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# Further Results on Robust Output-Feedback Dissipative Control of Markovian Jump Fuzzy Systems with Model Uncertainties

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**Abstract:** This paper investigates an improved criterion to synthesize dissipative observer-based controllers for Markovian jump fuzzy systems under model uncertainties. Since fuzzy-basis functions include some immeasurable state variable or uncertain parameters, there are differences in the fuzzy-basis functions between controller and plant, which is a mismatched phenomenon. This work presents the first attempt for applying double-fuzzy summation-based Lyapunov functions for the observer-based control scheme of the Markov jump fuzzy system regarding the mismatched phenomenon. To be specific, the dissipative conditions are formulated in terms of uncertain parameterized bilinear matrix inequalities. Based on the improved relaxation techniques, a linear-matrix-inequality (LMI)-based algorithm is proposed in the framework of sequence linear programming matrix method. The obtained observer-based controller ensures that the closed-loop system is stochastically stable, and the dissipative performances produce less conservative results compared to preceding works via two numerical examples.

**Keywords:** markov jump fuzzy systems; dissipative control; mismatched phenomenon; model uncertainties

**MSC:** 93C42; 93E15

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## 1. Introduction

The development of control engineering is faced with a class of hybrid systems with probabilistic sudden changes to their behavior, named the stochastic hybrid system. The systems have attracted a huge consideration from many control theorists due to their abilities in showing hybrid dynamics with probabilistic changes. Markov jump systems (MJSs) whose jumping parameters are governed by the Markov process belong to the class of the stochastic hybrid system, and have expressed great potential to represent random abrupt variations such as component fault or failures, sudden environmental changes, and changing subsystem interconnections. In the view of realistic problems, discrete-time MJSs have played important roles to implement digital experiments including network control systems [1–3], power systems [4–6] and communication systems [7,8].

The Takagi–Sugeno (T-S) fuzzy model is well known as an effective tool to describe nonlinear dynamics via an average sum of given linear models. Recent years have witnessed a massive increase of studies related to the systematic control design of nonlinear systems using The T–S fuzzy model [9–11] According to this trend, the T-S fuzzy model has been investigated intensively to cover various nonlinear control problems [12,13]. In many situations, all state variables are not fully measurable. The observer-based fuzzy control scheme needs to estimate FBFs and state variables, then establish fuzzy control laws [13–15]. When the premise variables of the T–S fuzzy system are related to the immeasurable state, that leads to a mismatched phenomenon between fuzzy-basis functions

(FBFs) in the plant and those of the controller, there have been fruitful works devoted to observer-based output-feedback control synthesis, such as stability and stabilization [16,17], $\mathcal{H}_\infty$ and dissipative control [18,19].

Over the past decade, the extensions of the T-S fuzzy model to MJSs has established the framework of Markov jump fuzzy systems (MJFSs), and particularly to the output-feedback control of MJFSs [20,21]. However, So far as we know, in the presence of model uncertainties, there has been little progress toward the output-feedback scheme with consideration to the mismatched phenomenon. Studies on [22] have used interval type 2 fuzzy MJFSs to deal with the mismatched phenomenon, while [21] presenting a sliding mode output-feedback with uncertain transition rates. The authors in [23] present a two-step LMI-based method to design dissipative output-feedback controllers for MJFS. To improve the dissipative performance, the work in [24] develops a single-step LMI-based method regarding sensor failures. Lately, relaxed results for observed-based controllers for discrete-time MJFSs have been investigated in [25] by nonparallel distributed compensation (non-PDC) scheme. However, a common limitation of the above studies is relaxed attempts to overcome the conservatism of the output-feedback scheme by a single-step or two-step LMI solution. As reported in [13], the two-step approach has much conservatism and sensitivity due to the weak selection of decision variables in the first step [23], while the single-step requires excessive use of free weighting matrices [24]. Thus, it is necessary to develop an innovative method based on the progress of relaxation techniques and modified Lyapunov functions.

Motivated by these discussions, this paper presents improved results of the output-feedback dissipative control of MJFSs with model uncertainties. By taking advantage of the mode-fuzzy-dependent Lyapunov functions in terms of a double-fuzzy summation, our work can obtain better computed dissipative performance compared to existing results. In short, besides proposing the dissipative observer-based controller for the discrete-time MJFSs regarding the model uncertainties and mismatched phenomenon, our contributions also contain:

- The model uncertainties and mismatch phenomenon entail difficulties in handling multiple parameterized matrix inequalities when deriving LMI-based dissipative conditions. Thus, a refined relaxation process with the sequence linear programming matrix method (SLPMM) is proposed to solve dissipative conditions by LMI-based algorithm.
- Apart from this, our work takes advantage of the double-fuzzy summation-based mode-fuzzy-dependent Lyapunov functions to relax the dissipative conditions. The Lyapunov function collaborates with the relaxation process to release less conservative LMI-based dissipative conditions compared to [13,23,24,26]. The results are verified through two illustrative examples.

In accordance with the contributions, this work can be applied to stabilize the nonlinear systems with jumping and certainties in system parameters, e.g., tracking control of unmanned ground vehicles over network communications with packet losses and stabilization power grids under sudden load changes.

The notations $X \geq Y$ and $X > Y$ mean that $X - Y$ is positive semi-definite and positive definite, respectively. In symmetric block matrices, the asterisk $(*)$ is used as an ellipsis for terms induced by symmetry. $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation; $\mathcal{L}_2[0,\infty)$ stands for the space of square summable sequences over $[0,\infty)$; $\mathbf{diag}(\cdot)$ stands for a diagonal matrix with diagonal entries; $\mathbf{col}(v_1, v_2, \cdots, v_n) = [v_1^T \; v_2^T \; \cdots \; v_n^T]^T$ for scalar or vector $v_i$; $\otimes$ denotes the Kronecker product; $\mathbf{He}\{\mathcal{P}\} = \mathcal{P} + \mathcal{P}^T$ for a square matrix $\mathcal{P}$; $\mathbb{N}_1 \setminus \mathbb{N}_2$ indicates the set of elements in the set $\mathbb{N}_1$, but not in the set $\mathbb{N}_2$; and $n(\mathbb{N})$ denotes the number of elements in set $\mathbb{N}$. For $\mathbb{N} = \{a_1, a_2, \cdots, a_s\}$, the following matrix expansion notation is used:

$$[\mathcal{M}_i]^{\mathbf{d}}_{i\in\mathbb{N}} = \mathbf{diag}(\mathcal{M}_{a_1}, \cdots, \mathcal{M}_{a_s}),$$
$$[\mathcal{M}_i]_{i\in\mathbb{N}} = \begin{bmatrix} \mathcal{M}_{a_1} \\ \vdots \\ \mathcal{M}_{a_s} \end{bmatrix}, \ \left[\mathcal{M}_{ij}\right]_{i,j\in\mathbb{N}} = \begin{bmatrix} \mathcal{M}_{a_1a_1} & \cdots & \mathcal{M}_{a_1a_s} \\ \vdots & \ddots & \vdots \\ \mathcal{M}_{a_sa_1} & \cdots & \mathcal{M}_{a_sa_s} \end{bmatrix}$$

where $\mathcal{M}_i$ and $\mathcal{M}_{ij}$ are real matrices with appropriate dimensions or scalar values.

The rest of the paper is sketched as follows. The next section presents problem statements and fundamental definitions of MJFSs, and the preceding useful results exploited in the paper. Section 3 includes control synthesis for LMI-based dissipative conditions of the concerned observer-based controller. The last section shows two numerical implementations to verify the validity and effectiveness of the proposed method.

## 2. Preliminaries

For a given complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, consider a discrete-time homogeneous Markov chain $\psi$ as a sequence of random variables $\psi_0$, $\psi_1$, ... whose values belong to a finite set of state $\mathbb{N}_\psi = \{1, 2, \cdots, s\}$ and satisfy Markov properties. Let $\pi_{pq} = \mathbf{Pr}\big(\psi_{k+1} = q|\psi_k = p\big)$ be a time-invariant one-step probability of jumping from state (or mode) $p$ to $q$. Accordingly, we have $\pi_{pq} \in [0, 1]$ and $\sum_{q=1}^{r} \pi_{pq} = 1$. Based on the definitions, let us consider a class of Markovian jump fuzzy systems (MJFSs) as follows:

$$\begin{cases} x_{k+1} = \big(A(\psi_k, \xi) + \Delta A(\psi_k, k)\big)x_k + \big(B(\psi_k, \xi) + \Delta B(\psi_k, k)\big)u_k + E(\psi_k, \xi)d_k, \\ z_k = G(\psi_k, \xi)x_k + H(\psi_k, \xi)u_k + J(\psi_k, \xi)d_k, \\ y_k = C(\psi_k, \xi)x_k + D(\psi_k, \xi)d_k, \end{cases} \tag{1}$$

in which $x_k \in \mathbb{R}^{n_x}$, $u_k \in \mathbb{R}^{n_u}$, $y_k \in \mathbb{R}^{n_y}$, $z_k \in \mathbb{R}^{n_z}$, and $d_k \in \mathbb{R}^{n_d}$ represent for the state variable, the control input, the measured output, the performance output, and the bounded-energy disturbance (belonging to $\mathcal{L}_2[0, \infty)$), respectively. In addition, $\psi_k$ is the discrete-time homogeneous Markov chain standing for sudden changes in system matrices $A_p(\xi), B_p(\xi), E_p(\xi), G_p(\xi), H_p(\xi), J_p(\xi)$ where

$$\begin{bmatrix} A & B & E \\ C & 0 & D \\ G & H & J \end{bmatrix}(\psi_k = p, \xi) = \begin{bmatrix} A_p(\xi) & B_p(\xi) & E_p(\xi) \\ C_p(\xi) & 0 & D_p(\xi) \\ G_p(\xi) & H_p(\xi) & J_p(\xi) \end{bmatrix} = \sum_{i=1}^{r} \xi_i \begin{bmatrix} A_{pi} & B_{pi} & E_{pi} \\ C_{pi} & 0 & D_{pi} \\ G_{pi} & H_{pi} & J_{pi} \end{bmatrix},$$

where $A_{pi}, B_{pi}, C_{pi}, D_{pi}, E_{pi}, G_{pi}, H_{pi},$ and $J_{pi}$ are constant system matrices with appropriate dimensions. To be more specific, $r$ indicates the number of fuzzy rules, and we denote the fuzzy-basis function vector as $\xi = \xi(\varrho(x_k))$ (or simply $\xi_k) = \big[\xi_1(\varrho(x_k)), \xi_2(\varrho(x_k)), \ldots, \xi_r(\varrho(x_k))\big]^T \in \mathbb{R}^r$ where $\varrho(x_k) = [\varrho_1(x_k), \varrho_1(x_k), \ldots, \varrho_d(x_k)]^T \in \mathbb{R}^d$ stands for premise variable. Please note that $\xi_i(\varrho(x_k))$ denotes the $i$th element of fuzzy-basis vector $\xi$ who fulfill $\sum_{i=1}^{r} \xi_i = 1$ and $\xi_i \in [0, 1]$ for all $i \in \mathbb{N}_\xi = \{1, 2, \cdots, r\}$.

In this paper, we assume that the model uncertainties $\Delta A(\psi_k, k)$ and $\Delta B(\psi_k, k)$ can be decomposed into matrix multiplications of the following forms:

$$\begin{cases} \Delta A(\psi_k = p, k) = \Delta A_p(k) = T_{a,p}U_a(k)Y_{a,p}, \\ \Delta B(\psi_k = p, k) = \Delta B_p(k) = T_{b,p}U_b(k)Y_{b,p} \end{cases} \tag{2}$$

where $T_{a,p}, T_{b,p}, Y_{a,p}$ and $Y_{b,p}$ are given constant matrices with appropriate dimensions; $U_a(k)$ and $U_b(k)$ are time-varying matrices with $U_a(k)U_a^T(k) \leq I$, $U_b(k)U_b^T(k) \leq I$.

Since the premise variable vector depends on several immeasurable state variables $x_k$ or on uncertain parameters, fuzzy control laws to be designed is impossible to share the same premise variables with the plant (1). In this light, we deal with the mismatched phenomenon by the observer-based fuzzy in the following form:

$$\begin{cases} \hat{x}_{k+1} = A_p(\hat{\xi})\hat{x}_k + B_p(\hat{\xi})u_k + L_p(\hat{\xi})\big(y_k - C_p(\hat{\xi})\hat{x}_k\big), \\ u_k = K_p(\hat{\xi})\hat{x}_k, \end{cases} \tag{3}$$

where $\psi_k = p$ and $\hat{x}_k \in \mathbb{R}^{n_x}$ stands for the observed state; $\hat{\xi} = \xi_i(\varrho(\hat{x}_k)) = \mathbf{col}(\xi_1(\varrho(\hat{x}_k)), \xi_2(\varrho(\hat{x}_k)), \cdots, \xi_r(\varrho(\hat{x}_k)))$ represents for the observed fuzzy-basis function vector calculated on the controller side based on observed states at time step $k$; $L_p(\hat{\xi})$ and $K_p(\hat{\xi})$ are the fuzzy-dependent matrices needed to be designed, respectively; and

$$A_p(\hat{\xi}) = \sum_{i=1}^{r} \hat{\xi}_i A_{pi},\ B_p(\hat{\xi}) = \sum_{i=1}^{r} \hat{\xi}_i B_{pi},\ C_p(\hat{\xi}) = \sum_{i=1}^{r} \hat{\xi}_i C_{pi}.$$

Furthermore, let $e_k = x_k - \hat{x}_k$, $\zeta_k = [\hat{x}_k^T, e_k^T]^T \in \mathbb{R}^{2n_x \times 2n_x}$, and $\tilde{\xi} = [\tilde{\xi}_1, \tilde{\xi}_2, \cdots, \tilde{\xi}_r]^T$ with $\tilde{\xi}_i = \xi_i - \hat{\xi}_i$, the closed-loop control system of (1) and (3) is represented as follows:

$$\begin{cases} \zeta_{k+1} = \bar{\mathbf{A}}_p(\tilde{\xi}, \xi, \hat{\xi})\zeta_k + \mathbf{E}_p(\xi, \hat{\xi})d_k, \\ z_k = \mathbf{G}_p(\xi, \hat{\xi})\zeta_k + J_p(\xi)d_k, \end{cases} \tag{4}$$

where $\bar{\mathbf{A}}_p(\tilde{\xi}, \xi, \hat{\xi}) = \mathbf{A}_p(\tilde{\xi}, \xi, \hat{\xi}) + \left[\begin{array}{c:c} 0 & 0 \\ \hdashline \Delta A_p(k) + \Delta B_p(k)K_p(\hat{\xi}) & \Delta A_p(k) \end{array}\right]$,

$$\begin{aligned} \mathbf{A}_p(\tilde{\xi}, \xi, \hat{\xi}) &= \left[\begin{array}{c:c} A_p(\hat{\xi}) + B_p(\hat{\xi})K_p(\hat{\xi}) + L_p(\hat{\xi})C_p(\tilde{\xi}) & L_p(\hat{\xi})C_p(\xi) \\ \hdashline A_p(\tilde{\xi}) + B_p(\tilde{\xi})K_p(\hat{\xi}) - L_p(\hat{\xi})C_p(\tilde{\xi}) & A_p(\xi) - L_p(\hat{\xi})C_p(\xi) \end{array}\right], \\ \mathbf{E}_p(\xi, \hat{\xi}) &= \left[\begin{array}{c} L_p(\hat{\xi})D_p(\xi) \\ \hdashline E_p(\xi) - L_p(\hat{\xi})D_p(\xi) \end{array}\right], \\ \mathbf{G}_p(\xi, \hat{\xi}) &= \left[\begin{array}{c:c} G_p(\xi) + H_p(\xi)K_p(\hat{\xi}) & G_p(\xi) \end{array}\right]. \end{aligned}$$

Before going ahead, this paper presents the following definitions for stochastic analyses.

**Definition 1** ([27,28]). *For $d_k \equiv 0$, the closed-loop system (4) is stochastically stable if for any $\zeta_0 = [\hat{x}_0^T, e_0^T]^T$ and $\phi_0$, the following inequality holds*

$$\mathbf{E}\left\{ \sum_{k=0}^{\infty} \|\zeta_k\|^2 \Big| \zeta_0, \phi_0 \right\} < \infty. \tag{5}$$

**Definition 2** ([29,30]). *For given real matrices $\mathcal{Z}$, $\mathcal{S}$ and $\mathcal{D}$ such that $\mathcal{Z} = -\mathcal{Z}_1^T \mathcal{Z}_1$, $\mathcal{Z}_1 \in \mathbb{R}^{n_q \times n_z}(n_q \leq n_z)$, $\mathcal{S} \in \mathbb{R}^{n_d \times n_z}$, and $\mathcal{D} = \mathcal{D}^T \in \mathbb{R}^{n_d \times n_d}$, let us define a quadratic energy supply rate as follows*

$$\begin{aligned} \mathcal{Q}(z_k, d_k) &= \begin{bmatrix} z_k \\ d_k \end{bmatrix}^T \begin{bmatrix} \mathcal{Z} & (*) \\ \mathcal{S} & \mathcal{D} \end{bmatrix} \begin{bmatrix} z_k \\ d_k \end{bmatrix} \\ &= \begin{bmatrix} z_k \\ d_k \end{bmatrix}^T \begin{bmatrix} \mathcal{Z} & (*) \\ \mathcal{S} & \mathcal{D} \end{bmatrix} \left[\begin{array}{cc} \mathbf{G}_p(\xi, \hat{\xi}) & J_p(\xi) \\ \hdashline 0 & I \end{array}\right] \begin{bmatrix} \zeta_k \\ d_k \end{bmatrix}. \end{aligned} \tag{6}$$

*Then, for $\zeta_0 \equiv 0$, system (4) is said to be $(\mathcal{Z}, \mathcal{S}, \mathcal{D})$-$\gamma$-dissipative if the following condition holds for $\gamma > 0$ and $T > 0$:*

$$\sum_{k=0}^{T} \mathbf{E}\Big\{ \mathcal{Q}(z_k, d_k) \Big\} \geq \gamma \sum_{k=0}^{T} \mathbf{E}\Big\{ d_k^T d_k \Big\}, \tag{7}$$

*where $\gamma$ stands for the dissipative performance level.*

**Remark 1.** *It follows [22,31] that there are two particular performances deduced from the $(\mathcal{Z},\mathcal{S},\mathcal{D})$-$\gamma$-dissipativity (7): **(i)** $\mathcal{H}_\infty$-performance by $\mathcal{Z}=-I$, $\mathcal{S}=0$, and $\mathcal{D}=(\gamma^2+\gamma)I$, **(ii)** passivity performance by $\mathcal{Z}=0$, $\mathcal{S}=I$, and $\mathcal{D}=2\gamma I$.*

The mismatch phenomenon here is the difference between fuzzy basic functions in the system model $\xi_i(\varrho(x_k))$ and the observed-based controller $\xi_i(\varrho(\hat{x}_k))$. The difference tends to ruin the stability of the closed-loop system (4) if it is not considered in the controller design. Thus, this paper aims to design the observed-based controller (3) that guarantees the stochastic stability and dissipative performance of the closed-loop system (4) with the following constraint:

$$-1\le \underline{\alpha}_i \le \xi_i(\varrho(x_k))-\xi_i(\varrho(\hat{x}_k))\le \bar{\alpha}_i\le 1,\ \forall i\in\mathbb{N}_\xi=\{1,2,\cdots,r\}, \tag{8}$$

where $\bar{\alpha}_i$ and $\underline{\alpha}_i$ are given scalars. Next, the following well-known lemmas are used

**Lemma 1** ([32])**.** *For any matrix $\mathcal{M}_{ij}=\mathcal{M}_{ij}^T$, the condition $0\le\sum_{i=1}^r\sum_{j=1}^r\xi_i\xi_j\mathcal{M}_{ij}$ holds if*

$$0\le\mathcal{M}_{ii},\ \forall i\in\mathbb{N}_\xi, \tag{9}$$

$$0\le\frac{1}{r-1}\mathcal{M}_{ii}+\frac{1}{2}(\mathcal{M}_{ij}+\mathcal{M}_{ji}),\ \forall(i,j)\in\mathbb{N}_\xi\times\mathbb{N}_\xi\setminus\{j\}. \tag{10}$$

**Lemma 2** ([33])**.** *Let real matrices $\mathcal{M}=\mathcal{M}^T$, $\mathcal{N}_1$, $\mathcal{N}_2$ and $U$ with appropriate dimensions and $UU^T\le I$. The inequality $0>\mathcal{A}+\mathbf{He}\{\mathcal{N}_1U\mathcal{N}_2\}$ is true if*

$$0>\begin{bmatrix}\mathcal{M}+\beta\mathcal{N}_1\mathcal{N}_1^T & (*)\\ \mathcal{N}_2 & -\beta I\end{bmatrix}. \tag{11}$$

## 3. Control Synthesis

To establish the dissipative condition of a closed-loop system (4), this paper considers a Lyapunov function in the following form:

$$V_k=V(\zeta_k,\psi_k)=\zeta_k^TP(\hat{\xi},\psi_k)\zeta_k, \tag{12}$$

where $P(\hat{\xi}_k,\psi_k=p)=P_p(\hat{\xi}_k)=P_p^T(\hat{\xi})>0$, the double-fuzzy summation $P_p(\hat{\xi})=\sum_{i=1}^r\sum_{j=1}^r\hat{\xi}_i\hat{\xi}_jP_{pij}$, and symmetric matrices $P_{pij}$. The Lyapunov function does not require $P_{pij}>0$ for all $(p,i,j)\in\mathbb{N}_\psi\times\mathbb{N}_\xi\times\mathbb{N}_\xi\setminus\{i\}$. The conditions can be relaxed by Lemma 1. Then, by letting $\hat{\xi}^+=\xi(\hat{\varrho}_{k+1})$ and $\mathbf{P}_p(\hat{\xi}^+)=\sum_{q=1}^{s}\pi_{pq}P_h(\hat{\xi}^+)$, we can obtain

$$\begin{aligned}\mathbf{E}\{\Delta V_k\}&=\mathbf{E}\big\{V(\zeta_{k+1},\psi_{k+1}=h\big|\psi_k=p)\big\}-V(\zeta_k,\psi_k=p)\\&=\zeta_{k+1}^T\mathbf{P}_p(\hat{\xi}^+)\big(\mathbf{A}_p(\tilde{\xi},\xi,\hat{\xi})\zeta_k+\mathbf{E}_p(\xi,\hat{\xi})d_k\big)-\zeta_k^TP_p(\hat{\xi})\zeta_k.\end{aligned} \tag{13}$$

**Lemma 3.** *Suppose that there exist symmetric matrices $0<P_p(\hat{\xi})\in\mathbb{R}^{2n_x\times2n_x}$ and $0<P_h(\hat{\xi}^+)\in\mathbb{R}^{2n_x\times2n_x}$ such that for all $p\in\mathbb{N}_\psi$:*

$$\begin{aligned}0>&\left[\begin{array}{c:c|c}-P_p(\hat{\xi}) & (*) & (*)\\ \hdashline -\mathcal{S}\mathbf{G}_p(\xi,\hat{\xi}) & -\mathbf{He}\{\mathcal{S}J_p(\xi)\}+\gamma I-\mathcal{D} & (*)\\ \hline \mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1J_p(\xi) & -I\end{array}\right]\\&+\begin{bmatrix}\bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi}) & 0\end{bmatrix}^T\mathbf{P}_p(\hat{\xi}^+)\begin{bmatrix}\bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi}) & 0\end{bmatrix}.\end{aligned} \tag{14}$$

*Then, closed-loop system (4) is stochastically stable and $(\mathcal{Z},\mathcal{S},\mathcal{D})$-$\gamma$-dissipative.*

**Proof.** The formulation (6) can be rearranged as follows

$$\mathcal{Q}(z_k,d_k)=\begin{bmatrix}\zeta_k\\ d_k\end{bmatrix}^T\left(\begin{bmatrix}0 & (*)\\ \mathcal{S}\mathbf{G}_p(\xi,\hat{\xi}) & \mathbf{He}\{\mathcal{S}J_p(\xi)\}+\mathcal{D}\end{bmatrix}\right.$$
$$\left.-\begin{bmatrix}\mathbf{G}_p^T(\xi,\hat{\xi})\mathcal{Z}_1^T\\ J_p^T(\xi)\mathcal{Z}_1^T\end{bmatrix}\begin{bmatrix}\mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1J_p(\xi)\end{bmatrix}\right)\begin{bmatrix}\zeta_k\\ d_k\end{bmatrix}.$$

Following (13), it yields

$$\mathbf{E}\Big\{\Delta V_k+\gamma d_k^Td_k-\mathcal{Q}(z_k,d_k)\Big\}=\bar{\zeta}_k^T\Psi_k\bar{\zeta}_k, \tag{15}$$

where $\bar{\zeta}_k=\mathbf{col}(\zeta_k,d_k)=\mathbf{col}(\hat{x}_k,e_k,d_k)$,

$$\begin{aligned}\Psi_k=&\begin{bmatrix}\bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi})\end{bmatrix}^T\mathbf{P}_p(\hat{\xi}^+)\begin{bmatrix}\bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi})\end{bmatrix}\\ &+\begin{bmatrix}\mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1J_p(\xi)\end{bmatrix}^T\begin{bmatrix}\mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1J_p(\xi)\end{bmatrix}\\ &+\begin{bmatrix}-P_p(\hat{\xi}) & (*)\\ -\mathcal{S}\mathbf{G}_p(\xi,\hat{\xi}) & -\mathbf{He}\{\mathcal{S}J_p(\xi)\}+\gamma I-\mathcal{D}\end{bmatrix}.\end{aligned} \tag{16}$$

Furthermore, from (15), it follows that $\sum_{k=0}^T\bar{\zeta}_k^T\Psi_k\bar{\zeta}_k=\sum_{k=0}^T\mathbf{E}\{\Delta V_k\}-\sum_{k=0}^T\mathbf{E}\{\mathcal{Q}(z_k,d_k)-\gamma d_k^Td_k\}=\mathbf{E}\{V_{T+1}-V_0\}-\sum_{k=0}^T\mathbf{E}\{\mathcal{Q}(z_k,d_k)\}-\gamma\sum_{k=0}^T\mathbf{E}\{d_k^Td_k\}$. As a result,

- for $d_k\equiv 0$, it follows from (13) that

$$\mathbf{E}\{\Delta V_k\}=\zeta_k^T\big(\bar{\mathbf{A}}_p^T(\tilde{\xi},\xi,\hat{\xi})\mathbf{P}_p(\hat{\xi}^+)\bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi})-P_p(\hat{\xi})\big)\zeta_k.$$

  Thus, condition $\Psi_k<0$ guarantees that $\mathbf{E}\{\Delta V_k\}<0$, i.e., $\mathbf{E}\{\Delta V_k\}\le-\varepsilon\|\zeta_k\|^2$ for a small scalar $\varepsilon>0$. Sum up the inequality from 0 to $T$, it holds that

$$\mathbf{E}\left\{\sum_{k=0}^T\|\zeta_k\|^2\Big|\zeta_0,\phi_0\right\}\le\frac{1}{\varepsilon}\mathbf{E}\{V_0\}<\infty,$$

  for all $T>0$, then, closed-loop system (4) with $d_k\equiv 0$ is stochastically stable by Definition 1.
- for $V_0=0$ (i.e., $x_0\equiv 0$), with the inequality $\Psi_k<0$, it has $\mathbf{E}\{V_{T+1}\}-\sum_{k=0}^T\mathbf{E}\{\mathcal{Q}(z_k,d_k)\}-\gamma\sum_{k=0}^T\mathbf{E}\{d_k^Td_k\}<0$ or $\sum_{k=0}^T\mathbf{E}\{\mathcal{Q}(z_k,d_k)\}-\gamma\sum_{k=0}^T\mathbf{E}\{d_k^Td_k\}>\mathbf{E}\{V_{T+1}\}\ge 0$.

With the two particular cases, $\Psi_k<0$ implies the stochastic stability and $(\mathcal{Z},\mathcal{S},\mathcal{D})$-$\gamma$-dissipative performance of the closed-loop system (4). Finally, the condition $0>\Psi_k$ can be converted into (14) according to the Schur's complement. □

The following lemma aims to address the encountered relaxation problem for Lemma 3 with fewer dimensions of slack matrix variables and the asymmetric range of mismatch level (8).

**Lemma 4.** *For given a double-parameterized LMI in the following form:*

$$\begin{aligned}0>\Phi_0&+\sum_{i=1}^r\xi_i\mathbf{He}\Big\{\Gamma_1^T\Phi_{1,i}\Gamma_2\Big\}+\sum_{i=1}^r\hat{\xi}_i\Phi_{2,i}\\ &+\sum_{i=1}^r\sum_{j=1}^r\xi_i\hat{\xi}_j\mathbf{He}\Big\{\Gamma_1^T\Phi_{3,ij}\Gamma_2\Big\}+\sum_{i=1}^r\sum_{j=1}^r\hat{\xi}_i\hat{\xi}_j\Phi_{4,ij}\end{aligned} \tag{17}$$

*subject to*

$$\underline{\alpha}_\ell\le\tilde{\xi}_\ell=\xi_\ell-\hat{\xi}_\ell\le\bar{\alpha}_\ell, \tag{18}$$

*where* $\Phi_0 \in \mathbb{R}^{p\times p}$, $\Phi_{1,i} \in \mathbb{R}^{n_1\times n_2}$, $\Phi_{2,i} \in \mathbb{R}^{p\times p}$, $\Phi_{3,ij} \in \mathbb{R}^{n_1\times n_2}$, *and* $\Phi_{4,ij} \in \mathbb{R}^{p\times p}$; $\Gamma_1 \in \mathbb{R}^{n_1\times p}$ *and* $\Gamma_2 \in \mathbb{R}^{n_2\times p}$ *are full rank matrices, the condition (17) subjected to (18) holds if there exist matrices* $S_{ij} = S_{ij}^T \in \mathbb{R}^{n_1\times n_1}$ *and* $N_i \in \mathbb{R}^{n_1\times n_2}$ *such that:*

$$0 > \bar{\Phi}_{ii}, \tag{19}$$

$$0 > \frac{1}{r-1}\bar{\Phi}_{ii} + \frac{1}{2}\left(\bar{\Phi}_{ij} + \bar{\Phi}_{ji}\right), \tag{20}$$

*for all* $(i,j) \in \mathbb{N}_\xi \times (\mathbb{N}_\xi \setminus \{i\})$, *where*

$$\bar{\Phi}_{ij} = \left[\begin{array}{c:c} \Phi_0 + \mathbf{He}\{\Gamma_1^T(\Phi_{1,i}+\Phi_{3,ij})\Gamma_2\} + \Phi_{2,i} + \Phi_{4,ij} + \sum_{\ell=1}^{r} \underline{\alpha}_\ell \bar{\alpha}_\ell \Omega_1^T S_{\ell i}\Gamma_1 & (*) \\ \hdashline \left[(\Phi_{1,\ell}+\Phi_{3,\ell i}+N_i)\Gamma_2 - \frac{1}{2}(\underline{\alpha}_\ell+\bar{\alpha}_\ell)S_{\ell i}\Gamma_1\right]_{\ell\in\mathbb{N}_\xi} & \left[S_{\ell i}\right]^{\mathbf{d}}_{\ell\in\mathbb{N}_\xi} \end{array}\right].$$

**Proof.** Since $\sum_{\ell=1}^{r} \tilde{\xi}_\ell = 0$, it stands that $\sum_{\ell=1}^{r}\sum_{i=1}^{r} \tilde{\xi}_\ell \hat{\xi}_i \mathbf{He}\{\Gamma_1^T N_i \Gamma_2\} = 0$ by which we can rewrite (17) as

$$0 > \Phi_0 + \mathbf{Z}(\hat{\xi}) + \mathbf{He}\left\{\sum_{\ell=1}^{r} \tilde{\xi}_\ell \Gamma_1^T \mathbf{Z}_\ell(\hat{\xi})\Gamma_2\right\}, \tag{21}$$

where $\mathbf{Z}(\hat{\xi}) = \sum_{i=1}^{r} \hat{\xi}_i\left(\mathbf{He}\{\Gamma_1^T\Phi_{1,i}\Gamma_2\} + \Phi_{2,i}\right) + \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\xi}_i\hat{\xi}_j\left(\mathbf{He}\{\Gamma_1^T\Phi_{3,ij}\Gamma_2\} + \Phi_{4,ij}\right)$, and $\mathbf{Z}_\ell(\hat{\xi}) = \Phi_\ell^{(1)} + \sum_{i=1}^{r} \hat{\xi}_i \Phi_{\ell i}^{(3)} + \sum_{i=1}^{r} \hat{\xi}_i N_i$. In accordance with the above expressions and

$$\mathbf{He}\left\{\sum_{\ell=1}^{r} \tilde{\xi}_\ell \Gamma_1^T \mathbf{Z}_\ell(\hat{\xi})\Gamma_2\right\} = \mathbf{He}\left\{(\tilde{\xi}\otimes\Gamma_1)^T \left[\mathbf{Z}_\ell(\hat{\xi})\Gamma_2\right]_{\ell\in\mathbb{N}_\xi}\right\},$$

the condition (21) is rearranged as

$$0 > \left[\begin{array}{c} I \\ \hdashline \tilde{\xi}\otimes\Gamma_1 \end{array}\right]^T \left[\begin{array}{c:c} \Phi_0 + \mathbf{Z}(\hat{\xi}) & (*) \\ \hdashline \left[\mathbf{Z}_\ell(\hat{\xi})\Gamma_2\right]_{\ell\in\mathbb{N}_\xi} & 0 \end{array}\right] \left[\begin{array}{c} I \\ \hdashline \tilde{\xi}\otimes\Gamma_1 \end{array}\right]. \tag{22}$$

Meanwhile, since (19) implies $S_{\ell i} = S_{\ell i}^T < 0$, it follows from (18) that

$$\begin{aligned} 0 &\le \sum_{i=1}^{r} \hat{\xi}_i \sum_{\ell=1}^{r} (\tilde{\xi}_\ell - \bar{\alpha}_\ell)(\tilde{\xi}_\ell - \underline{\alpha}_\ell)\Gamma_1^T S_{\ell i}\Gamma_1 \\ &= \left[\begin{array}{c} I \\ \hdashline \tilde{\xi}\otimes\Gamma_1 \end{array}\right]^T \left[\begin{array}{c:c} \sum_{i=1}^{r} \hat{\xi}_i \left(\sum_{\ell=1}^{r} \underline{\alpha}_\ell\bar{\alpha}_\ell \Gamma_1^T S_{\ell i}\Gamma_1\right) & (*) \\ \hdashline \left[-\frac{1}{2}\sum_{i=1}^{r} \hat{\xi}_i(\underline{\alpha}_\ell+\bar{\alpha}_\ell)S_{\ell i}\Gamma_1\right]_{\ell\in\mathbb{N}_\xi} & \left[\sum_{i=1}^{r}\hat{\xi}_i S_{\ell i}\right]^{\mathbf{d}}_{\ell\in\mathbb{N}_\xi} \end{array}\right] \left[\begin{array}{c} I \\ \hdashline \tilde{\xi}\otimes\Gamma_1 \end{array}\right]. \end{aligned} \tag{23}$$

Supported by the S-procedure, the combination of (22) with (23) ensures

$$\begin{aligned} 0 &> \left[\begin{array}{c:c} \Phi_0 + \mathbf{Z}(\hat{\xi}) + \sum_{i=1}^{r} \hat{\xi}_i \left(\sum_{\ell=1}^{r} \underline{\alpha}_\ell\bar{\alpha}_\ell \Gamma_1^T S_{\ell i}\Gamma_1\right) & (*) \\ \hdashline \left[\mathbf{Z}_\ell(\hat{\xi})\Gamma_2 - \frac{1}{2}\sum_{i=1}^{r} \hat{\xi}_i(\underline{\alpha}_\ell+\bar{\alpha}_\ell)S_{\ell i}\Gamma_1\right]_{\ell\in\mathbb{N}_\xi} & \left[\sum_{i=1}^{r}\hat{\xi}_i S_{\ell i}\right]^{\mathbf{d}}_{\ell\in\mathbb{N}_\xi} \end{array}\right] \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\xi}_i\hat{\xi}_j \bar{\Phi}_{ij}, \end{aligned} \tag{24}$$

and by Lemma 1, condition (19) implies (24). $\square$

**Remark 2.** *To deal with presence of two different types of parameters in (17) induced by the mismatch phenomenon, Lemma 4 presents a relaxation technique based on parameterized-LMIs given in Lemma 1 to avoid the excessive use of free slack matrix variables. Compared to other relaxation techniques for the mismatch phenomenon, our work concerns asymmetric range of mismatch level (18) and reduces dimensions of slack matrix variables by introducing constant matrices $\Gamma_1$ and $\Gamma_2$.*

With the help of Lemma 4, the following theorem presents a parameter-independent criteria from Lemma 3

**Theorem 1.** *Suppose that there exist scalars $\gamma > 0$ and $\beta$, matrices $0 < P_{pi} = P_{pi}^T \in \mathbb{R}^{2n_x \times 2n_x}$, $0 < X = X^T \in \mathbb{R}^{2n_x \times 2n_x}$, $0 < \bar{X} = \bar{X}^T \in \mathbb{R}^{2n_x \times 2n_x}$, $K_{pi} \in \mathbb{R}^{n_u \times n_x}$, $L_{pi} \in \mathbb{R}^{n_x \times n_y}$, $N_{pi} \in \mathbb{R}^{(2n_x+n_d)\times(2n_x+n_d+n_q)}$, and $S_{p\ell i} = S_{p\ell i}^T \in \mathbb{R}^{(2n_x+n_d)\times(2n_x+n_d)}$ such that the following inequalities hold for all $p \in \mathbb{N}_\psi$, $(m,i,j) \in \mathbb{N}_\xi \times \mathbb{N}_\xi \times \mathbb{N}_\xi \setminus \{i\}$:*

$$0 < P_{pii},\ 0 < \frac{1}{r-1}P_{pii} + \frac{1}{2}\left(P_{pij} + P_{pji}\right), \tag{25}$$

$$0 < X - \Lambda_{pii},\ 0 < \frac{r}{r-1}X - \frac{1}{r-1}\Lambda_{pii} - \frac{1}{2}\left(\Lambda_{pij} + \Lambda_{pji}\right), \tag{26}$$

$$0 > \bar{\Phi}_{pmii},\ 0 > \frac{1}{r-1}\bar{\Phi}_{pii} + \frac{1}{2}\left(\bar{\Phi}_{pij} + \bar{\Phi}_{pji}\right),\ \forall j \in \mathbb{N}_\xi \setminus \{i\}, \tag{27}$$

$$X\bar{X} = I, \tag{28}$$

*where* $\Lambda_{pij} = \sum_{q=1}^{s} \pi_{pq} P_{qij}$,

$$\bar{\Phi}_{pmij} = \left[\begin{array}{c:c} \Phi_p^{(0)} + \mathbf{He}\left\{\Gamma_1^T\left(\Phi_{pi}^{(1)} + \Phi_{pij}^{(3)}\right)\Gamma_2\right\} + \Phi_{pi}^{(2)} + \Phi_{pij}^{(4)} + \sum_{\ell=1}^{r} \underline{\alpha}_\ell \bar{\alpha}_\ell \Gamma_1^T S_{p\ell i}\Gamma_1 & (*) \\ \hdashline \left[\ \left(\Phi_{p\ell}^{(1)} + \Phi_{p\ell i}^{(3)} + N_{pi}\right)\Gamma_2 - \frac{1}{2}(\underline{\alpha}_\ell + \bar{\alpha}_\ell) S_{p\ell i}\Gamma_1\ \right]_{\ell \in \mathbb{N}_\xi} & \left[S_{p\ell i}\right]^{\mathbf{d}}_{\ell \in \mathbb{N}_\xi} \end{array}\right],$$

$$\Phi_p^{(0)} = \mathbf{diag}\left(0, \gamma I - \mathcal{D}, -I, -\bar{X} + \beta\,\mathbf{diag}\left(0, T_{a,p}T_{a,p}^T + T_{b,p}T_{b,p}^T\right), -\beta I\right),$$

$$\Phi_{pi}^{(1)} = \left[\begin{array}{c|c:cc} -G_{pi}^T\mathcal{S}^T & G_{pi}^T\mathcal{Z}_1^T & 0 & A_{pi}^T \\ -G_{pi}^T\mathcal{S}^T & G_{pi}^T\mathcal{Z}_1^T & 0 & A_{pi}^T \\ \hdashline -J_{pi}^T\mathcal{S}^T & J_{pi}^T\mathcal{Z}_1^T & 0 & E_{pi}^T \end{array}\right],\ \Phi_{pi}^{(2)} = \left[\begin{array}{c:c|c:c|c} 0 & 0 & 0 & (*) & (*) \\ \hdashline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hdashline \begin{bmatrix} A_{pi} & 0 \\ -A_{pi} & 0 \end{bmatrix} & 0 & 0 & 0 & 0 \\ \hline \begin{bmatrix} Y_{a,p} & Y_{a,p} \\ 0 & S_{b,q}K_{pi} \end{bmatrix} & 0 & 0 & 0 & 0 \end{array}\right],$$

$$\Phi_{pij}^{(3)} = \left[\begin{array}{c|c:cc} -K_{pj}^T H_{pi}^T \mathcal{S}^T & K_{pj}^T H_{pi}^T \mathcal{Z}_1^T & C_{pi}^T L_{pj}^T & K_{pj}^T B_{pi}^T - C_{pi}^T L_{pj}^T \\ 0 & 0 & C_{pi}^T L_{pj}^T & -C_{pi}^T L_{pj}^T \\ \hdashline 0 & 0 & D_{pi}^T L_{pj}^T & -D_{pi}^T L_{pj}^T \end{array}\right],$$

$$\Phi_{pij}^{(4)} = \left[\begin{array}{c:c|c:c|c} -P_{pij} & 0 & 0 & (*) & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ \hdashline \begin{bmatrix} B_{pi}K_{pj} - L_{pj}C_{pi} & 0 \\ -B_{pi}K_{pj} + L_{pj}C_{pi} & 0 \end{bmatrix} & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right],\ \Gamma_1^T = \left[\begin{array}{cc} I & 0 \\ 0 & I \\ 0 & 0 \\ \hdashline 0 & 0 \end{array}\right] \in \mathbb{R}^{n_1 \times (2n_x+n_d)},$$

$$\Gamma_2 = \left[\begin{array}{ccc:c} 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{array}\right] \in \mathbb{R}^{(n_d+n_q+n_x)\times n_1},\ n_1 = 4n_x + n_d + n_q.$$

*The closed-loop system (4) is $(\mathcal{Z},\mathcal{S},\mathcal{D})$-$\gamma$-dissipative with the following observer and control gains*

$$K_p(\hat{\xi}) = \sum_{i=1}^{r} \hat{\xi}_i K_{pi},\ L_p(\hat{\xi}) = \sum_{i=1}^{r} \hat{\xi}_i L_{pi}. \tag{29}$$

**Proof.** Following the definition of the Lyapunov function (12), $P_p(\hat{\xi}^+) = \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\xi}_i^+ \hat{\xi}_j^+ P_{pij}$ which in turn leads to $\Lambda_p(\hat{\xi}) = \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\xi}_i \hat{\xi}_j \Lambda_{pij}$. Then, by (25) and Lemma 1, it follows that $\Lambda_p(\hat{\xi}) > 0$ and $P_p(\hat{\xi}^+) > 0$. Furthermore, with the help of (26) and Lemma 1, it has $\sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\xi}_i^+ \hat{\xi}_j^+ (X - \Lambda_{pij}) > 0$ and then

$$\mathbf{P}_p(\hat{\xi}^+) = \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\xi}_i^+ \hat{\xi}_j^+ \Lambda_{pij} < X = \bar{X}^{-1}.$$

Thus, condition (14) satisfies if

$$\begin{aligned} 0 > {} & \left[\begin{array}{c:c|c} -P_p(\hat{\xi}) & (*) & (*) \\ \hdashline -\mathcal{S}\mathbf{G}_p(\xi,\hat{\xi}) & -\mathbf{He}\{\mathcal{S}J_p(\xi)\} + \gamma I - \mathcal{D} & (*) \\ \hline \mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1 J_p(\xi) & -I \end{array}\right] \\ & + \begin{bmatrix} \bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi}) & 0 \end{bmatrix}^T \bar{X}^{-1} \begin{bmatrix} \bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi}) & 0 \end{bmatrix}. \end{aligned} \tag{30}$$

Moreover, the inequality (30) is guaranteed by Schur's complement

$$\begin{aligned} 0 > {} & \left[\begin{array}{c:c|c:c} -P_p(\hat{\xi}) & (*) & (*) & (*) \\ \hdashline -\mathcal{S}\mathbf{G}_p(\xi,\hat{\xi}) & -\mathbf{He}\{\mathcal{S}J_p(\xi)\} + \gamma I - \mathcal{D} & (*) & (*) \\ \hline \mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1 J_p(\xi) & -I & (*) \\ \hdashline \bar{\mathbf{A}}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi}) & 0 & -\bar{X} \end{array}\right] \\ = {} & \left[\begin{array}{c:c|c:c} -P_p(\hat{\xi}) & (*) & (*) & (*) \\ \hdashline -\mathcal{S}\mathbf{G}_p(\xi,\hat{\xi}) & -\mathbf{He}\{\mathcal{S}J_p(\xi)\} + \gamma I - \mathcal{D} & (*) & 0 \\ \hline \mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1 J_p(\xi) & -I & (*) \\ \hdashline \mathbf{A}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi}) & 0 & -\bar{X} \end{array}\right] \\ & + \mathbf{He}\left\{ \left[\begin{array}{cc} 0 & 0 \\ \hline 0 & 0 \\ \hdashline 0 & 0 \\ \hline T_{a,p} & T_{b,p} \end{array}\right] \begin{bmatrix} U_a(k) & 0 \\ 0 & U_b(k) \end{bmatrix} \left[\begin{array}{cc} Y_{a,p}^T & 0 \\ Y_{a,p}^T & K_p^T(\hat{\xi}) Y_{b,p}^T \\ \hdashline 0 & 0 \\ \hline 0 & 0 \end{array}\right]^T \right\}. \end{aligned}$$

Then, buy using Lemma 2, the above inequality can be deduced from

$$0 > \left[\begin{array}{c:c|c:c|c} -P_p(\hat{\xi}) & (*) & (*) & (*) & (*) \\ \hdashline -\mathcal{S}\mathbf{G}_p(\xi,\hat{\xi}) & -\mathbf{He}\{\mathcal{S}J_p(\xi)\} + \gamma I - \mathcal{D} & (*) & (*) & (*) \\ \hline \mathcal{Z}_1\mathbf{G}_p(\xi,\hat{\xi}) & \mathcal{Z}_1 J_p(\xi) & -I & 0 & 0 \\ \hdashline \mathbf{A}_p(\tilde{\xi},\xi,\hat{\xi}) & \mathbf{E}_p(\xi,\hat{\xi}) & 0 & -\bar{X} + \beta\mathbf{T}_p & 0 \\ \hline \mathbf{U}_p(\hat{\xi}) & 0 & 0 & 0 & -\beta I \end{array}\right]. \tag{31}$$

where $\mathbf{Y}_p(\hat{\xi}) = \begin{bmatrix} Y_{a,p} & Y_{a,p} \\ 0 & Y_{b,p}K_p(\hat{\xi}) \end{bmatrix}$ and $\mathbf{T}_p = \mathbf{diag}\left(0, T_{a,p}T_{a,p}^T + T_{b,p}T_{b,p}^T\right)$. It can be rearranged in the form of (17) as follows:

$$\begin{aligned} 0 > {} & \Phi_p^{(0)} + \sum_{i=1}^{r} \xi_i \mathbf{He}\left\{\Gamma_1^T \Phi_{pi}^{(1)} \Gamma_2\right\} + \sum_{i=1}^{r} \hat{\xi}_i \Phi_{pi}^{(2)} \\ & + \sum_{i=1}^{r}\sum_{j=1}^{r} \xi_i \hat{\xi}_j \mathbf{He}\left\{\Gamma_1^T \Phi_{pij}^{(3)} \Gamma_2\right\} + \sum_{i=1}^{r}\sum_{j=1}^{r} \hat{\xi}_i \hat{\xi}_j \Phi_{pij}^{(4)}. \end{aligned} \tag{32}$$

In accordance with Lemma 4, the inequality (32) is ensured by (27) and (28). □

The following algorithm based on SLPMM [34] is presented to solve the set of conditions in Theorem 1.

**Remark 3.** *In contrast with the cone complementarity linearization (CCL) method [35], the SLPMM [34] can provide the non-decreasing sequence $\{\mathcal{J}_i\}_{i\in\mathbb{N}}$ and also point out feasibility of the problem. Consequently, we can define a terminal condition by giving a threshold for decrease of sequence $\{\mathcal{J}_i\}_{i\in\mathbb{N}}$ when the problem is infeasible.*

## 4. Illustrative Examples

The simulation part is carried out using MATLAB software, MathWorks, Inc., Seoul, Korea. The LMI problem (33) and (34) in Algorithm 1 are numerically solved by LMI solver in Robust Control Toolbox, MATLAB. To use the LMI solver, we program our code using the MATLAB script files in a computer with i7 CPU Intel and 16 GB RAM DDR4. The coding program can be found in https://github.com/thanhbinh91/Ro-OuFe-DissCtrl-MJFSs, accessed on 2 October 2022.

**Algorithm 1** SLPMM to solve Theorem 1

1: Initialize matrices $X_0$ and $\bar{X}_0$ that satisfy

$$\text{LMIs: (25)–(27) and} \begin{bmatrix} X_0 & (*) \\ I & \bar{X}_0 \end{bmatrix} \geq 0. \tag{33}$$

2: Chose a sufficiently small real number $\epsilon > 0$ for the error bound of the solution precision and $i = 0$. For given positive scalars $\beta > 0$ and $\gamma > 0$.

3: **for** $i = i + 1$ **do**

4: Find $P_{pij}$, $K_{pi}$, $L_{pi}$, $X^*$ and $\bar{X}^*$ by solving the optimization problem:

$$\mathcal{J}_i^* = \min \mathbf{Tr}\{X_i\bar{X} + \bar{X}_i X\}$$
$$\text{s.t. (25), (26), (27) and} \begin{bmatrix} X & (*) \\ I & \bar{X} \end{bmatrix} \geq 0. \tag{34}$$

5: **if** $\left|\mathcal{J}_i^* - 4n_x\right| < \epsilon$ **then**

6: **return** $P_{pij}$, $K_{pi}$, $L_{pi}$ as a solution of Theorem 1 with respect to performance $\gamma$ .

7: **end if**

8: Find $\sigma^* = \min_{\sigma\in[0,1]} \mathbf{Tr}\{(X_i+\sigma(X^*-X_i))(\bar{X}_i+\sigma(\bar{X}^*-\bar{X}_i))\}$.

9: **if** $\sigma^* \neq 0$ **then**

10: $X_{i+1} = (1-\sigma^*)X_i + \sigma^* X^*$, $\bar{X}_{i+1} = (1-\sigma^*)\bar{X}_i + \sigma^*\bar{X}^*$,

11: **else return** set of conditions in Theorem 1 is infeasible.

12: **end if**

13: **end for**

**Example 1 (Improved results).** *Without jumping parameter (no Markov process), let us consider the truck-trailer system, used in [13,26,36] with the sampling time $T_s$ = 2.0 [s], length between center of truck and trailer to connection point and maximum velocity $\ell_1$ = 5.5 [m] and $\ell_2$ = 2.8 [m], and maximum velocity $v$ = −1.0 [m/s].*

$$A_1 = \begin{bmatrix} 1 - \dfrac{vT_s}{\ell_1} & 0 & 0 \\ \dfrac{vT_s}{\ell_1} & 1 & 0 \\ \dfrac{(vT_s)^2}{\ell_1} & vT_s & 1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1 - \dfrac{vT_s}{\ell_1} & 0 & 0 \\ \dfrac{vT_s}{\ell_1} & 1 & 0 \\ \delta\dfrac{(vT_s)^2}{\ell_1} & \delta vT_s & 1 \end{bmatrix},\ B_1 = B_2 = \begin{bmatrix} \dfrac{vT_s}{\ell_2} \\ 0 \\ 0 \end{bmatrix},$$
$$E_1 = E_2 = \begin{bmatrix} 0 \\ 0.2 \\ 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$
$$D_1 = D_2 = 0,\ G_1 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix},\ G_2 = \begin{bmatrix} -0.1 & 0 & 0 \end{bmatrix},$$
$$H_1 = H_2 = -0.1,\ J_1 = 3,\ J_2 = -3, \tag{35}$$

*where $\delta = 0.01/\pi$. There are two fuzzy-basis functions defined as*

$$\xi_1(\varrho_k) = \begin{cases} \big(\sin(\varrho_k) - \delta\varrho_k\big)/\big((1-\delta)\varrho_k\big), & \varrho_k \neq 0, \\ 1, & \varrho_k = 0, \end{cases}$$
$$\xi_2(\varrho_k) = 1 - \xi_1(\varrho_k), \tag{36}$$

*where $\varrho_k$ is premise variable is established as follows:*

$$\varrho_k = x_{2,k} + \frac{v \cdot T_s}{2\ell_2} x_{1,k}.$$

*with $x_{1,k}$ and $x_{2,k}$ stands for sampling at time step $k$ of the angle difference between the truck and trailer, and the angle of trailer, respectively.*

The above setups aim at a particular case where the output-feedback controller is synthesized with the matched fuzzy-basis functions, i.e., no mismatched phenomenon ($\alpha_i \equiv 0$ set in (8)), to asymptotically stabilize the truck-trailer system (36). Accordingly, the comparison of the smallest $\mathcal{H}_\infty$ performance indices obtained by [12,13,26] and Theorem 1 is shown in Table 1. To create the comparison, LMI-based conditions in Theorem 1 are solved by Algorithm 1 with $\beta = 0.02$. It is shown in Table 1 that Theorem 1 provides much improved results (the lower the better) in comparison with preceding works [12,13,26]. For more details, Theorem 1 releases about 98%, 51% and 15% better $\mathcal{H}$-index than that of [12,13,26], respectively. With $\gamma_{\min} = 3.18$, Algorithm 1 provides the following solution

$$F_1 = \begin{bmatrix} 2.921 & -1.568 & 0.076 \end{bmatrix},\ F_2 = \begin{bmatrix} 2.152 & -0.510 & 0.034 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 0.9655 & -1.0692 & 0.2855 \\ -0.8570 & -0.6130 & 0.7821 \\ 0.2938 & -1.1399 & 0.5619 \end{bmatrix},\ L_2 = \begin{bmatrix} 0.9205 & -0.9312 & 0.2135 \\ -0.6855 & -0.1369 & 0.6408 \\ -0.2100 & -0.2520 & 0.4045 \end{bmatrix}.$$

In accordance with the following initial setups

$$\hat{x}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T,\ x_0 = \begin{bmatrix} 0.2 & -0.3 & 0.1 \end{bmatrix}^T,\ d_k = e^{-0.3k}\sin(k),\ \text{for } k \geq 0,$$

state behavior and control input are shown in Figure 1a–d, in which Figure 1a–c present the asymptotic convergence of $x_{1,k}$, $x_{2,k}$ and $x_{3,k}$. Moreover, the observed states $\hat{x}_{1,k}$, $\hat{x}_{2,k}$ and $\hat{x}_{3,k}$ asymptotically track the real $x_{1,k}$, $x_{2,k}$ and $x_{3,k}$, respectively. In addition, Figure 1d shows the behavior of control input that proves the well-defined control problem. Eventually, Figure 1 shows the availability and validity of the observer and controller gains designed by Theorem 1 for (35),

**Table 1.** A comparison of minimum $\mathcal{H}_\infty$-performance indices in Example 1 between several studies.

| Methods | [12] [Th. 3] | [26] [Th. 1] | [13] [Th. 3.4] | [23] [Th. 9] | [24] [Cor. 1] | Th. 1 |
|---|---|---|---|---|---|---|
| $\bar{\alpha}_i = \underline{\alpha}_i = 0$ | 6.27 | 4.77 | 3.63 | 3.54 | Infeasible | 3.18 |

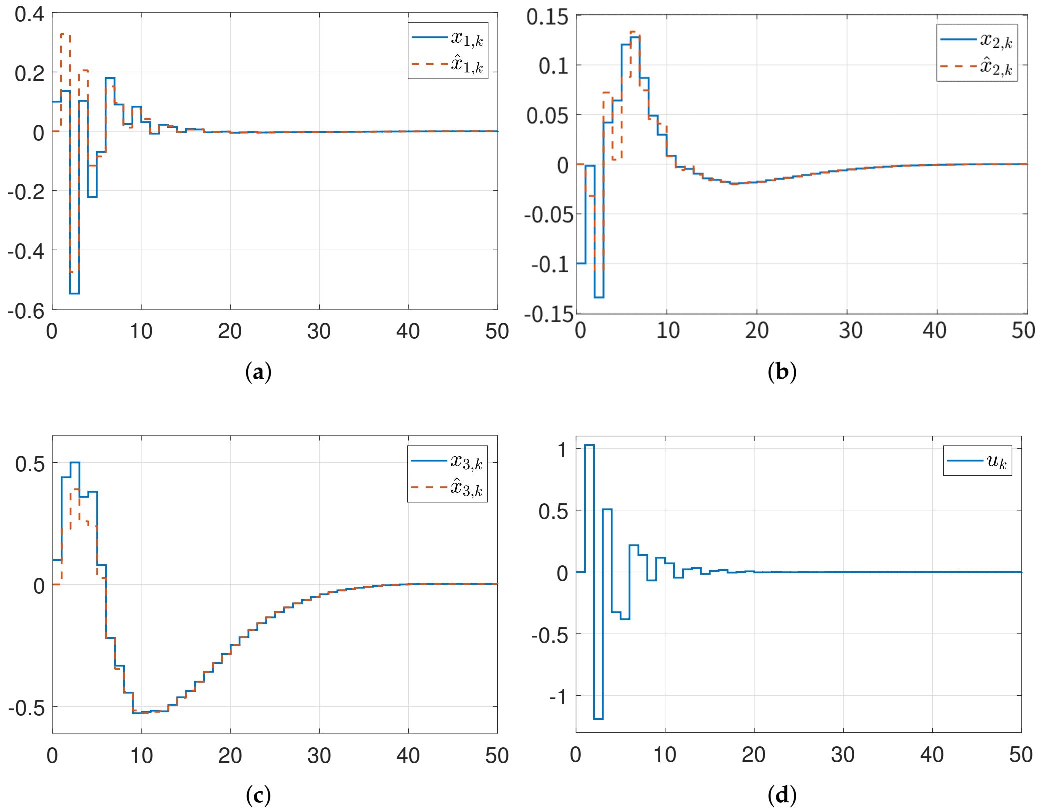


**Figure 1.** Time evolution of the truck-trailer system (35): (**a**–**c**) real and observed state and (**d**) control input.

**Example 2 (Relaxed practical example).** *Let us consider the following single-link robot arm system with plant mode $\psi(t) \in \mathbb{N}_\psi = \{1,2,3\}$, adopted in [37]:*

$$\begin{cases} \ddot{\varphi}(t) = -\dfrac{M(\psi(t))g_a\ell}{J(\psi(t))}\sin(\varphi(t)) - \dfrac{c_v\dot{\varphi}(t)}{J(\psi(t))} + \dfrac{1}{J(\psi(t))}u(t) + d(t), \\ y(t) = \varphi(t), \end{cases} \tag{37}$$

*where $\varphi(t)$, $\dot{\varphi}(t)$, $y(t)$, $u(t)$, and $d(t)$ stands for the angle, angular velocity, the controlled torque input, the load torque of the arm, and the measurement noise, respectively; and payload mass $M(\psi(t))$, inertia moment $J(\psi(t))$, arm length $\ell$ = 0.5 [m], the gravity acceleration $g_a$ = 9.81 [m/s$^2$], and viscous friction coefficient $c_v$ = 2.0 [N.s/m]. Then, by defining $x(t) = [x_1(t)\ x_2(t)]^T = [\varphi(t)\ \dot{\varphi}(t)]^T$ and $d(t)$ and performing the same process with the sampling time $T_s = 0.1$ as in [38,39], we can obtain the following discrete-time T-S fuzzy model for (37) with $p \in \mathbb{N}_\psi = \{1,2,3\}$:*

$$
\begin{aligned}
&A_{p1} = \begin{bmatrix} 1 & T_s \\ -\dfrac{T_s M_p g_a \ell}{J_p} & 1 - \dfrac{T_s c_v}{J_p} \end{bmatrix}, A_{p2} = \begin{bmatrix} 1 & T_s \\ -\dfrac{\delta T_s M_p g_a \ell}{J_p} & 1 - \dfrac{T_s c_v}{J_p} \end{bmatrix}, \\
&B_{p1} = B_{p2} = \begin{bmatrix} 0 \\ \frac{T_s}{J_p} \end{bmatrix}, E_{p1} = E_{p2} = \begin{bmatrix} 0 \\ T_s \end{bmatrix}, \\
&C_{p1} = C_{p2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{p1} = D_{p2} = \begin{bmatrix} 0 & 0.05 \end{bmatrix}, \\
&G_{p1} = G_{p2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, H_{p1} = H_{p2} = 0.1, J_{p1} = J_{p2} = 0,
\end{aligned}
$$

*where* $\delta = 0.01/\pi$, $M_1 = M(\psi(t) = 1) = 1.0$ *[kg]*, $M_2 = M(\psi(t) = 2) = 1.5$ *[kg]*, $M_3 = M(\psi(t) = 3) = 2.0$ *[kg]*, $J_1 = J(\psi(t) = 1) = 1.0$ *[kg.m/s$^2$]*, $J_2 = J(\psi(t) = 2) = 2.0$ *[kg.m/s$^2$], and* $J_3 = J(\psi(t) = 3) = 2.5$ *[kg.m/s$^2$]. In addition, for* $x_{1,k} \in (-\pi, \pi)$*, we define FBFs as*

$$
\begin{aligned}
&\xi_1(x_{1,k}) = \begin{cases} \dfrac{\sin(x_{1,k}) - \delta x_{1,k}}{(1-\delta)x_{1,k}}, & x_{1,k} \neq 0, \\ 1, & x_{1,k} = 0, \end{cases} \\
&\xi_2(x_{1,k}) = 1 - \xi_1(x_{1,k}),
\end{aligned}
$$

*and the mismatched FBFs were given by* $\hat{\xi}_1 = \xi_1(\hat{x}_{1,k})$ *and* $\hat{\xi}_2 = 1 - \hat{\xi}_1$.

Furthermore, the transition probabilities are chosen similarly [23]:

$$
[\pi_{pq}]_{p,q \in \mathbb{N}_\psi} = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.7 & 0.1 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}. \tag{38}
$$

Based on the setup as [23], a comparison of $(\mathcal{Z} = -0.01, \mathcal{D} = 5, \mathcal{S} = 0.2)$-dissipative and $\mathcal{H}_\infty$-performance indices obtained by Algorithm 1 and preceding studies, are shown in Table 2. Intuitively, Theorem 1 provides higher dissipative indices (the higher the better) compared to [23] and lower $\mathcal{H}_\infty$-indices compared to [23,24]. In particular, since mismatched level increases $\bar{\alpha}_i = -\underline{\alpha}_i = 0.1, 0.2$, our advantages are shown clearly, i.e., at $\bar{\alpha}_i = -\underline{\alpha}_i = 0.2$ [23] failed to obtain a solution and our result is 18% less than that of [24]. In the case where $\bar{\alpha}_i = -\underline{\alpha}_i = 0.2$, Theorem 1 provides a solution for dissipative performance at $\gamma_{\min} = 3.64$:

$$
\begin{aligned}
&\left[\begin{array}{c|c|c} F_{11} & F_{21} & F_{31} \\ \hline F_{12} & F_{22} & F_{32} \end{array}\right] = \left[\begin{array}{cc|cc|cc} 0.9701 & -1.5750 & 2.1238 & -6.2871 & 4.0832 & -7.5086 \\ \hline -2.7655 & -2.2250 & -5.4372 & -6.0615 & -7.0027 & -7.5363 \end{array}\right], \\
&\left[\begin{array}{c|c|c} L_{11} & L_{21} & L_{31} \\ \hline L_{12} & L_{22} & L_{32} \end{array}\right] = \left[\begin{array}{c|c|c} 1.2946 & 1.5475 & 1.8345 \\ -1.3860 & -1.0266 & -0.4478 \\ \hline 1.5122 & 1.4357 & 1.3936 \\ 0.4437 & 1.1962 & 1.2943 \end{array}\right].
\end{aligned}
$$

With $\hat{x}_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, $x_0 = \begin{bmatrix} 0.2 & -0.3 & 0.1 \end{bmatrix}^T$, and $d_k = e^{-0.4k}\sin(k)$, the time evolution of the single-link robot arm is shown in Figure 2. As can be seen in the Figure 2a,b, real state variables asymptotically converge, and the observed error converges to zero as time increases. Despite sudden changes in system mode, the closed-looped systems are asymptotic stable.

**Table 2.** Three performance levels for different mismatch phenomena $\bar{\alpha}_i = -\underline{\alpha}_i$ in (8).

| | Dissipativity | | | $\mathcal{H}_\infty$ Performance | | |
|---|---|---|---|---|---|---|
| | **Th. 1** | **[23]** | **[24]** | **Th. 1** | **[23]** | **[24]** |
| $\bar{\alpha}_i = -\underline{\alpha}_i = 0$ (matched) | 4.65 | 4.30 | - | 1.61 | 1.85 | 1.71 |
| $\bar{\alpha}_i = -\underline{\alpha}_i = 0.1$ | 4.38 | 2.89 | - | 2.42 | 5.13 | 3.64 |
| $\bar{\alpha}_i = -\underline{\alpha}_i = 0.2$ | 3.64 | Infeasible | - | 4.78 | Infeasible | 5.78 |

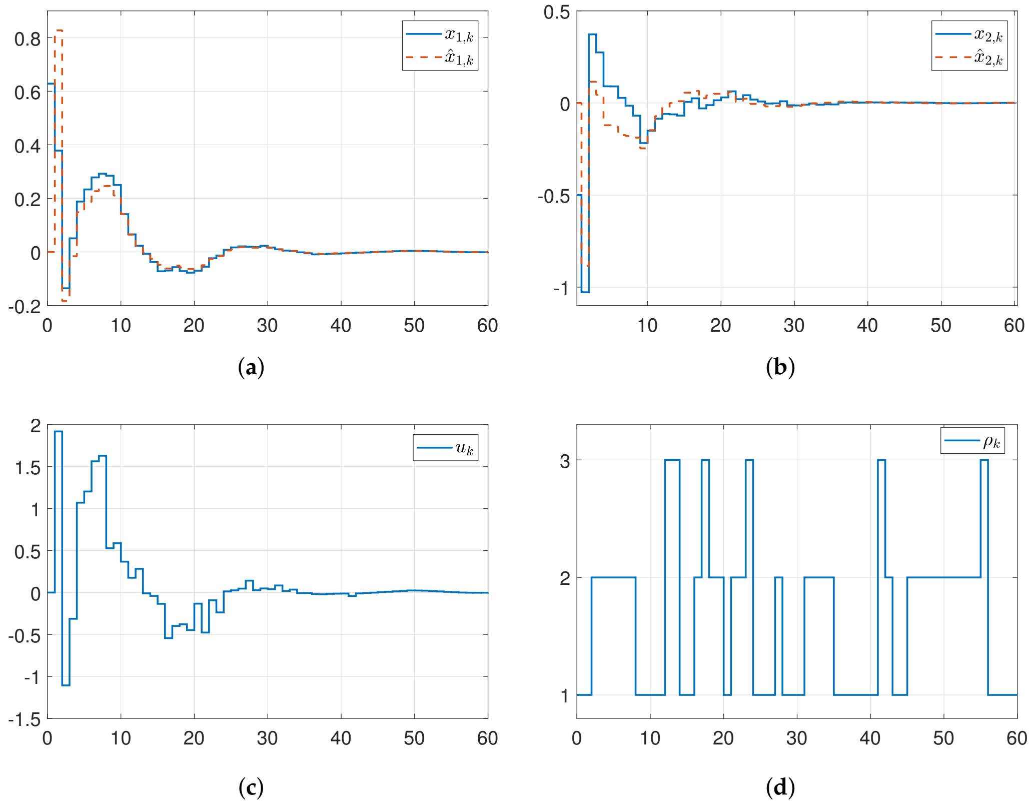


**Figure 2.** Time evolution of single-link robot arm (37): (**a**,**b**) real and observed state variables and (**c**) control input, (**d**) system mode.

## 5. Conclusions

This paper addresses the problem of observer-based dissipative control design for MJFSs under model uncertainties and a mismatched phenomenon entailed by the output-feedback scheme of fuzzy systems. The $(\mathcal{Z}, \mathcal{S}, \mathcal{D})$-dissipative conditions first were formulated in terms of multiple parameterized matrix inequalities. In light of proper relaxation techniques, the conditions were cast into parameter-independent bilinear matrix inequalities. Then we proposed an LMI-based algorithm to obtain the observer-based dissipative controller. The key success of our work is an achievement of much less conservative dissipative performance compared to other studies via the refined relaxation process and double-fuzzy summation Lyapunov function. The better results and validity of the LMI-based algorithm were verified via two numerical examples with different mismatch levels. In light of the success, future works should take asynchronous phenomena of operation mode between controller and plant into account to cover more realistic problems.

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