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Article

A Sustainable Green Supply Chain Model with Carbon Emissions for Defective Items under Learning in a Fuzzy Environment

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Abstract: Assuming the significance of sustainability, it is considered necessary to ensure the conservation of our natural resources, in addition to minimizing waste. To promote significant sustainable effects, factors including production, transportation, energy usage, product control management, etc., act as the chief supports of any modern supply chain model. The buyer performs the firsthand inspection and returns any defective items received from the customer to the vendor in a process that is known as first-level inspection. The vendor uses the policy of recovery product management to obtain greater profit. A concluding inspection is accomplished at the vendor's end in order to distinguish the returned item as belonging to one of four specific categories, namely re-workable, reusable, recyclable, and disposable, a process that is known as second-level inspection. Then, it is observed that some defective items are suitable for a secondary market, while some are reusable, and some can be disassembled to shape new derived products, and leftovers can be scrapped at the disposal cost. This ensures that we can meet our target to promote a cleaner drive with a lower percentage of carbon emissions, reducing the adverse effects of landfills. The activity of both players in this model is presented briefly in the flowchart shown in the abstract. Thus, our aim of product restoration is to promote best practices while maintaining economic value, with the ultimate goal of removing the surrounding waste with minimum financial costs. In this regard, it is assumed that the demand rate is precise in nature. The learning effect and fuzzy environment are also considered in the present model. The proposed model studies the impacts of learning and carbon emissions on an integrated green supply chain model for defective items in fuzzy environment and shortage conditions. We optimized the integrated total fuzzy profit with respect to the order quantity and shortages. We described the vendor's strategy and buyer's strategy through flowcharts for the proposed integrated supply chain model, and here, in the flowchart, R-R-R stands for re-workable, reusable, and recyclable. The demand rate was treated as a triangular fuzzy number. In this paper, a numerical example, sensitivity analysis, limitations, future scope, and conclusion are presented for the validation of the proposed model.

Keywords: optimization; learning effect; fuzzy environment; singed distance method; carbon emissions; supply chain approach; sustainability

MSC: 90-XX

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1. Introduction

In today's ubiquitous environment, sustainability has become a necessity for the creation of clean and green business. Considering the importance of sustainability, it is necessary to ensure the conservation of our natural resources, in addition to reducing waste. In order to promote a significant sustainable impact, factors including production, transportation, energy use, product control management, etc., serve as the main supports of any modern integrated green supply chain model. By observing their roles in the immediate landscape, we can connect them with sustainable policies for both vendors and buyers. In this model, the vendor manages the production of the items and provides the demanded lot to the buyer, according to the single setup and many more delivery strategies. In order to eliminate defective items, a screening process is completed at the vendor and buyer's ends, respectively. These defective items are kept in seclusion, and furthermore, permanent progress is made by asking the customer to return their used products and gain a rebate on their successive purchases. The buyer receives the used products from the customer, and the buyer returns these defective products to the vendor. The vendor inspects the defective-quality products received from the buyer and separates the defective-quality products on the basis of the quality of defective products. After that, it is determined that some imperfect-quality items are suitable for another business sectors, while some are reusable, some can be deconstructed to form new derivative products, and leftovers can be scrapped at the disposal cost. The supply chain system works well when the demand rate is deterministic and all the inventory parameters are controlled by the vendor and the buyer. However, in general, this is not really true, because some inventory parameters depend on the market demand. This ensures our goal of promoting a cleaner drive with a lower percentage of carbon emissions and minimizing the adverse impacts of landfills. The production of defective items in any industry is inescapable, regardless of the implementation of widely recognized techniques. Within the process of manufacturing the goods, there is still potential for a crash, which leads to the production of defective items along with perfect-quality items.

It is impractical for any manufacturing unit to adopt the responsibility of manufacturing items of a 100% perfect quality. There are many factors, including system machinery failure, poor workmanship, etc., that increase the chance of producing imperfect-quality items. Learning theory is beneficial where any work is in the repetition form. The learning effect and fuzzy environment are also assumed in the present model. In our study, an EOQ model with carbon emissions in a supply chain system, as well as shortages and product recovery management, was derived along with a numerical analysis, where the demand rate was treated as a triangular fuzzy number, and the holding and ordering costs were the function of shipment. We defuzzified the joint total fuzzy profit through the singed distance method. The whole paper divided into sections and subsections as follows: Section 1 offers an introduction and literature review; Section 2 explains the notation assumptions; Section 3 presents the basic definitions; Section 4 presents the description of the problems and mathematical formulation; and Section 5 presents the methodology of the optimization of the decision variable and contains subsections describing the solution method, a numerical example, sensitivity analysis, and the managerial insight and observations, which provide the results of the proposed model. Section 6 explains the conclusions of the model. Section 7 discusses the limitations and future scope of the present model. Section 8 presents the applications of the proposed study.

This segment provides an overview of a series of articles which are associated with the present study. Subsequently, to establish the place of the present study within the existing research knowledge, the available gaps are spot-lighted.

Salameh and Jaber [1] contributed their remarkable work in this aspect by considering the impacts that these defective items have using the inventory model and introduced the importance of screenings. Various prevailing studies have made fairly impractical presumptions about supply chain management, stating that shortages are not permitted. Indeed, shortages will occur with unanticipated demand or an irregular production capacity, and these occurrences will periodically influence the decisions of suppliers and retailers. Wee et al. [2] extended the model of Salameh and Jaber [1], where shortfalls were additionally applied in each cycle. The research of Salameh and Jaber [1] was extended by Eroglu and Ozdemir [3] for the consideration of defective-quality items under the condition of shortages. An inventory model was developed by Roy et al. [4] and Sarkar and Iqbal [5] for decaying items of a defective quality under inspection in a process where the defective items were treated as a random variable. An EOQ mathematical model was improved by Jaggi and Mittal [6] for decaying items of a defective quality under inspection in a process where the screening rate is faster than the demand rate. They further concluded that all the defective items are suitable for the secondary market and can be sold in that market at a price lower than the original market price.

This inevitable presence of imperfect-quality items in the inventory was researched further using possible realistic approaches. This research incorporated the proposal of many models which considered planned backorders, along with effective screening tests at the vendor's end, faulty production techniques, etc. The model of Salameh and Jaber [1] was resolved by Maddah and Jaber [7] for the expected whole worth per unit time using the very renowned theorem of Ross [8].

Relative increments in the levels of carbon emissions mainly occur because of the modes of transport through which they are produced. In order to maintain the standard emission norms, the index of carbon emissions must be checked by the organization so as to sustain their due quality standards and, thus, promote their brand value. Hua et al. [9] presented an inventory model based on a carbon footprint. In this vein, Howitt et al. [10] contributed to research through their work based on the CO2 emissions of the global space freight. Guereca et al. [11] discussed cleaner research for the institutes of Mexico based on a carbon footprint. Gurtu et al. [12] proposed an inventory model with the effect of the fuel cost in regard to carbon emissions. Sarkar et al. [13] studied the impacts of variable transportation and carbon emissions on the three-echelon supply chain model. Tiwari et al. [14] presented a sustainable inventory model for deteriorating defective items under carbon emissions. Sarkar et al. [15] explained the best approach by considering the carbon emissions of the supply chain. Thomas and Mishra [16] considered a sustainable supply chain model with waste reduction under carbon emissions for 3D printing and carbon minimization in some plastic industries.

Supply chain model management is helpful in identifying the best methods to apply in numerous industries. Each participant in a supply chain has the objective of fulfilling their tasks and obtaining the best outcomes of their processes. Various theories have previously been stated and proved. Sarkar et al. [17] proposed an SCM with inflation and a credit period for perishable items. Jaber and Goyal [18] explained a three-level supply chain model based on multiple players. Furthermore, Jaber et al. [19] extended a supply chain model, through learning, into a three-level supply chain model. Bazan et al. [20] described an SCM with greenhouse carbon emissions under energy utilization and applied a different approach. Aljazzar et al. [21] proposed a two-level SCM with credit financing for the purpose of strong coordination between the vendor and buyer.

In a recent scenario, Gautam and Khanna [22] derived an integrated SCM for the seller, as well as the buyer, which was sustainable, since it assumed the production of defective-quality items and carbon emissions. Later on, some researchers, such as Gautam et al. [23], Mashud et al. [24], and Rout et al. [25], proposed works with different strategies.

Alamari et al. [26] proposed an EOQ model with inflation and carbon emissions under the effect of learning for deteriorating items. This study was continued using the learning coefficient, as calculated in Khan et al. [27], reporting on the effects of learning and screening errors on the economic production model under supply chain and stochastic lead time demands.

Marchi et al. [28] presented an economic production model with the effects of the energy efficiency, production, reliability, and quality.

Afshari et al. [29] reported the impacts of learning and forgetting on the feasibility of adopting additive manufacturing in a supply chain model. Jaber and Peltokorpi [30] showed the impact of learning on the order quantity problem in regard to the production and group size. Masanta and Giri [31] proposed a closed-loop SCM with the effect of learning in an inspection process where the demand rate is a function of the price.

Jaggi et al. [32] presented an inventory model with a trade credit period and shortages based on a fuzzy concept and inspection of deteriorating items. In order to improve on previous research, Jaggi et al. [33] proposed a mathematical model with a fuzzy environment for deteriorating items under shortage, where the demand rate depends on time. Jaggi et al. [34] improved an EOQ model with a fuzzy environment and trade credit under the condition of shortages. Rout et al. [35] generalized an EOQ model with a fuzz-2 environment under the policy of a refill system. Patro et al. [36] explained an EOQ model with the influence of learning for imperfect-quality items in a fuzzy system. Bhavani et al. [37] presented a green EOQ model with shortages in a fuzzy environment. Jayaswal et al. [38] presented an EOQ with the effects of learning and a credit financing policy in a cloudy fuzzy environment.

In this light, we discussed the research gaps and studied a great deal of literature, described in the review provided above. Jayaswal et al.'s [38] study did not involve the formation of an integrated joint profit model. Considering this fact, the present study was framed by considering the need to develop an integrated model that used the approaches of learning and the fuzzy effect. Jayaswal et al. [39], described a fuzzy based inventory model with learning effect and credit policy under human learning and backorders. Wright [40] gave learning theory which is beneficial for ordering policy. Jayaswal and Mittal [41] presented an imperfect based inventory model with credit policy and inflationary condition under fuzzy environment.

Mittal and Sarkar [42] proposed a supply chain model with a credit policy for imperfect-quality items at a random energy price, where the global minimum cost was calculated for the supply chain model. In this vein, Wang et al. [43] worked on a closed-loop supply chain and also described competitive dual collecting in regard to consumer behavior. Using their model, Wang et al. [43] proposed a hybrid closed-loop supply chain model with competition concerning the reform of imperfect items and different types of product markets. Wang et al. [44] presented a supply chain model for Hybrid closed-loop with competition in recycling and product markets. The process of inspection for the separation of defective items through different approaches was briefly explained by the inventory model of Khanna et al. [45]. We selected some recent literature published between 2000 and 2022, as shown in Table 1. The idea of this proposed model is that it can fulfill the research gaps through a new approach. The present study discussed in this paper is shown at the bottom of Table 1. Our paper studies the impacts of leaning and carbon emissions on an integrated green supply chain model for defective items in a fuzzy environment. The present paper considers case studies of the seller–buyer supply chain model and reviews the available literature on joint inventory models, which were explained in order to manage the data. Consequently, to validate the proposed supply chain model, we constructed a dataset, following Hsu and Hsu [46] and Gautam et al. [23]. The introductory research in the area of defective goods was carried out by Rosenblatt and Lee [47], whose findings were later highlighted by many other scholars. Their research was based on the effects that are observed during the optimal production cycle time due to the production of imperfect products. Furthermore, Cardenas-Barron [48] made efforts to correct the possible mathematical modeling errors identified in the model of Salameh and Jaber [1]. The proposed study reviews the notion of managing the defective items using the best-known approaches, which, in turn, can be applied in an attempt to create cleaner, greener, and more sustainable surroundings. There are numerous industries that are working to make the best use of all the defective items, as well as the used items. This not only in the interest of the retailers but, instead, benefits the overall supply chain.

Table 1. Selected contributions.

Authors	Imperfect Items	SCM	Fuzzy Environment	Carbon Emissions	Learning
Salameh and Jaber [1]	√				
Tiwari et al. [14]	√			√	
Marchi et al. [28]	√			√	
Gautam et al. [23]	√	√			
Masanta and Giri [31]	√	√			√
Jayaswal et al. [38]	√		√		√
Our paper	√	√	√	√	√

2. Assumptions and Notations Used in the Model

Following are the assumptions and notations.

2.1. Notations

The notations and decision variables are shown in Appendix A.

2.2. Assumptions

We made some assumptions in regard to our proposed mathematical model, which are given below:

- ➢ It is considered that the buyer, customer, and vendor are involved in this supply chain model, where one type of item is used.
- ➢ No lead time is considered in this proposed model.
- ➢ The demand rate for the produced items is imprecise in nature.
- ➢ The demand function is taken as the triangular fuzzy number.
- ➢ The upper and lower fuzzy deviations of the demand rate follow the effect of learning.
- ➢ The buyer's holding cost is a decreasing function of the shipment, $H_1(n) = h_{0+}\frac{h_1}{n^{\mu}}$ and $H_2(n) = h_0 + \frac{h_2}{n^{\mu}}$, where $h_0,\ h_1$, and h_1 are the fixed holding cost, n is the shipment, and μ is the supporting parameter.
- ➢ The buyer's ordering cost is a decreasing function of the shipment, $A_c(n) = A_o + \frac{A_1}{n^{\mu}}$, where $A_0,\ A_1$ are the fixed ordering cost, n is the shipment, and μ is the supp-orting parameter.
- ➢ The process of manufacturing is controlled at the vendor's end, and the manufactured items are delivered at the buyer's end via multiple replacements without a first screening test. This leads to the delivery of a certain number of defective items, which follow a uniform distribution.
- ➢ This model assumes that the rate of demand is less than the rate of production.
- ➢ The buyer performs the first round of the inspection of the lot received from the vendor.
- ➢ The buyer provides the customers with perfect-quality items only. This implies that the rate of inspection is greater than the demand rate.
- ➢ To avert any incoming shortages while the inspection is taking place, the buyer limits α to follow $\alpha < \left(1 - \frac{D}{w}\right)$.
- ➢ All defective items segregated after the first round of inspection at the buyer's end are maintained up to the time of their upcoming procurement, and the cost involved in carrying these defective items is regarded as less than that involved in carrying perfect items.
- ➢ The last consumers return their used goods at the buyer's end in order to conduct a permanent operation, and these returned items collectively follow a uniform distribu-

tion. These items are returned by the vendor, along with the collection of imperfect-quality items.

- ➢ The communal effort implemented by the vendor and the buyer is proposed to be a modern policy and a cleaner and sustainable action, thus ensuring that no movement of empty vehicles occurs. From this point of view, the lot containing the imperfect-quality items and used goods, on behalf of the buyer, is sent back to the seller upon the delivery of the successive lot by the same vehicle. This means that the buyer is not responsible for paying any transportation costs and carbon emission costs when returning imperfect-quality and used items.
- ➢ It is considered that the carbon emissions are produced due to the multiple shipments and transportation. Here, we applied some carbon emission costs.
- ➢ The vendor applies the cost of the warranty for the imperfect-quality items returned by the buyer.
- ➢ It is assumed that the vendor uses the strategy of product recovery management, and its activities are in the flowchart abstract.
- ➢ Shortages are entirely backlogged at the buyer's end.
- ➢ The proposed model is solved using the concept of an integrated approach combining the cost components at the vendor's and buyer's ends.

2.3. Description of the Proposed Mathematical Model through a Flowchart

In this section, we describe all methodology and steps of the calculation of the proposed integrated supply chain model for the joint total fuzzy profit, as given in the Figures 1 and 2.

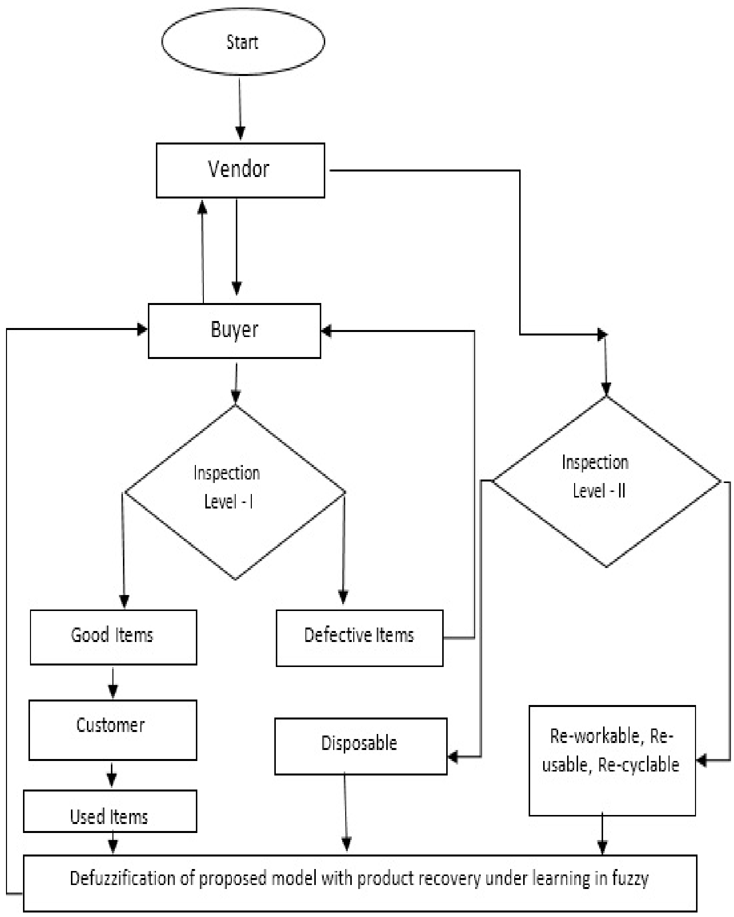

Figure 1. The activity of the vendor and buyer in a flowchart.

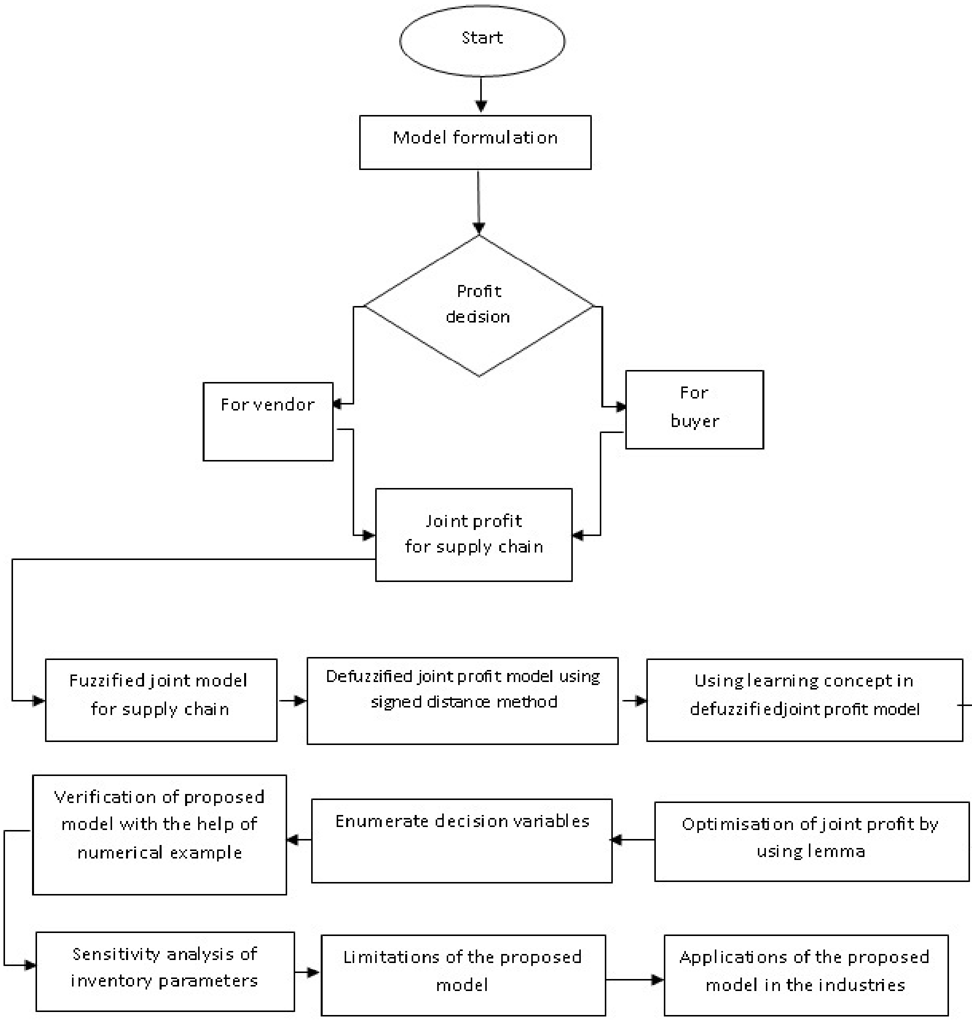

Figure 2. Presentation of the proposed model through a flow chart.

3. Some Basic Definition

There are some basic definitions which are highly important for the enlargement of the present study, and these are explained in Section 3.1.

3.1. Regarding the Fuzzy Concept

In this section, we provide some definitions that are very useful for the development of this model, which are given below:

Definition 1. *If R is a universal set and W is any set on R, then the fuzzy set of W on R is represented by $\widetilde{W}$, which, mathematically, can be written as $\widetilde{W} = \left\{ \left(r, \lambda_{\widetilde{W}}\left(\widetilde{r}\right) \right) : r \in R \right\}$, where $\lambda_{\widetilde{W}}$ represents a membership function, such that $\lambda_{\widetilde{W}} : R \to [0,1]$. The triplet (d_1, d_2, d_3) is used as the triangular fuzzy number, and this number should be associated with the condition $d_1 < d_2 < d_3$. The continuous membership function is defined below:*

$$\lambda_{\widetilde{W}} = \begin{cases} \dfrac{d - d_1}{d_2 - d_1} d_1 \le d \le d_2 \\ \dfrac{d_3 - d}{d_3 - d_2} d_2 \le d \le d_3 \\ 0 \text{ Otherwise} \end{cases}$$

Definition 2. *If c is any number and* $0 \in R$*, then the signed distance from c to* 0 *will be* $d(c, 0) = c$*, and if* $c < 0$*, then the signed distance from c to* 0 *will be* $d(-c, 0) = -c$*. Let it be assumed that* Ω *is the family of fuzzy sets* $\tilde{C}$ *defined on R. Then,* $\alpha - cut, C(\alpha) = [C_L(\alpha), C_U(\alpha)]$ *is* $\forall \alpha \epsilon [0, 1]$*, and* $C_L(\alpha)$ *and* $C_U(\alpha)$ *will be the continuous function of* α*. Then, we can write the value of* $C(\alpha)$*, which is shown below and shown in* Figure 3.

$$C(\alpha) = \bigcup_{0 \ll \alpha \ll 1} [C_L(\alpha)_\alpha, C_U(\alpha)_\alpha]$$

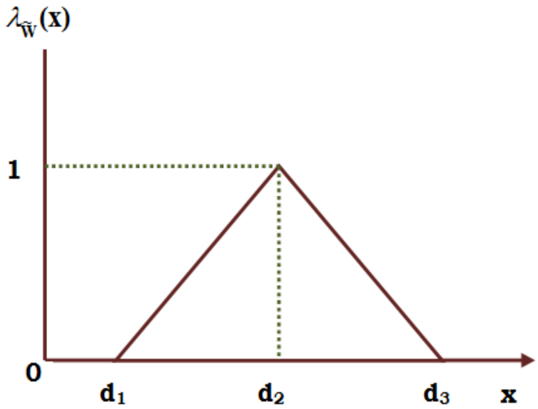

Figure 3. Membership function of a triangular fuzzy number.

Definition 3. *If* $\tilde{C}$ *is the member of* Ω*, then the signed distance from* $\tilde{C}$ *to* $\overset{\sim}{0}_1$ *is as given below:*

(i)

$$d(c, 0) = \frac{1}{2}\int_0^1 [C_L(\alpha) + C_U(\alpha)] d\alpha$$

Definition 4 *If* $\tilde{C} = (c_1, c_2, c_3)$ *is a triangular fuzzy number, then the* $\alpha - cut$ *of* $\tilde{C}$ *is* $C(\alpha) = [C_L(\alpha), C_U(\alpha)]$*, where* $C_L(\alpha) = c_1 + (c_2 - c_1)\alpha$ *and* $C_U(\alpha) = c_3 - (c_3 - c_2)\alpha$ *for* $\alpha \epsilon [0, 1]$*. The signed distance from* $\tilde{C}$ *to* $\overset{\sim}{0}_1$ *is:*

(ii)

$$d(\tilde{C}, 0) = \frac{(c_1 + 2c_2 + c_3)}{2}$$

3.2. Learning Curve

The learning (learning curve) demonstration is a statistical (geometric) development that expresses the falling cost necessary to achieve any cyclic process (operation). This concept expresses the notion that as the sum amount of the units produced doubles, the price per unit declines by a certain regular percentage. Wright [40] suggested that the learning concept (learning curve) is a power function formulation and is represented by $T_y = T_1 y^{-b}$, where T_y represents the time required to produce the $y_{n\ th}$ units, T_1 represents the time to produce the opening unit y, and b represents the learning slope and shown in the Figure 4.

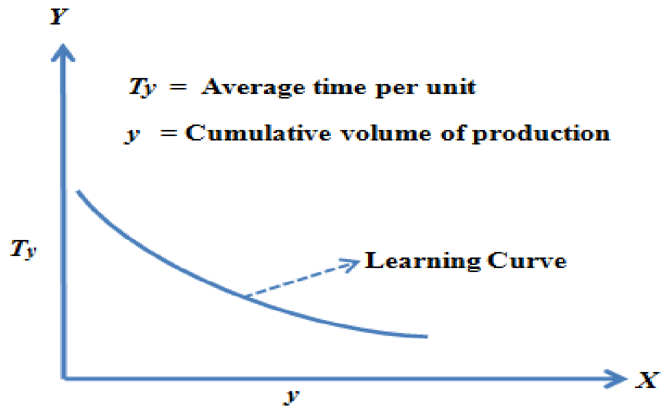

Figure 4. Wright's curve.

4. Mathematical Formulation

4.1. Theoretical Description

This section begins with the hypothetical explanation and meaning of the problem, following an individual and joint view of both the vendor and buyer, respectively. The problem is described in regard to the buyer, customer, and vendor for one kind of item in the supply chain model. The goods involved are proposed to take a fuzzy demand shape. The vendor is responsible for the production of the items, the sale operations are managed at the vendor's end, and the consumer then purchases the item, uses it, and returns it to the buyer. The activities of the vendor and buyer are in the flowchart contained in the abstract. The process starts when the buyer places the order, where the demand rate is imprecise in nature, and the lower and upper deviation of the demand follow the effect of learning, while the holding and ordering costs of the buyer are the function of the shipment. The vendor manufactures the quantity ordered by the buyer and, subsequently, delivers it to the purchaser through several deliveries. Carbon emissions are produced during the construction process and transportation. The delivered lot essentially contains defective items, which are identified and segregated by the buyer through a first round of inspection. A sustainable and clean campaign is inaugurated by supply chain researchers in an effort to achieve better product recovery. This drive encourages consumers to return all used items to the buyer in order to receive a rebate on their sequential purchases. The buyer is responsible for keeping the imperfect-quality products and used goods until the last of the shipment cycle and returns them, collectively, to the seller upon the reception of the successive lots. Defect formation can be found with various possibilities in the lots containing defective and used goods. Thus, to promote the full recovery of these products, another round of screening is encouraged on the vendor's side. Based on the circumstances of the goods, during the second round of inspection carried out by the vendor, the products are classified as re-workable items, reusable items, recyclable items, and disposable items, respectively. A re-workable product is of a good quality in nature and is sold in the secondary market. Reusable goods are not sufficient for trading in another business and are used to produce the derived goods. Those items that do not fit into one of these categories are labeled as recyclable. In the final step, those items that amount to scrap are classified as disposable.

4.2. Problem Description

Keeping in mind today's demand pattern, which does not ensure a compromise between the quality and quantity requirements for a particular type of item, the proposed models based on a single buyer, customer, and vendor for a single item were considered. It is initially assumed that the buyer considers a fuzzy annual rate of demand in D units. The required supply is expressed as nY units, which have to be managed by the vendor, and is delivered in n number of shipments, which are equal to Y units. In view of the inevitable defects in the manufactured lot, the demanded shipments may contain some defective items, which will lead to the development of warranty costs on the part of the vender. As soon as the shipment order is complete, an inspection of the lot is carried out at the buyer's end, and after the inspection, all the imperfect-quality items are isolated from the perfect-quality items. Let us assume that the defective percentage of a lot is α. By the end of the cycle, the total count of imperfect-quality items will be αnY. In addition, the buyer encourages all their consumers to return the items that they used. Since the quantity of items that are of a good quality items is β, the consumers will return βnY to the buyer by the end of the cycle, through which the buyer obtains a cost termed as the discount cost, which is nothing compared to the claim that was initially applied to the consumers to ensure a constant drive by returning their used items. The returned items tend to follow a uniform distribution. In the case of the returned defective items, the buyer uses them as a substitute to obtain an incentive cost from the vendor in order to supply the consumers with a rebate for each item that they returned after using it. The buyer tends to keep all these defective items, along with the used items, for one complete cycle, until the very end of the cycle and afterwards, when they return these isolated items to the vendor via the same transport vehicle that arrives to deliver the next shipment. This allows the vendor to include a warranty cost and an incentive cost on returned lots of items that contain defective and used items. At the vendor's end, a second inspection test of the lot of products returned by the buyer is carried out. The flowchart in the abstract explains the activities of the vendor and buyer in the supply chain. The fraction of re-worked items in the lot is $\eta_1\gamma nY$. The fraction of reused items in the lot is $\eta_2\gamma nY$. The fraction of recycled items in the lot is $\eta_3\gamma nY$. The fraction of disposable products in the lot is $\eta_4\gamma nY$. The present mathematical model was divided into two parts in the form of the vendor' strategy and the buyer's strategy, which are provided in the following sections.

4.2.1. Vendor's Strategy Model

In this section, the vendor's inventory incorporates two time phases. The former is the production phase (production time), and the other is the non-production phase (non-production time). The inventory at the vendor's end is illustrated in Figure 5 and the calculation of the holding cost has shown in the Figure 6.

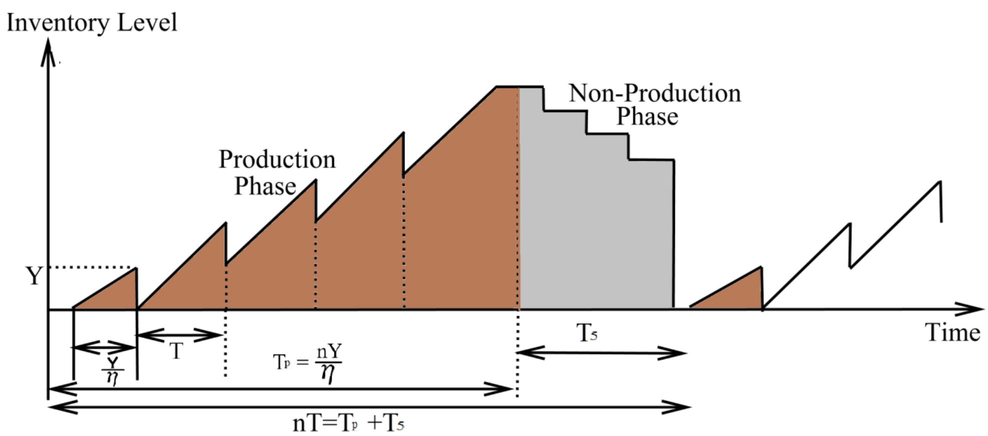

Figure 5. Representation of production and non-production systems in a supply chain model ***T***.

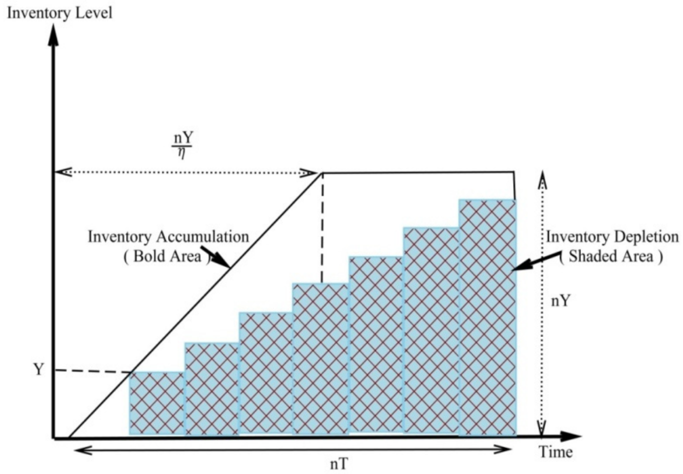

Figure 6. Explanation of the vendor's holding cost.

In reality, without any cost, no one can undertake production; therefore, some cost is required for production. The vendor incorporates the remaining costs during the production phase, defining such costs as the ordering cost (O_v), material and labor cost (ML_v), energy cost (EC_v), fixed transportation cost (FTC_v), variable transportation cost (VTC_v), holding cost (IHC_v), warranty cost (WC_v), incentive cost (IC_v), re-working cost (RC_v), reusing cost (REC_v), recycling cost (RECC_v), screening cost (SC_v), disposal cost (DC_v), carbon emission cost during the production phase (CEPC_v), carbon emission cost during transportation (CETC_v), and carbon emission cost during the disposal process (CEDC_v).

The total cost for vendor during the production process can be defined as shown in Equation (1).

Now, the vendor's total cost, (TC_v), is:

$$\begin{aligned} TC_v = {} & O_v + ML_v + EC_v + FTC_v + VTC_v + IHC_v + WC_v + IC_v + \mathrm{SC}_v + \mathrm{RC}_v + \mathrm{REC}_v + \mathrm{RECC}_v + \mathrm{DC}_v + \mathrm{CEPC}_v \\ & + \mathrm{CETC}_v + \mathrm{CEDC}_v \end{aligned} \tag{1}$$

Each cost component of the vendor's total cost (TC_v) is calculated, and these are given by Equations (2)–(16):

$$\text{Ordering cost}(O_v) = O_v \tag{2}$$

$$\text{Material and labor costs}(ML_v) = c_m \eta T_p \tag{3}$$

$$\text{Fixed transportation cost}(\mathrm{FTC}_v) = nF_t \tag{4}$$

$$\text{variable transportation cost}(\mathrm{VTC}_v) = nYV_t(1+\gamma) \tag{5}$$

$$\text{Holding cost}(\mathrm{IHC}_v) = H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{n(n-1)(1-\alpha)Y^2}{2D}\right] \tag{6}$$

$$\text{Warranty cost}(\mathrm{WC}_v) = w_c \alpha nY \tag{7}$$

$$\text{Incentive cost}(\mathrm{IC}_v) = i_c \beta nY \tag{8}$$

$$\text{Screening cost}(\mathrm{SC}_v) = I_2 \gamma nY \tag{9}$$

$$\text{Reworking cost }(\mathrm{RC}_v) = r_w \eta_1 \gamma nY \tag{10}$$

$$\text{Reusing cost}(\mathrm{REC}_v) = r_u \eta_2 \gamma nY \tag{11}$$

$$\text{Recycling cost}(\mathrm{RECC}_v) = r_c \eta_3 \gamma nY \tag{12}$$

$$\text{Disposal cost}(\mathrm{DC}_v) = d_w \eta_4 \gamma nY \tag{13}$$

$$\text{Carbon emissions during production phase}(\mathrm{CEPC}_v) = c_p nY \tag{14}$$

$$\text{Carbon emissions during transpotation}(\text{CETC}_v) = nc_{t_1} + nYc_{t_1}(1+\gamma) \quad (15)$$

$$\text{Carbon emissions during disposal process}(\text{CEDC}_v) = c_{t_2}\eta_4\gamma nY \quad (16)$$

$$\text{Energycost}(\text{EC}_v) = c_e T_p\left(\frac{\xi + K\eta}{nY}\right) \quad (17)$$

The value of each component cost obtained from Equations (2)–(17) is obtained by adding in Equation (1). Then, we get:

$$\begin{aligned} TC_v(n,Y) = {} & O_v + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{n(n-1)(1-\alpha)Y^2}{2D}\right] + w_c\alpha nY + i_c\beta nY \\ & + I_2\gamma nY + r_w\eta_1\gamma nY + r_u\eta_2\gamma nY + r_c\eta_3\gamma nY + d_w\eta_4\gamma nY + c_pnY + nc_{t_1} + nYc_{t_1}(1+\gamma) \\ & + c_{t_2}\eta_4\gamma nY + c_eT_p\left(\frac{\xi+K\eta}{nY}\right) \end{aligned} \quad (18)$$

The total revenue of the vender stems from different sources, such as the sale of perfect-quality products (=$c_1(1-\alpha)nY$), the sale of re-worked items which are sold in the secondary market at the reduced price $(= p_1\eta_1\gamma nY)$, the sale of reused items which are sold in different markets $(= p_2\eta_2\gamma nY)$, and the sale of re-cycled items which are sold in the primary market as raw materials $(= p_3\eta_3\gamma nY)$. The total revenue for the vender is the sum of all the revenues from the different sources, which is given below:

$$TR_v(n,Y) = c_1(1-\alpha)nY + p_1\eta_1\gamma nY + p_3\eta_3\gamma nY + p_2\eta_2\gamma nY \quad (19)$$

The total profit for the vender during the production process is:

$$TP_v(n,Y) = TR_v(n,Y) - TC_v(n,Y) \quad (20)$$

The values of $TR_v(n,Y)$ and $TC_v(n,Y)$, adding in Equation (20), are:

$$\begin{aligned} TP_v(n,Y) = {} & [c_1(1-\alpha)nY + p_1\eta_1\gamma nY + p_3\eta_3\gamma nY + p_2\eta_2\gamma nY] \\ & - \Bigg[O_v + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{n(n-1)(1-\alpha)Y^2}{2D}\right] + w_c\alpha nY \\ & + i_c\beta nY + I_2\gamma nY + r_w\eta_1\gamma nY + r_u\eta_2\gamma nY + r_c\eta_3\gamma nY + d_w\eta_4\gamma nY + c_pnY + nc_{t_1} + nYc_{t_1}(1+\gamma) \\ & + c_{t_2}\eta_4\gamma nY + c_eT_p\left(\frac{\xi+K\eta}{nY}\right)\Bigg] \end{aligned} \quad (21)$$

The total profit for the vender during the production process in a fuzzy environment, as obtained from Equation (21), is given below:

$$\begin{aligned} \widetilde{TP}_v(n,Y) = {} & [c_1(1-\alpha)nY + p_1\eta_1\gamma nY + p_3\eta_3\gamma nY + p_2\eta_2\gamma nY] \\ & - \Bigg[O + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{n(n-1)(1-\alpha)Y^2}{2\widetilde{D}}\right] + w_c\alpha nY \\ & + i_c\beta nY + I_2\gamma nY + r_w\eta_1\gamma nY + r_u\eta_2\gamma nY + r_c\eta_3\gamma nY + d_w\eta_4\gamma nY + c_pnY + nc_{t_1} + nYc_{t_1}(1+\gamma) \\ & + c_{t_2}\eta_4\gamma nY + c_eT_p\left(\frac{\xi+K\eta}{nY}\right)\Bigg] \end{aligned} \quad (22)$$

The total profit obtained from the Equation (22) for the vender during the production process can be defuzzified using the signed distance method. The signed distance between $\widetilde{TP}_v(n,Y)$ and $\widetilde{0}$ is defined below:

$$d\left(\widetilde{TP}_v(n,Y),\widetilde{0}\right) = [c_1(1-\alpha)nY + p_1\eta_1\gamma nY + p_3\eta_3\gamma nY + p_2\eta_2\gamma nY] - \left[O_v + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{n(n-1)(1-\alpha)Y^2}{2d\left(\widetilde{D}\widetilde{0}\right),}\right] + w_c\alpha nY + i_c\beta nY + I_2\gamma nY + r_w\eta_1\gamma nY + r_u\eta_2\gamma nY + r_c\eta_3\gamma nY + d_w\eta_4\gamma nY + c_pnY + nc_{t_1} + nYc_{t_1}(1+\gamma) + c_{t_2}\eta_4\gamma nY + c_eT_p\left(\frac{\xi + K\eta}{nY}\right)\right] \tag{23}$$

Here, we consider that $d\left(\widetilde{TP}_v(n,Y),\widetilde{0}\right) = \phi_1(n,Y)$ and use the definition of the signed distance concept:

$$\phi_1(n,Y) = [c_1(1-\alpha)nY + p_1\eta_1\gamma nY + p_3\eta_3\gamma nY + p_2\eta_2\gamma nY] - \left[O_v + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{2n(n-1)(1-\alpha)Y^2}{4D+\Delta_h{}^D-\Delta_l{}^D}\right] + w_c\alpha nY + i_c\beta nY + I_2\gamma nY + r_w\eta_1\gamma nY + r_u\eta_2\gamma nY + r_c\eta_3\gamma nY + d_w\eta_4\gamma nY + c_pnY + nc_{t_1} + nYc_{t_1}(1+\gamma) + c_{t_2}\eta_4\gamma nY + c_eT_p\left(\frac{\xi+K\eta}{nY}\right)\right] \tag{24}$$

4.2.2. Buyer's Strategy Model

In this section, we explain the activity of the buyer's policies from the vendor's point of view. As per the agreement contracted between the vendor and buyer, the vendor supplies Y units to the buyer, and the buyer receives Y units. The buyer's inventory includes the sales of defective and non-defective items and the screened Y units in the first round of screening. The buyer supplies the good-quality items to the customers. Moreover, shortages are allowed only under conditions of complete backlogging. A pictorial representation of the inventory at the buyer's end is shown in Figure 7 and for multiple orders have shown in the Figure 8. The buyer presents another strategy, where the customers are permitted to return their used products.

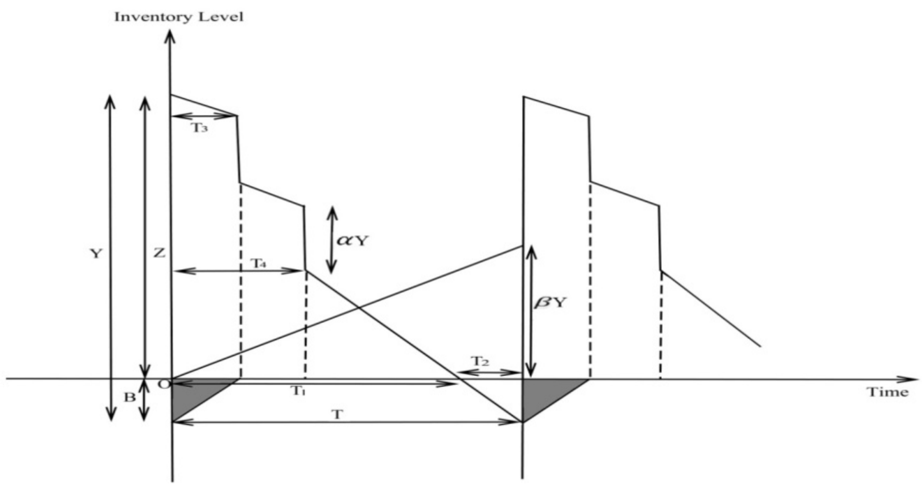

Figure 7. Buyer's inventory representation for time T.

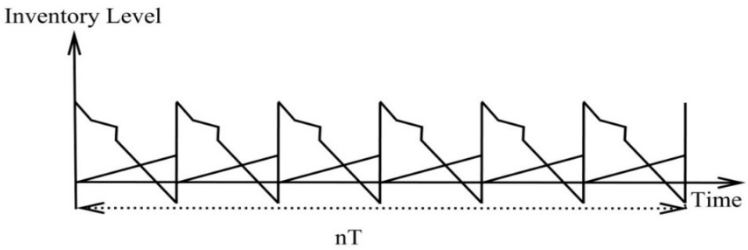

Figure 8. Buyer's inventory representation for nth cycle.

The screening time is given as:

$$T_4 = \frac{Y}{\omega} \tag{25}$$

The inventory level is completed at T_1, and its value is:

$$T_1 = \frac{Y(1-\alpha) - B}{D} \tag{26}$$

The time taken to create shortages after their accumulation, when an inventory is exhausted, is given as:

$$T_2 = \frac{B}{D} \tag{27}$$

The time taken to finish the backorders is given as:

$$T_3 = \frac{B}{w(1-\alpha) - D} \tag{28}$$

The inventory level after removing the backorders, which is equal *to* $Z = Y - B$, can be calculated, i.e., $T_3D + B$ after simplification (the calculation is shown in Appendix A), and we can write:

$$T_3D + B = \frac{BD}{w(1-\alpha) - D} + B = \frac{Bw(1-\alpha)}{w(1-\alpha) - D} \tag{29}$$

$$Z = \frac{Bw(1-\alpha)}{w(1-\alpha) - D} \tag{30}$$

The time taken for one shipment is given as $T = T_1 + T_2$

$$T = \frac{Y(1-\alpha) - B}{D} + \frac{B}{D} \tag{31}$$

The value of the cycle length using the expected approach based on the equation is:

$$E[T] = \frac{Y(1 - E[\alpha])}{D} \tag{32}$$

The total cost for the buyer is the sum of all the costs, including the ordering cost (O_b), screening cost (SC_b), purchase cost (PC), inventory carrying cost for good-quality items $(\mathrm{IHC}G_b)$, inventory carrying cost for defective items $(\mathrm{IHC}D_b)$, collection and handling cost of used items (CHC_b), shortage cost (SC_b), and the incentive cost (IC_b).

Now, the total cost for the buyer can be written as given below:

$$\mathrm{TC}_b(n, B, Y) = PC + O_b + SC_b + IHCG_b + IHCD_b + CHC_b + SC_b + IC_b \tag{33}$$

From Equation (33), each cost component can be calculated, and they are given below:

$$\text{Ordering cost } (O_b), A_c(n) = A_o + \frac{A_2}{n^{\mu}} \tag{34}$$

$$\text{Screening cost } (\mathrm{SC}_b) = I_1 nY \tag{35}$$

$$\text{Purchase cost (PC)} = c_1 nY$$

$$\text{Collection and handling cost of used items } (\mathrm{CHC}_b) = c_c \beta nY/2 \tag{36}$$

$$\text{Incentive cost } (\mathrm{IC}_b) = c_i \beta nY \tag{37}$$

- inventory carrying cost for good quality items ($\mathrm{IHC}G_b$)

$$\begin{aligned} IHCG_b = h_1(n) \Bigg[n \left\{ \frac{2Y(1-\alpha)(w(1-\alpha)-D) - wB(1-\alpha)}{2(w(1-\alpha)-D)} \right\} (T_3) \\ + \frac{n}{2} \left\{ \frac{2Y(1-\alpha)(w(1-\alpha)-D) - wB(1-\alpha)}{2(w(1-\alpha)-D)} \right\} (T_1 - T_3) \Bigg] \end{aligned} \tag{38}$$

where $h_1(n) = h_o + \dfrac{h_1}{n^\mu}$

- inventory carrying cost for defective items ($\mathrm{IHC}D_b$)

$$\mathrm{IHC}D_b = h_2(n) \left[\frac{n\alpha Y^2(1-\alpha)}{D} \right] \tag{39}$$

where $h_2(n) = h_o + \frac{h_2}{n^\mu}$

$$\text{shortage cost } (SRC_b) = s_c \left[\frac{nB^2}{2D} + \frac{nB^2}{2w\left(1 - \alpha - \frac{D}{w}\right)} \right] \tag{40}$$

Calculating the values of all the costs from Equations (34)–(40), adding in Equation (33), we get:

$$\begin{aligned} TC_b(n,B,Y) = \; & c_1 nY + A_o + \tfrac{A_2}{n^\mu} + I_1 nY + (h_o \\ & + \frac{h_1}{n^\mu}) \Bigg[n \left\{ \frac{2Y(1-\alpha)(w(1-\alpha)-D) - wB(1-\alpha)}{2(w(1-\alpha)-D)} \right\} (T_3) \\ & + \frac{n}{2} \left\{ \frac{2Y(1-\alpha)(w(1-\alpha)-D) - wB(1-\alpha)}{2(w(1-\alpha)-D)} \right\} (T_1 - T_3) \Bigg] + (h_o + \frac{h_2}{n^\mu}) \left[\frac{n\alpha Y^2(1-\alpha)}{D} \right] \\ & + c_c \beta nY/2 + s_c \left[\frac{nB^2}{2D} + \frac{nB^2}{2w\left(1 - \alpha - \dfrac{D}{w}\right)} \right] + c_i \beta nY \end{aligned} \tag{41}$$

The total revenue obtained by the buyer from different kinds of sources, such as the sale of good-quality items to the customers (=$c_2(1-\alpha)nY$), the vendor returning all the imperfect-quality items according to the type of warranty cost $(= w_c \alpha nY)$, and the vendor returning the used items $(= i_c \beta nY)$. The total revenue for the buyer is the sum of all the revenues from the different sources, which is given below:

$$TR_b(n,B,Y) = c_2(1-\alpha)nY + w_c \alpha nY + i_c \beta nY \tag{42}$$

The total profit for the buyer is:

$$TP_b(n,Y,B) = TR_b(n,Y,B) - TC_b(n,Y,B) \tag{43}$$

Calculating the values of $TR_b(n,B,Y)$ and $TC_b(n,B,Y)$ from Equations (42) and (41), replacing in Equation (43), we get:

$$\begin{aligned}
TP_b(n,Y,B) = \ & [c_2(1-\alpha)nY + w_c\alpha nY + i_c\beta nY] \\
& -\Big[c_1 nY + A_o + \tfrac{A_2}{n^\mu} + I_1 nY + (h_o \\
& + \frac{h_1}{n^\mu})\Big[n\Big\{\frac{2Y(1-\alpha)(w(1-\alpha)-D) - wB(1-\alpha)}{2(w(1-\alpha)-D)}\Big\}(T_3) \\
& + \frac{n}{2}\Big\{\frac{2Y(1-\alpha)(w(1-\alpha)-D) - wB(1-\alpha)}{2(w(1-\alpha)-D)}\Big\}(T_1 - T_3)\Big] + (h_o + \frac{h_2}{n^\mu})\Big[\frac{n\alpha Y^2(1-\alpha)}{D}\Big] \\
& + c_c\beta nY/2 + s_c\left[\frac{nB^2}{2D} + \frac{nB^2}{2w\left(1-\alpha-\frac{D}{w}\right)}\right] + c_i\beta nY\Big]
\end{aligned} \tag{44}$$

As per our assumption, the demand rate is imprecise in nature. Thus, the total profit for the buyer in a fuzzy environment, based on Equation (44), is represented by $\widetilde{TP}_b(n,Y,B)$, which is given below:

$$\begin{aligned}
\widetilde{TP}_b(n,Y,B) = \ & [c_2(1-\alpha)nY + w_c\alpha nY + i_c\beta nY] \\
& -\Big[c_1 nY + A_o + \tfrac{A_2}{n^\mu} + I_1 nY + (h_o \\
& + \tfrac{h_1}{n^\mu})\left[n\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-\widetilde{D}\right) - wB(1-\alpha)}{2\left(w(1-\alpha)-\widetilde{D}\right)}\right\}(T_3)\right. \\
& \left. + \tfrac{n}{2}\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-\widetilde{D}\right) - wB(1-\alpha)}{2\left(w(1-\alpha)-\widetilde{D}\right)}\right\}(T_1 - T_3)\right] + \left(h_o + \tfrac{h_2}{n^\mu}\right)\left[\frac{n\alpha Y^2(1-\alpha)}{\widetilde{D}}\right] \\
& + c_c\beta nY/2 + s_c\left[\frac{nB^2}{2\widetilde{D}} + \frac{nB^2}{2w\left(1-\alpha-\frac{\widetilde{D}}{w}\right)}\right] + c_i\beta nY\Big]
\end{aligned} \tag{45}$$

The total fuzzy profit based on Equation (45) for the buyer can be defuzzified using the signed distance method. The signed distance between $\widetilde{TP}_b(n,Y,B)$ and $\widetilde{0}$ is defined below:

$$\begin{aligned}
& d\left(\widetilde{TP}_b(n,Y,B),\widetilde{0}\right) \\
& = [c_2(1-\alpha)nY + w_c\alpha nY + i_c\beta nY] \\
& -\Big[c_1 nY + A_o + \tfrac{A_2}{n^\mu} + I_1 nY + (h_o \\
& + \tfrac{h_1}{n^\mu})\left[n\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-d\left(\widetilde{D},\widetilde{0}\right)\right) - wB(1-\alpha)}{2\left(w(1-\alpha)-d\left(\widetilde{D},\widetilde{0}\right)\right)}\right\}(T_3)\right. \\
& \left. + \tfrac{n}{2}\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-d\left(\widetilde{D},\widetilde{0}\right)\right) - wB(1-\alpha)}{2\left(w(1-\alpha)-d\left(\widetilde{D},\widetilde{0}\right)\right)}\right\}(T_1 - T_3)\right] + \left(h_o + \tfrac{h_2}{n^\mu}\right)\left[\frac{n\alpha Y^2(1-\alpha)}{d\left(\widetilde{D},\widetilde{0}\right)}\right] \\
& + c_c\beta nY/2 + s_c\left[\frac{nB^2}{2d\left(\widetilde{D},\widetilde{0}\right)} + \frac{nB^2}{2w\left(1-\alpha-\frac{d\left(\widetilde{D},\widetilde{0}\right)}{w}\right)}\right] + c_i\beta nY\Big]
\end{aligned} \tag{46}$$

Here, we consider that $d\left(\widetilde{TP}_b(n,Y,B),\widetilde{0}\right) = \phi_2(n,Y,B)$ and use the definition of the signed distance concept in Equation (46). Then, we get:

$$
\begin{aligned}
\phi_2(n,Y,B) = {} & [c_2(1-\alpha)nY + w_c\alpha nY + i_c\beta nY] \\
& -\Big[c_1 nY + A_o + \tfrac{A_2}{n^\mu} + I_1 nY + (h_o \\
& +\tfrac{h_1}{n^\mu})\left[n\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)-wB(1-\alpha)}{2\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)}\right\}(T_3)\right. \\
& \left.+\tfrac{n}{2}\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)-wB(1-\alpha)}{2\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)}\right\}(T_1-T_3)\right] + (h_o \\
& +\tfrac{h_2}{n^\mu})\left[\frac{4n\alpha Y^2(1-\alpha)}{4D+\Delta_h{}^D-\Delta_l{}^D}\right] + c_c\beta nY/2 + s_c\left[\frac{2nB^2}{4D+\Delta_h{}^D-\Delta_l{}^D} + \frac{nB^2}{2w\left(1-\alpha-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4w}\right)}\right] \\
& +c_i\beta nY\Big]
\end{aligned}
\tag{47}
$$

4.2.3. Integrated Model

In this case, we combined total the defuzzified profit of the vendor and buyer for the supply chain based on Equations (24) and (47), and it is represented by $\phi_3(n,Y,B)$. Then, we get:

$$\phi_3(n,Y,B) = \phi_1(n,Y,B) + \phi_2(n,Y,B)$$

$$
\begin{aligned}
\phi_3(n,Y,B) = {} & [c_1(1-\alpha)nY + p_1\eta_1\gamma nY + p_3\eta_3\gamma nY + p_2\eta_2\gamma nY] \\
& -\left[O + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{2n(n-1)(1-\alpha)Y^2}{4D+\Delta_h{}^D-\Delta_l{}^D}\right] + w_c\alpha nY\right. \\
& +i_c\beta nY + I_2\gamma nY + r_w\eta_1\gamma nY + r_u\eta_2\gamma nY + r_c\eta_3\gamma nY + d_w\eta_4\gamma nY + c_p nY + nc_{t_1} + nYc_{t_1}(1+\gamma) \\
& \left.+c_{t_2}\eta_4\gamma nY + c_e T_p\left(\frac{\xi+K\eta}{nY}\right)\right] + [c_2(1-\alpha)nY + w_c\alpha nY + i_c\beta nY] \\
& -\Big[c_1 nY + A_o + \tfrac{A_2}{n^\mu} + I_1 nY + (h_o \\
& +\tfrac{h_1}{n^\mu})\left[n\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)-wB(1-\alpha)}{2\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)}\right\}(T_3)\right. \\
& \left.+\tfrac{n}{2}\left\{\frac{2Y(1-\alpha)\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)-wB(1-\alpha)}{2\left(w(1-\alpha)-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)}\right\}(T_1-T_3)\right] + (h_o \\
& +\tfrac{h_2}{n^\mu})\left[\frac{4n\alpha Y^2(1-\alpha)}{4D+\Delta_h{}^D-\Delta_l{}^D}\right] + c_c\beta nY/2 + s_c\left[\frac{2nB^2}{4D+\Delta_h{}^D-\Delta_l{}^D} + \frac{nB^2}{2w\left(1-\alpha-\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4w}\right)}\right] \\
& +c_i\beta nY\Big]
\end{aligned}
\tag{48}
$$

Thus, the expected integrated defuzzified total fuzzy profit per unit time can be determined, and it is denoted by $\phi_4(n,Y,B)$. Then:

$$\phi_4(n,Y,B) = \frac{1}{E[T]}E[\phi_3(n,Y,B)] \tag{49}$$

The values of $E[\phi_3(n,Y,B)]$ and $E[T]$ are shown in Appendix A.

5. Integrated Model under Learning in a Fuzzy Environment

In this sequence, we move in the direction of learning shaped and governed by Wright [40], which is mathematically shown below:

$$S_n = S_{n1} n^{-l} \tag{50}$$

where S_n is the time for the nth order, S_{n1} is the initial time, and l is the learning factor. Using Equation (50) and the defined learning for the upper and lower triangular fuzzy numbers of the demand rate, we get:

$$\nabla^D_{h.i} = \begin{cases} \nabla^D_{h.1}, i = 1 \\ \nabla^D_{h.i}\left((i-1)\frac{365}{n}\right)^{-l}, i > 1 \end{cases} \tag{51}$$

$$\nabla^D_{l.i} = \begin{cases} \nabla^D_{l.1}, i = 1 \\ \nabla^D_{l.i}\left((i-1)\frac{365}{n}\right)^{-l}, i > 1 \end{cases} \tag{52}$$

Thus, the expected joint defuzzified total profit per unit time under learning in a fuzzy environment can be calculated using (49), (51), and (52), and it is denoted by $\phi_5(n, Y, B)$. Then:

$$\phi_5(n, Y, B) = \frac{1}{E_L[T]} E_L[\phi_3(n, Y, B)] \tag{53}$$

The values of $E_L[\phi_3(n, Y, B)]$ and $E_L[T]$ are shown in the Appendix A.

5.1. Solution Method

We used some useful lemma to identify the optimal values of the order quantity and backorders under learning in a fuzzy environment, and the statement and proof of the lemma are as given below:

Lemma 1. *The joint defuzzified total profit $\phi_5(n, Y, B)$ of the supply chain under learning in a fuzzy environment is concave.*

Proof. The conditions that must initially be satisfied for a specific value of the decision variable n are given as:

$$\frac{\partial \phi_5(n, Y, B)}{\partial Y} = 0 \tag{54}$$

and

$$\frac{\partial \phi_5(n, Y, B)}{\partial B} = 0 \tag{55}$$

Using Equations (53) and (54), we obtain the maximum value of the lot size Y and shortage units B, which, finally, are given below:

$$Y^*(n) = \sqrt{\left(\frac{\begin{array}{c}\frac{\left(A_o+\frac{A_2}{n^{\mu}}\right)\left(D+\frac{\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4}\right)}{n(1-E[\alpha])}+\frac{s_c B^2\left(D+\frac{\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4}\right)}{2(1-E[\alpha])}\left[+\frac{\begin{array}{c}\frac{4}{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}\\ 1\end{array}}{w\left(1-E[\alpha]-\frac{\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{w}\right)}\right]- \\ \frac{\left(h_0+\frac{h_1}{n^{\mu}}\right)B^2\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)\right)w}{\left[wE\left[(1-\alpha)^2\right]-\frac{\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)\right)(1-E[\alpha])}{4}\right]}+\frac{O_v\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)\right)}{4n(1-E[\alpha])}+\frac{F_t\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)\right)}{4(1-E[\alpha])}+\frac{c_{t_1}\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)\right)}{4(1-E[\alpha])}\end{array}}{\frac{H_c\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)\right)}{4(1-E[\alpha])}\left[\frac{1}{\eta}-\frac{n}{2\eta}+\frac{2(n-1)(1-E[\alpha])}{\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)\right)}\right]+\left(h_0+\frac{h_2}{n^{\lambda}}\right)\alpha+\left(h_0+\frac{h_1}{n^{\lambda}}\right)(1-E[\alpha])}\right)} \tag{56}$$

and

$$B^*(n) = \frac{\frac{\left(h_0+\frac{h_1}{n^{\lambda}}\right)\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)}{4}}{\left[\begin{array}{l} \frac{\left(h_0+\frac{h_1}{n^{\mu}}\right)w\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)}{4\left[YwE\left[(1-\alpha)^2\right]-\frac{\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)Y(1-E[\alpha])}{4}\right]} - \frac{\left(h_0+\frac{h_1}{n^{\mu}}\right)w^2}{\left[\begin{array}{l} w^2E\left[(1-\alpha)^2\right]+\frac{\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)^2}{16} \\ -\frac{w\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)}{2}(1-E[\alpha]) \end{array}\right]} \\ +\frac{s_c\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)}{8Y(1-E[\alpha])}\left[\frac{4}{\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)}+\frac{1}{\left(1-E[\alpha]--\frac{\left(4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla_{h.i}^{D}-\nabla_{l.i}^{D}\right)\right)}{4w}\right)}\right] \end{array}\right]} \tag{57}$$

Additionally, we calculated the maximum value of the shipment, given in relation to and satisfying [24]:

$$\begin{aligned} \phi_5(n+1, Y^*(n+1), B^*(n+1)) &\geq \phi_5(n^*, Y^*(n), B^*(n)) \\ &\leq \phi_5(n-1, Y^*(n-1), B^*(n-1)) \end{aligned} \tag{58}$$

The conditions required to satisfy the optimal condition are as follows: $\frac{\partial^2[\phi_5(n,Y,B)]}{\partial Y^2} < 0$, $\frac{\partial^2[\phi_5(n,Y,B)]}{\partial B^2} < 0$ and $\left(\frac{\partial^2[\phi_5(n,Y,B)]}{\partial Y^2}\right)\left(\frac{\partial^2[\phi_5(n,Y,B)]}{\partial B^2}\right) - \left(\frac{\partial^2[\phi_5(n,Y,B)]}{\partial Q \partial S}\right)\left(\frac{\partial^2[\phi_5(n,Y,B)]}{\partial S \partial Q}\right) > 0$. The concavity of the joint defuzzified total profit $\phi_5(n, Y, B)$ of the supply chain under learning in a fuzzy environment was proved with help of the concavity figures given in Figures 9–11. □

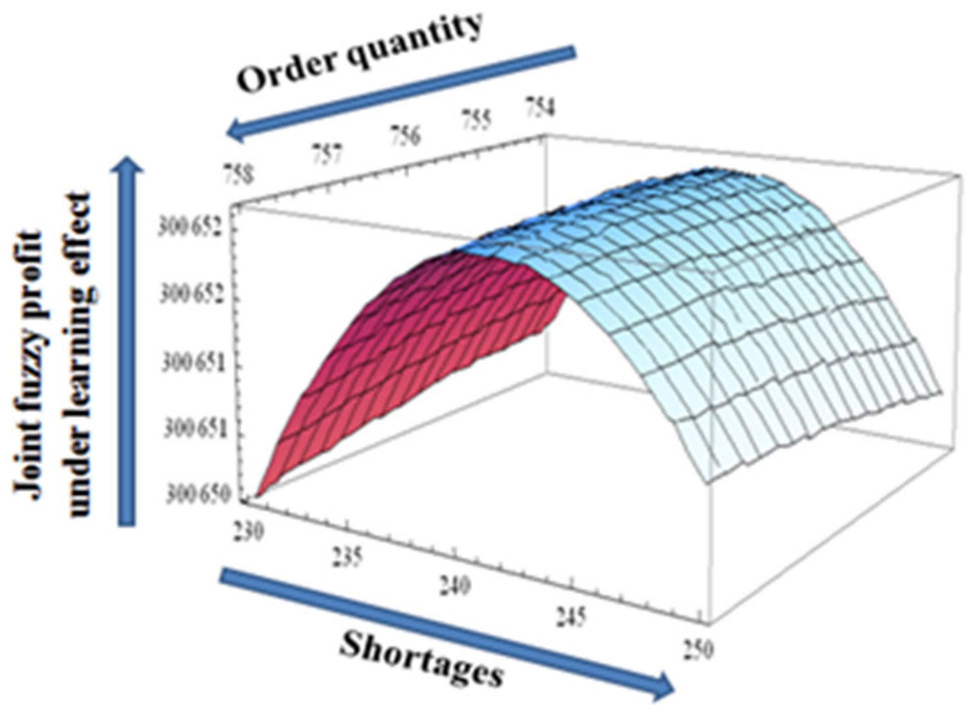

Figure 9. Concavity of the joint fuzzy profit with respect to the order quantity and shortages under learning in a fuzzy environment.

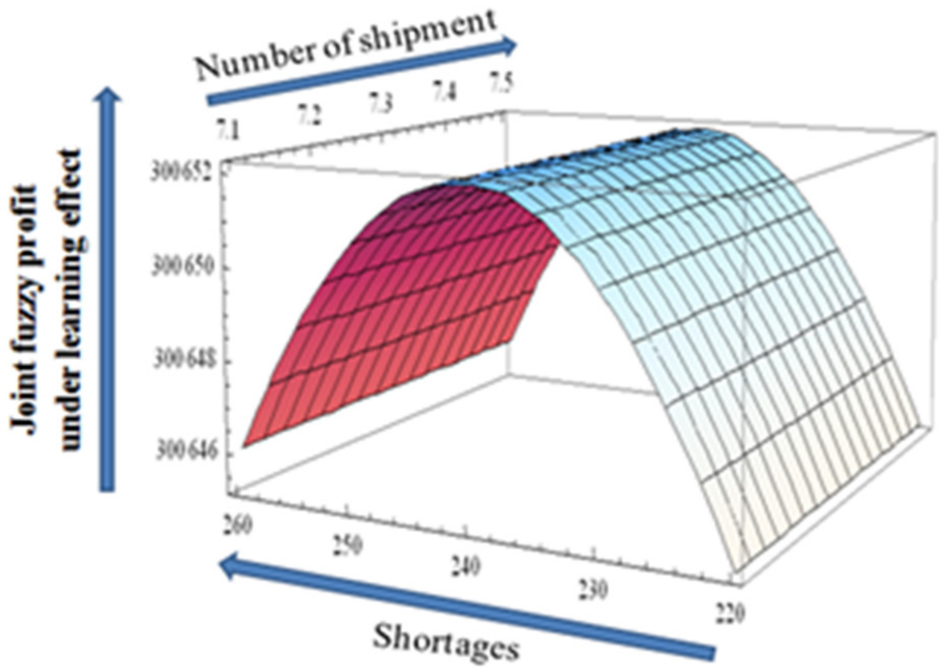

Figure 10. Concavity of the joint fuzzy profit with respect to the number of shipments and shortages under learning in a fuzzy environment.

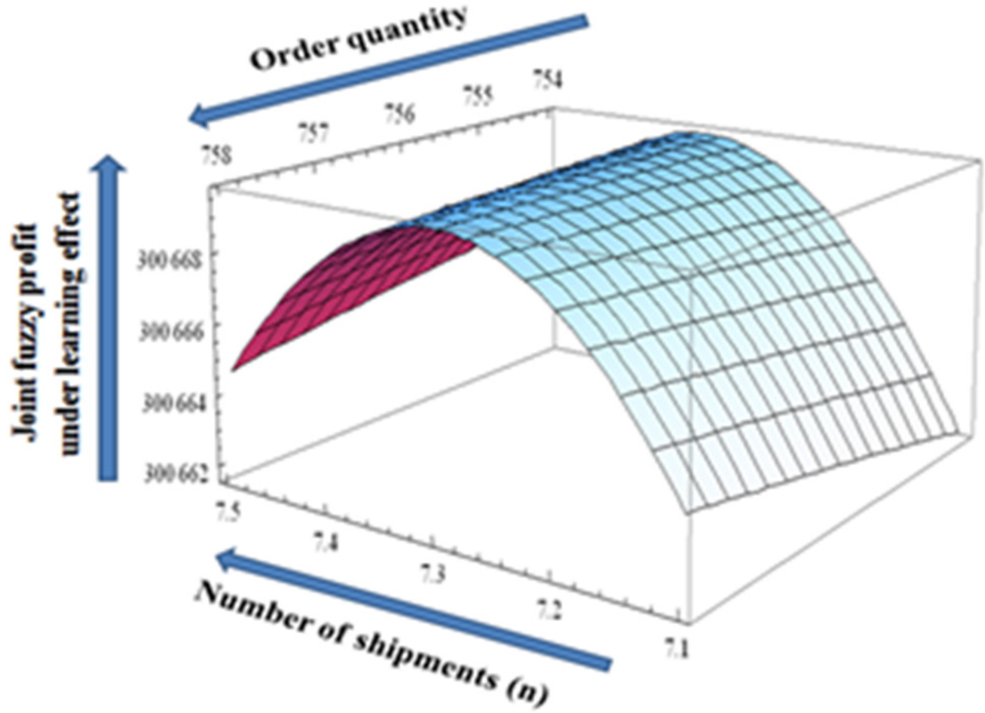

Figure 11. Concavity of the joint fuzzy profit with respect to the number of shipments and order quantity under learning in a fuzzy environment.

5.2. Numerical Analysis

For the justification of the proposed model, we took all the input inventory parameters from the works of some authors, including Salameh and Jaber [1], Gautam and Khanna [22], Jayaswal et al. [39], and Jayaswal et al. [41]. To execute the numerical analysis, all the inventory parameters, with notations, were collected in Table 2. The lot size (Y), shortages (B), and number of shipments are the decision variables, and the carbon dioxide and carbon footprint were not considered as dependent variables. Instead, they were discussed only to enable a better understanding of the carbon emission sources.

Table 2. Input parameter values used in the proposed model.

The Input Parameters for the Proposed Model	The Numerical Values of the Input Parameters	The Input Parameters for the Proposed Model	The Numerical Values of the Input Parameters	The Input Parameters for the Proposed Model	The Numerical Values of the Input Parameters
Production rate (η)	16,000 per units per year	Variable cost due to transportation (V_r)	USD 0.5 per unit	Variable ordering cost for buyer (A_0)	USD 200 per order
Ordering cost for vendor (O_v)	USD 500 per setup	Learning rate (b)	0.153	Purchase cost (c_1)	USD 35 per unit
Material and labor cost (c_m)	USD 30 per cycle	Variable cost due to carbon emissions (c_v)	USD 0.5 per unit	Fixed holding cost for good items (h_1)	USD 6 per unit per year
Energy cost for production (c_e)	USD 0.15 per kWh	Warranty cost (w_c)	USD 36 per defective unit	Fixed holding cost for defective items (h_2)	USD 2 per unit per year
Standard power system (ξ)	100 kW	Incentive unit cost (i_c)	USD 7 per used item	Screening cost for buyer (I_1)	USD 0.4 per unit
Constant (k)	10 kWh per unit	Fixed carbon emission cost due to disposed units (c_{t_2})	USD 5 per unit	Shortage unit cost (s_c)	USD 10 per unit per year
Fixed emission cost for production (c_p)	USD 4 per delivery	Re-worked unit cost (c_2)	USD 40 per unit	Collective cost for buyer ($\boldsymbol{c_c}$)	USD 1 per unit per year
Holding cost for vendor (H_c)	USD 4 unit per year	Re-worked product price (p_1)	USD 18 per unit	Incentive unit cost (C_i)	USD 4 per used item
Screening cost for vendor (I_2)	USD 0.6 per unit	Derived item price (p_2)	USD 22 per unit	Demand rate (D)	50,000 Units/year
Fraction of re-workable goods (η_1)	0.2	Recycled product price (p_3)	USD 28 per unit	Screening rate (w)	175,200 units/year
Fraction of reusable goods (η_2)	0.3	Re-worked unit cost (r_w)	USD 5 per unit	Expected defective percentage ($E[\alpha]$)	0.04
Fraction of re-cyclable goods (η_3)	0.4	Reused unit cost (r_u)	USD 10 per unit	Product recovery ($E[\beta]$)	0.4

Table 2. *Cont.*

The Input Parameters for the Proposed Model	The Numerical Values of the Input Parameters	The Input Parameters for the Proposed Model	The Numerical Values of the Input Parameters	The Input Parameters for the Proposed Model	The Numerical Values of the Input Parameters
Fraction of waste goods (η_4)	0.1	Recycled unit cost (r_c)	USD 15 per unit	Learning supporting parameter (μ)	0.02
Variable cost for transportation (F_r)	USD 25 per delivery	Disposed unit cost (d_w)	USD 8 per unit	Upper deviation demand rate $\left(\Delta_h^D\right)$	1000
Fixed holding cost for buyer (h_0)	USD 2 per unit per year	Fixed ordering cost for buyer (A_2)	USD 20 per order	Lower deviation demand rate $\left(\Delta_l^D\right)$	5000
Fixed emission cost due to disposed unit (c_{t_2})	USD 5 per unit	Fixed emission cost due to transportation (c_{t_1})	USD 1.5 per unit		

Continuing with our consideration that the defective proportion of the lot and the defective proportion of the used products follow the uniform probability distribution (UPD), the values are given below:

$$f(\alpha) = \begin{cases} 1/0.08, 0 \leq \alpha \leq 0.08 \\ 0, \text{Otherwise} \end{cases} \text{ and } f(\beta) = \begin{cases} 1/0.08, 0 \leq \beta \leq 0.08 \\ 0, \text{Otherwise} \end{cases}$$

Now, all the input parameters can be inserted into Equations (55) and (56), and using Equation (57), first of all, we obtain the optimal lot size and shortage units using the Mathematica software version(Mathematica 9.0, Wolfram Research, Champaign, IL, USA). The optimized values of the lot size, number of shipments, and shortages are:

$$Y^* = 775 \text{ units}, n^* = 7 \text{ and } B^* = 240 \text{ units}$$

Substituting the values of Y^*, n^*, and B^* in Equation (52), the total expected integrated fuzzy profit per unit time under the learning effect, $\phi_5(n^*, Y^*, B^*)$, for the given model is USD 300,652. In the absence of learning, the optimized values of the lot size, number of shipments, and shortages are $Y^* = 887$ units, $n^* = 10$ and$B^* = 300$ units. Substituting the values of Y^*, n^*, and B^* in Equation (51), the total expected integrated fuzzy profit per unit time, $\phi_4(n^*, Y^*, B^*)$, for the given model is USD 300,300. This model yields more profit (USD 300,652) under learning in a fuzzy environment through the product recovery process as compared with the traditional studies without product recovery (USD 131,920.88) and the study of Gautam and Khanna [22] with product recovery (USD 296,712.55). The learning in fuzzy environment concept gave positive effect in this this model has shown in Table 3.

Table 3. Representation of the comparison with and without the learning effect.

Models	Order Size Y (Units)	Backorder B (Units)	Joint Profit ($)
Present study without learning in a fuzzy environment ($\phi_4(n^*, Y^*, B^*)$)	887	300	300,300
Present study with learning in a fuzzy environment ($\phi_5(n^*, Y^*, B^*)$)	755	240	300,652

5.3. Sensitivity Analysis

In this section, we discuss the effects of the inventory parameters (shown in Table 2) on the decision variable and total integrated profit according to the change in their values. The sensitivity analysis of the present model is presented in Tables 4–22, and managerial insight is also discussed.

Table 4. Impact of the learning rate on the decision variable and joint total fuzzy profit.

Learning Rate (b)	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
0.100	7	776	247	301,706
0.120	7	766	247	301,277
0.140	7	759	247	300,887
0.150	7	756	247	300,705
0.151	7	756	240	300,687
0.152	7	756	240	300,670
0.153	7	755	240	300,652
0.154	7	755	240	300,652
0.155	7	755	240	300,612

Table 5. Impacts of upper and lower fuzzy deviations on the decision variables and joint fuzzy profit.

Upper Deviation Δ_h^D	Lower Deviation Δ_l^D	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
6000	3000	6	805	257	299,138
10,000	5000	7	755	240	300,652
20,000	10,000	9	652	203	304,527
30,000	15,000	11	547	172	308,537

Table 6. Impacts of the defective percentage parameters on the decision variables and joint fuzzy profit.

Defective Percentage $E[\alpha]$	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
0.01	6	740	260	300,780
0.02	7	745	252	300,698
0.03	7	749	245	300,674
0.04	7	755	240	300,652

Table 7. Impacts of the product recovery parameters on the decision variables and joint fuzzy profit.

Product Recovery E[β]	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
0.1	7	783	154	300,528
0.2	7	777	234	300,590
0.3	7	767	237	300,640
0.4	7	755	240	300,652

Table 8. Impact of the vendor's holding cost on the decision variable and joint fuzzy profit.

Holding Cost H_c	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
1	17	678	214	307,858
2	11	703	223	304,792
3	8	728	231	302,522
4	7	755	240	300,652

Table 9. Impact of the buyer's holding cost of the good items on the decision variable and joint fuzzy profit.

h_1	n	Y (Units)	B (Units)	Joint Fuzzy Profit (USD)
2	5	1020	214	301,539
3	6	919	223	301,267
4	6	849	230	301,036
5	6	796	236	300,838
6	7	755	240	300,652

Table 10. Impact of the buyer's holding cost of the defective items on the decision variables and joint fuzzy profit.

Buyer's Holding Cost of Defec-tive Items h_2	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
0.50	7	765	743	300,696
1.0	7	762	242	300,681
2.56	7	758	241	300,666
2.00	7	755	240	300,652

Table 11. Impact of the shortage cost on the decision variables and joint fuzzy profit.

Shortage Cost S_c	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
2	5	976	537	301,433
4	6	876	407	301,130
8	7	784	278	300,779
10	7	755	240	300,652

Transportation Parameters

Table 12. Impact of the fixed cost of transportation on the decision variables and joint fuzzy profit.

Fixed Cost of Transportation F_t	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
5	14	383	121	302,519
10	11	496	157	301,919
15	9	593	188	301,435
20	8	678	215	301,020
25	7	755	240	300,652

Table 13. Impact of the variable cost of transportation on the decision variables and joint fuzzy profit.

Variable Cost of Transportation V_t	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
0.1	7	743	236	331,024
0.2	7	746	237	323,431
0.3	7	749	238	315,838
0.4	7	752	239	308,245
0.5	7	755	240	300,652

Table 14. Impact of the fixed cost on the joint fuzzy profit.

Fixed Cost C_p	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
1	7	731	232	363,927
2	7	739	235	342,835
3	7	747	237	321,743
4	7	755	240	300,652
5	7	765	243	279,561
6	7	722	245	258,471
7	7	781	248	237,382
8	6	789	251	216,293

Carbon Emission

Table 15. Impact of the fixed carbon emission cost due to production on the decision variables and joint fuzzy profit.

Fixed Carbon Emissions Cost C_{t_1}	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
1	7	745	237	327,016
1.5	7	755	240	300,652
2	7	766	243	274,289
3	6	787	250	221,565

Table 16. Impact of the fixed carbon emission cost due to disposal on the decision variables and joint fuzzy profit.

C_{t_1}	n	Y (Units)	B (Units)	Joint Fuzzy Profit (USD)
1	7	745	237	327,016
1.5	7	755	240	300,652
2	7	766	243	274,289
3	6	787	250	221,565

Table 17. Impact of the vendor′s ordering cost on the decision variables and joint fuzzy profit.

Vendor′s Ordering Cost O_v	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
500	7	755	240	300,652
600	7	755	240	299,730
700	7	755	240	298,860
800	8	754	239	298,036
900	9	754	239	297,249

Table 18. Impact of the buyer′s ordering cost on the decision variables and joint fuzzy profit.

Buyer′s Ordering Cost O_b	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
200	7	755	240	300,652
300	7	751	239	299,766
400	8	748	237	298,930
500	8	745	236	297,324
600	9	742	235	297,383

Table 19. Impact of the material and labor cost on the decision variables and joint fuzzy profit.

Material and Labor Cost C_m	Number of Shipments n	Lot size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) $(\phi_5(n^*,Y^*,B^*))$
30	7	755	240	300,652
31	7	776	247	247,927
32	6	798	254	195,204
33	6	820	261	142,485
34	6	842	268	89,769
35	6	865	275	39,057

Table 20. Impact of the energy cost on the decision variables and joint fuzzy profit.

Energy Cost C_e	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
0.15	7	755	240	300,652
0.16	7	775	241	300,645
0.17	7	758	241	300,638
0.18	7	760	241	300,631
0.19	7	761	241	300,624

Table 21. Impact of the buyer's screening cost on the decision variables and joint fuzzy profit.

Buyer's Screening Cost I_1	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
0.4	7	755	240	300,652
0.5	7	757	241	297,324
0.6	7	759	241	290,107
0.7	7	762	242	284,834
0.8	7	764	243	279,541

Table 22. Impact of the vendor's screening cost on the decision variables and joint fuzzy profit.

Vendor's Screening Cost I_2	Number of Shipments n	Lot Size Y (Units)	Shortages B (Units)	Joint Fuzzy Profit (USD) ($\phi_5(n^*,Y^*,B^*)$)
0.6	7	755	240	300,652
0.7	7	756	240	298,332
0.8	7	757	241	296,012
0.9	7	758	241	293,692
1.0	7	769	241	291,372

5.4. Managerial Insights and Observations

From Table 4, we can see that if the rate of learning increases from 0.10 to 0.153, then the order quantity and total fuzzy profit decrease. After that, if the learning rate increases, then the order quantity, backorder quantity, and total fuzzy profit remain constant, while the number of shipments is constant. This means that the order quantity and backorder quantity are in a maturity situation. From Table 5, we can see that when the values of the upper deviation and lower deviation of the demand rate increase, the demand rate increases, and then the number of shipments and profit increase, but the order quantity and shortage unit decrease, while the other input parameters are constant.

The impacts of the defective percentage and defective percentage of the recovery product on the joint total fuzzy profit and decision variables can be described as follows. From Table 6, if the value of the percentage of defective items increases, then the number of shipments and order quantity increase, while the shortage units and total fuzzy profit decrease. In this regard, from the Table 7, we can observe that the defective percentage of the recovery product increases, and then the shortages and total fuzzy profit increase, while the order quantity decreases, but the number of shipments remains constant. From Table 8, we can see that if the vendor's holding cost increases, then the number of shipments and total fuzzy profit decrease, but the order quantity and shortages increase.

From Table 9, we can see that if the buyer's holding cost of the good-quality items increases, then the number of shipments and total fuzzy profit decrease, but the order quantity and shortages increase.

Table 10, we can see that if the buyer's holding cost of the imperfect-quality items increases, the lot size and total fuzzy profit decrease, but the shortage units increase, while the that numbers of shipment is approximately constant. It can easily be seen from Table 11 that when the value of the shortage cost increases, then the lot size, shortage units, and total fuzzy profit decrease, while the number of shipments becomes constant.

It can be analyzed from Table 12 and Figure 12 that when the value of the fixed transportation cost increases, then the lot size and shortage units increase, but the total fuzzy profit and number of shipments decrease. Similarly, as shown in Table 13 and Figure 13, when the value of the variable transportation cost increases, then the lot size and shortage units increase, but the total fuzzy profit increases, while the number of shipments remains unchanged. Table 14, show that when the value of the fixed unit cost increases, the lot size and shortages increase as the number of shipments and total fuzzy profit decrease. From the Figure 14, carbon emission cost due to production increases then total fuzzy profit increases. From the Figure 15, if lower and upper deviation increase then total fuzzy profit increases. Table 15, we can see that if the fixed carbon emission cost due production increases, then the number of shipment remains almost constant, and the total fuzzy profit decreases as the order lot size and shortages increase. From Table 16 and Figure 16, we can see that when the value of the carbon emission cost due to disposal increases, the order quantity increases, and total fuzzy profit decreases, while the number of shipments and shortages remain constant. We analyzed the effects of the vendor's ordering cost on the decision variables and total fuzzy profit, and it can easily be seen from the Table 17 that if the value of the vendor's ordering cost increases, then the number of shipments increases following some values of the vendor's ordering, whereas the shortages, lot size, and total fuzzy profit decrease. Observing the buyer's ordering cost, from Table 18, we can see that if the buyer's ordering cost increases, then the lot size, shortages, and total fuzzy profit decrease, and the number of shipments increases. From Table 19 and Figure 17, it is clear that if the value of the material and labor cost increases, then the order lot size and shortage units increase, but the joint total profit decreases, and the number of shipments remains constant. It can be observed from Table 20 and Figure 18 that when the value of the energy cost increases, then the total joint fuzzy profit increases, but the order quantity and shortages increase, whereas the number of shipments remains constant.

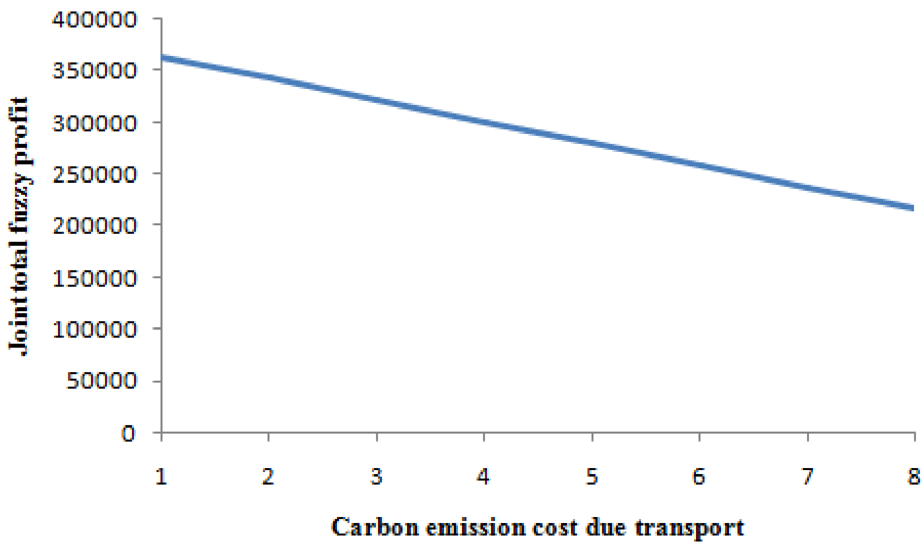

Figure 12. The carbon emission cost due to transportation vs. the total fuzzy profit.

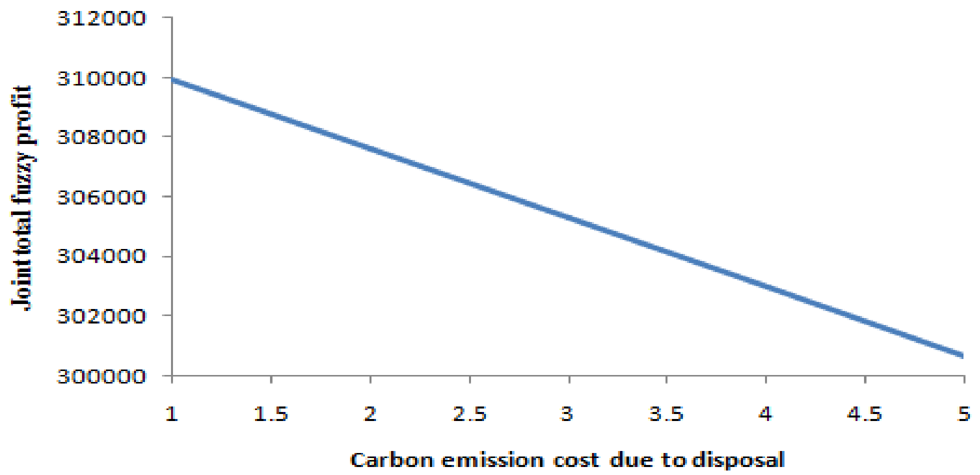

Figure 13. The carbon emission cost due to disposal vs. the total fuzzy profit.

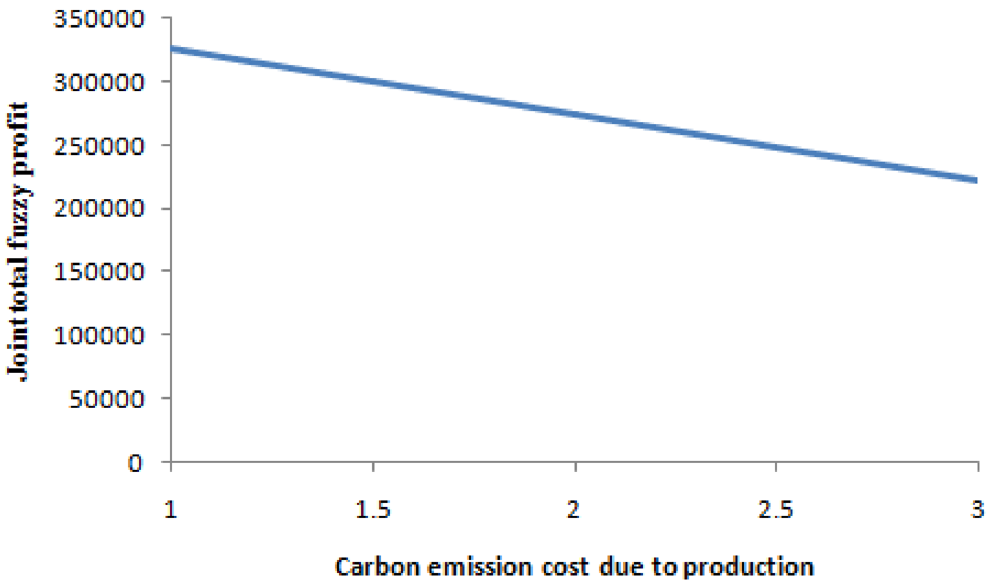

Figure 14. The carbon emission cost due to production vs total fuzzy profit.

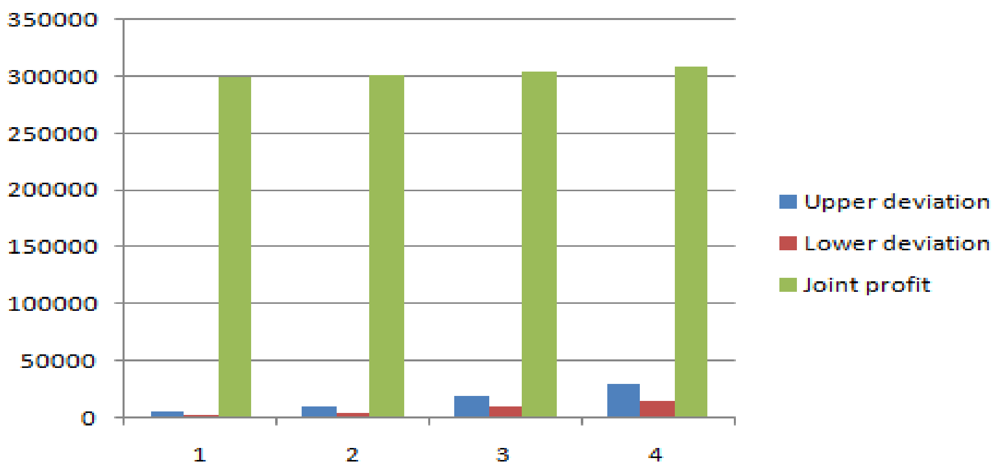

Figure 15. Upper and lower deviation of the demand rate vs. the joint total fuzzy profit.

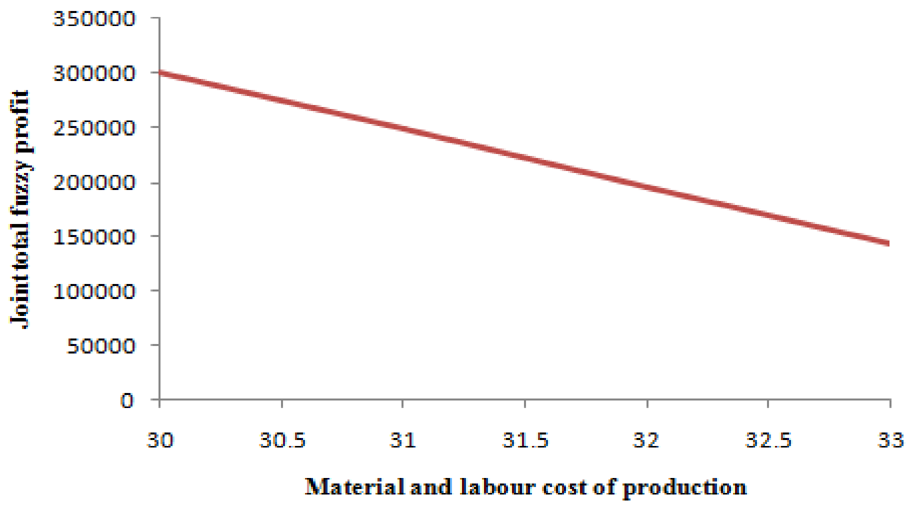

Figure 16. Material and labor cost vs. the joint total fuzzy profit.

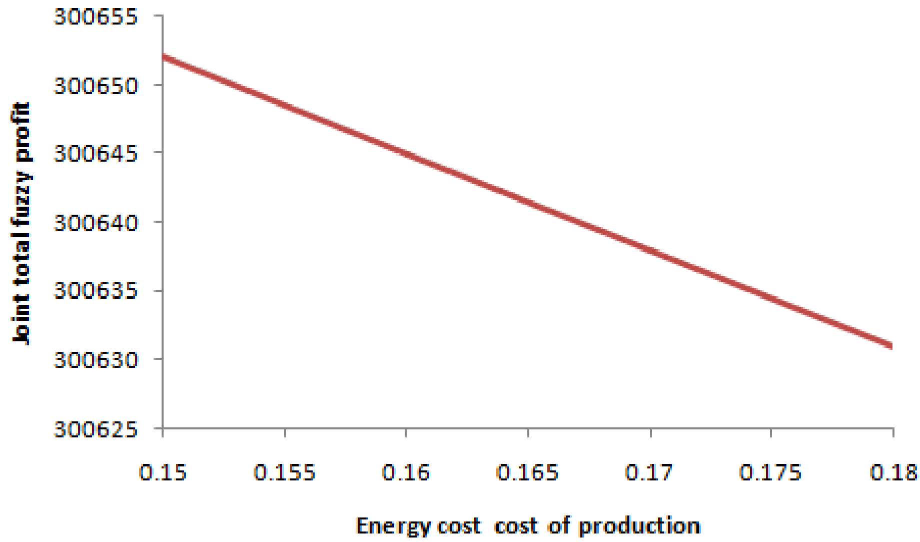

Figure 17. Energy cost of production vs. the joint total fuzzy profit.

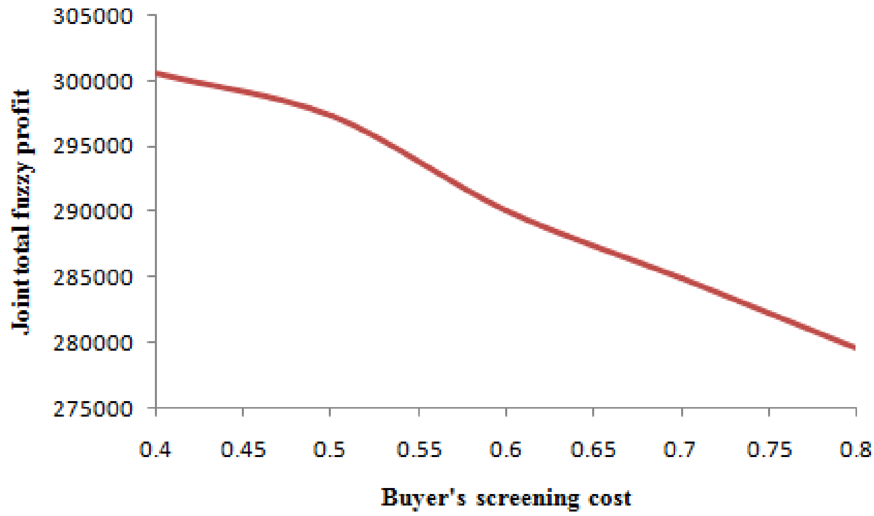

Figure 18. Buyer's screening cost vs. the joint total fuzzy profit.

It can easily be seen from Table 21 and Figure 18 that if the value of the buyer's screening cost increases, then the total joint fuzzy profit increases, but the order quantity and shortages increase, whereas the number of shipments remains constant. It can easily be seen from Table 22 and Figure 19 that if the value of the buyer's screening cost increases, then the total joint fuzzy profit increases, but the order quantity and shortages increase, whereas the number of shipments remains constant.

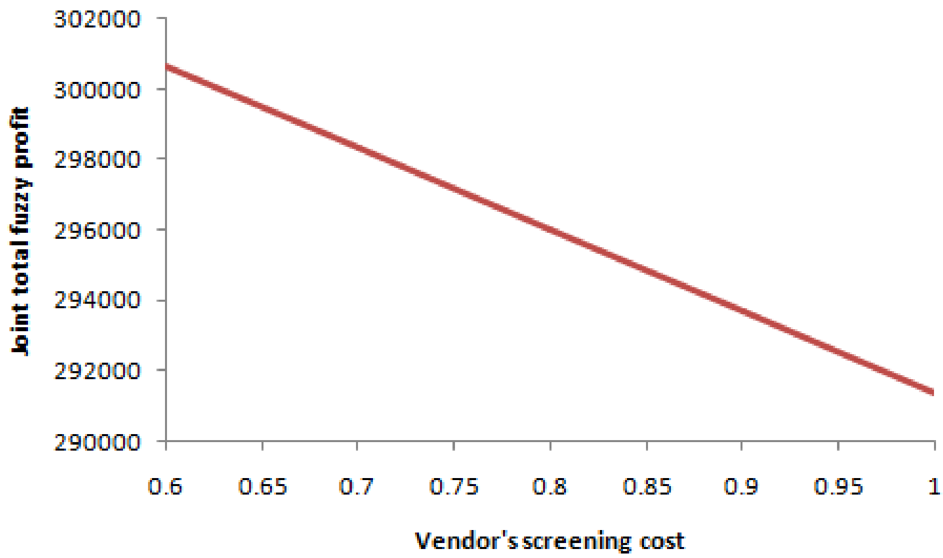

Figure 19. Vendor's screening cost vs. the joint total fuzzy profit.

6. Conclusions

In this paper, we analyzed the impacts of learning and carbon emissions on an integrated green supply chain model for defective items in a fuzzy environment. Our study revealed that several sustainable supply chain models would be helpful for both the vendor and buyer in cases where the demand rate takes the form of a triangular fuzzy number. From the managerial insight and observations, we obtained more information about the inventory parameters in regard to the decision variables and joint total fuzzy profit. This information is more beneficial for the supply chain players. The learning concept is a good decision maker in this model. The buyer wants a lesser order quantity obtained more frequently and to earn more profit. The vendor will yield less production when the demand rate is imprecise in nature, as this may pose a greater risk for sale. A joint model was formulated by taking the vendor's and buyer's strategies into account, respectively. The aim was to optimize the joint total profit $\phi_5(n, Y, B)$ with the effect of learning (b) in a fuzzy environment for the integrated supply chain value by simultaneously optimizing the number of shipments (n), order quantity value (Y), and the shortage amount (B). The formulated model was compared with and without learning in a fuzzy environment and is discussed in the Table 3. The results revealed demand deviation, i.e., when the values of the upper deviation and lower deviation of the demand rate and the demand rate increase, then the number of shipments and profit increase, but the order quantity and shortage units decrease, while the other input parameters are constant. The numerical analysis and sensitivity analysis were used to explore the model's viability. The present study could be developed with the credit financing policy and applied in the textile industry and in many science laboratories, and this study is also useful for omnichannel.

7. Limitations and Future Research Strategy of Our Present Study

The limitations and future scope of the present paper are explained in this section. The present proposed model is optimized for a supply chain where the rate of demand follows the triangular fuzzy number. Furthermore, the researcher can investigate new policies to manage waste and recovered items. The limitation of the model is that the inspection is performed at the vendor's and buyer's ends in the supply chain. The inspection process may have errors or human error, and the carbon emission cost, as well as the carbon emissions, can be considered as a decision variable in the newest version.

8. Applications of Our Present Study

The demand rate of any product is not fixed. In general, it varies according to time. By considering this concept, we studied the supply chain model in cases when the demand rate is imprecise in nature. The present work could be beneficial in the field of omnichannel environments where the demand rate is imprecise in nature and the buyer uses the strategy of product recovery management, performing the firsthand inspection of the lot received from the vendor. Online shopping on Amazon, Flipkart, and Snapdeal, etc., are good examples of the present research work's applications.

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Conflicts of Interest: The authors declare that there was no conflict of interest.

Appendix A

Appendix A.1 Notations and Assumptions

Notations

Y(Decision variable): Lot size (units)

B(Decision variable): Shortage inventory level of quantity (units)

$Z = Y - B$: Positive inventory level (units)

n(Decisionvariable): Number of shipments

D: Demand rate (units/year)

$\widetilde{D}$: Fuzzy demand rate (units/year)

Δ_h^D: Upper deviation of fuzzy demand rate (units/year)

Δ_l^D: Lower deviation of fuzzy demand rate (units/year)

b: Learning slope

μ: Learning supporting parameter

α: The proportion of defective products in a lot which are considered to obey a uniform distribution, with the probability density function (Pdf) $f(\alpha)$.

β: The proportion of defective items among the used products which are considered to obey a uniform distribution, with the probability density function (Pdf) $f(\beta)$.

$\gamma = \alpha + \beta$: The total proportion of defective items (in a lot and among the used products)

w: The rate of screening in the buyer model (units per year)

n: Number of shipments (integer)

$A_c(n)$: Buyer's ordering cost, which is a decreasing function of the shipment (n)

$H_1(n)$: Buyer's holding cost for the good items, which is a decreasing function of the shipment (n) (USD/unit/year)

$H_2(n)$: Buyer's holding cost for the defective items, which is a decreasing function of the shipment (n) (USD/unit/year)

I_1: The cost associated with the inspection of the unit on the buyer side (dollar per unit)

s_c: The cost associated with the shortage of units for the buyer (dollar per unit per year)

C_c: The cost associated with the collection of units for the buyer (dollar per unit per year)

C_i: The cost associated with the incentive unit for the buyer (dollar per unit per year)

T_1: The time associated with the inventory level, where the stock will be zero (years)

T_2: The time associated with the time required for the build-up shortage time (years)

T_3: The time associated with the finished shortage time (years)

T_4: The time associated with the inspection on the buyer side (in years)

η: The rate of production (unit per year)

O_v: The ordering cost associated with the shipment in the production phase for the vendor (dollar per shipment)

c_m : The cost associated with the material sources and labor work in the production phase for the vendor (dollar per cycle)

c_e : The cost associated with the energy in the production phase for the vendor (dollar cycle)

ξ: Standard power system when production starts (kW)

k: Variable component of the power consumption during production (kWh per unit)

c_p : The fixed cost associated with the carbon emissions in the production phase (dollar per disposed unit)

c_{t_1} : The fixed cost associated with the carbon emissions from the transportation (dollar per transport)

c_{t_2} : The fixed cost associated with the carbon emissions from disposed unit (dollar per disposed unit)

c_v: The variable cost associated with the carbon emission unit (dollar per unit) on the vendor side

V_t: The variable cost associated with the transportation (dollar per unit)

F_t: The fixed cost associated with the transportation (dollar per transport)

H_c: The cost associated with the holding unit of the vendor (dollar per unit per year)

I_2: The cost associated with the screened defective items on the vendor side (dollar per imperfect quality item)

r_w: The cost associated with the re-worked units (dollar per unit)

r_u: The cost associated with the reused units (dollar per unit)

r_c: The cost associated with the recycled unit (dollar per unit)

d_w: The cost associated with the disposed units (dollar per unit)

w_c: The cost associated with the warranty unit (dollar per imperfect unit)

i_c : The cost associated with the incentive unit (dollar per used unit)

p_1: The price associated with the re-worked product in the supply chain (dollar per re-worked unit)

p_2: The price associated with the derived items in the supply chain (dollar per derived unit)

p_3: The price associated with the recycled products in the supply chain (dollar per re-cycled unit)

T_p: Total production runtime (years)

T_n: Whole time period for the non-production phase (in years)

T: Time for one shipment (in years)

T_c: Whole cycle time (in years)

η_1: Fraction of re-workable goods with pdf$f(\eta_1)$

η_2: Fraction of reusable goods with pdf$f(\eta_2)$

η_3: Fraction of recyclable goods with pdf$f(\eta_3)$

η_4: Fraction of waste with the probability density function $f(\eta_4)$, where $\eta_1 + \eta_2 + \eta_3 + \eta_4$= 1, and pdf stands for the probability density function

$E[.]$: Expected value operator

$TC_v(n, Y)$: Total vendor cost (in USD)

$TR_v(n, Y)$: Total vendor revenue (in USD)

$TP_v(n, Y)$: Total vendor revenue (in USD)

$TC_b(n, Y)$: Total vendor cost (in USD)

$TR_b(n, Y)$: Total vendor revenue (in USD)

$TP_b(n, Y)$: Total vendor revenue (in USD)

$\phi_1(n, Y, B)$: Total defuzzified vendor profit (in USD)

$\phi_2(n, Y, B)$: Total defuzzified buyer profit (in USD)

$\phi_3(n, Y, B)$: Total joint defuzzified profit (in USD) for the supply chain

$\phi_4(n, Y, B)$: Total joint defuzzified profit (in USD) per unit time for the supply chain system

$\phi_5(n, Y, B)$: Joint total fuzzy profit per unit time for the supply chain system under learning in a fuzzy environment (in USD)

$\phi_5(n^*Y^*, B^*)$: Optimized joint total fuzzy profit per unit time for the supply chain system under learning in a fuzzy environment (in USD)

Appendix A.2 Mathematical Formulation

Due to the large size of the equation, we assumed some new notations in Section 4.2.3 and Equation (48), which is given below:

$$
\begin{aligned}
E[\phi_3(n,Y,B)] = \; & [c_1(1-E[\alpha])nY + p_1\eta_1 E[\gamma]nY + p_3\eta_3 E[\gamma]nY + p_2\eta_2 E[\gamma]nY] \\
& - \Bigg[O_v + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{2n(n-1)(1-E[\alpha])Y^2}{4D+\Delta_h{}^D-\Delta_l{}^D}\right] \\
& + w_c E[\alpha]nY + i_c E[\beta]nY + I_2 E[\gamma]nY + r_w\eta_1 E[\gamma]nY + r_u\eta_2 E[\gamma]nY + r_c\eta_3 E[\gamma]nY \\
& + d_w\eta_4 E[\gamma]nY + c_p nY + nc_{t_1} + nYc_{t_1}(1+E[\gamma]) + c_{t_2}\eta_4 E[\gamma]nY + c_e T_p\left(\frac{\xi+K\eta}{nY}\right)\Bigg] \\
& + [c_2(1-E[\alpha])nY + w_c E[\alpha]nY + i_c E[\beta]nY] \\
& - \Bigg[c_1 nY + A_o + \frac{A_2}{n^\mu} + I_1 nY + (h_o \\
& + \frac{h_1}{n^\mu}) \left[n \left\{ \frac{2Y(1-E[\alpha])\left(w(1-E[\alpha]) - -\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right) - wB(1-E[\alpha])}{2\left(w(1-E[\alpha]) - \frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)} \right\}(T_3) \right. \\
& \left. + \frac{n}{2}\left\{ \frac{2Y(1-E[\alpha])\left(w(1-E[\alpha]) - -\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right) - wB(1-E[\alpha])}{2\left(w(1-E[\alpha]) - \frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4}\right)} \right\}(T_1 - T_3) \right] + (h_o \\
& + \frac{h_2}{n^\mu}\Big) \left[\frac{4n\alpha Y^2(1-E[\alpha])}{4D+\Delta_h{}^D-\Delta_l{}^D}\right] + c_c E[\beta]nY/2 \\
& + s_c \left[\frac{2nB^2}{4D+\Delta_h{}^D-\Delta_l{}^D} + \frac{nB^2}{2w\left(1-E[\alpha] - -\frac{4D+\Delta_h{}^D-\Delta_l{}^D}{4w}\right)} \right] + c_i E[\beta]nY
\end{aligned}
$$

and $E[T] = \frac{4nY(1-E[\alpha])}{4D+\Delta_h{}^D-\Delta_l{}^D}$.

and in Equation (52):

$$
\begin{aligned}
E_L[\phi_3(n,Y,B)] \; & = [c_1(1-E[\alpha])nY + p_1\eta_1 E[\gamma]nY + p_3\eta_3 E[\gamma]nY + p_2\eta_2 E[\gamma]nY] \\
& - \Bigg[O + c_m\eta T_p + nF_t + nYV_t(1+\gamma) + H_c\left[\frac{nY^2}{\eta} - \frac{n^2Y^2}{\eta} + \frac{2n(n-1)(1-E[\alpha])Y^2}{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}\right] \\
& + w_c E[\alpha]nY + i_c E[\beta]nY + I_2 E[\gamma]nY + r_w\eta_1 E[\gamma]nY + r_u\eta_2 E[\gamma]nY + r_c\eta_3 E[\gamma]nY \\
& + d_w\eta_4 E[\gamma]nY + c_p nY + nc_{t_1} + nYc_{t_1}(1+E[\gamma]) + c_{t_2}\eta_4 E[\gamma]nY + c_e T_p\left(\frac{\xi+K\eta}{nY}\right)\Bigg] \\
& + [c_2(1-E[\alpha])nY + w_c E[\alpha]nY + i_c E[\beta]nY] \\
& - \Bigg[c_1 nY + A_o + \frac{A_2}{n^\mu} + I_1 nY + (h_o \\
& + \frac{h_1}{n^\mu}) \left[n \left\{ \frac{2Y(1-E[\alpha])\left(w(1-E[\alpha]) - -\frac{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4}\right) - wB(1-E[\alpha])}{2\left(w(1-E[\alpha]) - \frac{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4}\right)} \right\}(T_3) \right. \\
& \left. + \frac{n}{2}\left\{ \frac{2Y(1-E[\alpha])\left(w(1-E[\alpha]) - -\frac{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4}\right) - wB(1-E[\alpha])}{2\left(w(1-E[\alpha]) - \frac{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4}\right)} \right\}(T_1 - T_3) \right] \\
& + \left(h_o + \frac{h_2}{n^\mu}\right) \left[\frac{4n\alpha Y^2(1-E[\alpha])}{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}\right] + c_c E[\beta]nY/2 \\
& + s_c \left[\frac{2nB^2}{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)} + \frac{nB^2}{2w\left(1-E[\alpha] - -\frac{4D+\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4w}\right)} \right] \\
& + c_i E[\beta]nY\Bigg]
\end{aligned}
\tag{A1}
$$

and $E_L[T] = \frac{nY(1-E[\alpha])}{D+\frac{\left((i-1)\frac{365}{n}\right)^{-b}\left(\nabla^D_{h.i}-\nabla^D_{l.i}\right)}{4}}$.

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