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Pseudo-Conformal Sound Speed in the Core of Compact Stars

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Abstract: By implementing the putative "hadron-quark continuity" conjectured in QCD in terms of skyrmion-half-skyrmion topological change in an effective field theory for dense matter, we argue that (quasi-)baryons could "masquerade" deconfined quarks in the interior of compact stars. We interpret this phenomenon as a consequence of possible interplay between hidden scale symmetry and hidden local symmetry at high density. A surprising spin-off of the emerging symmetry that we call "pseudo-conformality" is that the long-standing puzzle of the quenched $g_A \approx 1$ in nuclei can be given a simple resolution by the way the hidden symmetries impact nuclear dynamics at low density.

Keywords: hidden symmetries in nuclei; quenched g_A; scale-chiral EFT; ultra dense matter; core structure of neutron stars; pseudo-conformal sound speed

1. The Approach: G*n*EFT

It was found in [1] that when two hidden symmetries presumed to be encoded in QCD, scale symmetry and flavor local symmetry, were suitably incorporated into chiral symmetric effective field theory to capture low-energy nuclear dynamics and extended to a chiral-scale symmetric EFT—coined as G*n*EFT in [2]—to address the equation of state of compact-star matter, the sound speed (SS) of massive stars came out precociously close to the conformal speed $v_s^2/c^2 = 1/3$ in the interior of the stars at a density of $n \sim (3-6)n_0$. The trace of the energy momentum tensor (TEMT) was found to be not zero (even in the chiral limit) at that density, so the speed cannot be truly conformal. It was therefore referred to—in the absence of a better terminology—as "pseudo-conformal." In this article, we explain how this surprising prediction could account for the possible presence of "deconfined quarks" in the core of compact stars updating recent developments confronting microscopic approaches anchored on QCD proper being discussed in the current literature [3–6]. It turns out that the notion of pseudo-conformality can figure equally importantly in nuclear physics at low density. In Appendix A, we show how the long-standing mystery of the quenched $g_A \simeq 1$ in Gamow–Teller transitions in nuclei can be resolved in G*n*EFT.

Given that the nuclear dynamics involved at non-asymptotic densities is intrinsically nonperturbative from the QCD point of view, the approach is inevitably anchored in effective field theory.

The hidden symmetries, suitably incorporated, involve "heavy degrees of freedom (HdFs)" that encompass the density range from the regime of normal nuclear matter density $n_0 \approx 0.16\ \text{fm}^{-3}$, where standard chiral EFT applies, to $\sim 7n_0$, where it is most likely broken down. In the framework of G*n*EFT that we have formulated, the hidden local symmetry is represented by the light-quark vector mesons $\mathcal{V} = (\rho, \omega)$ "conjectured" to be (Seiberg-)dual to the gluons [7]. It turns out to be significant that this local symmetry is "hidden" in the sense defined in [8,9]: they are to emerge from pion clouds as composite gauge bosons. For this phenomenon to take place, it is indispensable that there be the "vector manifestation (VM)" [10] at some high density with the vector mesons becoming massless. Where the VM density n_{VM} is located cannot be calculated in the effective theory. It turns out, however, that the sound velocity behaves qualitatively differently depending on whether n_{VM} is near or asymptotically higher than the density of the star.

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Our approach relies closely on that the scale symmetry enters via the "genuine dilaton (GD)" formulated by Crewther and Tunstall [11–13]. The GD is associated with an infrared (IR) fixed point at which the QCD β function vanishes, i.e., $\beta(\alpha_{\rm IR}) = 0$ (where α_s is the strong gauge coupling), realized in the Nambu–Goldstone mode with massless pions π and dilaton (that we denote as σ_d) with nonzero pion decay constant f_π and dilaton decay constant, f_{σ_d}, $f_\pi = f_{\sigma_d} \neq 0$. Most significantly for us, it supports massive matter fields at the IR fixed point. A somewhat similar idea was put forward in [14]. The difference, if any, between these two ideas, we believe, would not affect our arguments made for nuclear matter at non-asymptotic density.

It should be stressed that this GD IR structure appears to be drastically different [13,14] from the IR structure of the dilatonic Higgs models, which purport to go beyond the standard model. This, we believe, makes a basic difference that leads us to the pseudo-conformality, not the conformality [4]—which in our picture should set at a much higher density than in compact stars—that prevails in nuclear dynamics in a density from $\sim n_0$ to the densities relevant to compact stars and possibly beyond before the dilaton limit, defined below, sets in [15]. It should be remarked, however, that the "conformal dilaton phase" of [14] which borders between the conformal window and QCD could support the conformality discussed in [4]. How the QCD phase moves into the conformal phase, if present, is not clear.

The scale symmetry is incorporated into the HLS Lagrangian $\mathcal{L}_{\rm HLS}$ [8] with the "conformal compensator (CC)" field χ [16]

$$\chi = f_\chi e^{\sigma_d/f_\chi} \tag{1}$$

which linearly transforms both in mass and length scales. (Note that σ_d, like the pion in chiral symmetry transforms nonlinearly under scaling.) One can formally construct a scale-invariant Lagrangian from $\mathcal{L}_{\rm HLS}$ by multiplying a suitable power of the CC field χ such that the action is scale-invariant. The resulting scale-chiral Lagrangian can be written as

$$\mathcal{L}_{\chi\rm HLS} = \mathcal{L}_{\rm inv} + V \tag{2}$$

where *all* scale-symmetry-breaking terms, including quark mass terms, are included in the "dilaton potential" V. One can rewrite this in terms of the power counting in both chiral and scale symmetries with the trace anomaly taken into account [11,17] by implementing the standard power-counting in chiral symmetry [18] with the departure from the IR fixed point taken as

$$\Delta\alpha_s \sim O(p^2) \sim O(\partial^2). \tag{3}$$

The resulting scale-chiral expansion [17,19,20]—that we call $\chi\mathrm{PT}_{\sigma_d}$ expansion—is a lot more complicated than the standard chiral perturbation theory (SchiPT), but to the leading order (LO) in the scale-chiral expansion to which we will restrict, it turns out to be relatively simple.

To access baryonic matter, the baryon fields $\psi^\dagger = (p\ n)$ are coupled into $\mathcal{L}_{\chi\rm HLS}$. We will be dealing with $N_f = 2$ although the GD approach of [11] is for $N_f = 3$. While skipping details that can be found in the reviews [15], we briefly mention here that the HdFs brought in by the hidden symmetries are to play the role in G*n*EFT the putative "hadron-quark continuity" that plays the crucial role in going, in what we consider to be hadronic variables, from nuclear matter at low density to compact-star matter at high densities. We call the resulting Lagrangian $\mathcal{L}_{\psi\chi\rm HLS}$ with which G*n*EFT is formulated, as we will subsequently explain. In the scale-chiral power counting involving the vector mesons [10] and the scalar χ [17], this Lagrangian will then be of $O(\partial^n) \sim O(p^n)$ with $n = 1, 2, ...$ We will be mainly working with the leading order (LO) $n \leq 2$.

Now, instead of doing the systematic power expansion, highly successful in SchiPT, to high-order in $\chi\mathrm{PT}_{\sigma_d}$ expansion, which is in principle doable but cumbersome at best with uncontrollable paramers, we will instead develop an EFT that maps the resulting LO Lagrangian denoted $\mathcal{L}^{\rm LO}_{\psi\chi{\rm HLS}}$ to a density functional theory (DFT) built on the renormalization-group (RG) treatment of baryons on the Fermi sea along the line developed in condensed matter physics [21]. We apply the approach of how to go from a chiral Lagrangian (in LO with suitable BR-scaling taken into account) to the "Fermi-liquid fixed point (FLFP)" theory, first formulated in in [22], to the Lagrangian $\mathcal{L}^{\rm LO}_{\psi\chi{\rm HLS}}$. This is the principal tool in GnEFT that we will employ. Going beyond the FLFP can also be and will be done in "ring-diagram approximations" in V_{lowk} RG, as described in [1]. What is needed to access the compact-star densities in this GnEFT approach, not worked out in [22], is how the HdFs effectuate the putative hadron-quark continuity (HQC) at $n \gtrsim 2n_0$ in the Landau (fixed-point) parameters extracted from the Lagrangian.

In this approach, there is only one unique—not a hybrid—Lagrangian, an effective Landau(–Migdal) Lagrangian, that is to be valid over the whole range of involved densities with the fixed-point quasi-particle (QP) mass or Landau mass m_L, two-body QP–QP interaction parameters F, G, etc., built from $\mathcal{L}^{\rm LO}_{\psi\chi{\rm HLS}}$. It should be reminded to the readers that it is Migdal who reformulated Landau theory for nuclear processes [23]. Therefore from here on, by "Landau," we will mean "Landau–Migdal." We should point out that the Fermi-liquid structure described here focuses on bulk properties of matter, in particular, the equation of state (EoS), whereas in condensed matter physics, what is currently of high interest in the EFT of Fermi liquid is gapless Fermi-surface fluctuations, potentially leading to non-Fermi-liquid states [24].

The key points developed in [1] (refined in [15] with errors corrected) are as follows:

1. Up to the density of n_0, the parameters of the (Landau–Migdal) Lagrangian are controlled by the known properties of the normal nuclear matter. Treated at the mean-field level, which corresponds to what is known as the Landau Fermi-liquid fixed point (FLFP) approximation (with $\bar{N} = k_F/(\Lambda_{\rm FS} - k_F) \to \infty$) [21], it reproduces more or less all global properties of nuclear matter that are reliably described, to N^{3}LO, by SchiEFT. How the FLFP approximation fares in nature is aptly illustrated by the anomalous proton orbital gyromagnetic ratio δg_l^p, the quenched g_A in nuclei, and enhanced axial-charge transitions in heavy nuclei [22].
2. The effect of the putative hadron-quark (HQ) continuity is brought into the Lagrangian by what is given by the topology change at $n \gtrsim 2n_0$ from skyrmions to half-skyrmions when the topological baryons of the Lagrangian $\mathcal{L}_{\chi{\rm HLS}}$ are put on the crystal lattice [1]. Putting skyrmions on the crystal lattice to describe baryon matter is evidently a very poor procedure at low densities (e.g., at $\sim n_0$, which is in Fermi liquid), however, it is justified at high density and in the large N_c limit which underlies the Landau Fermi-liquid effective field theory. The precise densities involved cannot be pinned down in theory. However, the transition involves robust features that topology brings in. The most crucial feature for us is that at the HQ changeover, the condensate of the bilinear quark fields $(q^\dagger q)$, while non-zero locally and supporting chiral waves, goes to zero when space-averaged at a density of $n = n_{1/2} \gtrsim 2n_0$. However, the pion condensate f_π remains nonzero, thus the quark condensate is not an order parameter of the chiral phase transition. It somewhat resembles the "pseudo-gap" phenomenon in condensed matter physics [25].
3. There arise various, highly remarkable, consequences when this topology change is incorporated into the Lagrangian $\mathcal{L}_{\psi\chi{\rm HLS}}$ and treated at the mean field (FLFP) approximation.
   First: There is a cusp in the nuclear symmetry energy $E_{\rm sym}$ wrapped by the ρ-meson cloud at the skyrmion-to-half-skyrmion density $n_{1/2} \gtrsim 2n_0$. This brings in the soft-to-hard changeover in the EoS at that cross-over density [26] that naturally accounts for K_0, $E_{\rm sym}(n_0)$ and $L = 3n\frac{dE_{\rm sym}}{dn}|_{n=n_0}$ to be consistent with nature (e.g., [27], $K_0 \approx 240$ MeV, $E_{\rm sym}(n_0) \approx 31.7$ MeV, $L(n_0) = 57.7 \pm 19$ MeV.)

Second: The coupling between the dilaton χ and ω, both playing a crucial role in the in-medium nucleon mass, leads, in the chiral limit, to the extremely simple result for the TEMT for $n \gtrsim n_{1/2}$ [1]

$$\langle\theta_\mu^\mu\rangle = \epsilon - 3P = 4V_d(\langle\chi\rangle) - \langle\chi\rangle \frac{\partial V_d(\chi)}{\partial\chi}|_{\chi=\langle\chi\rangle}. \tag{4}$$

In this formula, the dilaton potential V_d contains also the baryon fields. In doing this calculation, it is imperative that the density dependence of the parameters inherited from matching with QCD (via relevant current correlators and the VEV with the dilaton field) be treated in accordance with the thermodynamic consistency [28]. Unless this is done correctly, one fails to arrive at (4) [1].

Third: $\langle\chi\rangle$ becomes density-independent for $n \gtrsim n_{1/2}$ only for high VM density, $n_{\rm VM} \gtrsim 25n_0$, and $\langle\chi\rangle \to cm_0$ (with c a constant), where m_0 is the chiral invariant nucleon mass. This result, due to a close interplay—with the ρ decoupled from the nucleons—between the dilaton χ and the ω, signals the emergence of parity doubling in the baryon spectrum [29,30]. In our approach, the parity doubling is not put in ab initio as is done in the literature. It emerges from the interactions. It was verified to remain valid when $\mathcal{O}(1/\bar{N})$ corrections to the FLFP approximation are made in the V_{lowk} formalism. That the TEMT (4) becomes density-independent for $n \gtrsim n_{1/2}$ is the key ingredient of the pseudo-conformality.

2. Predictions on Compact Stars

We now turn to the predictions on compact stars given in the theory GnEFT.

First, we look at the density dependence of the TEMT. One obtains a qualitatively correct answer by looking at the mean field (a.k.a., FLFP) approximation result (4). Taking the derivative with respect to density, we have

$$\frac{\partial}{\partial n}\langle\theta_\mu^\mu\rangle = \frac{\partial\epsilon(n)}{\partial n}\left(1 - 3\frac{v_s^2(n)}{c^2}\right) \tag{5}$$

where the sound speed v_s is inserted using $v_s^2/c^2 = \frac{\partial P(n)}{\partial n}/\frac{\partial\epsilon(n)}{\partial n}$. Since the condensate $\langle\chi\rangle$ changes as density changes for the density regime $n \lesssim n_{1/2}$, with $n_{1/2}$ being the topology change density, the TEMT changes with density. However, as noted, the dilaton condensate tends towards a density-independent constant $\propto m_0$, as n goes above $n_{1/2}$, the right-hand side of (5) will go to zero

$$\frac{\partial\epsilon(n)}{\partial n}\left(1 - 3\frac{v_s^2(n)}{c^2}\right) \to 0 \text{ for } n \to n_{1/2}. \tag{6}$$

There is no reason to expect $\frac{\partial\epsilon(n)}{\partial n} \to 0$ (e.g., no Lee–Wick state), hence, we arrive at what we call pseudo-conformal speed

$$v_{\rm pcs}^2/c^2 \to 1/3. \tag{7}$$

One arrives at the same result in the V_{lowk} calculation, going beyond the FLFP approximation [1]. What was found was that with the HQ changeover from below to above $n_{1/2}$, *strong nuclear correlations* intervene in the way that the TEMT varies over the changeover density, with a "huge" bump produced with the speed going towards the causality limit $v_s/c = 1$ and rapidly converging at an increasing density close to, though not necessarily on top of, $v_s^2/c^2 = 1/3$ [1]. This becomes notably more prominent if the changeover density is $n \gtrsim 4n_0$—in fact, at this $n_{1/2}$, even the causality is violated [15]. The mechanism for producing such a big bump may be indicative of how the changeover from hadronic degrees of freedom in EFT to quark–gluon degrees of freedom in nonperturbative QCD may be signaling the complexity involved in that region. (This resembles, curiously, how the HLS degrees of freedom (via the homogeneous or hidden Wess–Zumion term) wraps the cusp structure for the η' potential associated with the $U_A(1)$ anomaly [31]. It would be interesting to see whether there is any connection between the two phenomena.)

The detailed structure depends on the location of the changeover density. It turns out, however, to be qualitatively the same in the range of phenomenology places, say, at $2 \lesssim n_{1/2}/n_0 < 4$ [15]. The typical structure is illustrated in Figure 1 for $n_{1/2} = 2n_0$.

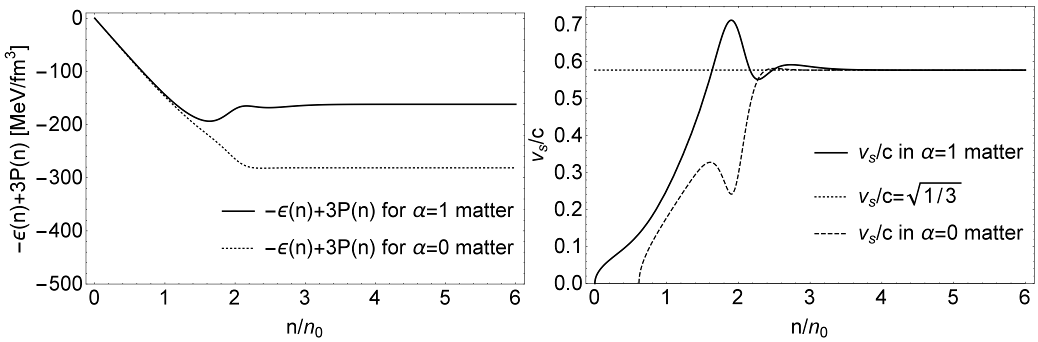

Figure 1. Left panel: $-\epsilon + 3P$; **Right panel**: sound speed vs. density for $\alpha = \frac{N-P}{N+P}$ where $N(P)$ is the number of neutrons (protons).

It is important to note that, in the range of densities involved in compact stars

$$\langle \theta^{\mu}_{\mu} \rangle > 0 \tag{8}$$

which gives

$$\Delta = \frac{1}{3} - \frac{P}{\epsilon} > 0 \tag{9}$$

going independently of density for $n \gtrsim n_{1/2}$. It can be seen in Figure 2 that it is parallel and very close, with small deviation, to the band generated with the "sound velocity interpolation method" used in [3].

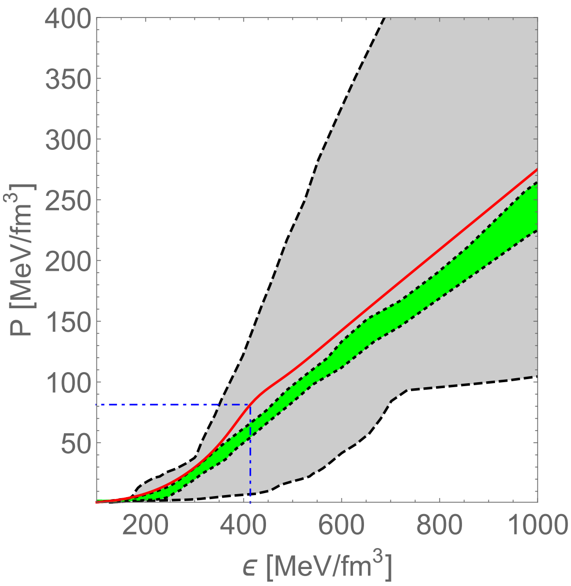

Figure 2. The predicted P/ϵ is compared with that generated with the sound velocity ("SV") interpolation method used in [3]. The gray band is from the causality and the green band is from the conformality. The red line is the GnEFT prediction [32]. The dash-dotted line indicates the location of the topology/HQ change.

It should of course be stressed that the approach to the pseudo-conformal speed $v_{\rm pcs}^2/c^2 \approx 1/3$ is *not* conformal $v_s^2/c^2 = 1/3$. As noted below, at a higher density, approaching what is called a "dilaton-limit fixed point (DLFP)" [33], where $\langle\chi\rangle$ goes to zero, the sound speed should approach the true conformal speed. We believe that the DLFP density should be close to the VM fixed point $n_{\rm VM} \gtrsim 25n_0$, way outside of the range of densities involved in the stars. It is difficult to directly relate what is described in this GnEFT with the analysis made in [4]. It is, however, tantalizing that the sound speed given in Figure 2 in [4] resembles the pseudo-conformal sound speed (Figure 1). It indicates that we are dealing here with strongly coupled (pseudo-)conformal matter.

There are further indications that the pseudo-conformal matter resembles "deconfined quark" matter. Let us look at the polytropic index defined by

$$\gamma = d\ln P/d\ln\epsilon. \tag{10}$$

Plotted in Figure 3 is the prediction for γ. It shows a large $\gamma \sim 3$ below the $n_{1/2}$ expected in nuclear matter, and drops below 1.75 at the topology change and then goes to near 1 at the core density $\sim 6n_0$ of the star. This reproduces what was identified as a signal for "deconfined quark matter" in [3]. However, there are basic differences between our system and what is described in [3]. In our theory, conformality is broken, though perhaps only slightly at a high density, in the system. There can also be fluctuations around $v_{pcs}^2/c^2 = 1/3$ coming from the effects by the anomalous dimension β'. Thus, the v_{pcs}^2/c^2 must fluctuate around $1/3$, not on top of it.

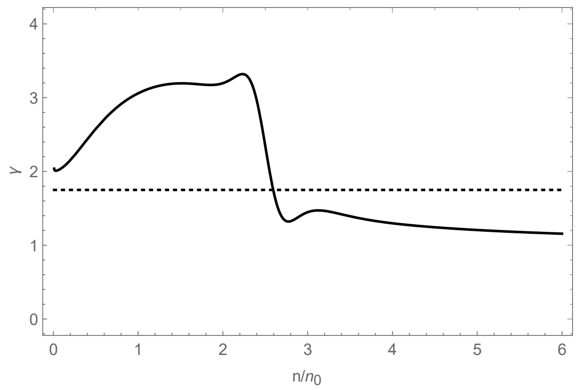

Figure 3. Density dependence of the polytropic index in neutron matter. Here, $n_{1/2} \approx 2.5n_0$ was taken, so the changeover—strong correlation—regions are shifted slightly upwards, but there is no qualitative difference.

3. Consistency with pQCD Top–Down?

The degrees of freedom that enter above the hadron–quark continuity simulated by the topology change with what appear to be fractionally charged baryonic quasiparticles are not so outlandish if one imagines strong nuclear correlations in action as in the way electrons behave in strongly correlated condensed matter systems. In fact, some *albeit* speculative but novel ideas along that line were recently discussed in [34,35].

Now, let us see how these – somewhat unorthodox – ideas can mesh with what is predicted by QCD.

There have been tour-de-force efforts to arrive at the EoS relevant to the core of compact stars in perturbative QCD coming as top–down. Some of the results obtained thus far appear to be quite relevant to what we obtained in GnEFT. Among them are the results of the pQCD calculation given in [6] that render feasible even a semi-quantitative comparison with the GnEFT prediction.

In Figure 4 (left panel) is shown the GnEFT prediction for $n_{1/2} \approx 3n_0$, corresponding to the hadron–quark crossover density. (Shown also is for $n_{1/2} = 4n_0$ which was ruled out in our scheme because the sound speed exceeds the causality limit.) The prediction for $n_{1/2} = 2n_0$ is slightly different from that of $n_{1/2} = 3n_0$ but they both are essentially indistinguishable with the uncertainty involved. What is characteristic of this prediction is that the cusp in the symmetry energy $E_{\rm sym}$, "wrapped" by ρ meson cloud with dropping mass, is first hardened by approaching $n_{1/2}$. This makes the symmetry energy at $\sim 2n_0$ bigger than what is given by standard chiral perturbation theory, then softened before reaching the density of the star core at $\sim 6n_0$; this feature is reflected in Figure 4 with $E_{\rm sym}$ bending after $n_{1/2}$, making the EoS softer toward the central density of the star. It is easy to see how this softening behavior sets in in the Fermi-liquid fixed point approximation in GnEFT. As noted, as one approaches the dilaton limit fixed point, the ρ meson decouples from the baryonic matter (before the VM fixed point) and the remaining (heavy meson) degrees of freedom, the scalar (dilaton σ_d), and the isoscalar vector meson (ω) interplay to keep the nucleon mass stay at $\sim m_0$. The quasiparticles involved are more or less free of interactions, hence preserving scale invariance (as also seen in a dense half-skyrmion simulation, Figure (11) (right panel) in [1]).

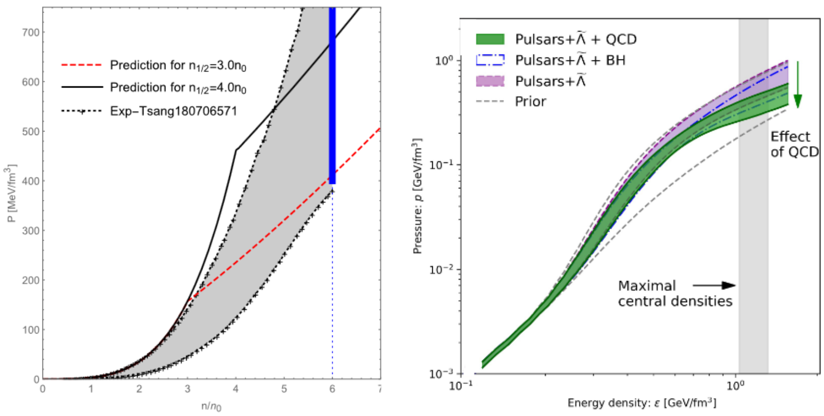

Figure 4. Left panel: The GnEFT prediction for $n_{1/2} = 3n_0$ [32]. $n_{1/2} = 4n_0$ seems to be ruled out as the sound speed violates the unitarity bound $v_s^2/c^2 = 1$. **Right panel**: The state-of-the-art pQCD calculation obtained in [6] indicating the effect of QCD. Note that the central density involved is $\sim 6n_0$ in both cases.

In Figure 4 (right panel) is plotted the pQCD result of [6]. Here, one sees the way the QCD asymptotically high-density calculations are propagated down to lower densities subjected to "thermodynamic consistency, stability and causality." The calculation provides rather stringent constraints, as indicated in the figure, to the EoS at densities relevant to the core of the star. The result clearly shows that pQCD softens the EoS of the most massive neutron stars going toward the central density of $\sim 6n_0$. Note that the stiffening and then softening in the EoS take place roughly at the same densities as in GnEFT. Precise matching would not make much sense given the approximations involved in both pQCD and GnEFT, but there is a clear qualitative consistency. It would be interesting to understand the stiffening–softening of the EoS in pQCD as in GnEFT, where the interplay of scale symmetry and hidden local symmetry is found to play a crucial role in the emerging parity-doubling symmetry.

4. Conclusions with a Bit of Speculation

In this brief review is recounted what has taken place, initiated in 2007, in the five-year "World-Class University Program" at Hanyang University in Seoul supported by the Korean Government, then continued at IBS (Korea) and at Jilin University in Changchun (China), with the objective of understanding ultradense matter stabie against gravitational collapse. This issue has currently become a hot topic in physics with the advent of gravitational waves. The resulting approach of GnEFT with only hadronic degrees of freedom, implemented with hidden symmetries of QCD with the parameters of the Lagrangian endowed with the vacuum sliding by density, leads to the results are globally consistent with the available data.

What is striking of this approach is that although no explicit QCD degrees of freedom are involved, the property of the core of massive stars predicted in this approach is uncannily similar to that of "deconfined quarks" with one apparent difference in the nature of the sound speed.

How can this be?

We have no convincing argument but one possible answer is this.

In the skyrmion–half-skyrmion crystal simulations of dense matter, one can imagine how the half-skyrmions can turn into fractionally charged baryons. Two half-skyrmions are confined by a pair of monopoles, so they are not separable. However, surprisingly, the bound two-1/2-skyrmions are found to propagate scale-invariantly, as observed in [1]. Suppose that it is feasible to liberate the two half-skyrmions by suppressing the monopoles. Then, it may be feasible to transform two half-skyrmions into three 1/3-charged objects [36]. One can then think of fractionally charged baryons populating the dense matter inside the core. Indeed, in condensed matter physics, with domain walls, there can be stacks of sheets containing deconfined fractionally charged objects behaving like "deconfined quarks" coming from the bulk in which the objects are confined [37]. In [38], highly speculative ideas along this line are entertained.

In conclusion, we propose that what is seen in the core of compact stars could be fractionalized quasiparticles masquerading "deconfined quarks." As a test, we offer the "duck test" [39]: *If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.*

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Appendix A. The Case of Quenched g_A in Light of Scale Symmetry

A most surprising spin-off of the emergence of hidden-scale symmetry in dense matter is that the same pseudo-conformality also permeates at low density in certain channels of nuclear interactions. There has been a long-standing mystery that the axial-vector coupling constant g_A in Gamow–Teller transitions in nuclei is observed to be quenched from the matter-free space value$g_A = 1.276$ to $g_A^{\rm obs} \approx 1$ [40]. This mystery which dates way back to the early 1970s [41] could be given a simple resolution in terms of emergent scale symmetry in the same approach, G*n*EFT, which is successively applicable at a high density to compact stars.

In G*n*EFT, the nuclear axial current coupling to the EW current is scale-invariant—modulo possible anomalous-dimension corrections [42]

$$J^j_{5\mu} = g_A \bar{\psi}\gamma\gamma_\mu \frac{\tau^j}{2}\psi \tag{A1}$$

where j is the isospin index and $\psi^\dagger = (p\ n)$ field. No conformal compensator field χ figures here in the construction of the Lagrangian. In the mean field of $\mathcal{L}_{\psi\chi{\rm HLS}}$ (a.k.a. in the Fermi-liquid fixed-point approximation), it turns out that one precisely reproduces the same Landau–Migdal (LM) $g_A^{\rm LM}$ for the quasiparticle, as was obtained in the simplified treatment made in [22]

$$g_A^{\rm LM} = g_A q^{\rm LM} = g_A\big(1 - \frac{1}{3}\Phi\tilde{F}_1^\pi\big)^{-2}. \tag{A2}$$

Here, $\Phi = f_\pi^*/f_\pi$ and q^{LM}, identified as the "g_A quenching factor," is a FLFP quantity with $O(\bar{N})$ corrections suppressed. The superscript $*$ stands for density dependence. Φ therefore represents the scaling of f_π as the vacuum changes at varying densities. This quantity was measured in deeply bound pionic atoms up to $\sim n_0$ [43]. Here, $\tilde{F}_1^\pi$ is the pion contribution to the Landau interaction parameter F_l, which is accurately calculable by standard chiral perturbation theory (SchiPT) for $n \lesssim 2n_0$. The product $\Phi\tilde{F}_1^\pi$ in (A2) turns out to be remarkably constant in the range $1/2 \lesssim n/n_0 \lesssim 1$, which covers the density range between light and heavy nuclei. As noted in [22], (A2) is effectively an in-medium Goldberger–Treiman relation anchored in a low-energy theorem which is satisfied in the vacuum to a few % of accuracy and could be equally accurate in nuclear matter, although high-order SchiPT calculations, as far as we are aware, have not been performed to date. We take (A2) to be of the same accuracy.

Remarkably, the quenching factor (A2) comes out, numerically, as

$$q^{\text{LM}} \approx 0.79 - 0.80 \tag{A3}$$

in the range of density involved—with a few % uncertainty—in medium and heavy nuclei. Thus, the effective g_A for the quasiparticle, g_A^{qpeff}, is

$$g_A^{qpeff} \approx g_A^{\text{LM}} \approx 1.0. \tag{A4}$$

It's surprising that what appears to be a solution to the long-standing puzzle in nuclear physics turns out to be deceptively simple.

The Formula (A2) corresponds to the single-decimation procedure in the Wilsonian renormalization-group flow calculation in the Landau fixed-point approximation [44]. This means that the path integral is performed for the Gamow–Teller amplitude from the cut-off Λ_{FS} above the Fermi sea, all the way to the top of the Fermi surface in the Fermi-liquid fixed point approximation (ignoring higher-order $1/\bar{N}$ terms in the V_{lowk} RG). The *full* Gamow–Teller response is given by the response of the single quasiparticle sitting on the Fermi surface multiplied by a constant that captures the *full* nuclear correlations

$$M_{\text{GT}} = q^{\text{LM}} g_A \big(\sum_j (\tau_j^\pm \sigma_j) \big)_{Q_f Q_i} \tag{A5}$$

where $Q_{i,f}$ stands for single quasiparticle states. As defined, it is q^{LM} that captures the *full* correlation effects. How this q^{LM} in the Fermi-liquid theory is related to the shell-model results is discussed in [42]. (In short, in terms of high-order nuclear perturbation calculations in shell-model space, this procedure would be equivalent to connecting, with Goldstone diagrams, the parent ground state to the daughter particle–hole state via a blob that contains all orders of particle–hole–bubble intermediate states with the intermediate energy going up to $\Delta E \sim 300$ MeV connected by the tensor forces.) Here, we assume that the quasiparticles involved are quasi-nucleons, ignoring resonances (e.g., Δs), etc. It is, in practice, difficult to identify the quasiparticle states $(Q_f Q_i)$ in Ferm-liquid theory with the corresponding shell-model states. Furthermore, the observation of $g_A^{\text{obs}} \approx 1$ is in light nuclei and the Fermi-liquid approach adopted is, however, more likely applicable to nuclear matter, so how to access shell-model states needs to be addressed.

Although one cannot make one-to-one correspondence between the shell model states and the Fermi liquid states, what appears to be the most suitable for mapping the FLFP result in Fermi-liquid theory to shell-model in heavy nuclei is the doubly magic closed-shell nuclei such as ^{100}Sn proposed in [42]. It is not quite clear how to experimentally zero-in on the lowest shell-model daughter state in ^{100}In that best corresponds to the quasi-particle–quasi-hole excitation on the Fermi surface. As such, certain uncertainty will remain that requires to be scrutinized in analyzing experimental results. One can, however, make an interesting observation as it stands from the presently available data on this transition.

What is involved is the strongly enhanced Gamow–Teller transition of a proton in the completely filled shell (i.e., $g_{9/2}$) in ^{100}Sn to a neutron in the empty shell (i.e., $g_{7/2}$) in ^{100}In with zero momentum transfer and $\sim$ zero MeV energy transfer. This involves a transition as close as possible in kinematics to the decay of a quasiproton to a quasineutron on the Fermi surface in the Landau–Migdal theory. The square of the Gamow–Teller transition matrix element going from the pure proton $g_{9/2}$ shell to the pure neutron $g_{7/2}$ shell, referred to as "extreme single-particle shell-model (ESPSM)" strength, is given by

$$\mathcal{B}_{\rm GT}^{\rm ESPSM} = (160/9). \tag{A6}$$

Suppose we have the exact wave functions—which are of course not yet available for the given parent and daughter states—and hence have the exact Gamow–Teller strength for the transition with the given scale-invariant Gamow–Teller operator that we write $\mathcal{B}_{\rm GT}^{\rm exact}$, the "exact" quenching factor will then be

$$q_{\rm th}^{\rm exact} = \left(\mathcal{B}_{\rm GT}^{\rm exact}/\mathcal{B}_{\rm GT}^{\rm ESPSM}\right)^{1/2}. \tag{A7}$$

Now, since we do not know the exact wave functions, we do not know what $\mathcal{B}_{\rm GT}^{\rm exact}$ in the shell-model is. However, nature, i.e., an accurately measured experiment, will give it. Let's call it $\mathcal{B}_{\rm GT}^{\rm nature}$. Measuring the GT strength will give us what the quenching factor is in nature. *It is the prediction of GnEFT at the FLFP approximation that $q_{\rm th}^{\rm exact}$ should be given by (A3).*

In order to see how this prediction fares with nature, we pick the measured transition strength to the single daughter state, i.e., the neutron in $g_{7/2}$. While not rigorously established, the transition seems to go to this state more than 95% in certain model calculations, so let us simply assume that it goes 100% and take what is measured in [45] corresponding to "nature" à la GSI

$$\mathcal{B}_{\rm GT}^{\rm nature} \approx 10. \tag{A8}$$

This would give the measured quenching factor—within the uncertainty involved (say, $\sim$5%),

$$q^{\rm nature} \approx 0.75 \tag{A9}$$

hence the quenched g_A is

$$g_A^{\rm nature} \approx 0.96. \tag{A10}$$

Thus, we have, within the possible uncertainty involved,

$$g_A^{\rm LM} \simeq g_A^{\rm nature}. \tag{A11}$$

We should underline two points here:

One is that $g_A^{\rm LM}$ represents the effect of *solely* nuclear correlations, not a *fundamental renormalization* in the given EFT of the coupling g_A as was suggested by some workers in the field. The nuclear correlations, as formulated, contain not only the single-particle operator but also many-body (meson-exchange) currents involving nucleon fields [46]. The constant $g_A^{\rm LM}$ is therefore not a renormalized constant in nuclear axial response functions that applies to non-superallowed GT transitions such as axial-charge (i.e., first forbidden) transitions, double-neutrino and neutrinoless Gamow–Teller transitions.

The other is that $g_A^{\rm LM} \to 1$ in nuclear matter is an intricate consequence of hidden scale symmetry emerging in a medium at low density.

Now, what can one say as density increases toward the IR fixed-point density? To see what happens at high density, we reparametrize the scale-chiral field as $\mathcal{Z} = U(\chi/f_\chi)$ where $U = e^{i\pi/f_\pi}$ is the chiral field and then let ${\rm Tr}(\mathcal{Z}\mathcal{Z}^\dagger) \to 0$ in $\mathcal{L}_{\psi\chi{\rm HLS}}$ treated in the mean-field. We do this to drive the system to the DLFP (dilaton-limit fixed point) density $n_{\rm dlfp} \sim n_{\rm VM} \gtrsim 25n_0$. In order to prevent singularities from developing, one is constrained to impose the conditions [33]

$$g_A \to 1,\ f_\pi \to f_\chi. \tag{A12}$$

Thus, the effective coupling constant for the quasiparticle, $g_A^{qp\rm eff} = 1$, is seen to maintain the same value from $n \sim n_0$ due to the nuclear correlations controlled by scale invariance to $n_{\rm dlfp} \sim n_{\rm VM} \gtrsim 25n_0$ at high density as $m_{\sigma_d} \sim \langle\chi\rangle \to 0$. We interpret this as a manifestation of the pseudo-conformal symmetry permeating from finite nuclei at low density to high compact-star density and to even higher densities approaching $n_{\rm dlfp} \gtrsim 25n_0$.

Appendix B. Note Added in Proof

The referees to this review raised several constructive issues in their comments. They do not directly address the thesis developed in this paper, but we find them most likely to be relevant to future development along the lines of the thinking adopted in this article. We would like to respond to their comments to the best we can.

There were three issues raised.

The first, raised by one of the referees, is the possible relevance of the color-flavor locked (CFL) state in the three-flavor QCD at high density. The question is: Given that the CFL state is conjectured to emerge at asymptotic density in QCD, what does the scenario developed in this article (anchored on the notion of hadron-quark (HQ) continuity) predict for the sound speed (SS) in the CFL phase? We cannot give a precise answer to this question in the framework of the pseudo-conformal notion, but it is very likely that the SS, v_s^2/c^2, is not conformal (that is $\neq 1/3$) in the CFL phase, given that the trace of the energy-momentum tensor (TEMT) is not likely to be zero even at the asymptotic density. This of course does not exclude the possibility of (so far unobserved) "quark stars," possessing drastically different equation of states (EoSs).

Briefly restated, the pseudo-conformal structure adopted in this review—which is anchored in the putative HQ continuity (with no phase transitions) via the topology change mediated by the hadronic HdFs governed by hidden symmetries—cannot be naively extended to too high a density. The highest density relevant in the given formalism is the vector manifestation density or the dilaton-limit fixed point density $\gtrsim 25n_0$. What is pertinent to the issue is the currently controversial question as to whether the Schäfer–Wilczek conjecture that underlies the HQ continuity at the asymptotic density is even valid. There are arguments [47], though not generally agreed upon in the field, that it is indeed *invalid*, according to which "the hadronic (e.g., nuclear) matter and the quark matter present must necessarily be separated by a phase transition as a function of density."

The second issue is the property of the compact-star SS deduced from a huge number of randomly generated EoSs that satisfy theoretical and observational constraints [48]. A number of non-trivial correlations in softness and stiffness of EoS at different density regimes reveal that the SS tends to decrease below the conformal limit in the core of the star, while it increases beyond conformal in outer layers. Although not ruled out by these results, the pseudo-conformal structure sound speed is found to deviate distinctively from them. It exhibits a maximum below the putative HQ transition density $n_{1/2}$, hence in the outer layer of the star, with the height dependent on the cross-over density, and converges *precociously* to v_s^2/c^2 near $1/3$ towards the star center. Within the approximations made, one expects certain degrees of fluctuations from $1/3$ but not as wildly varying as seen in the agnostic analysis. A similar behavior is also predicted in the microscopic description of [4], with the convergence pushed to higher densities than in the pseudo-conformal case. It has been noted in [26] that there seems to be little, if any, correlation between global star properties and the plethora of bump structures in the sound speed. Given that the cross-over region is the least well-controlled theoretically, EFT bottom–up and QCD top–down in density, this bump structure must be the hardest to resolve theoretically.

Finally, the third issue is what role the SS plays in the causal maximum mass limit of neutron stars in the framework of modified gravity such as, e.g., $f(R)$ [49]. Such an issue addresses whether, or in what way, modified gravity can make impacts on the structure of the EoSs that one extracts from the astrophysical observables. It appears that, for a given EoS, the speed of sound can even be considered as a *free variable* for the causal properties of compact stars. This would be a serious matter for zeroing-in on the ultradense structure of massive stars.

In this third category of issues, there is one that addresses whether the physics of sound speed in massive compact stars can be exploited as a test of string theory in the gravitational sector. In a post-Newtonian analysis of double-field theory (DFT) as such a test, it has been suggested [50] that the "Eddington–Robertson–Schiff parameter" $\gamma_{\rm PPN} \to 1$ in support of DFT could be extracted from the interior structure of the proton. In a private communication [51], J.-H. Park inquired to the author whether the vanishing of the TEMT in the core of massive compact stars discussed in the main text of this review could not be interpreted as the vanishing of the TEMT in the proton interior and hence provide a signal for or against DFT in the gravitational sector. The possible conformal speed $v_s^2/c^2 = 1/3$ [3–5] or the pseudo-conformal speed $v_{\rm pcs}^2/c^2 \approx 1/3$ could perhaps be translated to a quantity corresponding to $\gamma_{\rm PPN} \to 1$.

What seems tantalizing in the way that the pseudo-conformal speed is formulated is that a certain notion, say, pseudo-gap phenomenon, borrowed from condensed matter physics seems to overlap even with what is at issue in gravity theory.

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