$.

In summary, it can be obtained that $\theta_c^* > \theta_g^* > \theta_d^*$. The conclusion is that the cost-sharing contract for greening improvement can improve the green level compared with decentralized decision making. It is possible that consumers are more willing to buy green products, despite a high level of greenness leading to higher product prices. Therefore, the cost-sharing contract remains effective.

Proposition 5. *The optimal wholesale price meets the following condition: $w_g^* > w_d^*$, where w_g^* represents the wholesale price in the green cost-sharing model.*

Proof. The optimal wholesale prices for these models are:

$$w_d^* = \frac{4Ib^2cf - 2Ibcr^2 - \alpha^2bcf + 4Iabf - 2Iar^2}{(8Ibf - 4Ir^2 - \alpha^2f)b},$$

$$w_g^* = \frac{5\alpha^2bcf - 16b^2cfI + 8bcIr^2 + a\alpha^2f - 16abfI + 8aIr^2}{2b(3\alpha^2f - 16bfI + 8Ir^2)}.$$

□

Due to $w_g^* - w_d^* = \frac{\alpha^4f^2(a-bc)}{2(3f\alpha^2 - 16bfI + 8Ir^2)b(f\alpha^2 - 8bfI + 4Ir^2)} > 0$, it can be obtained that $w_g^* > w_d^*$.

As implied by the conclusion, the wholesale price of products under the cost-sharing contract for greening improvement is higher than under decentralized decision making. This may result from the increase in the green cost of the product that causes the wholesale price to rise.

Proposition 6. The optimal profit meets the following condition: $\pi_c^* > \pi_g^* > \pi_d^*$, where π_g^* represents the profit in the green cost-sharing model.

Proof. The optimal profits for these models are:

$$\pi_c^* = \frac{fI(a-bc)^2}{4Ibf - f\alpha^2 - 2Ir^2},$$

$$\pi_d^* = \frac{(a-bc)^2(-\alpha^2f + 12Ibf - 6Ir^2)fI}{(8Ibf - 4Ir^2 - \alpha^2f)^2},$$

$$\pi_g^* = \frac{f(a-bc)^2(\alpha^2f - 48bfI + 24Ir^2)}{8(3\alpha^2f - 16bfI + 8Ir^2)(2bf - r^2)}.$$

□

Due to $\pi_c^* - \pi_d^* = -\frac{4fI^3(a-bc)^2(2bf-r^2)^2}{(\alpha^2f-4bfI+2Ir^2)(\alpha^2f-8bfI+4Ir^2)^2}$, we have $\pi_c^* > \pi_d^*$.

Due to $\pi_c^* - \pi_g^* = -\frac{f(a-bc)^2(\alpha^4f^2-4\alpha^2bf^2I+2\alpha^2fIr^2-64b^2f^2I^2+64bfI^2r^2-16I^2r^4)}{8(3\alpha^2f-16bfI+8Ir^2)(2bf-r^2)(\alpha^2f-4bfI+2Ir^2)}$ and $bf \gg r^2$, we have $\pi_c^* > \pi_g^*$.

Due to $\pi_g^* - \pi_d^* = \frac{(a-bc)^2f^3\alpha^4(\alpha^2f-16bfI+8Ir^2)}{8(\alpha^2f-8bfI+4Ir^2)^2(3\alpha^2f-16bfI+8Ir^2)(2bf-r^2)} > 0$, we have $\pi_g^* > \pi_d^*$.

The conclusion is that the optimal profit of supply chain is higher under the cost-sharing contract for greening improvement than under decentralized decision making. This is possibly attributed to the manufacturer investing more costs in product greening improvements under the green cost-sharing contract, which stimulates the purchasing desire of customers and increases market demand.

6. Numerical Experiment

From the previous section, it can be seen that despite some relationships determined between parameters, it remains difficult to draw comparison in the complex formula of ϕ . To further reveal the impact of parameters on the supply chain, numerical experiments are conducted on the models in this section. The values of parameters are set within a feasible range to ensure the effectiveness of numerical experiments (refer to [23]). The parameter settings are as follows: $a = 1000$, $b = 50$, $c = 6$, $r = 1$, $I = 40$, $\alpha = 40$. The main focus of study is the impact of sales effort cost coefficient f on other decision variables. According to a series of numerical tests, the impact of f on decision variables is minimal when f exceeds 0.5. Therefore, f ranges between 0 and 0.5.

Through numerical experiments, it can be found out that the cost-sharing ratio always discards one of the solutions when there are two solutions, as further explained in previous

sections. As shown in Figures 2–8, the sales effort cost coefficient has a diminishing impact on decision variables. According to Figures 2 and 4, the retail prices and green levels of green products are higher when promotional cost and green improvement cost are shared than under decentralized decision making. As indicated by this result, the manufacturer and the retailer are more willing to improve the green level of products through cost sharing, but it also causes the sales prices of products to rise. As shown in Figure 3, the wholesale price can be effectively reduced in the bargaining process of sharing promotional cost, which reduces the costs for the retailer. Figure 4 shows that the sales effort coefficient has no significant impact on the green level of the product. However, green cost-sharing contracts can still improve the green level to a certain extent when compared to the decentralized model. According to Figure 5, the retailer can incur lower promotion cost under the effect of cost-sharing contracts than under the centralized mode. In addition, the promotion coefficient of the retailer has a significant impact on promotion cost when it ranges between 0.1 and 0.3. As indicated by Figures 6 and 7, the green cost-sharing bargaining model puts the retailer at a disadvantage, whereas the manufacturer benefits most under this model. This is possibly attributed to the fact that the retailer shares a proportion of the cost of green improvement, which erodes their profits. Therefore, the retailer may be reluctant to bargain on the proportion of green cost sharing. Meanwhile, it can be seen that the profit loss of the retailer is insignificant, which leads to the limited impact. Furthermore, it is evident that cost-sharing contracts create an advantage for the manufacturer. As shown in Figure 8, the green cost-sharing bargaining model can improve the overall profit of the supply chain. According to Figure 9, the retailer can reduce the burden of promotion cost and increase the burden of green improvement cost under the bargaining model, while the sales effort coefficient barely has any effect on them.

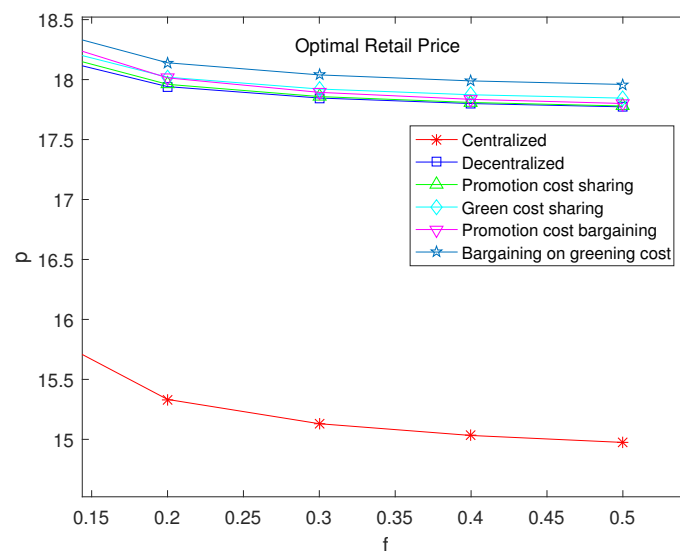
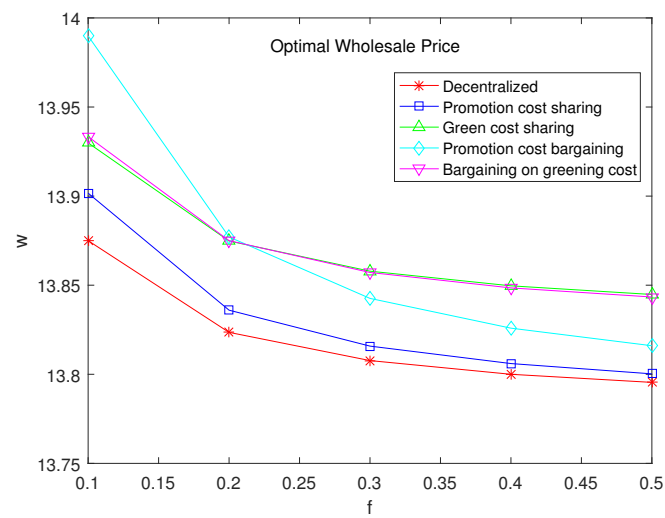
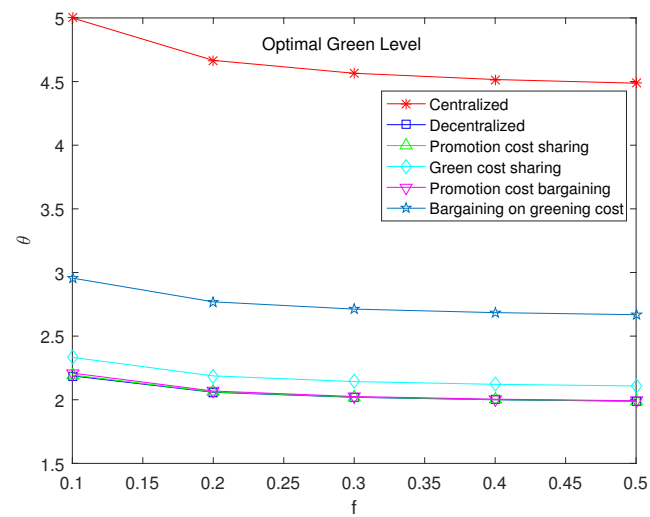
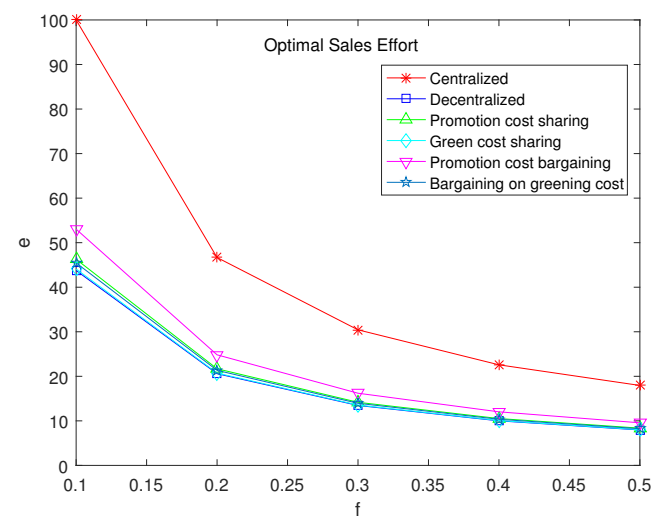
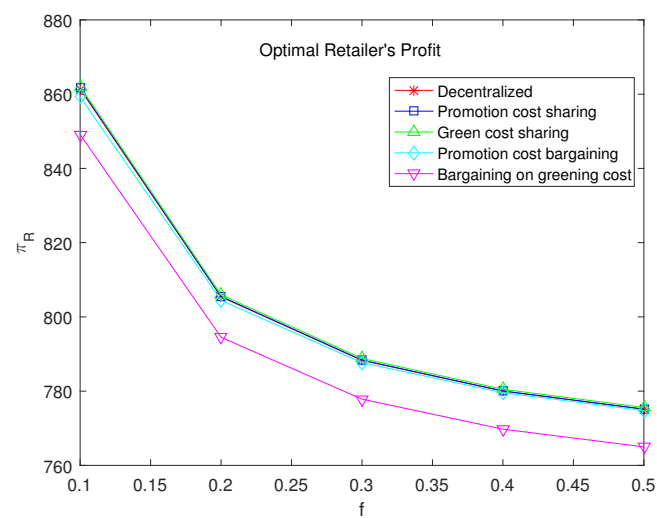
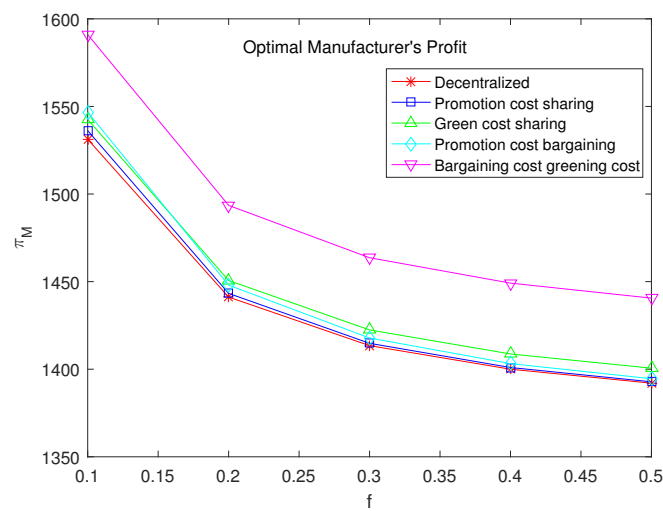
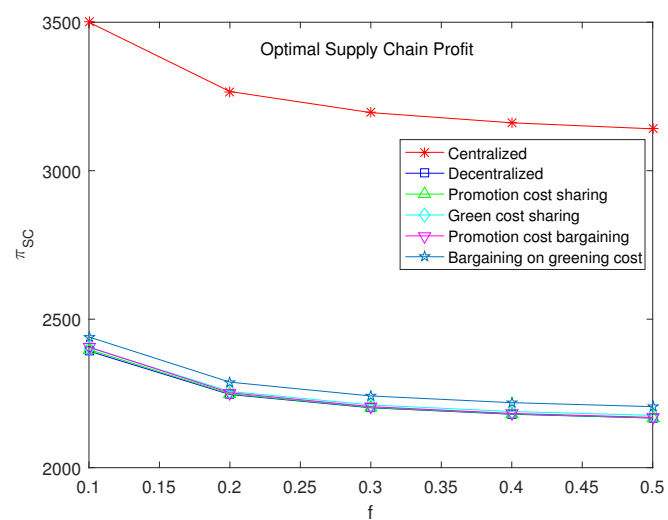


Figure 2. Optimal retail price vs. f .

Our research finding is that the proposed cost-sharing contracts are effective in improving the green level of products and increasing the profits generated for the manufacturer and the entire supply chain with minimal impact on retailer profits. Additionally, cost-sharing contracts can further reduce the promotional costs incurred to retailers, enhance their promotional enthusiasm, and recommend better green products to customers. In general, the two cost-sharing contracts are effective in coordinating the conflicts among members. Notably, this article is subjected to some limitations. There is no consideration given to the quality effort cost of manufacturers, and the information among supply chain members is symmetrical. Consequently, the model is relatively idealized and unfit for fully reflecting the complex situation in the real world.

Figure 3. Optimal wholesale price vs. f .Figure 4. Optimal green level vs. f .Figure 5. Optimal sales effort vs. f .

Figure 6. Optimal retailer's profit vs. f .Figure 7. Optimal manufacturer's profit vs. f .Figure 8. Optimal supply chain profit vs. f .

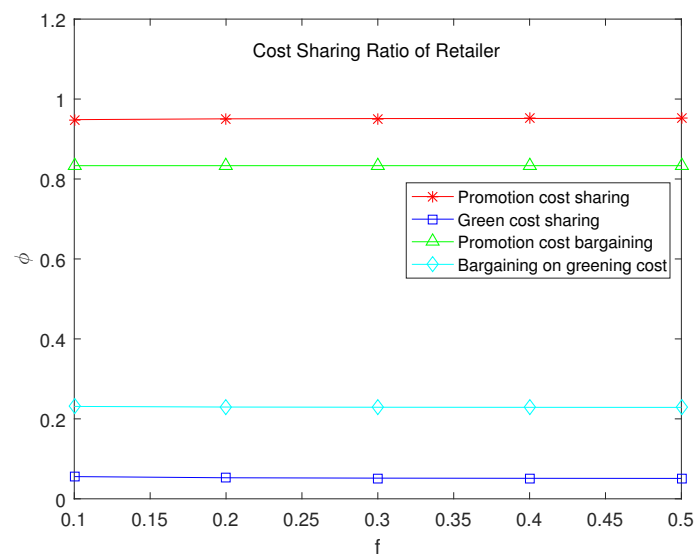


Figure 9. Optimal cost-sharing ratio of retailer vs. f .

7. Actual Case Analysis and Managerial Implications

In this section, the model established earlier is used to resolve a practical problem. As the demand of consumers for green and environmentally friendly products rises, the green detergent market is anticipated to boom. With a gradual increase in the awareness and willingness of consumers to purchase green detergents, market demand increases constantly. In the past few years, green detergents have accounted for over 20% of the fabric detergent market. In the future, the green detergent market will continue expanding with the introduction of stricter environmental regulations by the government and the improvement of consumer environmental awareness. In this section, the pricing decision of a green and environmentally friendly detergent produced by a certain manufacturer is analyzed. Based on the results of previous theoretical analysis, the problem is solved under decentralized decision making and green cost-sharing negotiation and a comparative analysis is conducted.

The parameter settings are as follows: $a = 10000$, $b = 200$, $c = 10$, $r = 5$, $I = 50$, $\alpha = 50$, $f = 0.1$. On this basis, the decision variables and profits are obtained under decentralized decision making as follows:

$$p = 60.91, w = 31.82, \theta = 14.55, e = 1450$$

$$\pi_r = 63471, \pi_m = 116363, \pi_{sc} = 179834$$

Then, the decision variables and profits are obtained under the negotiation of green cost sharing:

$$p = 61.78, w = 31.90, \theta = 19.24, e = 1493$$

$$\pi_r = 62790, \pi_m = 119494, \pi_{sc} = 182285$$

As revealed by the comparison, manufacturers have significantly higher profits under the negotiation of green cost sharing, and the total profit of the supply chain increases. It suggests that the green cost-sharing contract plays a role in significantly improving the green level of the product when the retail price is basically unchanged.

According to the research findings of this article, there are some key management insights gained as follows. Centralized decision making consistently generates the best supply chain profit, while contract models increase supply chain profits relative to decentralized models. It is possibly difficult to achieve a win-win situation, but it is feasible to improve the green level of products and increase supply chain profits. For the man-

ufacturers with weaker negotiation capabilities, it is a necessity to invest more in green improvement for competing with non-green products.

8. Conclusions

In this paper, a two-echelon green product supply chain is studied that consists of a manufacturer and a retailer. To study the enthusiasm of supply chain members, the promotion effort of the retailer in the problem is considered. The demand function is defined as a linear function of sales price, green level, and promotion effort. The impact of sales effort on the supply chain is explored by establishing centralized and decentralized game models, with the optimal strategies determined. Furthermore, two cost-sharing contracts are proposed to coordinate the conflicts among supply chain members. According to the results, the proposed cost-sharing contracts can improve the green level of products, reduce promotional costs for the retailer, and increase the profits generated for supply chain members. In addition, the manufacturer gains more profits under the green cost-sharing contract than under other models. It indicates that this contract can incentivize the manufacturer to produce better products to give back to consumers. Despite no win–win outcome achieved, it remains possible to improve the green level and overall profit of the supply chain.

However, there are some limitations facing this paper. It ignores the quality effort cost of manufacturers, and the information among supply chain members is symmetrical. As a result, the model is relatively idealized and unfit for fully reflecting the complex situation in the real world. In the future, more effective contracts can be used to increase the profit of the retailer and achieve a win–win situation for the members of the supply chain. In this paper, only deterministic parameters are considered, which is relatively preliminary. In future research, stochastic demand functions and more complex situations can be considered, such as more factors, multiple manufacturers and retailers, and multi-level green supply chain problems.

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