mathematics

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Intuitionistic Fuzzy Metric-like Spaces and Fixed-Point Results

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Abstract: The objective of this paper is to describe the concept of intuitionistic fuzzy metric-like spaces. This space is an extension of metric-like spaces and fuzzy metric spaces, and intuitionistic fuzzy metric spaces. We discuss convergence sequences, contractive mapping and some fixed-point theorems in intuitionistic fuzzy metric-like space. We also give explanations, examples and counterexamples to validate the superiority of these results. Our results provide a substantial extension of several important results from fuzzy metric-like spaces.

Keywords: intuitionistic fuzzy set; metric-like spaces; intuitionistic fuzzy metric-like spaces; fixed-point theory

MSC: 03B52; 47H10; 54H25

1. Introduction

Metric spaces, obtained with the help of a metric function defined on a set, are prominent spaces in mathematics, especially in topology. They provide a powerful tool for generalizing some properties on arbitrary sets. Metric spaces were studied originally on classical sets by Maurice Frechet in 1906. Later, many generalizations of the concept of metric spaces were made on the different types of sets. In addition, different metric functions have been attained. It would not be wrong to separate these concepts as a set and metric functions. One of these generalizations about sets was constructed on fuzzy sets. The notion of fuzzy sets was first introduced by computer scientist Zadeh [1] in 1965 to introduce a new definition of approach to uncertain data. The main difference between a classical set and fuzzy set is that a fuzzy set allows the gradual assessment of the membership of the elements in a set. This approach has made a prominent improvement in dealing with situations that have fields such as engineering and natural sciences involving uncertainty and undesirability. After that, another generalization was obtained by combining fuzzy sets and metric concepts, called a fuzzy metric space [2]. George and Veeramani ([3,4]) modified the concept of fuzzy metric spaces in the sense of Kramosil and Michálek by using continuous t-norms, and they obtained a stronger version of the fuzzy metric space. They obtained a Hausdorff topology and the first countable topology on modified fuzzy metric spaces. Grabiec [5] defined the fuzzy version of the Banach contraction principle in fuzzy metric spaces given by Kramosil and Michalek. Gregori et al. [6] gave some examples and applications for fuzzy metric. Although the concept of the fuzzy set was initially sufficient to cope with uncertainty, modelling the problems of the world we live in with fuzzy sets has started to be insufficient. The study that fills this gap in the literature was resolved by Atanassov [7] in 1986. Atanassov introduced the intuitionistic fuzzy sets. These sets compared with fuzzy sets provide more flexible study possibilities for dealing with uncertain situations since an intuitionistic fuzzy set includes both membership degrees and non-membership degrees of the element that belongs in a set. Later, Park [8] introduced the concept of intuitionistic fuzzy metric spaces inspired by the idea of Atanassov's intuitionistic fuzzy sets. Many developments have been studied on fuzzy metric spaces, such

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Citation: Onbaşıoğlu, Ş.; Pazar Varol, B. Intuitionistic Fuzzy Metric-like Spaces and Fixed-Point Results. *Mathematics* **2023**, *11*, 1902. https://doi.org/10.3390/math11081902

Academic Editors: Vishal Gupta and Pooja Dhawan

Received: 23 March 2023
Revised: 11 April 2023
Accepted: 13 April 2023
Published: 17 April 2023

as fixed-point theorems ([9–13]) and convergence ([14–16]). Same structures also have been investigated on intuitionistic fuzzy metric spaces ([17–21]). Furthermore, we see in recent papers that studies on these structures will preserve popularity ([22–27]).

Matthews [28] introduced the notion of a partial metric space and Harandi [29] introduced metric-like space as a generalization of a partial metric space and metric space. Harandi also studied some fixed-point results in such spaces. Both of these metrics are presented on classical sets. By a metric-like space, we mean a pair (X, σ), where X is a nonempty set and $\sigma : X \times X \to IR$ satisfies all conditions of a metric except that $\sigma(x, x)$ may be different from zero for $x \in X$. In 2014, Shukla et al. [30] introduced the fuzzy version of the metric-like space in the literature. Thus, this notion generalized the concept of fuzzy metric spaces given by George and Veeramani. They proved some fixed-point results for fuzzy contractive mappings on fuzzy metric-like spaces. Such concepts may form a future frame for extending the already established fixed-point results of the fuzzy metric to the metric-like structure.

This paper aims to present the concept of "intuitionistic fuzzy metric-like space" by using the approach in [29] and study fixed-point theorems for contractive mappings in intuitionistic fuzzy metric-like spaces. Intuitionistic fuzzy metric spaces and intuitionistic fuzzy metric-like spaces are two different mathematical frameworks. Both of them can be used to model distances and measure similarity or dissimilarity between objects, depending on the nature and degree of uncertainty or ambiguity in data. The results obtained in both approaches provide accurate results in their own nature. When the results in the intuitionistic fuzzy metric spaces obtained with the metric structure are changed with the metric-like structure, we attain a more flexible working environment due to the metric-like structure's feature because in these spaces, even if the objects are the same, it is taken into account that the distance between them can be different when they are evaluated according to a certain parameter. For this reason, we aim to fill this gap in the literature by combining these structures which are compatible with existing approaches in the literature.

The structure of the paper is as follows. After the preliminaries, in Section 3, the concept of intuitionistic fuzzy metric-like space is defined and this notion is explained with the help of intelligible examples. In addition, the concept of the convergent sequence is given in intuitionistic fuzzy metric-like spaces and all these definitions, theorems and examples are presented in detail. Section 4 concerns the constructing and proving of common fixed-point theorems in intuitionistic fuzzy metric-like space. The obtained results are compatible with existing approaches in the literature.

2. Preliminaries

In this section, we give some basic definitions and notions to explain the main results. Throughout the paper, by ∞ we mean $+\infty$; IR and IN will denote the set of all real numbers and the set of all positive integer numbers, respectively.

Definition 1 ([29])**.** *Let $X \neq \emptyset$. A mapping $\sigma : X \times X \to IR^+$ is called metric-like on X if the following hold:*

(ML1) $\sigma(x, y) = 0 \Rightarrow x = y$;
(ML2) $\sigma(x, y) = \sigma(y, x)$;
(ML3) $\sigma(x, z) \leq \sigma(x, y) + \sigma(y, z)$.
The pair (X, σ) is called a metric-like space on X.

Definition 2 ([7])**.** *An intuitionistic fuzzy set A is defined by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote membership and non-membership functions, respectively. $\mu_A(x)$ and $\nu_A(x)$ are membership and non-membership degrees of each element $x \in X$ to the intuitionistic fuzzy set A and $\mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.*

Definition 3 ([31])**.** *A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous t-norm if $*$ satisfies the following:*

(1) $a * 1 = a, \ \forall a \in [0,1]$;
(2) $a * b = b * a$ *and* $a * (b * c) = (a * b) * c \ \forall a, b, c \in [0,1]$;
(3) *If* $a \leq c$ *and* $b \leq d$, *then* $a * b \leq c * d, \ \forall a, b, c, d \in [0,1]$;
(4) $*$ *is continuous.*

Definition 4 ([31]). *A binary operation* $\diamond : [0,1] \times [0,1] \to [0,1]$ *is called a continuous t-conorm if* $\diamond$ *satisfies the following:*

(1) $a \diamond 0 = a, \ \forall a \in [0,1]$;
(2) $a \diamond b = b \diamond a$ *and* $a \diamond (b \diamond c) = (a \diamond b) \diamond c \ \forall a, b, c \in [0,1]$;
(3) *If* $a \leq c, \ b \leq d$, *then* $a \diamond b \leq c \diamond d, \ \forall a, b, c, d \in [0,1]$;
(4) $\diamond$ *is continuous.*

Note that $a * b = min\{a, b\}$, $a \diamond b = max\{a, b\}$, $a * b = ab$ and $a \diamond b = min\{a + b, 1\}$ are basic examples of continuous t-norms and continuous t-conorms for all $a, b \in [0,1]$.

From the previous two definitions, we see that if $r_1 > r_2$, then there exist $r_3, r_4 \in (0,1)$ such that $r_1 * r_3 \geq r_2$ and $r_2 \diamond r_4 \leq r_1$.

Definition 5 ([3]). *Let* $X \neq \emptyset$. *Assume a triplet* $(X, M, *)$ *where* $*$ *is a continuous t-norm and* M *is a fuzzy set on* $X \times X \times (0, \infty)$. *If* $(X, M, *)$ *satisfies the following conditions for all* $x, y, z \in X$ *and* $t, s > 0$;
(FM1) $M(x, y, t) > 0$,
(FM2) $M(x, y, t) = 1$ *if and only if* $x = y$,
(FM3) $M(x, y, t) = M(y, x, t)$,
(FM4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(FM5) $M(x, y, .) : (0, \infty) \to [0,1]$ *is continuous,*

Then $(X, M, *)$ is called fuzzy metric space. M with $*$ is called fuzzy metric on X.

Definition 6 ([30]). *Let* $X \neq \emptyset$. *A triplet* $(X, F, *)$ *is called fuzzy metric-like space (for short, FMLS) if* $*$ *is a continuous t-norm and* F *is a fuzzy set on* $X \times X \times (0, \infty)$ *satisfy the following conditions for all* $x, y, z \in X$ *and* $t, s > 0$;
(FML1) $F(x, y, t) > 0$,
(FML2) $F(x, y, t) = 1 \Rightarrow x = y$,
(FML3) $F(x, y, t) = F(y, x, t)$,
(FML4) $F(x, y, t) * F(y, z, s) \leq F(x, z, t + s)$,
(FML5) $F(x, y, .) : (0, \infty) \to [0,1]$ *is continuous.*

Definition 7 ([8]). *Let* M *and* N *be fuzzy sets on* $X^2 \times (0, \infty)$, $*$ *be a continuous t-norm,* $\diamond$ *be a continuous t-conorm. If* M *and* N *satisfy the following conditions, we say that* (M, N) *is intuitionistic fuzzy metric on* X:
(IFM1) $M(x, y, t) + N(x, y, t) \leq 1$,
(IFM2) $M(x, y, t) > 0$,
(IFM3) $M(x, y, t) = 1$ *if and only if* $x = y$,
(IFM4) $M(x, y, t) = M(y, x, t)$,
(IFM5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(IFM6) $M(x, y, .) : (0, \infty) \to (0,1]$ *is continuous,*
(IFM7) $N(x, y, t) < 1$,
(IFMF8) $N(x, y, t) = 0$ *if and only if* $x = y$,
(IFM9) $N(x, y, t) = N(y, x, t)$,
(IFM10) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$,
(IFM11) $N(x, y, .) : (0, \infty) \to (0,1]$ *is continuous.*

A five-tuple $(X, M, N, *, \diamond)$ is called intuitionistic fuzzy metric space (for short, IFMS).

The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 1. *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space, then $(X,M,*)$ is a fuzzy metric space. Conversely, if $(X,M,*)$ is a fuzzy metric space, then $(X,M,1-M,*,\diamond)$ is an intuitionistic fuzzy metric space, where $a\diamond b = 1-((1-a)*(1-b))$, $\forall a,b\in[0,1]$.*

Definition 8 ([8])**.** *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $t>0, r\in(0,1)$ and $x\in X$. The set $B_x(r,t)=\{y\in X : M(x,y,t)>1-r,\ N(x,y,t)<r\}$ is said to be an open ball with center x, radius r with respect to t.*

$\{B_x(r,t) : x\in X, r\in(0,1), t>0\}$ generates a topology $\tau_{(M,N)}$ called the (M,N) topology.

Definition 9 ([8])**.** *Let $(X,M,N,*,\diamond)$ be an intuitionistic fuzzy metric space and $(x_n)\subset X$ be a sequence.*

(i) (x_n) *is called convergent to x if for all $t>0$ and $r\in(0,1)$ there exists $n_0\in IN$ such that $M(x_n,x,t)>1-r$, $N(x_n,x,t)<r$ for all $n\geq n_0$. ($M(x_n,x,t)\to 1$ and $N(x_n,x,t)\to 0$ as $n\to\infty$ for each $t>0$).*
It is denoted by $x_n\to x$ as $n\to\infty$.

(ii) (x_n) *is called Cauchy sequence if for $t>0$ and $r\in(0,1)$, there exists $n_0\in IN$ such that $M(x_n,x_m,t)>1-r$, $N(x_n,x_m,t)<r$ for all $n,m\geq n_0$.*

(iii) $(X,M,N,*,\diamond)$ *is called (M,N)-complete if every Cauchy sequence is convergent.*

3. Intuitionistic Fuzzy Metric-like Space

In this section, we introduce the intuitionistic fuzzy metric-like spaces and study some properties of them to support the structure. We give detailed examples and also define convergent sequences in intuitionistic fuzzy metric-like spaces.

Definition 10. *A five-tuple $(X,F,G,*,\diamond)$ is called intuitionistic fuzzy metric-like space (for short, IFMLS) if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and F,G are fuzzy sets on $X^2\times(0,\infty)$ satisfying the following conditions, for all $x,y,z\in X$ and $s,t>0$;*

(IFML1) $F(x,y,t)+G(x,y,t)\leq 1,$
(IFML2) $F(x,y,t)>0,$
(IFML3) $F(x,y,t)=1\Rightarrow x=y,$
(IFML4) $F(x,y,t)=F(y,x,t),$
(IFML5) $F(x,y,t)*F(y,z,s)\leq F(x,z,t+s),$
(IFML6) $F(x,y,.) : (0,\infty)\to(0,1]$ *is continuous,*
(IFML7) $G(x,y,t)<1,$
(IFML8) $G(x,y,t)=0\Rightarrow x=y,$
(IFML9) $G(x,y,t)=G(y,x,t),$
(IFML10) $G(x,y,t)\diamond G(y,z,s)\geq G(x,z,t+s),$
(IFML11) $G(x,y,.) : (0,\infty)\to[0,1)$ *is continuous.*

F and G are called an intuitionistic fuzzy metric-like on X with $*$ and $\diamond$.

If we compared the definition of IFMS and IFMLS according to the condition (IFML3) and (IFML8), we observe that in an IFMLS, $F(x,x,t)$ may be less than from 1 and $G(x,x,t)$ may be greater than from 0.

Every IFMS is IFMLS with unit self distance, that is, with $F(x,x,t)=1$ and $G(x,x,t)=0$ for all $t>0$, $x\in X$.

In the conditions (IFM3) and (IFM8), we see that when $x=y$, the degrees of nearness and the degree of non-nearness of x and y are 1 and 0, respectively. However, the conditions (IFML3) and (IFML8) indicate that when $x=y$, the value of $F(x,x,t)$ may be less than '1' and the value of $G(x,x,t)$ may be greater than '0'.

Definition 11. *Let $(X, F, G, *, \diamond)$ be an IFMLS. For $x \in X$, $r \in (0,1)$, $t > 0$, we define the open ball with center x, radius r with respect to t like $B(x,r,t) = \{y \in X : F(x,y,t) > 1-r,\ G(x,y,t) < r\}$.*

Therefore, $T_{(F,G)} = \{W \subset X : \forall x \in W\ \exists r \in (0,1),\ t > 0$ such that $B(x,r,t) \subset W\}$ is a topology on X.

Remark 2.

(1) If $(X, F, G, *, \diamond)$ is an IFMLS, then $(X, F, *)$ is an FMLS in the sense of Shukra et al. [30].
(2) Every FMLS $(X, F, *)$ is an IFMLS of the form $(X, F, 1-F, *, \diamond)$, where $a \diamond b = 1 - [(1-a) * (1-b)]$ for all $a, b \in [0,1]$.

Lemma 1. *Let (X, σ) be a metric-like space and $s, t > 0$. The following inequality holds $\frac{\sigma(x,z)}{(t+s)^n} \leq max\{\frac{\sigma(x,y)}{t^n}, \frac{\sigma(y,z)}{s^n}\}$, for all $n \geq 1$.*

Proof. We separate three cases:

(1) $\sigma(x,z) \leq \sigma(x,y)$;
(2) $\sigma(x,z) \leq \sigma(y,z)$;
(3) $\sigma(x,z) > \sigma(x,y)$ and $\sigma(x,z) > \sigma(y,z)$.

The inequality is obvious in cases (1) and (2). Assume (3) is satisfied. Then, $\sigma(x,z) \leq \sigma(x,y) + \sigma(y,z)$. Without loss of generality we can suppose that $\sigma(x,z) = \sigma(x,y) + \sigma(y,z)$. Since $\sigma(x,z) > \sigma(x,y)$, there exist $\beta \in (0,1)$ such that $\sigma(x,y) = \beta\sigma(x,z)$. Then, we get $\sigma(y,z) = (1-\beta)\sigma(x,z)$. Hence, the inequality in Lemma 1 becomes

$$\frac{\sigma(x,z)}{(t+s)^n} \leq max\{\frac{\beta\sigma(x,z)}{t^n}, \frac{1-\beta}{\sigma}(x,z)s^n\}$$

and we need to show that $\frac{1}{(t+s)^n} \leq max\{\frac{\beta}{t^n}, \frac{1-\beta}{s^n}\}$. To do this, consider the functions $f(\beta) = \frac{t^n}{\beta}$ and $g(\beta) = \frac{s^n}{1-\beta}$ which are decreasing and increasing, respectively. The largest value of $min\{\frac{t^n}{\beta}, \frac{s^n}{1-\beta}\}$ is $t^n + s^n$ that is taken when $f(\beta) = g(\beta)$ where $\beta = \frac{t^n}{t^n+s^n}$. Then, $(t+s)^n \geq t^n + s^n = f(\frac{t^n}{t^n+s^n}) \geq min\{\frac{t^n}{\beta}, \frac{s^n}{1-\beta}\}$ implies $\frac{1}{(t+s)^n} \leq \frac{1}{t^n+s^n} \leq max\{\frac{\beta}{t^n}, \frac{1-\beta}{s^n}\}$.

If $\sigma(x,z) < \sigma(x,y) + \sigma(y,z)$, then there exists $\lambda \in (0,1)$ such that $\frac{1}{\lambda}\sigma(x,z) = \sigma(x,y) + \sigma(y,z)$. Moreover, $\sigma(x,z) > \sigma(x,y)$ and $\sigma(x,z) > \sigma(y,z)$ imply $\frac{1}{\lambda}\sigma(x,z) > \sigma(x,y)$ and $\frac{1}{\lambda}\sigma(x,z) > \sigma(y,z)$.

Therefore, from the above case, we obtain $\frac{1}{\lambda}\frac{\sigma(x,z)}{(t+s)^n} \leq max\{\frac{\sigma(x,y)}{t^n}, \frac{\sigma(y,z)}{s^n}\}$ which implies $\frac{\sigma(x,z)}{(t+s)^n} \leq max\{\frac{\sigma(x,y)}{t^n}, \frac{\sigma(y,z)}{s^n}\}$. □

Proposition 1. *Let (X, σ) be any metric-like space. Then, the five-tuple $(X, F, G, *, \diamond)$ is an IFMLS, where $a * b = min\{a, b\}$ and $a \diamond b = max\{a, b\}$ for all $a, b \in [0,1]$ and F, G are given by $F(x,y,t) = \frac{h.t^n}{h.t^n + m.\sigma(x,y)}$, $G(x,y,t) = \frac{\sigma(x,y)}{h.t^n + m.\sigma(x,y)}$ for all $x, y \in X$, $t > 0$, where $h \in IR^+, m > 0, n \geqslant 1$.*

Proof. (IFML1)–(IFML4) are clear. For (IFML5), let $x, y, z \in X$, $t, s > 0$, $h \in IR^+$ and $n \geq 1$. By the Lemma 1, we have

$$\begin{aligned}
&1 + \tfrac{m\sigma(x,z)}{h(t+s)^n} \leq max\{1 + \tfrac{m\sigma(x,y)}{ht^n}, 1 + \tfrac{m\sigma(y,z)}{hs^n}\}.\\
&\Rightarrow \tfrac{h(t+s)^n + m\sigma(x,z)}{h(t+s)^n} \leq max\{\tfrac{ht^n + m\sigma(x,y)}{ht^n}, \tfrac{hs^n + m\sigma(y,z)}{hs^n}\}.\\
&\Rightarrow \tfrac{h(t+s)^n}{h(t+s)^n + m\sigma(x,z)} \geq min\{\tfrac{ht^n}{ht^n + m\sigma(x,y)}, \tfrac{hs^n}{hs^n + m\sigma(y,z)}\}\\
&\Rightarrow F(x,z,t+s) \geq F(x,y,t) * F(y,z,s).
\end{aligned}$$

For (IFML6), $\lim_{t \to t_0} F(x,y,t) = \lim_{t \to t_0} \frac{ht^n}{ht^n + m\sigma(x,y)} = \frac{ht_0{}^n}{ht_0{}^n + m\sigma(x,y)} = F(x,y,t_0)$, so F is continuous.

(IFML7)–(IFML9) and (IFML11) are clear. Now, we need to show that $G(x,y,t) \diamond G(y,z,s) \geq G(x,z,t+s)$, i.e.,

$$\frac{m\sigma(x,z)}{h(t+s)^n + m\sigma(x,z)} \leq max\{\frac{m\sigma(x,y)}{ht^n + m\sigma(x,y)}, \frac{m\sigma(y,z)}{hs^n + m\sigma(y,z)}\}.$$

By the Lemma 1, we have

$$\begin{aligned}
&1 + \tfrac{m\sigma(x,z)}{h(t+s)^n} \leq max\{1 + \tfrac{m\sigma(x,y)}{ht^n}, 1 + \tfrac{m\sigma(y,z)}{hs^n}\}.\\
&\Rightarrow \tfrac{h(t+s)^n + m\sigma(x,z)}{h(t+s)^n} \leq max\{\tfrac{ht^n + m\sigma(x,y)}{ht^n}, \tfrac{hs^n + m\sigma(y,z)}{hs^n}\}.\\
&\Rightarrow \tfrac{h(t+s)^n}{h(t+s)^n + m\sigma(x,z)} \geq min\{\tfrac{ht^n}{ht^n + m\sigma(x,y)}, \tfrac{hs^n}{hs^n + m\sigma(y,z)}\}.\\
&\Rightarrow 1 - \tfrac{h(t+s)^n}{h(t+s)^n + m\sigma(x,z)} \leq 1 - min\{\tfrac{ht^n}{ht^n + m\sigma(x,y)}, \tfrac{hs^n}{hs^n + m\sigma(y,z)}\}.\\
&\Rightarrow \tfrac{m\sigma(x,z)}{h(t+s)^n + m\sigma(x,z)} \leq max\{\tfrac{m\sigma(x,y)}{ht^n + m\sigma(x,y)}, \tfrac{m\sigma(y,z)}{hs^n + m\sigma(y,z)}\}.\\
&\Rightarrow G(x,z,t+s) \leq max\{G(x,y,t), G(y,z,s)\}.\\
&\Rightarrow G(x,z,t+s) \leq G(x,y,t) \diamond G(y,z,s).
\end{aligned}$$

□

Remark 3. *Proposition 1 holds even with the t-norm $a * b = a.b$ and $a \diamond b = min\{1, a+b\}$.*

Remark 4. *By the above proposition, we see that every metric-like space induces an IFMLS. For $h = n = m = 1$, the induced intuitionistic fuzzy metric-like space $(X, F, G, *, \diamond)$ is called the standard intuitionistic fuzzy metric-like space, where $F(x,y,t) = \frac{t}{t+\sigma(x,y)}$, $G(x,y,t) = \frac{\sigma(x,y)}{t+\sigma(x,y)}$ for all $x, y \in X$, $t > 0$.*

Example 1. *Let $X = IR^+$, $h \in IR^+$ and $m > 0$. Let $a * b = ab$ and $a \diamond b = min\{a+b, 1\}$ for all $a, b \in [0,1]$. Define the fuzzy sets F and G in $X^2 \times (0, \infty)$ by $F(x,y,t) = \frac{ht}{ht + m(max\{x,y\})}$ and $G(x,y,t) = \frac{max\{x,y\}}{ht + m(max\{x,y\})}$ for all $x, y \in X$ and $t > 0$.*

We know that $\sigma(x,y) = max\{x,y\}$ is metric-like on X for all $x, y \in X$. Hence, $(X, F, G, *, \diamond)$ is an IFMLS by Remark 3, but it is not an intuitionistic fuzzy metric space, as $F(x,x,t) = \frac{ht}{ht+mx} \neq 1$ and $G(x,y,t) = \frac{x}{ht+m} \neq 0$ for all $x, y \in X$ and $t > 0$.

Proposition 2. *Let $\sigma(x,y)$ be any metric-like space on X. Then, the five-tuple $(X, F, G, *, \diamond)$ is an IFMLS, where $a * b = a.b$ and $a \diamond b = min\{1, a+b\}$ for all $a, b \in [0,1]$ and the fuzzy sets F, G are defined by $F(x,y,t) = e^{\frac{-\sigma(x,y)}{t^n}}$, $G(x,y,t) = 1 - e^{\frac{-\sigma(x,y)}{t^n}}$ for all $x, y \in X$, $t > 0$, where $n \geqslant 1$.*

Remark 5. *The proposition 2 holds even with the t-norm $a * b = min\{a, b\}$ and the t-conorm $a \diamond b = max\{a, b\}$.*

Example 2. *Let $X = IR^+$, $a * b = ab$ and $a \diamond b = min\{1, a+b\}$. Define the fuzzy sets F and G in $X^2 \times (0, \infty)$ by $F(x,y,t) = \frac{1}{e^{\frac{max\{x,y\}}{t}}}$ and $G(x,y,t) = \frac{e^{\frac{max\{x,y\}}{t}} - 1}{e^{\frac{max\{x,y\}}{t}}}$ for all $x, y \in X$ and $t > 0$ Figure 1.*

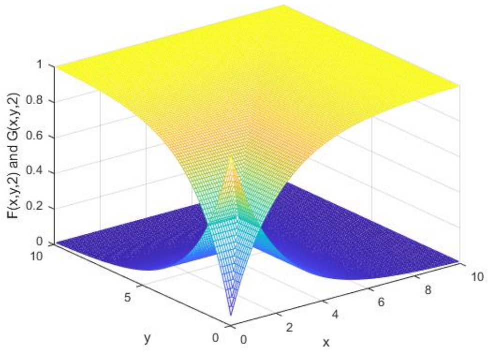

Figure 1. The graphical behavior of the F and G with $t = 2$ and $x, y \in (0, 10]$ in which the blue color depicts behavior of F and the yellow color depicts behavior of G.

We know that $\sigma(x,y) = max\{x,y\}$ is a metric-like on X for all $x, y \in X$. Hence, $(X, F, G, *, \diamond)$ is an IFMLS by Remark 3, but it is not an intuitionistic fuzzy metric space as $F(x,x,t) = \frac{1}{e^{\frac{x}{t}}} \neq 1$, $G(x,x,t) = \frac{e^{\frac{x}{t}}-1}{e^{\frac{x}{t}}} \neq 0$ for all $x > 0$ and $t > 0$ Figure 2.

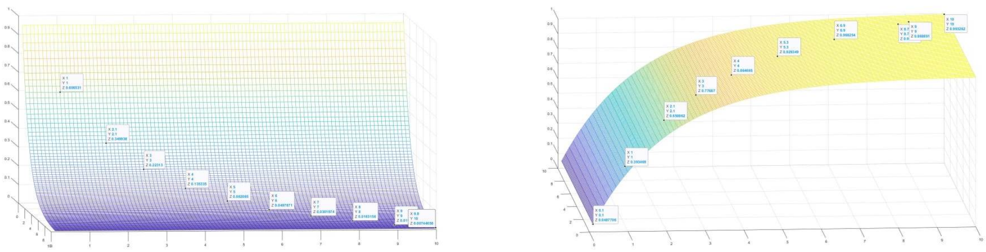

Figure 2. The graphical behavior of the $F(x,x,2)$ and $G(x,x,2)$, respectively.

Example 3. *Let $X = IN$, $a * b = ab$ and $a \diamond b = min\{1, a+b\}$. Define the fuzzy sets F and G in $X^2 \times (0, \infty)$ by*

$$F(x,y,t) = \begin{cases} \frac{x}{y^2}, & x \leq y; \\ \frac{y}{x^2}, & y \leq x. \end{cases}$$
$$G(x,y,t) = \begin{cases} \frac{y^2-x}{y^2}, & x \leq y; \\ \frac{x^2-y}{x^2}, & y \leq x. \end{cases}$$

for all $x, y \in X$.

Then, $(X, F, G, *, \diamond)$ is an IFMLS, but it is not IFMS as $F(x,x,t) = \frac{1}{x} \neq 1$ and $G(x,x,t) = \frac{1}{x} \neq 0$ for all $x > 1$, $t > 0$. Now, let it show that $(X, F, G, *, \diamond)$ is an IFMLS:

(IFML1)–(IFML4) are clear.

(IFML5) Let $x, y, z \in X$, $t, s > 0$ and suppose $x \leq y \leq z$, then $F(x,y,t) * F(y,z,s) = \frac{x}{y^2} \cdot \frac{y}{z^2} = \frac{x}{yz^2} \leq \frac{x}{z^2} = F(x,z,t+s)$. We obtain the same condition for other cases.

(IFML6) Let it show that $F(x,y,.) : (0,\infty) \to (0,1]$ is continuous. Let $x, y \in IN$ and $t > 0$.

If $x \leq y$, then $\lim_{t \to t_0} F(x,y,t) = \lim_{t \to t_0} \frac{x}{y^2} = \frac{x}{y^2} = F(x,y,t_0)$ and

If $y \leq x$, then $\lim_{t \to t_0} F(x,y,t) = \lim_{t \to t_0} \frac{y}{x^2} = \frac{y}{x^2} = F(x,y,t_0)$.

(IFML7) We have $F(x,y,t) + G(x,y,t) \leq 1$, then $G(x,y,t) \leq 1 - F(x,y,t)$. Since $F(x,y,t) > 0$, we obtain $G(x,y,t) < 1$.

(IFML8) Let $G(x,y,t) = 0$.

If $x \leq y$, then $\frac{y^2-x}{y^2} = 0$, so $x = y = 1$.

If $y \leq x$, then $\frac{x^2-y}{x^2} = 0$, so $x = y = 1$.

(IFML9) Obvious.

(IFML10) Let $x, y, z \in X$, $t, s > 0$ and suppose $x \leq y \leq z$. If the t-conorm is 1, we get the result. If the t-conorm is the other part, then we have $y - x \leq y^2 - x$.

$$\begin{aligned}
&\Rightarrow\ y^2(y-x) \leq z^2(y-x) \leq z^2(y^2-x)\\
&\Rightarrow\ y^3 - xy^2 \leq z^2y^2 - xz^2\\
&\Rightarrow\ \frac{y^3-xy^2}{z^2y^2} \leq \frac{z^2y^2-xz^2}{z^2y^2}\\
&\Rightarrow\ \frac{y-x}{z^2} \leq 1 - \frac{x}{y^2}\\
&\Rightarrow\ \frac{y}{z^2} \leq 1 - \frac{x}{y^2} + \frac{x}{z^2}\\
&\Rightarrow\ \frac{z^2-x}{z^2} \leq \frac{y^2-x}{y^2} + \frac{z^2-y}{z^2}.
\end{aligned}$$

Hence, $G(x, z, t+s) \leq G(x, y, t) \diamond G(y, z, s)$.
We obtain the same condition for other cases.
(IFML11) Obvious.

Remark 6. *In the above example, if we define $*$ by $a * b = max\{0, a+b-1\}$ and $\diamond$ by $a \diamond b = a + b - ab$, then we get IFMLS again, but if we define $*$ by $a * b = min\{a, b\}$ and $\diamond$ by $a \diamond b = max\{a, b\}$, (F, G) is not IFML.*

Definition 12. *Let $(X, F, G, *, \diamond)$ be an IFMLS.*

(a) *A sequence (x_n) in X is called convergent to $x \in X$ if $\lim_{n\to\infty} F(x_n, x, t) = F(x, x, t)$ and $\lim_{n\to\infty} G(x_n, x, t) = G(x, x, t)$ for all $t > 0$.*
(b) *A sequence (x_n) in X is called Cauchy sequence if $\lim_{n\to\infty} F(x_{n+p}, x_n, t)$ and $\lim_{n\to\infty} G(x_{n+p}, x_n, t)$ exist and finite for all $t > 0$, $p \geq 1$.*
(c) *$(X, F, G, *, \diamond)$ is called complete if every Cauchy sequence (x_n) in X converges to some $x \in X$ such that*
$\lim_{n\to\infty} F(x_n, x, t) = F(x, x, t) = \lim_{n\to\infty} F(x_{n+p}, x_n, t)$ and
$\lim_{n\to\infty} G(x_n, x, t) = G(x, x, t) = \lim_{n\to\infty} G(x_{n+p}, x_n, t)$ for all $t > 0$, $p \geq 1$.

Remark 7. *In an IFMLS, the limit of a convergent sequence may not be unique. Consider Example 1 with $m = k = h = 1$. Define a sequence (x_n) in X by $(x_n) = (1 + \frac{n}{n-1})$ for all $n \in IN$.*

If $x \geq 2$, then $\lim_{n\to\infty} F(x_n, x, t) = \lim_{n\to\infty} \frac{t}{t+max\{x_n,x\}} = \lim_{n\to\infty} \frac{t}{t+x} = F(x, x, t)$ and $\lim_{n\to\infty} G(x_n, x, t) = \lim_{n\to\infty} \frac{max\{x_n,x\}}{t+max\{x_n,x\}} = \lim_{n\to\infty} \frac{t}{t+x} = G(x, x, t)$ for all $t > 0$. Hence, the sequence x_n converges to all $x \in X$ with $x \geq 2$.

Remark 8. *In IFMLS, a convergent sequence may not be a Cauchy sequence. Again consider Example 1 with $m = k = h = 1$. Define a sequence (x_n) in X by $(x_n) = (-1)^n$ for all $n \in IN$.*

If $x \geq 1$, then $\lim_{n\to\infty} F(x_n, x, t) = \lim_{n\to\infty} \frac{t}{t+max\{x_n,x\}} = \lim_{n\to\infty} \frac{t}{t+x} = F(x, x, t)$ and $\lim_{n\to\infty} G(x_n, x, t) = \lim_{n\to\infty} \frac{max\{x_n,x\}}{t+max\{x_n,x\}} = \lim_{n\to\infty} \frac{t}{t+x} = G(x, x, t)$ for all $t > 0$. Hence, a sequence (x_n) converges to all $x \in X$ with $x \geq 1$, but it is not a Cauchy sequence as $\lim_{n\to\infty} F(x_n, x_{n+p}, t)$ and $G(x_n, x_{n+p}, t)$ do not exist.

4. Fixed-Point Results

In this section, we first describe the contraction mappings in IFMLS and provide some supporting examples.

Definition 13. *Let $(X, F, G, *, \diamond)$ be an IFMLS. A mapping $T : X \to X$ is called an intuitionistic fuzzy contractive if there exists $\lambda \in (0, 1)$ such that $\frac{1}{F(T(x),T(y),t)} - 1 \leq \lambda.[\frac{1}{F(x,y,t)} - 1]$ and $G(T(x), T(y), t) \leq \lambda.G(x, y, t)$ for all $x, y \in X$ and $t > 0$. Here, λ is called the intuitionistic fuzzy constant of T.*

Theorem 1. *Let $(X, F, G, *, \diamond)$ be a complete intuitionistic fuzzy metric-like space and $T : X \to X$ an intuitionistic fuzzy contractive mapping with intuitionistic fuzzy contractive constant λ, then T has a unique fixed point $a \in X$ and $F(a, a, t) = 1,\ G(a, a, t) = 0$ for all $t > 0$.*

Proof. For an arbitrary $x_0 \in X$, define a sequence $(x_n) \subset X$ by $x_1 = T(x_0), x_2 = T(x_1), \ldots,$ $x_n = T(x_{n-1})$ for all $n \in IN$. If $x_n = x_{n-1}$ for some $n \in IN$, then x_n is a fixed point of T. Now, assume that $x_n \neq x_{n-1}$ for all $n \in IN$. For $t > 0$ and $n \in IN$; we get following from Definition 13:

$$\frac{1}{F(x_n, x_{n+1}, t)} - 1 = \frac{1}{F(T(x_{n-1}), T(x_n), t)} - 1 \leq \lambda[\frac{1}{F(x_{n-1}, x_n, t)} - 1] \leq \frac{\lambda}{F(x_{n-1}, x_n, t)} - \lambda].$$

Take $F(x_n, x_{n+1}, t) = F_n(t)$ and $1 - \lambda = k$, then we have that $\frac{1}{F_n(t)} \leq \frac{\lambda}{F_{n-1}(t)} + k$ for all $t > 0$.

Continuing in the above inequality, we get

$$\frac{1}{F_n(t)} \leq \frac{\lambda^n}{f_0(t)} + \lambda^{n-1}k + \lambda^{n-2}k + \ldots + k \leq \frac{\lambda^n}{F_0(t)} + (\lambda^{n-1} + \lambda^{n-2} + \ldots + 1)k \leq \frac{\lambda^n}{F_0(t)} + 1 - \lambda^n, \tag{1}$$

then, we obtain $\frac{1}{\frac{\lambda^n}{F_0(t)} + 1 - \lambda^n} \leq F_n(t)$ for all $t > 0, n \in IN$.

Now, for $p \geq 1$ and $n \in IN$, we get

$$\begin{aligned} F(x_{n+p}, x_n, t) &\geq F(x_n, x_{n+1}, \tfrac{t}{2}) * F(x_{n+1}, x_{n+p}, \tfrac{t}{2}) \\ &\geq F(x_n, x_{n+1}, \tfrac{t}{2}) * F(x_{n+1}, x_{n+2}, \tfrac{t}{2^2}) * F(x_{n+2}, x_{n+p}, \tfrac{t}{2^2}) \\ &\geq F(x_n, x_{n+1}, \tfrac{t}{2}) * F(x_{n+1}, x_{n+2}, \tfrac{t}{2^2}) * \ldots \\ &* F(x_{n+p-2}, x_{n+p-1}, \tfrac{t}{2^{p-1}}) * F(x_{n+p-1}, x_{n+p}, \tfrac{t}{2^{p-1}}) \\ &= F_n(\tfrac{t}{2}) * F_{n+1}(\tfrac{t}{2^2}) * \ldots * F_{n+p-2}(\tfrac{t}{2^{p-1}}) * F_{n+p-1}(\tfrac{t}{2^{p-1}}) \end{aligned}$$

By using (1) in the above inequality, we obtain

$$\begin{aligned} F(x_{n+p}, x_n, t) &\geq \frac{1}{\frac{\lambda^n}{F_0(\frac{t}{2})} + 1 - \lambda^n} * \frac{1}{\frac{\lambda^{n+1}}{F_0(\frac{t}{2^2})} + 1 - \lambda^{n+1}} * \ldots * \frac{1}{\frac{\lambda^{n+p-1}}{F_0(\frac{t}{2^{p-1}})} + 1 - \lambda^{n+p-1}} \\ &\geq \frac{1}{\frac{\lambda^n}{F_0(\frac{t}{2})} + 1} * \frac{1}{\frac{\lambda^{n+1}}{F_0(\frac{t}{2^2})} + 1} * \ldots * \frac{1}{\frac{\lambda^{n+p-1}}{F_0(\frac{t}{2^{p-1}})} + 1}. \end{aligned}$$

Here, $\lambda \in (0, 1)$, using the properties of continuous t-norm we have from the above expression that $\lim_{n\to\infty} F(x_{n+p}, x_n, t) = 1$ for all $t > 0,\ p \geq 1$.

For any $n \in IN$ and $t > 0$, similarly we obtain from Definition 13 that $G(T(x_n), T(x_{n+1}), t) \leq \lambda G(x_n, x_{n+1}, t)$. Then, $G(x_{n+1}, x_{n+2}, t) = G(T(x_n), T(x_{n+1}), t) \leq \lambda G(x_n, x_{n+1}, t)$.

Setting, $G(x_{+1}, x_{n+2}, t) = G_n(t)$ and $1 - \lambda = k$, it follows from the above inequality that $G_n(t) \leq \lambda G_{n-1}(t) = (1 - k)G_{n-1}(t) = G_{n-1}(t) - kG_{n-1}(t)$.

From the applications of the above inequality, we have

$$\begin{aligned} G_n(t) &\leq G_{n-1}(t) - kG_{n-1}(t) \\ &\leq (1-k)G_{n-2}(t) - (1-k)kG_{n-2}(t)). \\ &\leq (1-k)^2 G_{n-3}(t) - (1-k)^2 kG_{n-3}(t)). \\ &\leq (1-k)^{n-1} G_0(t) - (1-k)^{n-1} kG_0(t)). \\ &= (1-k)^{n-1}[G_0(t) - .G_0(t)]. \\ &= \lambda^{n-1}[G_0(t) - kG_0(t)]. \end{aligned} \tag{2}$$

Then, we get $G_n(t) \leq \lambda^{n-1}[G_0(t) - k.G_0(t)]$ for all $t > 0, n \in IN$.

Now, for $p \geq 1$ and $n \in IN$, we get

$$\begin{array}{rcl} G(x_{n+p},x_n,t) & \leq & G(x_n,x_{n+1},\frac{t}{2})\diamond G(x_{n+1},x_{n+p},\frac{t}{2}) \\ & \leq & G(x_n,x_{n+1},\frac{t}{2})\diamond G(x_{n+1},x_{n+2},\frac{t}{2^2})\diamond G(x_{n+2},x_{n+p},\frac{t}{2^2}) \\ & \leq & G(x_n,x_{n+1},\frac{t}{2})\diamond G(x_{n+1},x_{n+2}\frac{t}{2^2})\diamond\ldots \\ & \diamond & G(x_{n+p-2},x_{n+p-1},\frac{t}{2^{p-1}})\diamond G(x_{n+p-1},x_{n+p},\frac{t}{2^{p-1}}). \end{array}$$

Using (2) in the above inequality, we have

$$\begin{array}{rcl} G(x_{n+p},x_n,t) & \leq & G_{n-1}(\frac{t}{2})\diamond G_n(\frac{t}{2^2})\diamond\ldots\diamond G_{n+p-3}(\frac{t}{2^{p-1}})\diamond G_{n+p-2}(\frac{t}{2^{p-1}}) \\ & \leq & \lambda^{n-2}[G_0(\frac{t}{2})-k.G_0(\frac{t}{2})]\diamond\lambda^{n-1}[G_0(\frac{t}{2^2})-k.G_0(\frac{t}{2^2})]\diamond\ldots \\ & \diamond & \lambda^{n+p-4}[G_0(\frac{t}{2^{p-1}})-k.G_0(\frac{t}{2^{p-1}})]\diamond\lambda^{n+p-3}[G_0(\frac{t}{2^{p-1}})-kG_0(\frac{t}{2^{p-1}})]. \end{array}$$

Here, $\lambda\in(0,1)$, using the properties of continuous t-conorm, we obtain from the above expression that $\lim_{n\to\infty}G(x_{n+p},x_n,t)=0$ for all $t>0,\ p\geq 1$.

Therefore, since $\lim_{n\to\infty}F(x_{n+p},x_n,t)=1$ and $\lim_{n\to\infty}G(x_{n+p},x_n,t)=0$ for all $t>0$, $p\geq 1$, (x_n) is Cauchy sequence in $(X,F,G,*,\diamond)$.

Since $(X,F,G,*,\diamond)$ is a complete intuitionistic fuzzy metric-like space, there exists $a\in X$ such that

$$\lim_{n\to\infty}F(x_n,a,t)=\lim_{n\to\infty}F(x_{n+p},x_n,t)=F(a,a,t)=1. \tag{3}$$

and

$$\lim_{n\to\infty}G(x_n,a,t)=\lim_{n\to\infty}G(x_{n+p},x_n,t)=G(a,a,t)=0 \text{ for all } t>0,\ p\geq 1. \tag{4}$$

Now, we prove that a is a fixed point for T. For this, we obtain from Definition 13 that $\frac{1}{F(T(x_n),T(a),t)}-1\leq\lambda.[\frac{1}{F(x_n,a,t)}-1]=\frac{\lambda}{F(x_n,a,t)}-\lambda$, $\frac{1}{\frac{\lambda}{F(x_n,a,t)}+1-\lambda}\leq F(T(x_n),T(a),t)$ and $G(T(x_n),T(a),t)\leq\lambda.G(x_n,a,t)$.

Using the above inequalities, we obtain

$$\begin{array}{rcl} F(a,T(a),t) & \geq & F(a,x_{n+1},\frac{t}{2})*F(x_{n+1},T(a),\frac{t}{2}) \\ & = & F(a,x_{n+1},\frac{t}{2})*F(T(x_n),T(a),\frac{t}{2}) \\ & \geq & F(a,x_{n+1},\frac{t}{2})*\frac{1}{\frac{\lambda}{F(x_n,a,\frac{t}{2})}+1-\lambda}. \end{array}$$

and

$$\begin{array}{rcl} G(a,T(a),t) & \leq & G(a,x_{n+1},\frac{t}{2})\diamond G(x_{n+1},T(a),\frac{t}{2}) \\ & = & G(a,x_{n+1},\frac{t}{2})\diamond G(T(x_n),T(a),\frac{t}{2}) \\ & \leq & G(a,x_{n+1},\frac{t}{2})\diamond\lambda.G(x_n,a,t). \end{array}$$

Taking limit as $n\to\infty$ and using (3) and (4) in the above inequalities, we get $F(a,T(a),t)=1$ and $G(a,T(a),t)=0$, that is $T(a)=a$. Hence, a is a fixed point of T and $F(a,a,t)=1$ and $G(a,a,t)=0$ for all $t>0$.

We investigate the uniqueness of the fixed point a of T. Let b be another fixed point of T, such that $F(a,b,t)<1$ and $G(a,b,t)>0$ for some $t>0$; it follows from the Definition 13 that $\frac{1}{F(a,b,t)}-1=\frac{1}{F(T(a),T(b),t)}-1\leq\lambda.[\frac{1}{F(a,b,t)}-1]<\frac{1}{F(a,b,t)}-1$ and $G(a,b,t)=G(T(a),T(b),t)\leq\lambda.G(a,b,t)<G(a,b,t)$, a contradiction.

Hence, we must have $F(a,b,t)=1$ and $G(a,b,t)=0$ for all $t>0$ and therefore, $a=b$. □

Example 4. *Let $X=[0,2]$. $*$ and $\diamond$ respectively defined as $a*b=a.b$ and by $a\diamond b=max\{a,b\}$ and intuitionistic fuzzy sets F,G in $X^2\times(0,\infty)$ given as $F(x,y,t)=e^{\frac{-(max\{x,y\})}{t}}$, $G(x,y,t)=1-e^{\frac{-(max\{x,y\})}{t}}$ for all $x,y\in X,\ t>0$. Then, $(X,F,G,*,\diamond)$ is a complete IFMLS.*

If $T:X\to X$ is given by

$$T(x) = \begin{cases} 0, & x = 1; \\ \frac{x}{2}, & x \in [0,1), \\ \frac{x}{4}, & x \in (1,2] \end{cases}$$

Then, we have nine cases:
Case 1: If $x = y = 1$, then $T(x) = T(y) = 0$.
Case 2: If $x = 1$ and $y \in [0,1)$, then $T(x) = 0,\ T(y) = \frac{y}{2}$.
Case 3: If $x = 1$ and $y \in (1,2]$, then $T(x) = 0,\ T(y) = \frac{y}{4}$.
Case 4: If $x \in [0,1)$ and $y \in (1,2]$, then $T(x) = \frac{x}{2},\ T(y) = \frac{y}{4}$.
Case 5: If $x \in [0,1)$ and $y \in [0,1)$, then $T(x) = \frac{x}{2},\ T(y) = \frac{y}{2}$.
Case 6: If $x \in [0,1)$ and $y = 1$, then $T(x) = \frac{x}{2},\ T(y) = 0$.
Case 7: If $x \in (1,2]$ and $y = 1$, then $T(x) = \frac{x}{4},\ T(y) = 0$.
Case 8: If $x \in (1,2]$ and $y \in (1,2]$, then $T(x) = \frac{x}{4},\ T(y) = \frac{y}{4}$.
Case 9: If $x \in (1,2]$ and $y \in [0,1)$, then $T(x) = \frac{x}{4},\ T(y) = \frac{y}{2}$.
All the above cases hold the intuitionistic fuzzy contractive given in the Definition 13. Therefore, T is an intuitionistic contractive mapping with $\lambda \in [\frac{1}{2}, 1)$. So, the conditions of Theorem 1 hold. Moreover, 0 is the unique fixed point of T and $F(0,0,t) = 1$ and $G(0,0,t) = 0$ for all $t > 0$.

If we take an intuitionistic fuzzy metric like on X as follows: $F(x,y,t) = e^{\frac{-(1+d(x,y))}{t}}$ and $G(x,y,t) = 1 - e^{\frac{-(1+d(x,y))}{t}}$ for all $x, y \in X,\ t > 0$, then T is not an intuitionistic fuzzy contractive mapping with respect to this contractive mapping. Here, (X,d) is a classical metric space and $\sigma(x,y) = 1 + d(x,y)$ is metric-like on X for all $x, y \in X$. Let $d(x,y) = |x-y|$ and $x = y = 0$, hence $\frac{1}{F(T(x),T(y),t)} - 1 \leq \lambda.[\frac{1}{F(x,y,t)} - 1] \Rightarrow \frac{1}{e^{\frac{-(1+|T(x)-T(y)|)}{t}}} - 1 \leq \lambda.[\frac{1}{e^{\frac{-(1+|x-y|)}{t}}} - 1] \Rightarrow e^{\frac{1+|T(x)-T(y)|}{t}} - 1 \leq \lambda.[e^{\frac{1+|x-y|}{t}} - 1] \Rightarrow e^{\frac{1}{t}} - 1 \leq \lambda.[e^{\frac{1}{t}} - 1] \Rightarrow 1 \leq \lambda$, there is no $\lambda \in (0,1)$ satisfying the above inequality.

Corollary 1. *Let $(X, F, G, *, \diamond)$ be a complete IFMLS and $T : X \to X$ be a mapping that satisfies the following inequalities; $\frac{1}{F(T^n(x),T^n(y),t)} - 1 \leq \lambda.[\frac{1}{F(x,y,t)} - 1]$ and $G(T^n(x), T^n(y), t) \leq \lambda.G(x,y,t)$ for some positive integer n and for all $x, y \in X,\ t > 0$, where $\lambda \in (0,1)$. Then, T has a unique fixed point $a \in X$ and $F(a,a,t) = 1,\ G(a,a,t) = 0$ for all $t > 0$.*

Proof. $a \in X$ is the unique fixed point of T^n and $F(a,a,t) = 1,\ G(a,a,t) = 0$ for all $t > 0$ from Theorem 4.1. Since $T^n(T(a)) = T(T^n(a) = T(a))$, $T(a)$ is also a fixed point of T^n and therefore, the fixed point of T is unique. □

Theorem 2. *Let $(X, F, G, *, \diamond)$ be an IFMLS and $T : X \to X$ be a intuitionistic fuzzy contractive mapping with contractive constant λ. Suppose that there exists $a \in X$ such that $(F(a, T(a), t) \geq F(x, T(x), t)$ and $G(a, T(a), t) \leq G(x, T(x), t)$ for all $x \in X$ and $t > 0$, then a becomes a unique fixed point of T and $F(a,a,t) = 1,\ G(a,a,t) = 0$ for all $t > 0$.*

Proof. Let $F_x(t) = F(x, T(x), t)$ and $G_x(t) = G(x, T(x), t)$ for all $x \in X$ and $t > 0$. By hypothesis, $F_a(t) \geq F_x(t)$ and $G_a(t) \leq G_x(t)$ for all $x \in X$ and $t > 0$. We suggest that $F(a, T(a), t) = 1$ and $G(a, T(a), t) = 0$ for all $t > 0$.

In fact, if $F_a(t) = F(a, T(a), t) < 1$ and $G_a(t) = G(a, T(a), t) > 0$ for some $t > 0$, then by Definition 13, we get

$$\begin{aligned} \frac{1}{F_{T(a)}(t)} - 1 &= \frac{1}{F(T(a), T(T(a)), t)} - 1 \\ &\leq \lambda.[\frac{1}{F(a, T(a), t)} - 1] \\ &= \lambda.[\frac{1}{F_a(t)} - 1] \\ &< \frac{1}{F_a(t)} - 1. \end{aligned}$$

and similarly $G_{T(a)}(t) = G(T(a), T(T(a)), t) \leq \lambda.G(a, T(a), t) = \lambda.G_a(t) < G_a(t)$.

Hence, $F_a(t) < F_{T(a)}(t),\ G_{T(a)}(t) < G_a(t),\ T(a) \in X$, a contradiction.

Then, we obtain $F_x(t) = F(a, T(a), t) = 1$ and $G_x(t) = G(a, T(a), t) = 0$ for all $t > 0$, so $T(a) = a$. By a similar way in the proof of Theorem 1, we can see that the fixed point of T is unique. If $F(a, a, t) < 1$ and $G(a, a, t) > 0$ for some $t > 0$, by the Definition 13 we get $\frac{1}{F(a,a,t)} - 1 = \frac{1}{F(T(a),T(a),t)} - 1 \leq \lambda.[\frac{1}{F(a,a,t)} - 1] < \frac{1}{F(a,a,t)} - 1$ and $G(a, a, t) = G(T(a), T(a), t) \leq \lambda.G(a, a, t) < G(a, a, t)$, a contradiction.

Hence, $F(a, a, t) = 1,\ G(a, a, t) = 0$. □

Example 5. *Let $X = [0,1] \cap Q$ and define continuous t-norm and continuous t-conorm as $a * b = max\{a + b - 1, 0\}$ and $a \diamond b = min\{a + b, 1\}$, respectively. In addition, the fuzzy sets F, G are defined as $F(x, y, t) = 1 - \frac{max\{x,y\}}{1+t}$ and $G(x, y, t) = \frac{max\{x,y\}}{1+t}$ for all $x, y \in X,\ t > 0$. Then, $(X, F, G, *, \diamond)$ is an IFMLS, but it is not complete.*

Define $T : X \to X$ as

$$T(x) = \begin{cases} \frac{x}{4}, & if\ x \in [0, \frac{1}{2}] \cap Q; \\ \frac{x}{2}, & x \in (\frac{1}{2}, 1] \cap Q \end{cases};$$

then, T is an intuitionistic fuzzy contractive mapping with intuitionistic fuzzy constant $\lambda \in [\frac{1}{2}, 1)$. Here, $F(0, T(0), t) = F(0, 0, t) = 1 \geq F(x, T(x), t)$ and $G(0, T(0), t) = G(0, 0, t) = 0 \leq G(x, T(x), t)$ for all $x \in X,\ t > 0$. Hence, all conditions of Theorem 2 are satisfied and 0 is the unique fixed point of T.

We have eight cases:

Case 1: If $x = y = 0$, then $T(x) = T(y) = 0$.

Case 2: If $x = 0,\ y = 1$, then $T(x) = 0,\ T(y) = \frac{1}{2}$.

Case 3: If $x = 1,\ y = 0$, then $T(x) = \frac{1}{2},\ T(y) = 0$.

Case 4: If $x = y = 1$, then $T(x) = \frac{1}{2},\ T(y) = \frac{1}{2}$.

Case 5: If $x \in (0, \frac{1}{2}) \cap Q,\ y \in (0, \frac{1}{2}) \cap Q$, then $T(x) = \frac{x}{4},\ T(y) = \frac{y}{4}$.

Case 6: If $x \in (0, \frac{1}{2}) \cap Q,\ y \in (\frac{1}{2}, 1) \cap Q$, then $T(x) = \frac{x}{4},\ T(y) = \frac{y}{2}$.

Case 7: If $x \in (\frac{1}{2}, 1) \cap Q,\ y \in (\frac{1}{2}, 1) \cap Q$, then $T(x) = \frac{x}{2},\ T(y) = \frac{y}{2}$.

Case 8: If $x \in (\frac{1}{2}, 1) \cap Q,\ y \in (0, \frac{1}{2}) \cap Q$, then $T(x) = \frac{x}{2},\ T(y) = \frac{y}{4}$.

All the above cases satisfy the IFMLS contraction in Definition 13. Hence, T is an intuitionistic fuzzy contractive mapping with contractive constant $\lambda \in [\frac{1}{2}, 1)$.

Moreover, $F(0, T(0), t) = 1 - \frac{max\{0,0\}}{1+t} = 1 \geq F(x, T(x), t)$ and $G(0, T(0), t) = \frac{max\{0,0\}}{1+t} = 0 \leq G(x, T(x), t)$ for all $x \in X,\ t > 0$ is satisfied and "0" is the unique fixed point of T and $F(0, 0, t) = 1,\ G(0, 0, t) = 1$ for all $t > 0$.

Theorem 3. *Let $(X, F, G, *, \diamond)$ be a complete IFMLS such that $\lim_{t\to\infty} F(x, y, t) = 1$ and $\lim_{t\to\infty} G(x, y, t) = 0$ for all $x, y \in X$ and $T : X \to X$ a mapping satisfying the conditions $F(T(x), T(y), kt) \geq F(x, y, t)$ and $G(T(x), T(y), kt) \leq G(x, y, t)$ for all $x, y \in X,\ t > 0$ where $k \in (0, 1)$. Then, T has a unique fixed point $a \in X$ and $F(a, a, t) = 1,\ G(a, a, t) = 0$ for all $t > 0$.*

Proof. Let $(X, F, G, *, \diamond)$ be a complete IFMLS. For an arbitrary $x_0 \in X$, define a sequence (x_n) in X by $x_1 = T(x_0) =, x_2 = T^2(x_0) = T(x_1), \ldots, x_n = T^n(x_0) = T(x_{n-1})$ for all $n \in IN$.

If $x_n = x_{n-1}$ for some $n \in IN$, then x_n is a fixed point of T. We suppose that $x_n \neq x_{n-1}$ for all $n \in IN$. For $t > 0$ and $n \in IN$, we have from conditions in the hypothesis that $F(x_n, x_{n+1}, t) = F(T(x_{n-1}), T(x_n), t) \geq F(x_{n-1}, x_n, \frac{t}{k})$ and $G(x_n, x_{n+1}, t) = G(T(x_{n-1}), T(x_n), t) \leq G(x_{n-1}, x_n, \frac{t}{k})$ for all $n \in IN$ and $t > 0$.

Let $F(x_n, x_{n+1}, t) = F_n(t)$ and $G(x_n, x_{n+1}, t) = G_n(t))$ and apply the above expression repeatedly; then, we deduce that

$$\begin{aligned} F_n(t) = F(x_n, x_{n+1}, t) \geq F(x_n, x_{n+1}, kt) = F(T(x_{n-1}), T(x_n), kt) \geq F(x_{n-1}, x_n, t) = \\ F(T(x_{n-2}), T(x_{n-1}), t) \geq F(x_{n-2}, x_{n-1}, \tfrac{t}{k}) \geq \ldots \geq F(x_0, x_1, \tfrac{t}{k^n}). \end{aligned} \tag{5}$$

So $F_n(t) \geq F_0(\frac{t}{k^n})$.

And similarly $G_n(t) \leq G_0(\frac{t}{k^n})$ for all $n \in IN$ and $t > 0$.

If $n \in IN$ and $p \geq 1$, then we get

$$\begin{array}{rcl} F(x_{n+p}, x_n, t) & \geq & F(x_n, x_{n+1}, \frac{t}{2}) * F(x_{n+1}, x_{n+p}, \frac{t}{2}) \\ & \geq & F(x_n, x_{n+1}, \frac{t}{2}) * F(x_{n+1}, x_{n+2}, \frac{t}{2^2}) * F(x_{n+2}, x_{n+p}, \frac{t}{2^2}) \\ & \geq & F(x_n, x_{n+1}, \frac{t}{2}) * F(x_{n+1}, x_{n+2}, \frac{t}{2^2}) * \ldots \\ & * & F(x_{n+p-2}, x_{n+p-1}, \frac{t}{2^{p-1}}) * F(x_{n+p-1}, x_{n+p}, \frac{t}{2^{p-1}}) \\ & = & F_n(\frac{t}{2}) * F_{n+1}(\frac{t}{2^2}) * \ldots * F_{n+p-2}(\frac{t}{2^{p-1}}) * F_{n+p-1}(\frac{t}{2^{p-1}}). \end{array} \tag{6}$$

and

$$\begin{array}{rcl} G(x_{n+p}, x_n, t) & \leq & G(x_n, x_{n+1}, \frac{t}{2}) \diamond G(x_{n+1}, x_{n+p}, \frac{t}{2}) \\ & \leq & G(x_n, x_{n+1}, \frac{t}{2}) \diamond G(x_{n+1}, x_{n+2}, \frac{t}{2^2}) \diamond G(x_{n+2}, x_{n+p}, \frac{t}{2^2}) \\ & \leq & G(x_n, x_{n+1}, \frac{t}{2}) \diamond G(x_{n+1}, x_{n+2}, \frac{t}{2^2}) \diamond \ldots \\ & \diamond & G(x_{n+p-2}, x_{n+p-1}, \frac{t}{2^{p-1}}) \diamond G(x_{n+p-1}, x_{n+p}, \frac{t}{2^{p-1}}) \\ & = & G_n(\frac{t}{2}) \diamond G_{n+1}(\frac{t}{2^2}) \diamond \ldots \diamond G_{n+p-2}(\frac{t}{2^{p-1}}) \diamond G_{n+p-1}(\frac{t}{2^{p-1}}). \end{array}$$

By using (5) and (6) in the above inequality, we get $F(x_{n+p}, x_n, t) \geq F_0(\frac{t}{2k^n}) * F_0(\frac{t}{2^2k^{n+1}}) * \ldots * F_0(\frac{t}{2^{p-1}k^{n+p-1}})$ and $G(x_{n+p}, x_n, t) \leq G_0(\frac{t}{2k^n}) \diamond G_0(\frac{t}{2^2k^{n+1}}) \diamond \ldots \diamond G_0(\frac{t}{2^{p-1}k^{n+p-1}})$.

Since $0 < k < 1$, $\lim_{t\to\infty} F(x, y, t) = 1$ and $\lim_{t\to\infty} G(x, y, t) = 0$ for all $x, y \in X$ and by the properties of continuous t-norm and t-conorm we obtain from the above expression that $\lim_{n\to\infty} F(x_{n+p}, x_n, t) = 1 * 1 * \ldots * 1 = 1$ and $\lim_{n\to\infty} G(x_{n+p}, x_n, t) = 0 \diamond 0 \diamond \ldots \diamond 0 = 0$ for all $t > 0,\ p \geq 1$.

Hence, (x_n) is a Cauchy sequence in $(X, F, G, *, \diamond)$. Since, $(X, F, G, *, \diamond)$ is a complete IFMLS, there exists $a \in X$ such that

$$\lim_{n\to\infty} F(x_n, a, t) = \lim_{n\to\infty} F(x_{n+p}, x_n, t) = F(a, a, t) = 1. \tag{7}$$

$$\lim_{n\to\infty} G(x_n, a, t) = \lim_{n\to\infty} G(x_{n+p}, x_n, t) = G(a, a, t) = 0 \text{ for all } t > 0,\ p \geq 1. \tag{8}$$

Now, we derive that $a \in X$ is a fixed point of T. To demonstrate this, we continue as below for all $n \in IN$ and $t > 0$; we obtain from the hypothesis that

$$\begin{array}{rcl} F(a, T(a), t) & \geq & F(a, x_{n+1}, \frac{t}{2}) * F(x_{n+1}, T(a), \frac{t}{2}) \\ & = & F(a, x_{n+1}, \frac{t}{2}) * F(T(x_n), T(a), \frac{t}{2}) \\ & \geq & F(a, x_{n+1}, \frac{t}{2}) * F(x_n, a, \frac{t}{2k}) \end{array}$$

and

$$\begin{array}{rcl} G(a, T(a), t) & \leq & G(a, x_{n+1}, \frac{t}{2}) \diamond G(x_{n+1}, T(a), \frac{t}{2}) \\ & = & G(a, x_{n+1}, \frac{t}{2}) \diamond G(T(x_n), T(a), \frac{t}{2}) \\ & \leq & G(a, x_{n+1}, \frac{t}{2}) \diamond G(x_n, a, \frac{t}{2k}). \end{array}$$

Now, limit as $n \to \infty$ and by (7) and (8), we get $F(a, T(a), t) = 1$ and $G(a, T(a), t) = 0$. Hence, a is a fixed point of T and $F(a, a, t) = 1$ and $G(a, a, t) = 0$, $\forall t > 0$.

To show the uniqueness of the fixed point, let b be another fixed point of T. Using the conditions of the hypothesis, we get

$$F(a, b, t) = F(T(a), T(b), t) \geq F(a, b, \frac{t}{k}),$$

$$G(a, b, t) = G(T(a), T(b), t) \leq G(a, b, \frac{t}{k}).$$

That is, $F(a, b, t) \geq F(a, b, \frac{t}{k})$ and $G(a, b, t) \leq G(a, b, \frac{t}{k})$, for all $t > 0$.

Since the above inequality holds for all $t > 0$, we get $F(a, b, t) \geq F(a, b, \frac{t}{k^n})$ and $G(a, b, t) \leq G(a, b, \frac{t}{k^n})$, for all $n \in IN$.

Now, take the limit as $n \to \infty$ and use $\lim_{t\to\infty} F(x,y,t) = 1$, $\lim_{t\to\infty} G(x,y,t) = 0$ for all $x,y \in X$; we obtain $F(a,b,t) = 1$, $G(a,b,t) = 0$ and so $a = b$. Hence, the fixed point is unique. $\square$

With the following example, we see that the conditions $\lim_{t\to\infty} F(x,y,t) = 1$, $\lim_{t\to\infty} G(x,y,t) = 0$, for all $x,y \in X$ in Theorem 3 are essential. If we do not have these conditions, we lose the unique fixed point of T.

Example 6. *Let $X = \{0,1\}$ and $m > 2$ be a fixed natural number.*

Define $*$ by $a * b = max\{a+b-1, 0\}$ and $\diamond$ by $a \diamond b = a + b - ab$ and the fuzzy sets F, G in $X^2 \times (0,\infty)$ by $F(1,0,t) = F(0,1,t) = 1 - \frac{1}{m}$, $F(0,0,t) = F(1,1,t) = 1 - \frac{2}{m}$ and $G(1,0,t) = G(0,1,t) = \frac{1}{m}$, $G(0,0,t) = G(1,1,t) = \frac{2}{m}$.

Then, $(X, F, G, *, \diamond)$ is a complete intuitionistic fuzzy metric-like space. Let $T : X \to X$ be a mapping defined by $T(0) = 1$, $T(1) = 0$. Hence, all the conditions Theorem 3, except $\lim_{t\to\infty} F(x,y,t) = 1$ and $\lim_{t\to\infty} G(x,y,t) = 0$ for all $x,y \in X$, are satisfied with arbitrary $0 < k < 1$. Therefore, T has no fixed point in X.

5. Conclusions and Future Works

In this paper, we presented the concept of intuitionistic fuzzy metric-like space and gave the results of the fixed-point theory, which is an important issue in applications. This study is the extended form of fuzzy metric-like spaces [30]. Because the result of the paper allows further development of the theory and practice of fuzzy mathematics, our study is useful and interesting as a theoretical aspect. This study can be used to solve the problems of uncertainty. Our results may provide a new motivation to researchers to develop the area of fixed-point theory in this new setting. This study can be extended in different structures such as intuitionistic fuzzy b-metric like spaces, etc. Furthermore, one can study whether versions of fixed-point results already established in (intuitionistic) fuzzy metrics remain valid in the intuitionistic fuzzy metric-like context.

For future applied works, these obtained results can provide a deeper understanding of the structure of intuitionistic fuzzy metric spaces. Moreover, these results can open up new opportunities and provide new approaches for their applications in various fields such as mathematical modelling, decision making, pattern recognition, image processing and data analysis, which are developing. In this way, researchers could engage with papers [32,33], obtain more profound predictive models and discuss their results.

Author Contributions: Conceptualization, B.P.V.; Software, B.P.V.; Formal analysis, Ş.O.; Investigation, Ş.O.; Resources, Ş.O.; Data curation, Ş.O.; Writing—original draft, B.P.V.; Supervision, B.P.V.; Project administration, B.P.V. All authors contributed equally in writing this article. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The experimental data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest: The authors declare no conflict of interest.

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