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Review

A Review of Intelligent Airfoil Aerodynamic Optimization Methods Based on Data-Driven Advanced Models

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Abstract: With the rapid development of artificial intelligence technology, data-driven advanced models have provided new ideas and means for airfoil aerodynamic optimization. As the advanced models update and iterate, many useful explorations and attempts have been made by researchers on the integrated application of artificial intelligence and airfoil aerodynamic optimization. In this paper, many critical aerodynamic optimization steps where data-driven advanced models are employed are reviewed. These steps include geometric parameterization, aerodynamic solving and performance evaluation, and model optimization. In this way, the improvements in the airfoil aerodynamic optimization area led by data-driven advanced models are introduced. These improvements involve more accurate global description of airfoil, faster prediction of aerodynamic performance, and more intelligent optimization modeling. Finally, the challenges and prospect of applying data-driven advanced models to aerodynamic optimization are discussed.

Keywords: aerodynamic optimization; advanced model; artificial intelligence; data driven

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1. Introduction

Aerodynamic optimization is the basis of commercial aircraft design, which is essential for reducing cost and environmental impact [1]. As a key component of commercial airplanes, an excellent aerodynamic configuration of the airfoil plays a crucial role in saving fuel consumption, reducing pollution emissions and improving performance. Over the years, aerodynamicists have accumulated a wealth of experience in aerodynamic optimization, and aerodynamic optimization methods for airfoils have continued to develop.

In the early stage, aerodynamic design was limited to "cut-and-try", which resulted in low design efficiency [2]. In the 1980s, with the development of computational technology, aerodynamic design methods based on Computational Fluid Dynamics (CFD) began to develop [3]. CFD, wind tunnel testing, and theoretical analysis have gradually become the main approaches for aerodynamic design, which advance the progress of airfoil aerodynamic optimization. However, in actual engineering design, wind tunnel tests are expensive and theoretical analysis is less applicable when dealing with complex engineering problems, so CFD has gradually become the main method for aerodynamic analysis. With powerful high-performance computing (HPC) resources, CFD-based aerodynamic design advances the airfoil geometry optimization [4], greatly shortens the airfoil design cycle, and improves the aerodynamic performance of airfoil.

The basic element of airfoil aerodynamic optimization mainly includes design object, design objective, design constraints, and design method [5]. Design object refers to the aerodynamic geometric configuration. Design objective refers to the expected aerodynamic performance, flow field characteristics, etc. Design constraints illustrate the interdependence and constraints between the design variables of the design object, and design methods provide the strategies and means for achieving the design objective. The previous aerodynamic optimization is mainly based on the gradient algorithm. Jameson [6] integrates the control theory into the optimization process, which reduces the computational effort of the gradient calculation. However, the gradient algorithm has weak global optimization ability and is not applicable to the optimization of complex problems. Therefore, modern optimization algorithms based on group intelligence have been applied in aerodynamic design. The representative algorithms include Genetic Algorithm (GA) [7], Particle Swarm Algorithm (PSO) [8], Simulated Annealing Algorithm (SAA) [9], etc. These algorithms are highly adaptive to optimization and have better global optimization capability. With the improvement of aerodynamic design requirements, aerodynamic optimization has evolved from the conventional single-variable optimization to multi-variable joint optimization. Multi-objective optimization algorithms, such as Nondominated Sorting Genetic Algorithm-II (NSGA-II) [10] and Pareto Archived Evolution Strategy (PAES) [11], have begun to develop rapidly. Subsequently, Genichi Taguchi [12] introduced the concept of robust design, and researchers have developed robust design methods by applying uncertainty analysis methods to aerodynamic optimization [13–17]. The appropriate utilization of optimization strategies is beneficial to increase the efficiency, robustness, etc. of airfoil aerodynamic optimization.

The airfoil aerodynamic optimization includes many disciplines, and its design efficiency is widely constrained by the development level of parameterization, numerical methods, nonlinear mapping, optimization methods, etc. Aerodynamicists have gradually integrated artificial intelligence algorithms into aerodynamic optimization, greatly improving design accuracy and efficiency. The goal of artificial intelligence is to complete the simulation of human intelligence, and the realization relies on multidisciplinary cooperation including computers, mathematics, statistics, and data analysis, etc. In 2006, Hinton et al. [18] proposed the deep learning algorithm, the integration of artificial intelligence technology and big data guided a promising direction of machine learning. Advanced models based on artificial intelligence have been developed as tools for solving physical problems by learning from data [19], not simply by mechanical memorization but from a deep understanding of knowledge and mind construction.

Data is the foundation for constructing advanced models. If sufficient data are provided, well-trained advanced models can accurately predict and describe underlying physical phenomena without solving complex physical governing equations [19]. In recent years, aerodynamicists have integrated data-driven methods with artificial intelligence techniques, thus contributing to the rapid development of the "fourth paradigm" in current aerodynamic research [20]. In the process of airfoil aerodynamic optimization, data-driven advanced models have been widely employed, including rapid prediction of flow field [21–30], super-resolution reconstruction [31–41], differential equation solution [42–49], and grid generation based on artificial intelligence model [50–55]. Sufficient data acquisition and advanced model construction are highly essential in the optimal design, having a significant impact on the accuracy and efficiency of airfoil aerodynamic optimization.

A review of the main applications of data-driven advanced models in the critical steps of airfoil aerodynamic optimization is presented. Figure 1 illustrates the critical steps of aerodynamic optimization using advanced models introduced in this paper. The main content is divided into three sections, including geometric parameterization of airfoil, aerodynamic solving and performance mapping, and optimization model. Finally, the application of data-driven advanced models in aerodynamic design is summarized, and an outlook for future research is presented.

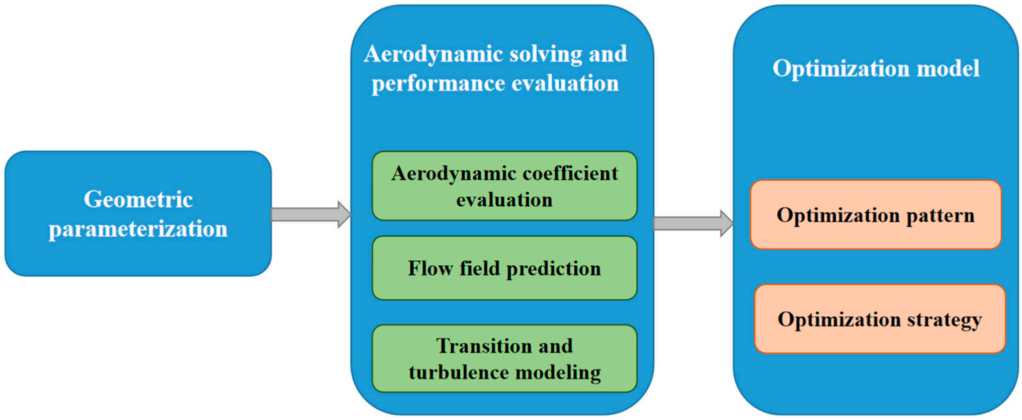


Figure 1. The critical steps of aerodynamic optimization using advanced models introduced in this paper.

2. Data-Driven Advanced Model in Critical Steps of Airfoil Aerodynamic Optimization

In recent years, with the increasing application of artificial intelligence (AI) technology in airfoil aerodynamic optimization, data-driven advanced models play a crucial role in various processes of optimization. The shape of the configuration is described by geometric parameterization. The appropriate design variables are determined, and smooth geometrical curves or surfaces are presented as a function. Geometric parameterization is the basis for the airfoil aerodynamic optimization [56]. Aerodynamic solving and performance mapping is applied to evaluate the aerodynamic performance of configuration, and the efficiency is greatly improved by constructing surrogate models [57]. The optimization model is the core of airfoil aerodynamic optimization, and its efficiency and capability of optimization search play a key role in the optimization system [58]. In this section, the application of data-driven advanced models in the above critical steps of airfoil aerodynamic optimization is presented with comments on their performance and contributions.

2.1. Geometric Parameterization

The accuracy and efficiency of the geometric profile description has a crucial impact on the quality of the optimization results. The method of geometrical description is also called geometric parameterization [59]. Various parametric methods influence the design space and determine the design boundaries of the optimization. In general, fewer geometric parameters are expected to achieve wider design space for efficient parametric methods. In order to meet the geometric control requirements of configurations in aerodynamic optimization, parametric methods need to provide accurate modeling and flexible geometric updating capabilities, with relevant adjustments based on aerodynamic design requirements and model complexity.

Geometric parameterization methods can be basically classified into two categories [56]. In first category are descriptive parameterization methods, which can directly achieve the description of the target geometry by given parameters, including PARSEC parametric section (PARSEC) [60], class function/shape function transformation (CST) [61], B-Splines [62], and non-uniform rational B-spline (NURBS) [63]. In the second category are geometrically modified parametric methods, which achieve the description of the target geometry by applying perturbations to the basic geometry, including Hicks–Henne (H–H) [64] and free-form deformation (FFD) [65]. The above conventional geometric parametric methods frequently employed an extensive set of design variables to ensure that the optimal design was encompassed within the prescribed design space. Nevertheless, such an approach can result in the curse of dimensionality and compromise design efficiency, which makes CFD solution and optimization difficult to merge. In choosing geometric dimensionality reduction techniques, there exists a trade-off between the precision of the global representation and the computational burden. Reducing the number of design parameters to improve the feasibility of an optimal design may inadvertently constrain the potential design space of the airfoil. Therefore, geometric parametric method is extremely essential in the aerodynamic optimization of airfoils.

Artificial intelligence tools provide a novel approach for geometric parameterization. Intelligent tools efficiently extract low-dimensional latent variables from a high-dimensional aerodynamic shape design space to ensure global geometric control and reconstruction accuracy. The effective parametric methods based on advanced models are mainly of two types, including modal parametric methods and geometric filtering methods [4]. Modal parametric models are employed to improve the geometric efficiency by coupling design variables. Geometric filtering models are employed to reduce the design space by adding geometric constraints. The aerodynamic geometry is controlled by derived modes applied in modal parametric methods. Orthogonal modes were extracted from the supercritical airfoil in the initial modal parametric method. Robinson and Keane [66] presented an approach to numerically derive a set of geometrically orthogonal base functions which provide a more concise representation of supercritical sections compared to analytically derived ones. Then principal component analysis (PCA)-based modal parameterization was gradually developed. Wang et al. [67] adopted PCA to perform dimensionality reduction on the geometry of high-lift device (HLD), and conducted the designing for robustness of stall lift using the proposed model. Poole et al. [68] applied singular value decomposition to a set of airfoil shapes for extracting airfoil deformation 'modes', which is used to mathematically define the optimal number of degrees of freedom. These modes were combined with the radial basis function (RBF)-based control point technique for unified shape parameterization, surface control and mesh deformation, linked to a parallel feasible sequential quadratic programming optimizer. Li et al. [69] suggested utilizing PCA to extract camber-thickness modes. Camber-thickness modes are the modes of airfoil camber and thickness lines. They are more intuitive and practical for airfoil shape optimization. Subsequently, deep learning techniques, such as variational encoders (VAE) [70] and generative adversarial networks (GANs) [71], have been explored for their capacity to acquire low-dimensional representations from intricate and widely distributed data. The data-driven advanced models for geometric parameterization have been developed. Chen et al. [72] introduced a novel parametric approach, utilizing Bézier-GAN, that incorporates latent code and noise variables to encode primary and secondary geometry variations. This method effectively reduced the design space by filtering out impractical airfoil shapes using intelligent techniques. The results demonstrated that the utilization of the Bézier-GAN parametric method expedites convergence and attains superior design results. Du et al. [73] proposed an intelligent parametric approach by combining B-Spline and a Generative Adversarial Network (GAN). The BSpline-GAN model was trained to generate airfoil geometries, effectively reducing the initial design space while preserving an adequate range of design flexibility. Wang et al. [74] introduced a method that combines autoencoders (AE) with powerful non-linear data-dimensionality reduction capabilities, along with class function/shape function transformation (CST), for deep manifold learning-assisted geometric multiple dimensionality reduction. The process depicted in Figure 2 involves the extraction of low-dimensional latent variables from a high-dimensional design space. These latent variables are then utilized to achieve a parametric representation of high-dimensional manifolds through the application of manifold learning techniques. U_α is the manifold satisfying geometric constraint α, U_β is the manifold satisfying geometric constraint β. U_α and U_β are open sets. For the two open set, two mappings exist, $\varphi_\alpha : U_\alpha \to \mathbb{R}^n$, $\varphi_\beta : U_\beta \to \mathbb{R}^m$. The transformations $\varphi_{\alpha\beta} = \varphi_\beta \varphi_\alpha^{-1}$ and $\varphi_{\beta\alpha} = \varphi_\alpha \varphi_\beta^{-1}$ refer to the transformation from the manifold to the coordinates and from local coordinates to the manifold, respectively. In comparison to conventional parametric methods, the proposed geometric dimensionality reduction technique enhances the precision and efficiency of geometric reconstruction and aerodynamic evaluation. These studies significantly advance the efficacy of parametric methods in airfoil applications, making notable contributions to the advancement of geometric dimensionality reduction methods combined with deep learning technology.

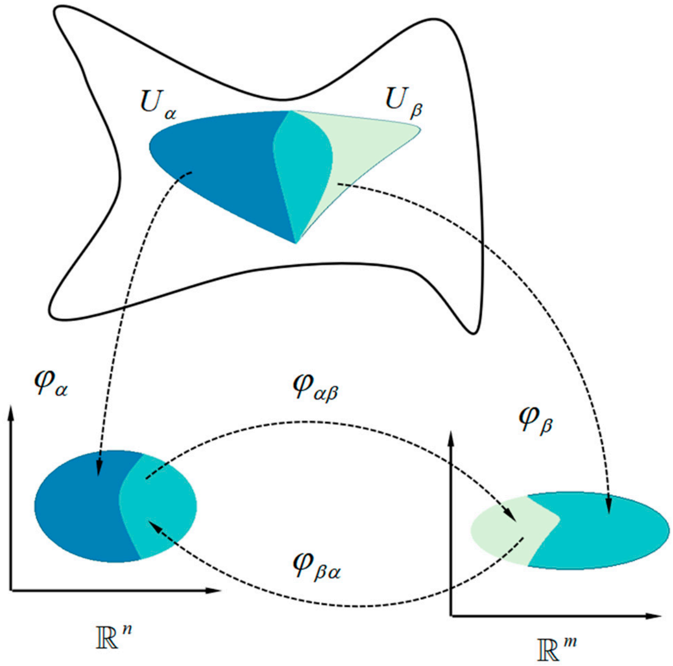


Figure 2. The concept of manifold learning applied in geometric parameterization [74].

On the other hand, geometric filtering methods are also the essential processes of geometric description. Geometric filtering evaluates the anomalies of the samples by defining a constraint function, which excludes the anomalous regions and reduces the design space [4]. Different to conventional geometric constraints, geometric filtering constraints are not subject to defined equations. Therefore, the data-driven advanced model are needed to be constructed [75]. Employing deep learning techniques, Li et al. [76] constructed a universal validation model to identify geometric anomalies in airfoils, as illustrated in Figure 3. Through ensuring that irregular shapes are not included in the training data, this method enhances the effectiveness of surrogate-based optimization by improving the precision of surrogate models. Moreover, this method benefits the development of precise and universally applicable data-driven aerodynamic models for interactive design optimization. The results validated the effectiveness of integrating data-driven advanced geometric filtering techniques in airfoil aerodynamic optimization.

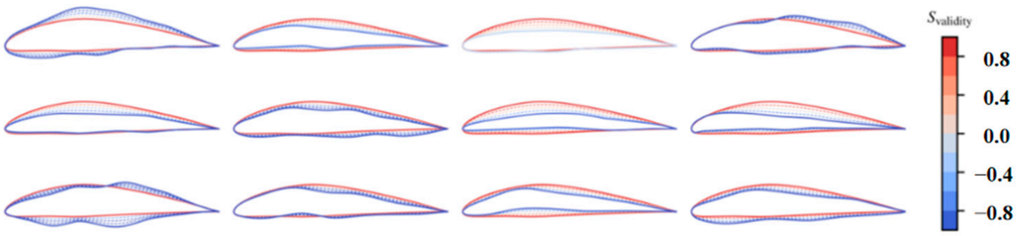


Figure 3. Geometric abnormality detection of airfoils by deep-learning model [76].

2.2. *Aerodynamic Solving and Performance Evaluation*

Aerodynamic solving and performance evaluation is essential in the aerodynamic optimization of airfoil. In the early stage, aerodynamic performance evaluation was based on CFD or wind tunnel experiments [77]. Especially in aerodynamic design, the evaluation of aerodynamic target and constraint functions corresponding to a large number of different airfoils tends to be costly.

Since the 21st century, the requirements and constraint complexity of aircraft design have gradually increased, and a complex coupling relationship between various disciplines also exists. Although traditional CFD methods can perform high-precision numerical simulations, the computation for data acquisition is much more expensive and time-consuming in airfoil aerodynamic optimization. Therefore, the surrogate model has emerged and gradually been developed as an essential branch and key technology of aerodynamic optimization [78].

The application of surrogate models can improve the efficiency of optimization design as well as reduce the complexity of optimization, and it is beneficial to filter out numerical noise and realize parallel optimization design [79]. It is worth stating that, although surrogate models can improve the performances in terms of simulation speed, they are bounded by the data that generated them and the characteristics of the model algorithm (trade-off speed/accuracy, danger associated extrapolation, predictive capabilities, etc.). In the early stage, a surrogate model is constructed based on sample point data that predict results similar to the results of the original model (numerical or experimental analysis) [80]. In addition, when it is difficult to express the objective function with an explicit function, a surrogate model can also be employed to express the objective function. The principle is as follows. Firstly, the design variables of the problem $x = (x_1, x_2, \cdots, x_n)^T$ and the upper and lower boundary of the variables (x_l, x_u) are determined. The locations of sample points $X_i = \left(x_1^{(i)}, x_2^{(i)}, \cdots, x_m^{(i)}\right)$ are determined by sampling and the response $Y_i = \left(y_1^{(i)}, y_2^{(i)}, \cdots, y_m^{(i)}\right)$ at the sample points X_i is obtained by numerical calculation. Finally, a surrogate model is constructed based on the dataset X and the response Y to realize the output $\hat{y} = f(x)$ at any x in the variable space. In order to improve the accuracy of model, the number of samples needs to be increased accordingly, but this also leads to an increase in computational cost. With the deep integration of AI technology and aerodynamic optimization, the data-driven surrogate model has demonstrated to be effective in reducing cost and improving accuracy [4]. In terms of aerodynamic solution and performance evaluation, the applications of data-driven advanced models can be classified into two categories: one is the performance prediction model with a limited set of scalar outputs, including aerodynamic coefficient prediction and flow field prediction; the other is the surrogate model, to enhance the performance of the flow field solution, which responses the function of time or a dimensionality comparable with the number of vertexes in the mesh (or both), including transition model, turbulence model, and boundary function model. In this section, the applications of data-driven advanced models in aerodynamic solving and performance evaluation are summarized. To facilitate readers in easily finding their interested targets and typical advanced models, we have compiled them along with their corresponding references, as shown in Table 1.

Table 1. A compliment of advanced models for aerodynamic solving and performance evaluation and their corresponding references.

Target	Typical Advanced Models	References
Aerodynamic coefficient evaluation	Response surface method	Ahn et al. [81] Giunta et al. [82]
	Kriging model	Han et al. [83,84]
	ANN/DNN	Oktay et al. [85] Wang et al. [67] Bouhlel et al. [86] Li and Zhang et al. [87]
	CNN	Zhang et al. [88] Yu et al. [89] Bakar et al. [90]
	Physical informed machine learning	Sun et al. [91]

Table 1. *Cont.*

Target	Typical Advanced Models	References
Flow field prediction	ANN/DNN	Renganathan et al. [28]
	CNN	Bhatnagar et al. [29]
	LSTM	Mohan and Gaitonde [23]
	Physical informed machine learning	Raissi et al. [45,46]
	Field inversion and machine learning	Holland et al. [92]
	VAE	Wang et al. [93]
	GAN	Wu et al. [94]
	GCN	Lan et al. [95]
Transition modeling and turbulence modeling	ANN/DNN	Tieghi et al. [96] Wang et al. [97,98]
	CNN	Zafar et al. [99]
	Physical informed machine learning	Wang et al. [100]
	Field inversion and machine learning	Yang et al. [101] Zhang et al. [102]
	Symbolic regression	Zhao et al. [103] Wu and Zhang et al. [104]

2.2.1. Aerodynamic Coefficient Evaluation

CFD solution provides abundant flow field information in the computational domain; however, aerodynamic design is generally based on aerodynamic performance indicators. Therefore, it is significant to construct the evaluation model of aerodynamic performance.

Traditional aerodynamic performance modeling is implemented based on classical surrogate models. The surrogate model is constructed by fitting a prediction function between the geometry design variables and the aerodynamic coefficients using training data generated by high-fidelity aerodynamic analyses. Traditional methods for training surrogate models include polynomial response surface method, support vector regression method, and the kriging model. Ahn et al. [81] applied the response surface method to the transonic airfoil design problem with another optimization method. The objective function and constraints of this method are modeled by quadratic polynomials, and the response surface is constructed by Navier–Stokes analysis in the transonic region. Giunta et al. [82] developed an aerodynamic response surface model for lift-induced volumetric wave drag and supersonic drag in wing design and employed the model to optimize the High Speed Civil Transport(HSCT) wing. Han et al. [83,84] used the kriging model approach to model the aerodynamic characteristics of the airfoil and wing, respectively, and then combined the gradient information obtained by the adjoint optimization method in efficient aerodynamic design. The traditional surrogate model has adjustable parameters; however, the main issue is that it is not suitable for handling large-scale training data, so it is usually trained with a small amount of data in a relatively limited space [4].

To address the above issue, machine learning-based methods for training aerodynamic surrogate models with neural networks have been developed to deal with large-scale training data efficiently. Artificial neural networks (ANN) were first applied to construct surrogate models of geometric–aerodynamic performance characteristics. With its economical computational effort and accurate generalization capability, the ANN surrogate model provides a feasible method for fast research and optimal solution in the aerodynamic design [67,85–87,105,106]. Oktay et al. [85] trained the model with the drag coefficient data obtained from the accumulated experimental results from the wind tunnel tests, and established the model to evaluate the accurate values of parameters of geometry relative to the input drag coefficient. Wang et al. [67] trained an inverse design model based on principal component analysis–artificial neural network (PCA–ANN) for stall-lift robust design considering aerodynamic constraints. Bouhlel et al. [86] employed a gradient-enhanced artificial neural network to train the aerodynamic surrogate model of the airfoil at subsonic and transonic conditions. The result of optimal airfoil is similar to that of high-fidelity CFD optimization. Li and Zhang [87] utilized 135,108 wing samples with different aerodynamic shapes, flight speeds, and flight altitudes. They trained the above data to construct an ANN-based aerodynamic analysis model for wing shape design. It is verified that the average relative errors of drag, lift, and pitch moments with the results by high-fidelity CFD are within 0.4%. The above results show that the ANN model can achieve aerodynamic evaluation with high accuracy on the basis of the large-scale database.

Artificial neural networks essentially analyze the algorithmic laws between input and output parameters through the connected neurons, and cannot locally sense and learn from the nodes in the flow field. With the development of machine learning techniques and aerodynamic evaluation, convolutional neural networks (CNN) with feature learning capability are applied to finely identify flow field images and perform the hierarchical learning of aerodynamic characteristics [88–90,107–109]. Zhang et al. [88] developed an appropriate CNN structure for varying flow conditions and geometric configurations, which could predict airfoil's lift coefficients under different Mach numbers, Reynolds numbers, and various attack angles. Yu et al. [89] proposed an enhanced deep CNN method by introducing the "feature enhance-image" strategy for airfoil images. Bakar et al. [90] presented a framework for designing low Reynolds number airfoils based on a CNN-based aerodynamic coefficient prediction model. The trained model is validated that can reproduce the authentic Pareto front in a very short time compared with other models. Powerful CNN models demonstrate promise in predicting aerodynamic coefficients based on geometric coordinates without relying on shape parameterization.

Besides, the introduction of physics-informed neural networks (PINNs) ensures prediction models adhere to the fundamental physics, with the improvement of prediction accuracy. PINNs possess extrapolation capabilities, which can mitigate the curse of dimensionality in airfoil aerodynamic optimization. Sun et al. [91] applied PINNs to concurrently model and optimize the airfoil flow, with the aim of maximizing its lift-to-drag ratio. PINN is intrinsically differentiable, allowing for the computation of gradients of the lift-to-drag ratio relative to the airfoil shape parameters. The airfoil aerodynamic optimization offers greater efficiency than non-gradient-based algorithms, without requiring the derivation of an adjoint code.

2.2.2. Flow Field Prediction

Aerodynamic coefficient prediction for configurations based on surrogate models could only be applied to analyze the specified aerodynamic performance in aerodynamic optimization. However, in order to study the flow mechanism, the prediction of flow field, such as pressure distribution and velocity distribution, is of great significance. It is important research to determine the flow field detail information of airfoil efficiently and accurately. Advanced models based on artificial intelligence can seek and learn hidden feature information from big data to predict the future behavior of complex nonlinear systems, providing a fast and alternative solution for solving complex and time-consuming N-S equations.

Flow field prediction involves the prediction of physical parameters for a large number of grid points within the flow field, thus dimensionality reduction is an essential step in flow field prediction. Mohan and Gaitonde [23] built reduced order modeling (ROM) by using long short-term memory (LSTM) to model the key physics/features of a flow-field without computing the full Navier–Stokes (NS) equations. In order to overcome heavy offline costs incurred by proper orthogonal decomposition-reduced order modeling (POD-ROMs) in constructing the reduced operators and adapting to parametric changes, Renganathan et al. [28] proposed a machine learning-based approach based on deep neural networks to learn the nonlinear dependence of the reduced state on high-dimensional parameters with modest training dataset sizes. This model was employed in prediction of flow field of RAE2822 airfoil under inviscid transonic flight conditions. Singh et al. [110] developed a modeling paradigm to augment the prediction of turbulence by machine learning utilizing limited data generated from physical experiments, and applied the methodology to turbulent flows over airfoils involving flow separation. The neural networks-augmented Spalart Allmaras (NN-augmented SA) model has great performance in predicting the pressure distribution of surface.

With the development of deep learning models, an increasing number of advanced models, e.g., convolutional neural networks (CNN), variational autoencoder (VAE), generative adversarial networks (GAN), and graph convolutional network (GCN), have been applied to flow field modeling and prediction. Zuo et al. [111] proposed a data-driven method based on CNN and multi-head perceptron to predict the incompressible laminar steady sparse flow field around the airfoils. The authors considered that the multi-head perceptron can achieve better prediction results than multi-layer perceptron for sparse flow field. Bhatnagar et al. [29] proposed a CNN-based approximation model for flow field prediction, and explored the effectiveness of the network structure in predicting flow fields with different airfoil shapes, angles of attack, and Reynolds numbers. Kashefi et al. [22] presented a novel PointNet-based framework for flow field predictions in irregular domains. Grid vertices in CFD domain were viewed as point clouds and used as inputs to a neural network, which learns an end-to-end mapping between spatial positions and CFD quantities. In the work by Wang et al. [93], a VAE network was designed to extract informative features from the flow fields. In order to predict the flow fields under high Reynolds number, multi-layer perceptron (MLP) is connected with the decoder of VAE. The effectiveness of this model was verified by achieving accurate predictions over the shock. Wu et al. [94] introduced a novel data-augmented GAN for rapid and precise flow filed prediction. This approach enables effective adaptation to flow field prediction tasks even with sparse data. Wu et al. [77] also leveraged the property of GAN combined with CNN to directly establish a one-to-one mapping between a parameterized supercritical airfoil and its corresponding transonic flow field profile over the parametric space. Lan et al. [95] presented a novel framework to predict cascades flow fields, utilizing GCN and point clouds to enhance prediction performance, as shown in Figure 4. It has been proved that the innovative framework can reconstruct the internal flow field at a high speed on a large-scale point cloud, while maintaining the accuracy of the prediction.

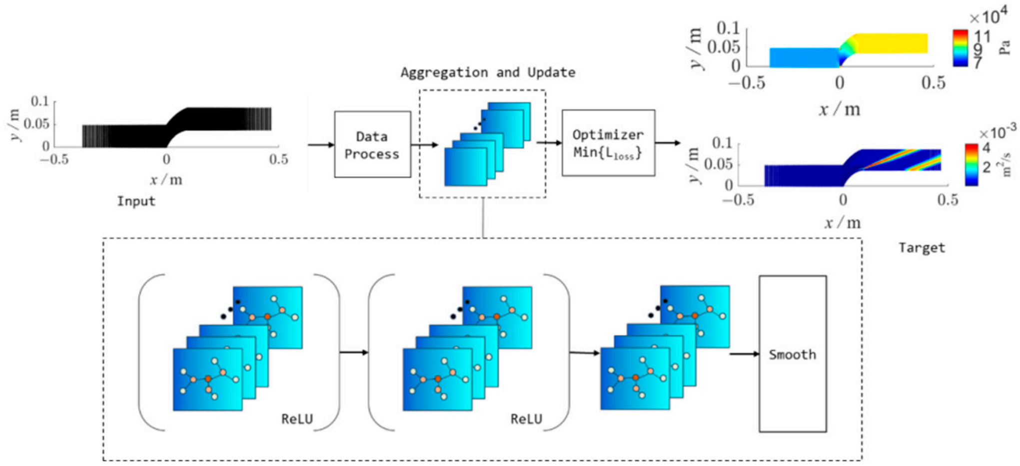


Figure 4. Using GCN and point clouds to enhance prediction performance [95].

Physical information performs an important role in flow field prediction, and data-driven models that fully incorporate physical constraints improve the accuracy and generalization of flow field prediction. Holland et al. [92] applied the Field Inversion and Machine Learning (FIML) approach to embed the discrepancies into functional forms reconstructed by machine learning within physical models. This model was demonstrated to improve

Reynolds-averaged Navier–Stokes (RANS) predictions for airfoils at high angles of attack. Raissi et al. [45] introduced PINN to solve the partial differential equations of flow field. The neural network was trained with specific flow field data and used the governing equations as constraints without boundary conditions to realize the prediction of the flow field in a specific region.

2.2.3. Transition Modeling and Turbulence Modeling

Simulation of complex flow phenomena and analysis of flow field mechanisms have been focused by researchers. According to the report "CFD Vision 2030 Study" by NASA, RANS method will remain the main numerical simulation method for CFD until 2030 [112]. Based on RANS, transition modeling and turbulence modeling are essential for accurate simulation of the flow field and the analysis of typical flow phenomena.

Boundary layer transition usually refers to the development of boundary layer flow from laminar flow to turbulence, which is a strongly nonlinear evolutionary process with the coupled influence of multiple factors [113]. Therefore, the complex transition mechanism has achieved much attention, and the calculation and prediction of the transition is the key factor in the design of the aircraft. In recent years, with the development of algorithms and the accumulation of transition data, boundary layer transition modeling methods incorporating machine learning have gradually developed [99,101,102,114–117]. Transition theories, models, and methods through artificial intelligence have been established to promote the intelligent application of fluid mechanics to overcome the lack of theory and experience. Zafar et al. [99] proposed a CNN-based model to construct a transition model. This method extracted integral quantities from the velocity profiles, and then fully connected layers were used to map the extracted integral quantities, along with frequency and Reynolds number, to the output (amplification ratio). Li et al. [114] trained a model to identify the turbulent/non-turbulent interface in the flow past a circular cylinder by machine learning method. Meng et al. [115] validated the effectiveness of prediction neural networks-model of transition location in three-dimensional (3D) hypersonic boundary layers. Besides the application of machine learning model for transition location determination, it is also critical to improve the predictive performance of conventional transition models by constructing advanced models for Reynolds-averaged simulations. Yang et al. [101] improved the four-equation k-omega-gamma-A(r) transition model by field inversion and machine learning. The regularizing ensemble Kalman filtering (EnKF) was employed to obtain the distributions of space-varied correction terms. Additionally, a mapping was established from the mean flow variables to the correction terms. Zhang et al. [102] developed a data-driven framework to enhance the prediction capability of the original gamma-Re(theta) transition model for the hypersonic boundary layer transition. The results demonstrated a significant improvement in the performance of transition prediction. The data-driven framework successfully enabled accurate determination of the boundary layer transition onset location and the length of the transition zone.

The capturing of turbulence phenomena is an essential means of flow mechanism analysis. The conventional turbulence models have difficulty achieving better prediction accuracy in complex flows such as large separations and unsteady conditions. In recent years, machine learning-based turbulence modeling has made great progress in improving the performance of traditional turbulence models and directly constructing surrogate models for Reynolds stress models [100,103,104,118–120]. Ling et al. [119] presented a model for the Reynolds stress anisotropy tensor employing deep neural networks based on high-fidelity simulation data, which constructed a multiplicative layer with an invariant tensor basis to embed Galilean invariance into the predicted anisotropy tensor. The improved turbulence model was demonstrated significant improvement compared with linear and nonlinear eddy viscosity model. Wang et al. [100] proposed a physics-informed machine learning approach for reconstructing discrepancies in RANS modeled Reynolds stresses based on DNS data and evaluated its effectiveness. Zhu et al. [120] between the turbulent eddy viscosity and the mean flow variables by neural networks, which improved the accuracy and generalization capability of turbulence models based on machine learning. Zhao et al. [103] introduced the gene-expression programming (GEP) to develop turbulence models based on CFD-driven training. The candidate EASM-like models were explicitly given via symbolic regression in GEP. Wu and Zhang [104] modified a Shear Stress Transport (SST) turbulence model using flow field inversion and symbolic regression, which provided a generalizable and interpretable data-driven approach for turbulence modeling based on the advanced model.

In order to analyze complex flows, in addition to transition modeling and turbulence modeling, near-wall modeling through the construction of wall functions is also utilized to assist the numerical simulations. The application of neural networks to construct data-driven wall surrogate models can achieve a compromise between accuracy and solution efficiency. Tieghi et al. [96] constructed a data-driven wall-function to k-epsilon simulations of a 2D periodic hill and a modified compressor cascade National Advisory Committee for Aeronautics (NACA) airfoil with a sinusoidal leading edge. The wall-function was trained by the multilayer perceptron ANN to obtain turbulent production and dissipation values near the walls. In our team's previous work, an ANN-based wall modification model was applied to perform multi-scale numerical simulations of aeronautical configurations with micro/nano-scale surface structures. For a realistic aeronautical configuration with micro-texture, using massive grids to describe the flow within the boundary layer makes the simulation of multi-scale flow field unfeasible. Wang et al. [97,98] proposed a novel aerodynamic solution strategy based on the wall modification model by machine learning to perform multi-scale numerical simulations. Flow features of micro-textured surface are provided by high-fidelity surface flow data acquired through Lattice Boltzmann Method (LBM) simulation. The wall modification model is constructed to reproduce the behavior of microflow near the micro-textured surface. The ANN-based wall modification model was applied in the simulation of airfoil and compressor cascade shown in Figure 5. The results indicate that the micro-textured surface structure has the effect of reducing the flow loss and the wall modification model was validated through numerical and experimental study.

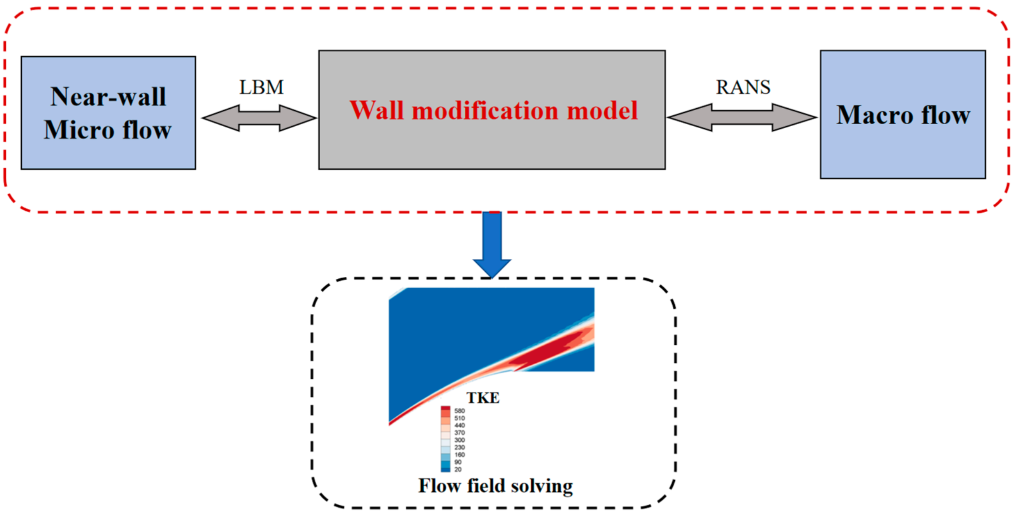


Figure 5. The framework of ANN-based wall modification model for multi-scale numerical simulations of compressor cascade with micro/nano-scale surface structures.

2.3. Optimization Model

The optimization model is an essential component of the airfoil aerodynamic optimization. After completing the geometrical parametric modeling and the aerodynamic performance evaluation of the flow field, it is necessary to establish an appropriate optimization model to design the airfoil. Advanced models support new optimization architectures and contribute to improving the performance of traditionally optimal design. In this section, optimization pattern and optimization strategy are introduced.

2.3.1. Optimization Pattern

In general, the airfoil aerodynamic optimization includes two optimization patterns, respectively, direct design and inverse design [121]. The optimization objectives of the direct design are mostly flow field integral features (e.g., lift, drag, moment, etc.), and are also single flow field features (e.g., laminar flow length, shock wave intensity, drag divergence velocity, etc.). The aerodynamic geometry that satisfies the objectives and constraints is obtained through optimization iterations. The optimization objectives of inverse design methods are mostly flow field features (e.g., pressure distribution, isentropic Mach number distribution, etc.). Through the parametric method, the geometry is continuously adjusted to obtain the aerodynamic shape that is closest to the target feature distribution of flow field.

In the direct optimization, the aerodynamic optimization process can be divided into gradient-based optimization and non-gradient-based optimization. The adjoint method is a typical gradient-based optimization algorithm, and the direction of searching optimization can be obtained directly by solving the adjoint equations. Tanabi et al. [122] proposed an adjoint-based optimization framework that can robustly optimize the shape of the airfoil based on variable parameters of the airfoil and flight conditions. Chen et al. [123] developed an adjoint-based robust optimization design framework of laminar airfoil subject to uncertainties in flight conditions for subsonic and transonic conditions. The results indicated that the adjoint-based optimization can improve the ability of the laminar airfoil under uncertain disturbance of flight conditions. Compared with the adjoint method in aerodynamic optimization, non-gradient optimization algorithms are more widely used in engineering problems. Non-gradient optimization is not able to directly obtain the gradient of the optimization objective with respect to the design variables, making the efficiency more dependent on the performance of the optimization algorithm. Non-gradient optimization has been developed since 1970s. First, aerodynamic evaluation was carried out based on small disturbance equation and velocity potential flow [124–127]. In the 1980s, modern optimization algorithms represented by particle swarm algorithm [8], genetic algorithm [7], and simulated annealing algorithm [9] gradually began to be used in aerodynamic optimization. In recent years, machine learning has started to be applied to improve the performance and efficiency of optimization. Owoyele et al. [128] proposed a novel design optimization approach that employs an ensemble of machine learning algorithm. Compared with genetic algorithm, the proposed method reduced the number of function evaluations needed to reach the global optimum, and thereby reduced time-to-design by 80%. Song et al. [129] presented a machine learning-based algorithm that can achieve much better aerodynamic performance and much shorter simulation time for the same airfoil optimization problem compared with the traditional genetic algorithm method.

Different from the direct optimization, the inverse design method usually requires the given desired aerodynamic parameters (e.g., pressure coefficient distribution, velocity distribution, etc.). The aerodynamic configuration that satisfies the given flow field features are obtained by solving the flow control equations step by step. The inverse design method is efficient and favorable for engineering when the aerodynamic constraints are given. Sun et al. [130] introduced an applicable airfoil/wing inverse design method with the help of ANN and airfoil/wing database, and provided the verification of the applicability of the approach. Sekar et al. [131] proposed an approach to perform the inverse design of airfoils using CNN. Due to the excellent capability of airfoil feature extraction, the pressure coefficient distribution was as the input to CNN, and the airfoil geometry was the output. However, even though the inverse design approach is efficient, it requires giving the excepted flow field distribution in advance, which makes it difficult to guarantee that the target distribution is the optimal solution. Therefore, researchers have improved the aerodynamic inverse design method to address this problem. Zhang et al. [132] determined the derivatives of the design target by the adjoint method in the airfoil inverse design.

Sketching a rough expectation of the pressure distribution law can greatly improve the efficiency of the initial stage of optimal. Wang et al. [133] employed the conditional variational autoencoder (CVAE) and Wasserstein GAN (WGAN) to generate the target wall Mach distribution, and a deep neural network was used for nonlinear mapping to obtain an airfoil geometry corresponding to the target wall Mach distribution. Yang et al. [134] used a VAE and MLP to generate realistic target distributions, respectively, and predicts the quantities of interest and shape parameters from the generated distribution. The target distribution was then determined in the inverse optimization. Wang et al. [135] proposed an inverse design framework based on improved GAN shown in Figure 6. The GAN model was trained using a database of pressure distributions obtained from CFD, and a large number of samples were generated using a well-trained generator to search for the optimal pressure distribution. Double ANN models were built to evaluate the aerodynamic performance of the target distribution and obtain the geometry corresponding to the target distribution. The results showed that the inverse design framework with the improved GAN model achieved better optimal results compared with the traditional method.

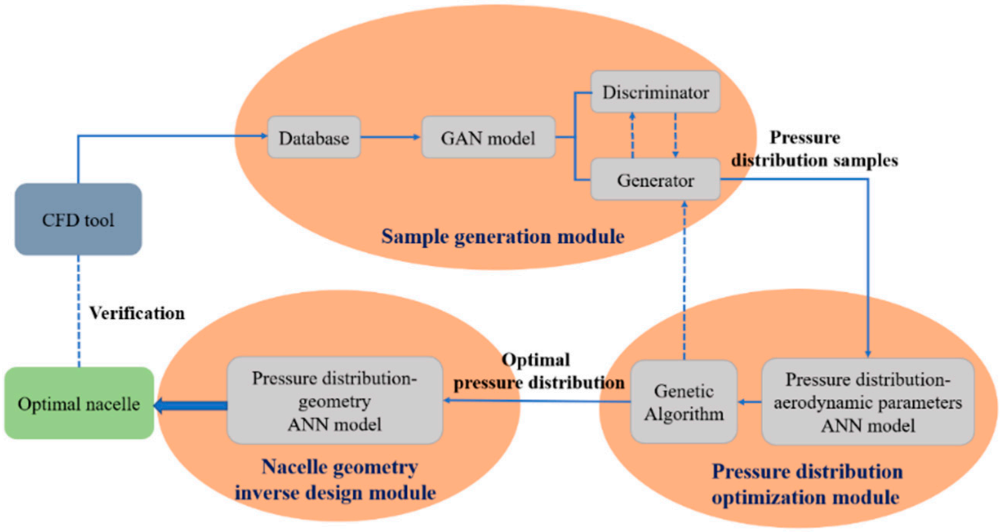


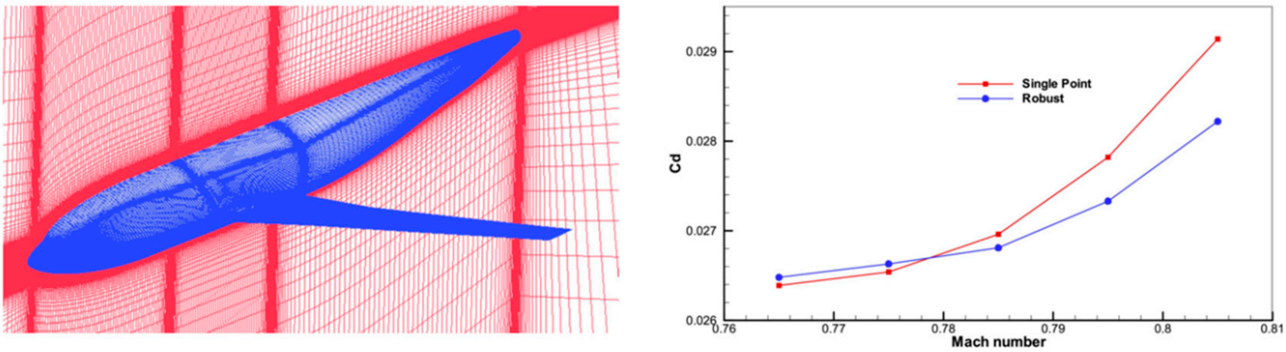
Figure 6. The framework of an inverse design based on improved GAN [135].

2.3.2. Optimization Strategy

The optimization strategy has a significant impact on determining the aerodynamic optimization model, and a suitable design strategy can improve the efficiency and accuracy of the design problem. In this section, several typical optimization strategies are introduced.

In order to enhance the optimization efficiency, a two-wheel optimization strategy is applied for the airfoil aerodynamic design. The first round of optimization provides an initial solution and does not require high accuracy. The second round of optimization is based on the results of the first round of optimization and minor adjustments are conducted to obtain the object with better performance. Xing et al. [136] obtained supercritical airfoil with better laminar performance using a genetic algorithm based on a surrogate model. The proportion of laminar flow region is expected to improve while satisfying the lift coefficient, lift-to-drag ratio, airfoil thickness, and airfoil leading edge radius. Due to the slow convergence of the genetic algorithm and the large amount of computation, the goal of this round is just to provide a better initial solution for the second round of optimization. The goal of the second round of optimization is to minimize or eliminate the shock wave while ensuring the proportion of the laminar flow region, so as to further improve the lift-to-drag ratio of the airfoil.

Multi-point optimization strategy is an effective method to improve the robustness of airfoil aerodynamic optimization design. Hicks and Vanderplaats [137] found that a single-point optimization of the drag at a design point is not enough to guarantee the drag performance near the design point. A two-point optimal design or a multi-point optimal design can address this issue, making the optimal design more robust. The authors set the drag at the design Mach number as the objective function and the drag coefficients at the non-design Mach number are defined as constraints, thus realizing multi-point aerodynamic optimization. In the optimal design process, a series of constraints are needed to be imposed. The mathematical description of constraints and design indicators is still difficult. Ye et al. [138] proposed a multi-point optimization strategy, which can make up for the insufficiency of constraints and design indicators difficult to be described mathematically, so as to improve the efficiency and accuracy of optimal design. Multi-point optimization can make the design more robust to some extent. However, this method relies strongly on the selection of design points and weights. Wu et al. [139] mathematically demonstrated that it is necessary to ensure that the number of design points for a multi-point optimization is greater than the number of design variables characterizing the airfoil, otherwise the deterioration of the airfoil performance in the off-design state will not be avoided. On the other hand, due to the cost of aerodynamic analysis, the number of design points cannot be too large. Robust optimization is a concept appropriate for aerodynamic design. Robust optimization requires both improving the performance of the configuration and reducing the sensitivity of the configuration to uncertainties, allowing the configuration to provide excellent performance and stability in the region of variation of stochastic factors. This method is generally applied to conditions where there are uncertain parameters or disturbances. Tao et al. [13] proposed an improved PSO (particle swarm optimization) algorithm in the robust optimization of a wing at drag divergence Mach number. Compared with the results of single-point optimization, the robust optimization reduced drag coefficients of the wing at both cruise Mach numbers and non-designed Mach numbers, as shown in Figure 7.



(a) (b)

Figure 7. The results of robust optimization: (**a**) The geometry of a wing; (**b**) The comparison of results of drag coefficient between single-point optimization and robust optimization [13].

In engineering issues, most of the aerodynamic optimization is multi-objective optimization problem. Multiple objectives do not have the same properties and are even in conflict with each other. For example, some objectives are continuous and differentiable functions, while others are discontinuous or discrete functions. The objective of multi-objective optimization is to obtain a set of approximate Pareto optimal solutions. The Nash equilibrium theory [140] is an effective approach for solving multi-objective optimization in aerodynamics. Based on the concept of competitive games, the solution of a multi-objective optimization problem can be considered as a Nash equilibrium. Tang et al. [141] proposed a Nash equilibrium strategy in aerodynamic optimization and applied it to optimize the drag coefficient of a wing–body fusion at transonic speeds while maintaining a constant lift coefficient. The results showed that the Nash strategy was able to effectively couple local and global optimization without increasing the computational cost. The drag of the wing–body fusion body was significantly reduced while keeping the lift constant. The geometry and pressure distributions of the initial and Nash-optimized wings are indicated in Figure 8. A significant shock wave reduction was achieved on the upper airfoil of the Nash-optimized wing. Han et al. [142] studied the method for multi-objective optimization when gradients were available. In this method, multi-objective evolutionary algorithm (MOEA/D) was combined with gradient-enhanced kriging (GEK), and the effectiveness of the proposed method was validated for a transonic airfoil through experimental studies.

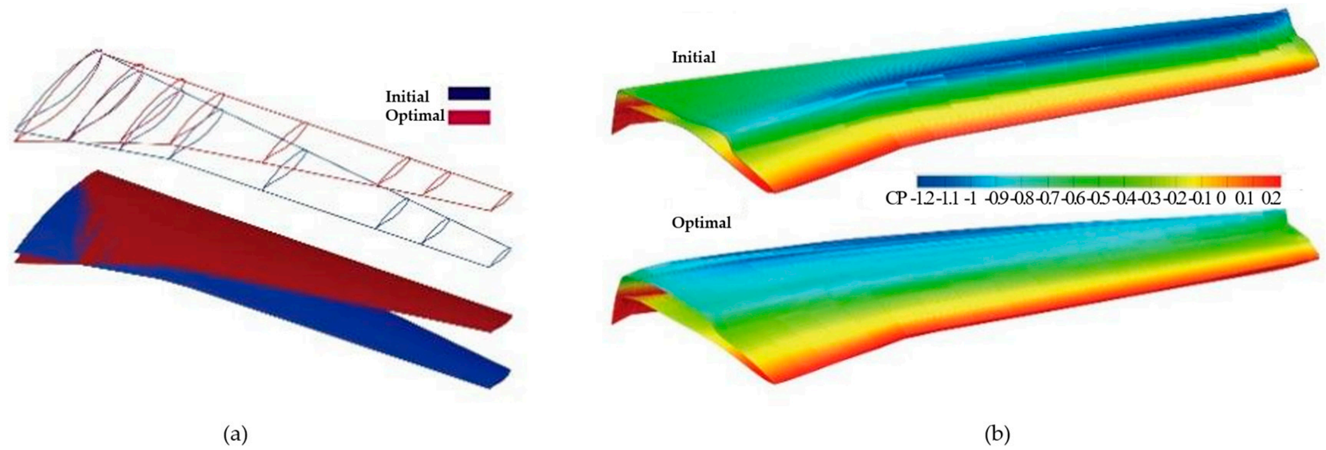


Figure 8. The results of multi-objective optimization based on Nash-equilibrium strategy: (**a**) The geometry of a wing; (**b**) The results of pressure coefficient distribution [141].

3. Conclusions

In this paper, the application of advanced models to airfoil aerodynamic optimization has been reviewed and discussed. Data-driven advanced models have been employed successfully in critical steps of aerodynamic optimization including geometric parameterization, aerodynamic solving and performance evaluation, and optimization model. With the evolution of artificial intelligence technology, advanced models have gradually developed from small-scale expert knowledge models to machine learning models. Advanced models have developed from neural networks at the beginning, to multi-layer neural networks based on deep learning, convolutional neural networks, transformer models, and then to the current GPT models. The development and refinement of advanced models has promoted the process of integration between artificial intelligence and aerodynamics. Based on the updating and iteration of advanced models, the surrogate and optimization models are provided with more superior performance, thus making it possible to solve complex aerodynamic optimization problems efficiently.

With respect to the further application of data-driven advanced models in aerodynamic optimization, the following prospects are proposed and suggested:

(a) Expanding the database for aerodynamic modeling. A large and sufficient amount of data is the basis for aerodynamic modeling. There is no sufficient publicly available dataset with abundant flow field characteristics for aerodynamic optimization. The quantity and quality of data limit the further development and application of models. In order to address the above problem, it is recommended to develop aerodynamic modeling strategies applicable for small-scale data, e.g., data augmentation and meta-learning-based modeling methods. The construction of a large-scale aerodynamic database might also be enhanced, and the data fusion of wind tunnel test, flight test, and numerical simulation data could be considered.

(b) Improving the interpretability and generalization of advanced models. Most of the current aerodynamic models are "black box" models, making it hard for researchers to understand the learning principle and process in the network. Improving the interpretability of advanced models and transforming models from the original "black box" to "gray box" or even "white box" will help to enhance the understanding of aerodynamics and realize the update of knowledge in the progress of optimization. On the other hand, improving the generalizability of advanced models is also significant for the expansion of application scenes and the improvement of optimization design efficiency.

(c) Enhancing the ability of advanced models to solve 3D complex configuration optimization. The "dimensional disasters" by three-dimensional complex configurations are a great challenge for optimization design. Compared to aerodynamic optimization of 2D configurations, the difficulty of each process of 3D complex configuration optimization increases significantly. Most of the current advanced models focus on improving the efficiency of solving existing problems, and advanced models should be developed to address the unsolved issue by the traditional methods, especially to facilitate the aerodynamic optimization of 3D complex configurations.

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