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Deformation Prediction Model of Existing Tunnel Structures with Equivalent Layered Method–Peck Coupled under Multiple Factors

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Abstract: The existing tunnel structure, the new underpass tunnel structure and the rock strata in the area of influence of the crossover tunnel are interacting systems that are affected by various factors, such as dynamic and static excavation loads and dynamic and static train loads. The existing theoretical models for the deformation prediction of existing tunnels lack the synergistic analysis of dynamic and static loads on both existing and new tunnels. Based on the theory of the current layer method and Peck's empirical formula, this paper considers the stiffness of existing tunnels, the stiffness of new tunnels, the loads of excavation methods and the loads of existing tunnels. The results show that a theoretical model for the prediction of the deformation of double-lane highway tunnels underneath existing railroad tunnels with the coupling of the current layer method and Peck under multiple factors is constructed; a modified Peck settlement formula for the base plate of the existing tunnels is put forward; and, through numerical calculations and monitoring data for validation and optimization, it is proved that the theoretical model is applicable to the excavation of tunnels underneath mountainous areas mined by the blasting method.

Keywords: deformation prediction model; multiple factors; equivalent layered method; Peck; underpass tunnel; existing tunnel

MSC: 74-10

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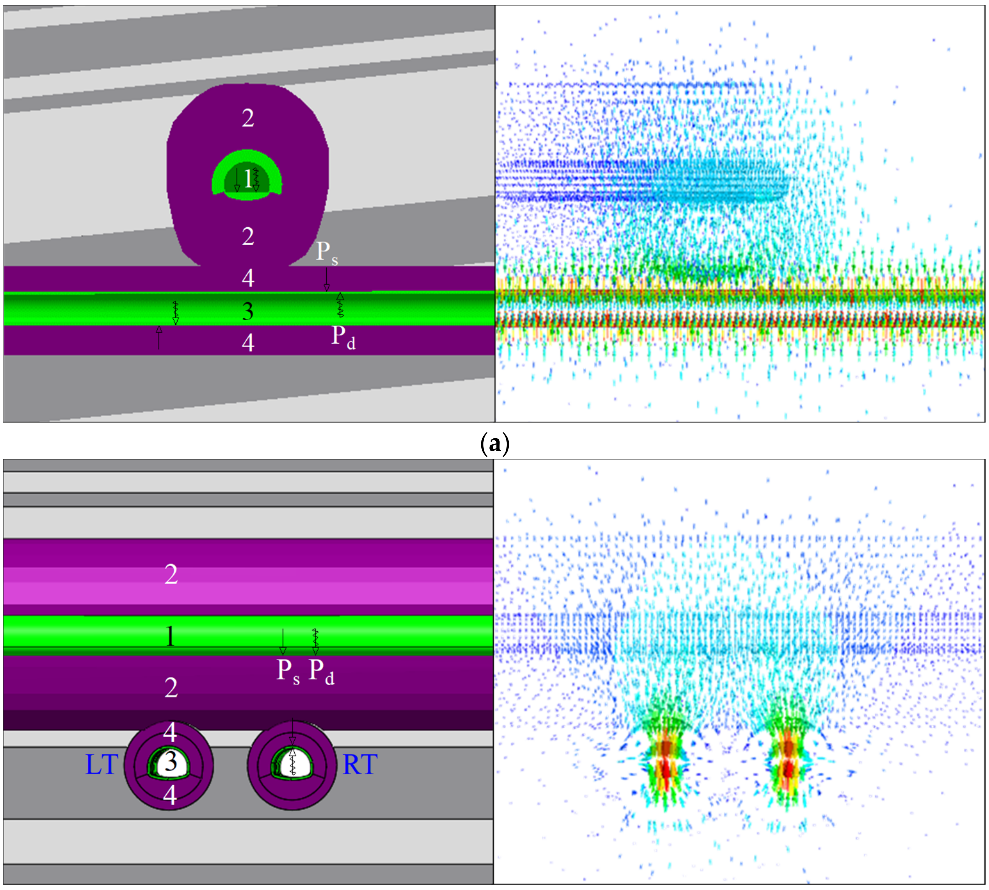
1. Introduction

With the sustained high-speed and stable development of economic construction and national livelihood, large-scale and high-quality land transportation infrastructure construction has entered a rapid development stage [1]. Road and railroad tunnels are an important part of the transportation system [2]. With the increase in the number of tunnels, regarding the spatial scope and addressing conditions of special areas, in the space of the overlapping tunnel project, new single-line or multi-line tunnels through the existing tunnels will become more and more common [3,4]. The construction of underpass tunnels will disturb the overlying rock layer and the existing tunnel structure to a certain extent, which will cause the rock layer to move and the existing tunnel structure to settle, resulting in great safety hazards [5,6]. Therefore, the scientific and reasonable prediction of the deformation of existing tunnels and strata caused by the construction of underpass tunnels is of great theoretical and engineering significance for the design and construction of underpass tunnels and for ensuring the safety of existing underground structures. Numerous scholars have carried out research studies, and the commonly used methods include theoretical analysis [7–9], numerical simulation [10,11] and model tests [12,13]. Numerical calculations and model test calculations use large-scale models, which can maximize the engineering scale, and are both characterized by high simulation, but are labor-intensive, material-intensive and time-consuming, while the dependence of test conditions and parameter selection is prominent. The theoretical analysis method can obtain results succinctly and quickly to evaluate the effect of excavation on the force and deformation of adjacent existing tunnels, but it requires generalization of the engineering model and multiple assumptions. In order to accurately and quickly evaluate the impact of tunnel excavation on the overlying existing tunnel structure, combined research means of the three methods are mostly used to optimize the theoretical analysis model through physical tests and numerical calculations, providing a theoretical method for the rapid evaluation of similar projects.

Zhou [14] proposed the most representative formula (or Gaussian curve) for predicting the surface settlement of tunnel excavation. Kong [15] and Mair et al. [16] proposed that the settlement pattern of the overlying rock and soil layers triggered by tunnel excavation can also be described by a Gaussian curve, and gave a formula for the settlement width parameter. Lin et al. [17] summarized the coefficients of the settlement slot kaudo coefficients for different rock and soil layers, and gave a formula for the settlement width parameter. Based on Peck's formula and elastic equivalence theory, Zhou et al. [18] proposed a method to evaluate the impact of new tunnels on existing tunnel excavation in the environment of composite strata. Vorster et al. [19] and Marshall et al. [20] modified the Gaussian curve formula. Sagaseta [21] obtained an analytical solution for soil displacement induced by uniform convergence of circular holes at any point in the formation based on the virtual mirror method and the theory of elasto dynamics. Verruijt et al. [22] obtained an analytical solution for the geotechnical displacement considering both the uniformly converging deformation of the tunnel and the ellipticalized deformation of the tunnel lining. Loganathan [23] established a spatial distribution function of the loss in the geotechnical formation and gave a semianalytical solution for the geotechnical displacement induced by the excavation of a tunnel. Li et al. [24] and Liu et al. [25] proposed an improved model for calculating the geotechnical displacement and deformation induced by new tunnel construction based on the Loganathan–Poulos solution. Shi et al. [26] applied Winkler's elastic foundation beam method to solve the impacts of new tunnels on existing pipelines, based on which the two-parameter Pasternak model [27], Hetenyi model [28] and Vlasov model [29] and three-parameter Kerr model [30] were modified. Klar et al. [31] predicted the geotechnical deformation based on the Peck's formula or normal distribution curve, and analyzed the deformation of the geotechnical layer as the displacement load above the existing tunnel. Liu et al. [32] combined the Pasternak foundation model and the energy variational method to investigate the longitudinal mechanical response of the existing shield tunnels triggered by the underpassing of the new tunnels, and Zhang et al. [33] analyzed the impact of the new tunnels on the existing tunnels under different working conditions by applying the Winkler foundation theory. Xu et al. [34] established a model to predict the impact of the excavation of the new tunnels on the surface settlement under multifactorial stratigraphy based on the equivalent stratification method and the theory of the stochastic medium model. After that, Xu [35] derived the tunnel foundation loss based on the Sagaseta formula and equivalent stratification method and calculated the surface settlement caused by the excavation of new tunnels. Zhou [36] and others used the equivalent stratification method to simplify the composite rock layer, introduced the mud consolidation equation to correct the deformation generated by different factors and analyzed the deformation effects of the excavation of new tunnels on existing tunnels.

Excavation of the tunnel underneath causes stress release in the rock strata, and the deformation and stress are transmitted to the existing tunnel structure through the rock strata, inducing settlement. The condition of the rock layer where the tunnel is located and its excavation method are the controlling factors for the construction of the new tunnel and the deformation of the existing tunnel structure. Repeated dynamic and static loads from trains running in existing tunnels for a long period of time induce the deterioration of the mechanical parameters of the rock layer in which the tunnel is located, which is a secondary factor causing the deformation of the existing structure. Under the dynamic and static loads of the tunnel excavation and the existing tunnel trains, the stresses and deformations of the rock strata and the existing tunnel structure in the affected areas of the tunnel and the crossover tunnel project are dynamic changes. The mechanical properties of the rock strata in the influence area of the cross-tunneling project are affected by the repeated unloading and loading of the dynamic and static excavation loads, and the repeated dynamic and static train loads. The underpass tunnel, the rock layer in the influence area of the cross-tunneling project and the existing tunnel structure constitute a multi-structure, multifactor interaction system, as shown in Figure 1. Therefore, it is necessary to consider the stiffness of the existing tunnel, the stiffness of the new tunnel, the excavation method load, the existing tunnel load and other factors.



(**b**)

Figure 1. Schematic diagram of the location of the overlying multi-layered geotechnical layers of the tunnel. (**a**) Axial section of the new tunnel. (**b**) Axial section of the existing tunnel: 1 is the existing tunnel structure, 2 is the area of the rock layer affected by the dynamic and static loads of the existing tunnel, 3 is the structure of the tunnel underneath, 4 is the area affected by the dynamic and static loads of the tunnel excavation underneath, P_s is the static load and P_d is the dynamic and static load.

At present, regarding domestic and foreign research, although considerable progress has been made, the results are mainly concentrated on the subway shield excavation and single-line drilling and blasting method of excavation under the project. For the two-lane large cross-section highway tunnels diagonally under the existing railroad tunnel project, there are not many research cases; for the dynamic and static loads in the excavation process of the new tunnels, dynamic and static loads of the existing tunnels are mostly analyzed individually, and the results of the synergistic analysis need to be perfected. In view of this, the stiffness of the existing tunnels is equated by the local layer method; the influence of the underpass first excavated tunnel on the settlement is optimized by the correction factor of the influence of the structural stiffness of the existing tunnel on the width of the settlement tank; the degradation of the rock parameters in the influence area of the cross-tunneling project based on the Hoek–Brown criterion characterizes the extent of the influence of the excavation method and dynamic and static loads. A theoretical model for the prediction of the excavation deformation of a two-lane highway tunnel underneath an existing railroad tunnel is coupled with Peck's multifactors when the layer method is proposed.

2. Calculation Method

2.1. Peck Model for Predicting Settlement of Existing Tunnel Structures in Under-Tunnel Excavation

The surface settlement curve caused by the excavation of the two-lane tunnel is shown in Figure 2 [15].

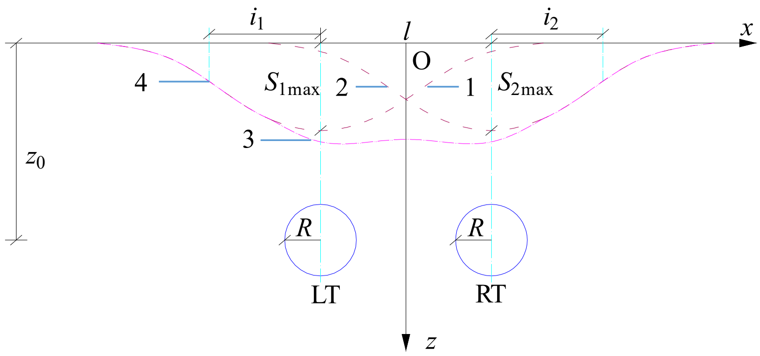


Figure 2. Peck prediction curve for surface settlement in two-lane tunnel excavation: 1 is the surface settlement curve caused by the excavation of the left-lane tunnel (LT), 2 is the surface settlement curve caused by the excavation of the right-lane tunnel (RT), 3 is the surface settlement curve caused by the superposition of the two-lane tunnel excavation and 4 is the inflection point of the curve.

The Peck curve equation for surface settlement in a two-lane parallel tunnel excavation is as follows:

$$S(x) = \frac{A_1 V_1}{i_1\sqrt{2\pi}} exp\left(-\frac{(x-0.5l)^2}{2{i_1}^2}\right) + \frac{A_2 V_2}{i_2\sqrt{2\pi}} exp\left(-\frac{(x+0.5l)^2}{2{i_2}^2}\right) \tag{1}$$

where A_1 and A_2 are the cross-sectional area of the twin tunnels; V_1 and V_2 are the rate of stratum loss caused by the construction of the twin tunnels; i_1 and i_2 are the respective widths of the sinkholes of the twin tunnels; and l is the spacing between the centers of the twin tunnels.

Mair et al. [16] corrected the width of sinkholes to obtain expressions for calculating the width of sinkholes i at different depths z locations:

$$i = K\,(z_0 - z) \tag{2}$$

where K is the width coefficient of the sinkhole and $z0$ is the excavation tunnel burial depth.

Zhou [14] proposed the formula for K:

$$K = 0.5(z_0/R)^{1-n} \tag{3}$$

where n is the adjustment factor and n = 0.8 to 1.0.

Zhou [18] et al. concluded that the deformation of existing tunnels can be predicted using Peck's formula with a corrected ground loss rate and sinkhole width coefficient.

$$V_{et} = \lambda^{\alpha}\lambda^{g}V \tag{4}$$

where V_{et} is the stratum loss rate of the settlement curve of the existing tunnel; λ^{α} is the geometric correction coefficient of the angle between the existing tunnel and the new tunnel on the stratum loss rate of the settlement channel, where $\lambda^{\alpha} = 1/cos\alpha$ and α is the angle between the new tunnel and the existing tunnel; λ^g is the discount factor for the rate of stratum loss by the construction protection measures of the existing tunnel (value range 0 to 1.0).

$$K_{et} = \eta^d \eta^m K \tag{5}$$

where K_{et} is the coefficient of the width of the settlement channel of the existing tunnel; η^d is the correction coefficient considering the influence of the depth of the existing tunnel on the width of the settlement channel, where $\eta^d = [1 - a\,(z/z_0)]/(1 - z/z_0)$ and a is the parameter considering the lithological condition of the ground layer (the range of values is 0~1); η^m is the correction coefficient considering the influence of the structural stiffness of the existing tunnel on the width of the settlement channel (the range of values is 1.05~1.6), where $\eta^m = 0.7M^{0.20}$ and M is the shear stiffness of the existing tunnel section.

The settlement calculation formula for a single-line tunnel with a diagonal intersection underneath the existing tunnel is as follows:

$$S(x) = \frac{AV_{et}}{i_{et}\sqrt{2\pi}} exp\left(-\frac{x^2}{2{i_{et}}^2}\right) \tag{6}$$

The settlement calculation formula for a two-lane tunnel crossing diagonally under an existing tunnel is as follows:

$$S(x) = \frac{A_1 V_{1et}}{i_{1et}\sqrt{2\pi}} exp\left(-\frac{(x-0.5l_{et})^2}{2{i_{1et}}^2}\right) + \frac{A_2 V_{2et}}{i_{2et}\sqrt{2\pi}} exp\left(-\frac{(x+0.5l_{et})^2}{2{i_{2et}}^2}\right) \tag{7}$$

where i_{et} is the width of the sinkhole of the existing tunnel, where $i_{et} = K_{et}(z_0 - z)$; l_{et} is the diagonal spacing of the two-lane tunnel along the direction of the existing tunnel, where $l_{et} = l/\sin\alpha$.

2.2. Theory of Multi-Level Equivalent Layer Method Conversion

Figure 3 shows the schematic diagram of the conversion calculation of the layer method for the multi-layer geotechnical materials with different properties over the tunnel. The thickness of the i layer is h_i, modulus of elasticity is E_i, Poisson's ratio is μ_i and density is ρ_i, and the propagation path of the vibration wave under the action of concentrated force P is shown in Figure 3.

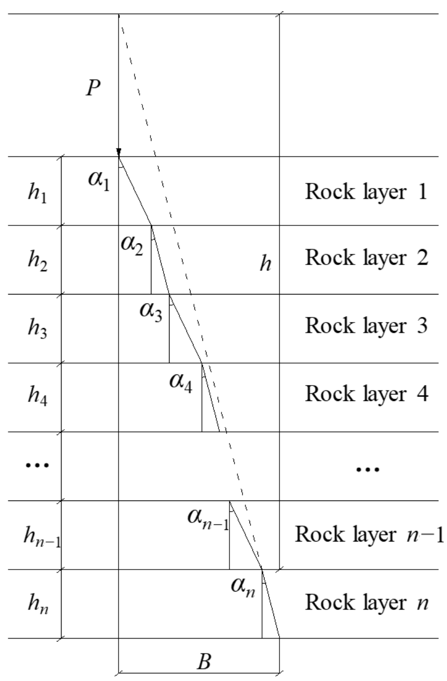


Figure 3. Multiple layers of geotechnical materials with different properties overlying the tunnel.

The vibration wave propagates in the first rock layer at a small angle of α_1 with respect to the vertical direction, refracts at the upper interface of the second rock layer, propagates in the second rock layer at a small angle of α_2 with respect to the vertical direction, refracts once at the interface of the rock layers of each of the two layers and propagates in the i layer at the refraction angle α_i.

The vibrating wave is at a depth h_n in the n rock layer; then,

$$B = \sum_{i=1}^{n} h_i tan\alpha_i \approx \sum_{i=1}^{n} h_i \alpha_i \quad (8)$$

There is an equivalent transformation of each rock layer with different properties into the same rock layer with a vertical distance of (h + h_n) and width of B. h is the sum of the equivalent thicknesses of the 1~$n-1$ layers. B is as follows:

$$B = (h + h_n) \tan \alpha_n \approx (h + h_n)\alpha_n \quad (9)$$

According to Equations (8) and (9),

$$h = \frac{1}{\alpha_n} \sum_{i=1}^{n-1} h_i \alpha_i = \frac{1}{v_n} \sum_{i=1}^{n-1} h_i v_i \quad (10)$$

Further available,

$$h = \left(\rho_n / E_n\right)^a \sum_{i=1}^{n-1} h_i (E_i / \rho_i)^a \quad (11)$$

Let the coordinates of any position of the actual stratum be (x', z'), where x' is the horizontal distance of the position from the tunnel axis and z' is the vertical distance of the position from the ground surface, as shown in Figure 4. Through the theory of the current layer method, other geotechnical materials with different properties overlying the tunnel are converted to the same properties as those of the geotechnical layer where the tunnel is located, and the converted coordinates are (x, z).

$$x = x'$$
$$z = \begin{cases} z'(E_1\rho_n/E_n\rho_1)^a, z' \le h_1 \\ h_1(E_1\rho_n/E_n\rho_1)^a + (z' - h_1)(E_1\rho_n/E_n\rho_1)^a, h_1 < z' \le (h_1 + h_2) \\ \sum_{i=1}^{k-1} h_i(E_i\rho_n/E_n\rho_i)^a + \left(z' - \sum_{i=1}^{k-1} h_i\right)(E_k\rho_n/E_n\rho_k)^a, \sum_{i=1}^{k-1} h_i < z' \le \sum_{i=1}^{k} h_i, 2 < k \le n \\ \sum_{i=1}^{n-1} h_i(E_i\rho_n/E_n\rho_i)^a + \left(z' - \sum_{i=1}^{n-1} h_i\right), z' > \sum_{i=1}^{n-1} h_i \end{cases} \quad (12)$$

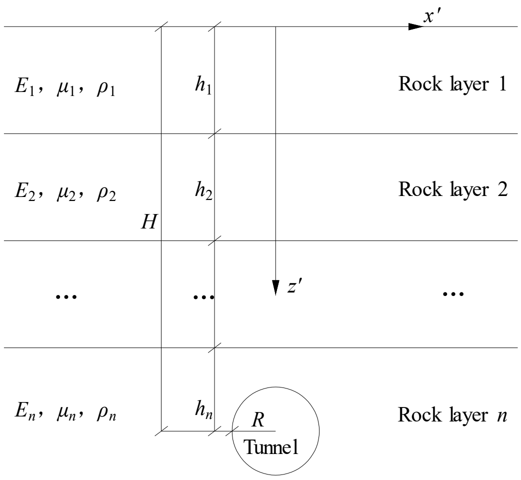


Figure 4. Schematic diagram of the location of the multi-layered soil and rock layers overlying the tunnel. Perform coordinate conversion.

By coordinate transformation, the problem is transformed into a problem of solving a deformed analytical solution in a homogeneous stratum.

2.3. *Structural Deformation Prediction Model for Existing Tunnels When Layer Method–Peck Coupling Is Used*

2.3.1. Equivalent Transformation of the Rock Layer Method for Multifactors

Figure 5 shows the schematic diagram of the equivalent conversion of the cross-tunnel project rock layer by the area equivalence method. Assuming that the existing tunnel is converted to a circular diameter of z_2 by the area equivalence method, it is equivalent to a rock layer of the same thickness with the same flexural rigidity (ETRL), and the modulus of elasticity, Poisson's ratio and density of the equivalent rock layer are E_2, μ_2 and ρ_2, respectively. The modulus of elasticity, Poisson's ratio and density of the rock layer where the tunnel is underneath (UTRL) are E_3, μ_3 and ρ_3, respectively, with a height of z_3. The modulus of elasticity, Poisson's ratio and density of the rock layer in the area affected by dynamic and static loads (DRL-I) are $E_3{}'$, $\mu_3{}'$ and $\rho_3{}'$, and the height is $z_{3\mathrm{I}}$; the modulus of elasticity, Poisson's ratio and density of the rock layer in the area affected by dynamic and static loads in the existing tunnels (DRL-II) are $E_2{}'$, $\mu_2{}'$,and $\rho_2{}'$, and the heights of the upper rock layer (DRL-II-U) and the lower one (DRL-II-D) are $z_{2\text{II-U}}$.The height of the upper rock layer (DRL-II-D) is $z_{2\text{II-D}}$. The modulus of elasticity, Poisson's ratio and density of the rock layer overlying the existing tunnel (ORL) are E_1, μ_1 and ρ_1, respectively, and the height of the ORL is z_1.

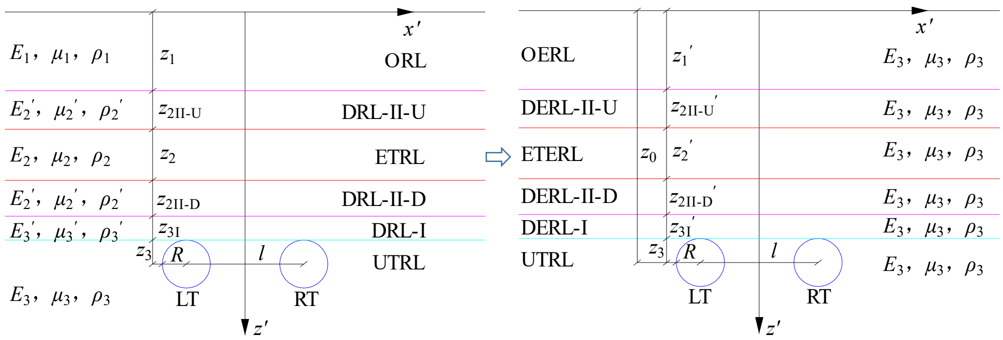


Figure 5. Schematic diagram of the equivalent transformation of the rock layer method for cross-tunneling projects.

Based on the principle of the current layer method, the overlying rock layer of the existing tunnel, the rock layer of the dynamic and static load influence area of the cross-tunneling project and the rock layer of the following tunnel (UTRL) as the current layer are equivalently transformed into mean rock layers, as shown in Figure 5. The height of the transformed overlying equivalent rock layer (OERL) is $z_1{}'$, the height of the upper equivalent rock layer in the area of dynamic and static load influence of the existing tunnels (DERL-II-U) is $z_{2\text{II-U}}{}'$, the height of the lower equivalent rock layer in the area of dynamic and static load influence of the existing tunnels (DERL-II-D) is $z_{2\text{II-D}}{}'$ and the height of the equivalent rock layer in the existing tunnels (ETERL) is $z_2{}'$. The height of the equivalent rock layer (DERL-I) in the area affected by the excavation method and static and dynamic loads is $z_{3\mathrm{I}}{}'$. The coordinates of the equivalent transformed rock layer using the local layer method are set as (x, z) and the coordinates are as follows:

$$
\begin{aligned}
&x = x' \\
&z = \begin{cases}
z_1 (E_1\rho_3/E_3\rho_1)^a, z' \le z_1' \\
z_1' + (z' - z_1')(E_2{}'\rho_3/E_3\rho_2{}')^a, z_1' < z' \le (z_1' + z_{2\text{II-U}}{}') \\
z_1' + z_{2\text{II-U}}{}' + (z' - z_1' - z_{2\text{II-U}}{}')(E_2\rho_3/E_3\rho_2)^a, z_1' + z_{2\text{II-U}}{}' < z' \le (z_1' + z_{2\text{II-U}}{}' + z_2') \\
z_1' + z_{2\text{II-U}}{}' + z_2' + (z' - z_1' - z_{2\text{II-U}}{}' - z_2')(E_2{}'\rho_3/E_3\rho_2{}')^a, \\
z_1' + z_{2\text{II-U}}{}' + z_2' < z' \le (z_1' + z_{2\text{II-U}}{}' + z_2' + z_{2\text{II-D}}{}') \\
z_1' + z_{2\text{II-U}}{}' + z_2' + z_{2\text{II-D}}{}' + (z' - z_1' - z_{2\text{II-U}}{}' - z_2' - z_{2\text{II-D}}{}')(E_3{}'\rho_3/E_3\rho_3{}')^a, \\
z_1' + z_{2\text{II-U}}{}' + z_2' + z_{2\text{II-D}}{}' < z' \le (z_1' + z_{2\text{II-U}}{}' + z_2' + z_{2\text{II-D}}{}' + z_{3I}{}') \\
(z' - z_1' - z_{2\text{II-U}}{}' - z_2' - z_{2\text{II-D}}{}' - z_{3I}{}'), \\
(z_1' + z_{2\text{II-U}}{}' + z_2' + z_{2\text{II-D}}{}' + z_{3I}{}') < z' \le (z_1' + z_{2\text{II-U}}{}' + z_2' + z_{2\text{II-D}}{}' + z_{3I}{}' + z_3)
\end{cases}
\end{aligned}
\tag{13}
$$

2.3.2. Modification of Peck's Prediction Model under Multifactorial

The transformed equivalent coordinate relationship, Equation (13), is combined with Peck's formula for the settlement calculation of two-lane tunnel diagonally intersecting underneath an existing tunnel to form a theoretical model for the prediction of deformation in the excavation of a two-lane highway tunnel underneath an existing railroad tunnel by multifactorial analysis when the layer method is coupled with Peck, as detailed in Equation (14).

$$\begin{cases} S(x) = \frac{A_1 V_{1et}}{i_{1et}\sqrt{2\pi}} exp\left(-\frac{(x-0.5l_{et})^2}{2{i_{1et}}^2}\right) + \frac{A_2 V_{2et}}{i_{2et}\sqrt{2\pi}} exp\left(-\frac{(x+0.5l_{et})^2}{2{i_{2et}}^2}\right) \\ l_{et} = l/sina \\ i_{1et} = 0.5(z_0/R)^{1-n}(z_0 - z)\eta_1^m[1 - a(z/z_0)]/(1 - z/z_0) \\ i_{2et} = 0.5(z_0/R)^{1-n}(z_0 - z)\eta_2^m[1 - a(z/z_0)]/(1 - z/z_0) \\ V_{1et} = (1/cosa)\lambda_1^g V_1 \\ V_{2et} = (1/cosa)\lambda_2^g V_2 \\ z_0 = z_1' + z_{2\text{II-U}}{}' + z_2' + z_{2\text{II-D}}{}' + z_{3I}{}' + z_3 \end{cases} \tag{14}$$

By adjusting the correction factor η^m for the effect of the structural stiffness of the existing tunnel on the width of the sinkhole, the difference in the structural settlement of the existing tunnel caused by the underpassing of the left and right tunnels is optimized.

3. Results and Discussion

3.1. Project Examples and Parameter Values

Taking the twin-lane Wanshoushan road tunnel under the Hurong Wanshoushan railroad tunnel project as the background [37], the spatial relationship between the underpassing twin-lane tunnel and the existing tunnel and its rock formation are shown in Figure 6.

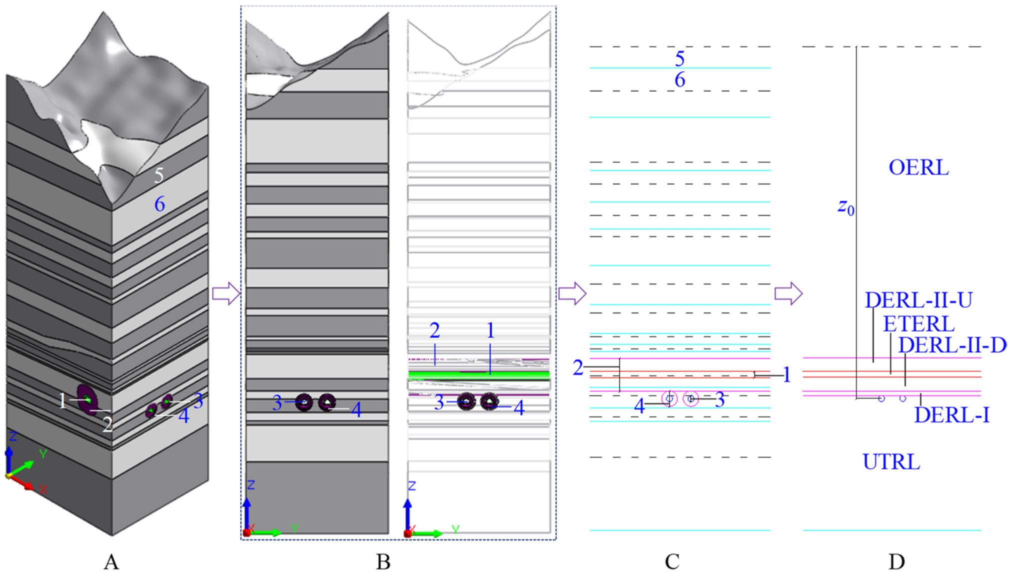


Figure 6. Spatial relationship and rock layer diagram of the underpassing two-lane tunnel and the existing tunnel: (**A**) is the three-dimensional spatial relationship, (**B**) is the X-direction side view, (**C**) is the two-dimensional rock layer and spatial relationship and (**D**) is the spatial relationship transformed by the layer method; 1 is the structure of the existing tunnel, 2 is the dynamic and static load influence area of the existing tunnel, 3 is the structure of the underpassing tunnel, 4 is the dynamic and static load influence area of the excavation of the underpassing tunnel, 5 is the mudstone and 6 is the malmstone.

(1) Calculation of Equivalent Modulus of Elasticity and Density Parameters for Existing Tunnels

The cross-tunnel structure consists of initial and secondary lining supports, whose combined modulus of elasticity $\overline{E}$ and density $\overline{\rho}$ are calculated by Equations (15) and (16), and the equivalent modulus of elasticity E and density of the rock layer are calculated by Equations (17) and (18).

$$\overline{E} = \frac{t_1 E_1 + t_2 E_2}{t_1 + t_2} \tag{15}$$

$$E\frac{\pi D^3}{64} = \overline{E}\frac{\pi D^3 - \pi D'^3}{64} \tag{16}$$

$$E\frac{\pi D^3}{64} = \overline{E}\frac{\pi D^3 - \pi D'^3}{64} \tag{17}$$

$$\rho \pi R^2 = \overline{\rho}\left(\pi R^2 - \pi R'^2\right) \tag{18}$$

where t_1 is the thickness of the initial support, t_2 is the thickness of the secondary lining, E_1 is the modulus of elasticity of the initial support, E_2 is the modulus of elasticity of the secondary lining, D is the equivalent diameter of the tunnel excavation, D' is the equivalent diameter of the net section after the tunnel support, ρ_1 is the density of the initial support, ρ_2 is the density of the secondary lining, R is the equivalent radius of the tunnel excavation and R' is the equivalent radius of the net section after the tunnel support.

The excavated area of the existing tunnel is 124.98 m^2, the clear area is 99.49 m^2 and the radius is 6.31 m and 5.63 m according to the area equivalent; the excavated area of the new tunnel is 110.31 m^2, the clear area is 76.85 m^2 and the radius is 5.93 m and 4.95 m according to the area equivalent; the thickness of the initial support of the existing tunnel is 23 cm and the thickness of the secondary lining is 45 cm; the thickness of the initial support of the new tunnel is 28 cm and the thickness of the secondary lining is 70 cm; the corresponding modulus of elasticity and density parameters are taken from the literature [37], and the modulus of elasticity of the equivalent rock layer of the existing tunnel is 8.51 GPa and the density is 5.19 kN·m^{-3} according to Equations (16) and (17).

(2) Parameters related to equivalent rock transformation in cross tunnels

The double-lane Wanshoushan Highway Tunnel under the Hurong Wanshoushan Railway Tunnel Project is a mudstone- and malmstone-interbedded rock layer, the new tunnel is located in the rock layer and the height of the overburden rock layer information is shown in Table 1; the mechanical parameters of the rock layer are taken from the literature [37]. The mudstone where the lower tunnel is located is the current layer and the theoretical method proposed in the previous section is used to transform the malmstone and the existing tunnel into the current layer, with a current layer index a of 0.33. The height of the dynamic and static loading zone of the existing and new tunnels is based on the literature [37], ignoring the width of the overlapping zone of influence. The relationship between the layers considering the stiffness of the existing tunnel and the dynamic and static loads of the cross-tunnel project is shown in Figure 7. The modulus of elasticity and density parameters of each rock layer considering the effects of dynamic and static loading of the existing tunnel and the cross-tunneling project are shown in Table 2, and the parameters of the influence area are taken from the results of the literature [38].

Table 1. X-direction side-view 2D rock layer with equivalent rock layer height parameters.

Rock Layer (Top to Bottom)	Height (m)	Equivalent Height (m)	Remarks
Mudstone	31.6	31.6	Area equivalent
Malmstone	35.5	47.6	
Mudstone	41.5	41.5	
Malmstone	68.9	92.3	
Mudstone	14.0	14.0	
Malmstone	21.2	28.4	

Table 1. *Cont.*

Rock Layer (Top to Bottom)	Height (m)	Equivalent Height (m)	Remarks
Mudstone	27.3	27.3	
Malmstone	20.4	27.3	
Mudstone	22.4	22.4	
Malmstone	10.0	13.4	
Mudstone	46.3	46.3	
Malmstone	30.0	40.2	
Mudstone	32.0	32.0	
Malmstone	11.0	14.7	
Mudstone	32.3	32.3	
Malmstone	5.6	7.5	
Mudstone	10.6	10.6	
Malmstone	7.4	9.9	
Mudstone	3.9	3.9	
Malmstone	37.5	50.3	Existing tunnel formations
Mudstone	18.3	18.9	
Malmstone	12.1	16.6	
Mudstone	20.6	20.6	Rock formation of the lower tunnel

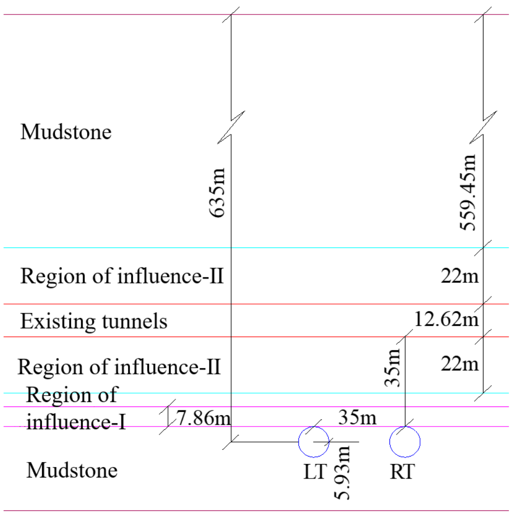


Figure 7. Relationship between the rock strata considering the effect of the stiffness of the existing tunnel and the dynamic and static loads of the cross-tunneling project.

Table 2. Formation parameters taking into account the influence of the stiffness of the existing tunnel and the dynamic and static loads of the cross-tunneling project.

Rock Stratum	Modulus of Elasticity (GPa)	Density (kN·m^{-3})
Mudstone	2.15	23.89
Existing Tunnels	8.51	5.19
Impact Area-I (Blasting)	1.10	23.89
Area of Influence-I (Static)	1.47	23.89
Area of Influence-II (Blasting)	1.32	23.89
Area of Influence-II (Static)	1.32	23.89

Combined with Figure 8 and Table 2 data, using the theoretical method proposed in the previous section of the existing tunnel, the impact of the regional rock layer where the layer occurs transforms the calculation, where the layer of the index a takes 0.33. This is derived from the consideration of the stiffness of the existing tunnel and the cross-tunnel engineering dynamic and static load impact of the rock layer when the layers have a relationship, as shown in Figure 8. The new tunnel depth is 647.17 m (blasting method excavation) and 647.80 m (static method excavation), and the cross-tunnel spacing is 30.17 m (blasting method excavation) and 30.80 m (static method excavation).

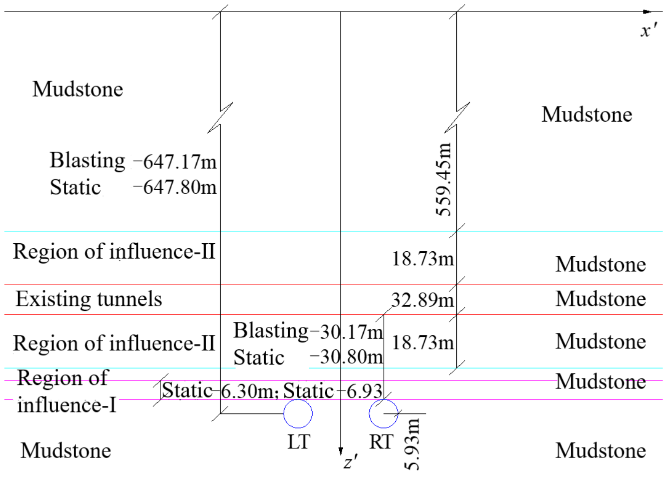


Figure 8. Stratigraphy of the rock layers taking into account the effect of the stiffness of the existing tunnel and the static and dynamic loads of the cross-tunneling project.

3.2. Theoretical Calculation Results

In the project example, the angle between the new tunnel and the existing tunnel is 61°, $\alpha = 61°$ [37]. The numerical calculation conditions are upper and lower step blasting method excavation (UPBM), upper and lower step static method excavation (UPSM), CD blasting method excavation (CDBM) and CD static method excavation (CDSM) [37]. The parameters related to Formulas (2)–(7) of the Peck settlement formula are optimized by the numerical calculation results, which are shown in Figure 9, and the parameters related to the calculation of the settlement of the base plate of the existing tunnel for different working conditions are finally determined as shown in Table 3. Only η^m is different for the left and right lines of the same working condition, and the other parameters take the same value.

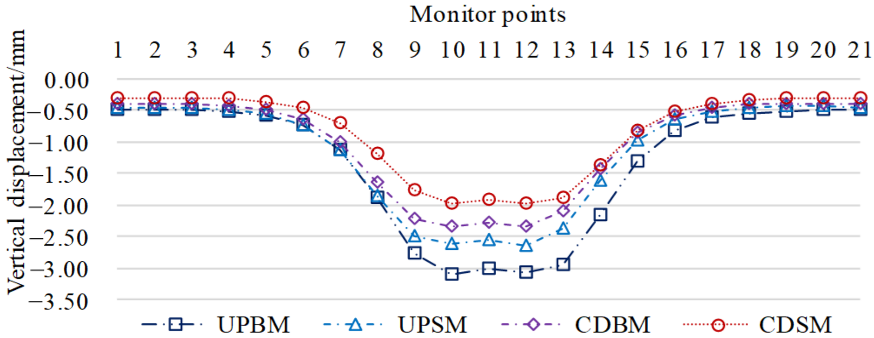


Figure 9. Displacement and settlement curves of the existing tunnel base plate under different calculation conditions.

Table 3. Parameters related to Peck settlement calculation for different working conditions of existing tunnels.

Working Condition	z_0/m	z/m	l/m	R/m	n	λ^{α}	λ^{g}	V/%	η^d	η_1^m	η_2^m
UPBM	641.17	617.00	35.00	5.93	0.99	2.06	0.20	0.230	1.06	1.06	1.05
UPSM	641.80	617.00	35.00	5.93	0.99	2.06	0.20	0.193	1.06	1.05	1.06
CDBM	641.17	617.00	35.00	5.93	0.99	2.06	0.20	0.173	1.06	1.05	1.06
CDSM	641.80	617.00	35.00	5.93	0.99	2.06	0.20	0.148	1.06	1.06	1.05

Substitute the parameters of Table 3 into Equations (2)–(7) to obtain the settlement value of Peck's bottom plate of existing tunnels under different calculation conditions, and extract the values of the monitoring points corresponding to the numerical simulation settlement curves to generate the settlement curves as shown in Figure 10.

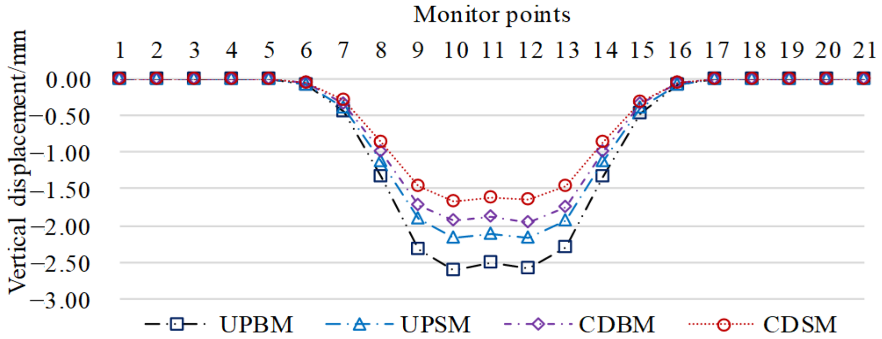


Figure 10. Calculation curves of Peck's settlement at monitoring points of existing tunnel base plate under different calculation conditions.

3.3. Comparison and Optimization

(1) Comparison and optimization of numerical and theoretical calculations

Comparing the results of numerical simulation and theoretical calculation for different calculation conditions, the error between the two is mainly caused by the phenomenon of the overall uniform settlement of the existing tunnel floor during numerical calculation; based on this phenomenon, Equation (7) is optimized by increasing the value of overall settlement S_{os}, and the revised formula for calculating the Peck settlement of the existing tunnel floor is as follows:

$$S(x) = S_{os} + \frac{A_1 V_{1et}}{i_{1et}\sqrt{2\pi}} exp\left(-\frac{(x-0.5l_{et})^2}{2{i_{1et}}^2}\right) + \frac{A_2 V_{2et}}{i_{2et}\sqrt{2\pi}} exp\left(-\frac{(x+0.5l_{et})^2}{2{i_{2et}}^2}\right) \quad (19)$$

Overall settlement S_{os} values are determined by numerical calculations and field monitoring data. In this chapter, the S_{os} values are determined by numerical calculation results, and the S_{os} is 0.50 mm for the UPBM working condition, 0.45 mm for the UPSM working condition, 0.40 mm for the CDBM working condition and 0.31 mm for the CDSM working condition. The calculated curves of the modified Peck settlement for the base plate of the existing tunnel for different working conditions are shown in Figure 11.

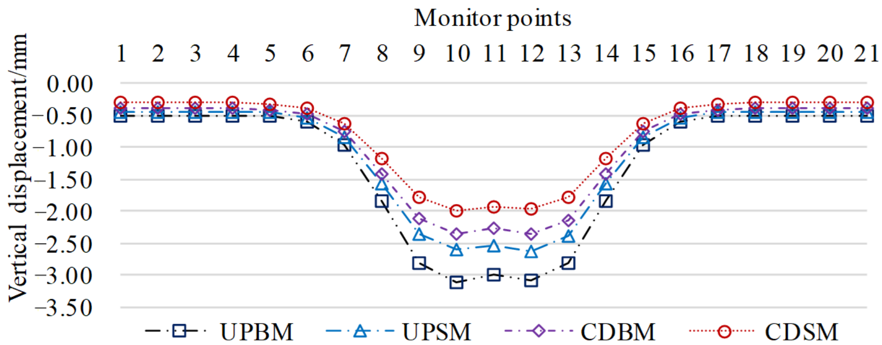


Figure 11. Modified Peck settlement calculation curve for existing tunnel base plate monitoring points under different calculation conditions.

The data comparison is based on the numerical calculation result data, as shown in Figure 12. The data error of the maximum value of the four calculation conditions is less than 5%, and the data error of the inflection point area is less than 15%, which indicates that the settlement of the base plate of the existing tunnel is in accordance with the Peck settlement calculation formula.

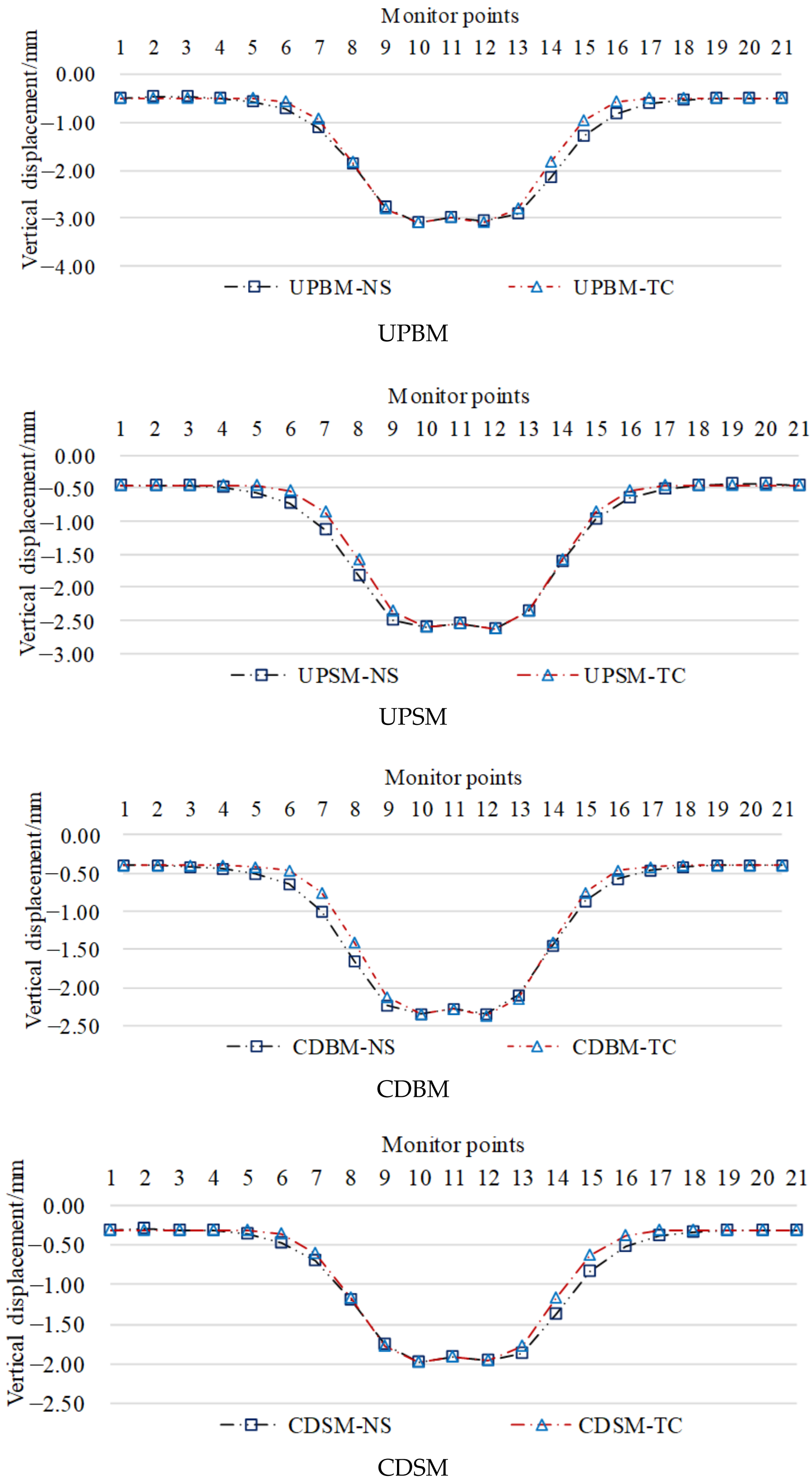


Figure 12. Comparison of numerical calculations and Peck's theoretical calculations of the settlement of the base plate of existing tunnels under different working conditions.

(2) Monitoring data and theoretical calculations compared to verify

Project examples of the new tunnel left and right lines and the existing tunnel crossing point before and after the 60 m excavation section process, as well as the existing tunnel floor settlement monitoring, are shown. Existing tunnel settlement monitoring points are arranged at the root of the left wall of the existing tunnel; following through the center line of the double tunnel and the left wall of the existing tunnel line intersection; and for the center line to the left and right every 10 m, resulting in a total of 27 measuring points and a total length of 260 m.

In order to facilitate the comparison and verification, using the modified Peck settlement formula for the base plate of the existing tunnel, the settlement curve of the base plate of the existing tunnel at 10 m intervals (CDBM-CT) is derived, and the comparison with the settlement monitoring curve of the existing tunnel (CDBM-MD) is shown in Figure 13. The error of the extreme value data is less than 3%, and the error of the data in the inflection point area is less than 14%, which verify that the modified theoretical model of Peck settlement calculation can predict the excavation deformation of double-lane highway tunnels underneath the existing railroad tunnels more accurately.

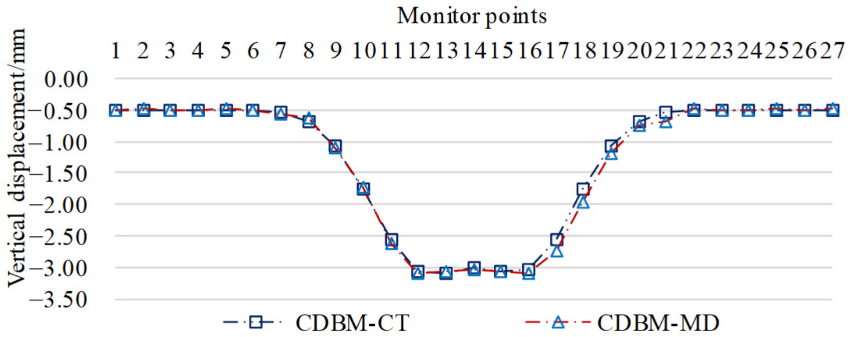


Figure 13. Comparison curve between the settlement monitoring data of the base plate of the existing tunnel and Peck's theoretical calculation data.

(3) Limitations

This paper proposes a theoretical model of deformation prediction for the tunnel excavation of two-lane highway tunnels under existing railroads coupled with Peck's multifactors and puts forward a modified Peck's settlement formula for the base plate of existing tunnels, which are both applicable to mountainous areas under the tunnel excavation project using the blasting method. However, the paper only combines the analysis of the double-lane Wanshoushan highway tunnel under the Hurong Wanshoushan Railway Tunnel Project, and it is necessary to modify the model and the calculation formula with more specific engineering examples in the future.

4. Conclusions

(1) Based on the theory of the current layer method, factors such as the stiffness of the existing tunnel, the stiffness of the new tunnel, the load of the excavation method and the load of the existing tunnel are introduced, and, by adjusting the correction coefficient η^m for the influence of the structural stiffness of the existing tunnel on the width of the sinkhole to optimize the difference in the structural settlement of the existing tunnel caused by the tunnel passing through the left and right tunnels, the theoretical model for predicting deformation of the tunnel excavation of a two-lane highway tunnel passing under the existing railroad under the current layer method coupled with the Peck method with multifactors is constructed as a deformation prediction theoretical model.

(2) Considering the overall uniform settlement of the existing tunnel floor, the overall settlement S_{os} parameter is added, and the modified Peck settlement formula for the existing tunnel floor is proposed.

(3) The Peck settlement formula was corrected by numerical calculation, and the field monitoring data were compared with the corrected Peck settlement formula. The error of the extreme value data was less than 3%, and the error of the data in the inflection point area was less than 14%, which verified the validity of the corrected theoretical model of Peck settlement calculation.

(4) Among the four excavation conditions, the CD static method has the lowest impact on the settlement of the base plate of the existing tunnel. The S_{os} is 0.50 mm for UPBM, 0.45 mm for UPSM, 0.40 mm for CDBM and 0.31 mm for CDSM.

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