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Fuzzy Evaluation Model for Products with Multifunctional Quality Characteristics: Case Study on Eco-Friendly Yarn

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Abstract: Numerous advanced industrial countries emphasize green environmental protection alongside athletic healthcare. Many world-renowned sports brands are actively developing highly functional, environmentally friendly, and aesthetically pleasing products. For example, in the production of sports shoes, the eco-friendly yarn process is one of the important processes. This process involves multiple crucial larger-the-better quality characteristics closely tied to the functionality of sports shoes. Facing green environmental regulations and external competitors, it is evidently an imperative issue for enterprises to consider how to improve the quality of newly developed products, increase product value, and lower rates of both rework and scrap to accomplish the goals of saving energy and minimizing waste. Aiming to solve this problem, this study proposed a fuzzy evaluation model for products with multifunctional quality characteristics to assist the sporting goods manufacturing industry in evaluating whether all functional quality characteristics of its products meet the required quality level. This study first utilized the larger-the-better Six Sigma quality index concerning environmental protection for evaluation and then proposed product evaluation indicators for the eco-friendly yarn. Since the parameters of these indicators have not yet been determined, sample data need to be used for estimation. Enterprises require rapid response, so that the sample size is relatively small. Sampling error will increase the risk of misjudgment. Therefore, taking suggestions from previous studies, this study constructed the fuzzy evaluation model based on confidence intervals of quality indicators for the eco-friendly yarn. This method incorporated previous experience with data, thereby enhancing assessment accuracy.

Keywords: larger-the-better; Six Sigma quality index; green environmental protection; upper confidence limit; fuzzy testing model

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1. Introduction

The rising awareness of green environmental protection is relatively significant, and the concept of athletic healthcare is also highly valued in numerous countries with advanced industries. Therefore, many well-known sports brands are proactively developing new products highlighting high functionality, environmental friendliness, and pleasing appearance. In addition, plenty of research has also indicated that good process quality can not only improve product yield and product value but also lower rates of rework and scrap [1–3]. Since enterprises are confronting the regulatory pressure of green environmental protection and competition from external rivals, they need to contemplate how to boost the quality of newly developed products, increase product value, and minimize rates of both rework and scrap to reach the goals of conserving energy and reducing waste. Evidently, this is a crucial matter. In order to solve this problem, this study proposes a fuzzy evaluation model for products that have multiple functional quality characteristics to help the sporting goods manufacturing industry evaluate whether all functional quality characteristics of its products meet the required quality level. In addition, this model can improve the quality characteristics to meet quality standards, thereby enhancing product value and industrial competitiveness.

Furthermore, in the production of sports shoes, the eco-friendly yarn process is one of the vital processes. Regarding the eco-friendly yarn, strength, pulling force, and yellowing resistance are three essential quality characteristics, two of which are closely related to the functionality of sports shoes. Additionally, these three important quality characteristics all fall into the category of the large-the-better (LTB) quality characteristics. Borgoni et al. [4] and Sanchez-Marquez et al. [5] have demonstrated that the process capability index is a convenient and effective tool for evaluating product quality. As noted by Wang et al. [6], many statisticians and quality engineers have invested in doing research on process capability indices to raise products' process quality. Targeting the LTB quality characteristics, Chen et al. [7] modified the process capability index recommended by Kane [8]. The revised LTB Six Sigma quality index is presented as follows:

$$Q_{PL} = \frac{\mu - LSL}{\sigma}, \tag{1}$$

where μ represents process mean, σ denotes process standard deviation, and LSL stands for lower specification limit.

The process distribution built on the Taguchi loss function usually obeys the normal distribution [9], which is called the normal process in this paper. Therefore, this study proposes a relevant evaluation model under the premise of normal manufacturing processes. As normality is assumed, the process yield p is defined as:

$$Yield\% = p(X \geq LSL) = p\left(Z \geq -\frac{\mu - LSL}{\sigma}\right) = \Phi(Q_{PL}), \tag{2}$$

where $Z = (X - \mu)/\sigma$ is viewed as a standard normal distribution, denoted by $N(0, 1)$. According to Equation (1), when the process mean exceeds the lower specification limit (LSL), or the process standard deviation is smaller, then the Six Sigma quality index tends to be larger. It is evident that the Six Sigma quality index can depict the process quality level as well as maintain a direct relationship with the process yield. Therefore, this study utilizes this index as an evaluation tool for the process quality of eco-friendly yarn.

Ying et al. [10] and Pearn et al. [11] have pointed out that as the process quality for each quality characteristic attains the required quality level, it is guaranteed that the process quality of the product can satisfy customer requirements. For this reason, this study integrates the LTB Six Sigma quality indices to evaluate three important quality characteristics for the eco-friendly yarn and then proposes a product quality index as an evaluation tool for the product quality. Since the product quality index is composed of evaluation indicators of all individual quality characteristics, the evaluation indicators for all individual quality characteristics must exceed the required product quality index. Only when the Six Sigma quality indices of all individual quality characteristics attain the desired level can the final product's quality be ensured to satisfy the requirements for quality standards [12–15]. Subsequently, building upon the desired product quality level, we define the required Six Sigma quality index for each individual quality characteristic. Now that these indices have unidentified parameters, we must estimate them using sample data [16]. According to some studies, since companies focus on rapid response, the sample size of sample data is usually not large. As a result, there is a growing risk of wrong judgment due to large sampling errors [17–19]. Hence, following suggestions from some studies and building upon the upper confidence limit, we come up with a fuzzy evaluation model for the product quality index of eco-friendly yarn in this paper [13,16,19,20]. Based on the above-mentioned, some studies are working on establishing process capability evaluation rules for products with multiple quality characteristics, whereas there is a lack of research on products with multiple LTB quality characteristics. Consequently, the model proposed by this study can fix this gap. In addition to this advantage, the proposed fuzzy evaluation model offers the following benefits:

1 The LTB Six Sigma quality index is utilized as an evaluation tool, not only having a one-to-one mathematical relation with the yield rate but also fully reflecting the Six Sigma quality level [7,13].
2 Based on the upper confidence limit, the risk that sampling errors may lead to misjudgment can be diminished [20,21].
3 When integrated with past accumulated data and experience, this model can boost evaluation accuracy [22].

We structure the remaining sections of this paper as follows. In Section 2, we introduce the larger-the-better (LTB) Six Sigma quality indicators for three key quality characteristics of eco-friendly yarn. Additionally, we integrate these three LTB Six Sigma quality indicators into a product quality index for eco-friendly yarn as well as explore the values of these indicators. In the meantime, we derive the mathematical relationship between the required Six Sigma quality index for each quality characteristic and the required product quality index for the eco-friendly yarn. In Section 3, we deduce three estimators for the LTB Six Sigma quality indices based on sample data as well as incorporate these three estimators into an estimator of the product quality index. Next, the $100(1-\alpha)\%$ upper confidence limits of these three LTB Six Sigma quality indicators are derived, and the required Six Sigma quality index for each quality characteristic derived from Section 2 is adopted as the required value for testing. In addition, the $100(1-\alpha)\%$ upper confidence limits are employed to develop a fuzzy testing model for the Six Sigma quality index. In Section 4, a case study is adopted to demonstrate how to apply the proposed fuzzy testing model. Finally, in Section 5, conclusions are made.

2. Required Values of Quality Evaluation Indices and Upper Confidence Limits

As mentioned earlier, green environmental protection and athletic healthcare are two issues highly valued by numerous countries with advanced industries. Plenty of well-known sports brands are actively developing new products that are functional, environmentally friendly, and beautiful. In the face of regulatory requirements for green environmental protection and many external competitors, it is a crucial task for enterprises to raise the quality of newly developed products, increase product value, and diminish rates of both rework and scrap so as to achieve the goals of energy conservation and waste reduction. During the production of sports shoes, the manufacturing process of eco-friendly yarn is one of the essential processes. Firstly, a receiving inspection mainly focuses on the analysis of TPU particles, and a melt index of the material is employed as a testing item. Then, this index is utilized to determine which parameters need to be used in the subsequent processing of raw materials. Appropriate parameters, including cooling water temperature, dryer temperature, and the rotation speed of the stretching machine, are established. Next, a series of processing steps, such as mixing, dehumidification and drying, yarn extrusion, cooling and shaping, stretching, lubricant roller application, winding into coils, and finished product inspection, are carried out. Finally, inspections are performed before shipments, in order to ensure that the characteristics of eco-friendly yarn meet the product quality required by customers.

Eco-friendly yarn has three important larger-the-better (LTB) quality characteristics: pulling force, strength, and yellowing resistance. Among them, pulling force and strength are necessary functions for a pair of high-quality sports shoes. Yellowing resistance is least relevant to functionality, whereas it is related to the beauty and brand image of the shoes. Consequently, they are all crucial quality characteristics. The lower specification limits of these three key LTB quality characteristics are displayed in the following Table 1:

Table 1. Three key LTB quality characteristics and the lower specification limits for eco-friendly yarn.

Quality Characteristic	*LSL*	Unit
1. Pulling force	LSL_1 = 0.270	kgf
2. Strength	LSL_2 = 1.70	g/D
3.Yellowing resistance	LSL_3 = 3	level

The LTB Six Sigma quality index, employed to evaluate these three important quality characteristics, is denoted by the following equation:

$$Q_{PLj} = \frac{\mu_j - LSL_j}{\sigma_j}, \tag{3}$$

where j = 1, 2, 3. As mentioned earlier, as long as the process quality of these three important quality characteristics reaches the required process quality level, the product quality of eco-friendly yarn can be ensured. According to Equation (2), the process yield of quality characteristic h is $Yield\% = p_j = \Phi(Q_{Pj})$. Additionally, based on Chen et al. [20], the product yield of eco-friendly yarn is:

$$p_T = \prod_{j=1}^{3} \Phi(Q_{PLj}). \tag{4}$$

Therefore, the product quality index of eco-friendly yarn is defined as follows:

$$Q_{PL}^T = \Phi^{-1}\left(\prod_{j=1}^{3} \Phi(Q_{PLj})\right). \tag{5}$$

According to Equations (4) and (5), we have

$$p_T = \prod_{j=1}^{3} p_j = \prod_{j=1}^{3} \Phi(Q_{PLj}) = \Phi\left(Q_{PL}^T\right). \tag{6}$$

Obviously, concerning eco-friendly yarn, there is a one-to-one mathematical relationship between its product quality index and its product yield. As mentioned above, when the required quality index of eco-friendly yarn is v, the required quality evaluation index v' for each quality characteristic must exceed v. When the process quality for each quality characteristic attains the required level, then the eco-friendly yarn is considered a quality product. Accordingly, Equation (5) can be rewritten as

$$v' = \Phi^{-1}\left(\sqrt[3]{\Phi(v)}\right). \tag{7}$$

The required values of quality indices and product indices are shown in Table 2.

Table 2. The required values of quality indices and product indices.

Quality Level	Q_{PL}^T	Q_{PLj}
6	6	6.176
5	5	5.208
4	4	4.253
3	3	3.320

The 100(1 − α)% upper confidence limit of the quality evaluation index Q_{PLj} is calculated in this paper. Suppose $\left(X_{j,1}, \cdots, X_{j,i}, \cdots, X_{j,n}\right)$ is a random sample of the quality characteristic j, where j = 1, 2, 3. The sample mean and the sample standard deviation of the quality characteristic j are expressed respectively as follows:

$$\overline{X}_j = \frac{1}{n} \times \sum_{i=1}^{n} X_{j,i} \tag{8}$$

and

$$S_j = \sqrt{\frac{1}{n-1} \times \sum_{i=1}^{n} \left(X_{j,i} - \overline{X}_j\right)^2}. \tag{9}$$

Therefore, the natural estimator of the quality evaluation index can be defined as

$$Q^*_{PLj} = \frac{\overline{X}_j - LSL_j}{S_j}. \tag{10}$$

As normality is assumed, let random variable T_j be

$$T_j = \frac{\sqrt{n}\left[\left(\overline{X}_j - LSL_j\right) - \left(\mu_j - LSL_j\right)\right]}{s_j} = \sqrt{n}\left(Q^*_{PLj} - Q_{PLj}\left(\frac{\sigma_j}{S_j}\right)\right). \tag{11}$$

Then T_j has a t-distribution with $n-1$ degrees of freedom, written as t_{n-1}. Thus,

$$\begin{aligned} & p\left\{T_j \geq -t_{\alpha/2;n-1}\right\} = 1-\alpha/2 \\ \Rightarrow \quad & p\left\{\sqrt{n}\left(Q^*_{PLj} - Q_{PLj}\left(\tfrac{\sigma_j}{S_j}\right)\right) \geq -t_{\alpha/2;n-1}\right\} = 1-\alpha/2, \\ \Rightarrow \quad & p\left\{Q_{PLj} \leq \left(Q^*_{PLj} + \tfrac{t_{\alpha/2;n-1}}{\sqrt{n}}\right)\left(\tfrac{S_j}{\sigma_j}\right)\right\} = 1-\alpha/2 \end{aligned} \tag{12}$$

where $t_{\alpha/2;n-1}$ represents the upper $\alpha/2$ quantile of t-distribution with $n-1$ degrees of freedom. Similarly, let random variable K_j be

$$K_j = \frac{(n-1)S_j^2}{\sigma_j^2}. \tag{13}$$

Then random variable K_j has a Chi-square distribution with $n-1$ degrees of freedom, denoted by χ^2_{n-1}. Thus,

$$\begin{aligned} & p\left\{K_j \leq \chi^2_{1-\alpha/2;n-1}\right\} = 1-\alpha/2 \\ \Rightarrow \quad & p\left\{\tfrac{(n-1)S_j^2}{\sigma_j^2} \leq \chi^2_{1-\alpha/2;n-1}\right\} = 1-\alpha/2, \\ \Rightarrow \quad & p\left\{\tfrac{S_j}{\sigma_j} \leq \sqrt{\tfrac{\chi^2_{1-\alpha/2;n-1}}{n-1}}\right\} = 1-\alpha/2 \end{aligned} \tag{14}$$

where $\chi^2_{1-\alpha/2;n-1}$ represents the lower $1-\alpha/2$ quantile of χ^2_{n-1}. Aiming to obtain the $(1-\alpha)100\%$ upper confidence limits of the quality evaluation index Q_{PLj}, we illustrate two events as follows:

$$A_j = \left(Q_{PLj} \leq \left(Q^*_{PLj} + \frac{t_{\alpha/2;n-1}}{\sqrt{n}}\right)\left(\frac{S_j}{\sigma_j}\right)\right) \tag{15}$$

and

$$B_j = \left(\frac{S_j}{\sigma_j} \leq \sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}}\right). \tag{16}$$

Obviously, $p(A_j) = p(B_j) = 1 - \alpha/2$ and $p\left(A_j^c\right) = p\left(B_j^c\right) = \alpha/2$, where event A_j^c is the compliment of event A_j, and event B_j^c is the compliment of event B_j. Based on DeMorgan's rule and Boole's inequality [23], we have

$$p(A_j \cap B_j) \geq 1 - p\left(A_j^c\right) - p\left(B_j^c\right) = 1 - \alpha. \tag{17}$$

According to Equations (15)–(17), we have

$$p\left(Q_{PLj} \leq \left(Q^*_{PLj} + \frac{t_{\alpha/2;n-1}}{\sqrt{n}}\right)\left(\frac{S_j}{\sigma_j}\right), \frac{S_j}{\sigma_j} \leq \sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}}\right) \geq 1 - \alpha. \tag{18}$$

Consequently, we have $p(Q_{PLj} \leq UQ_{PLj}) \geq 1 - \alpha$, where UQ_{PLj} represents the $(1-\alpha)100\%$ upper confidence limit of the quality evaluation index Q_{PLj}, as shown below:

$$UQ_{PLj} = \left(Q^*_{PLj} + \frac{t_{\alpha/2;n-1}}{\sqrt{n}}\right)\sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}}. \tag{19}$$

3. Fuzzy Hypotheses for Testing

According to Equation (7), as the required index of the product quality is designated as v, the required index of quality evaluation for each quality characteristic is v', where $v' = \Phi^{-1}\left(\sqrt[3]{\Phi(v)}\right)$. Clearly, when the values of the three individual quality evaluation indices exceed or equal v', then the values of the product quality indices also exceed or equal v. Accordingly, to determine whether the value of each individual quality evaluation index exceeds or equals v', the hypotheses for testing are listed below:

H_0: $Q_{PLj} \geq v'$ *(The individual quality meets the required level).*

H_1: $Q_{PLj} < v'$ *(The individual quality does not meet the required level).*

Suppose $(x_{j,1}, \cdots, x_{j,i}, \cdots, x_{j,n})$ represents the observed values of $(X_{j,1}, \cdots, X_{j,i}, \cdots, X_{j,n})$. The observed values of the sample mean $\overline{X}_j$ and the sample standard deviation S_j are respectively defined as follows:

$$\overline{x}_j = \frac{1}{n} \times \sum_{i=1}^{n} x_{j,i} \tag{20}$$

and

$$s_j = \sqrt{\frac{1}{n-1} \times \sum_{i=1}^{n} (x_{j,i} - \overline{x}_j)^2}. \tag{21}$$

Thus, the observed value of estimator Q^*_{PLj} for each quality evaluation index is denoted by

$$Q^*_{PLj0} = \frac{\overline{x}_j - LSL_j}{3s_j}. \tag{22}$$

The observed value of the $100(1-\alpha)\%$ upper confidence limit UQ_{PLj} is denoted by

$$UQ_{PLj0} = \left(Q^*_{PLj0} + \frac{t_{\alpha/2;n-1}}{\sqrt{n}}\right)\sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}}. \tag{23}$$

Similar to Yu et al. [21], this study developed the fuzzy testing method building upon Q^*_{PLj0}. According to Equation (23), the α-cuts of the triangular fuzzy number $\widetilde{Q}^*_{PLj0}$ is acquired and displayed as

$$\widetilde{Q}^*_{PLj0}[\alpha] = \begin{cases} [UQ_{PLj0}(1), UQ_{PLj0}(\alpha)], & for\ 0.01 \leq \alpha \leq 1 \\ [UQ_{PLj0}(1), UQ_{PLj0}(0.01)], & for\ 0 \leq \alpha \leq 0.01 \end{cases}, \tag{24}$$

where

$$UQ_{PLj0}(1) = Q^*_{PLj0}\sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}; \quad (25)$$

$$UQ_{PLj0}(\alpha) = \left(Q^*_{PLj0} + \frac{t_{\alpha/2;n-1}}{\sqrt{n}}\right)\sqrt{\frac{\chi^2_{1-\alpha/2;n-1}}{n-1}}. \quad (26)$$

Based on Lo et al. [20], a half-triangular fuzzy number is represented as $\widetilde{Q}^*_{PLj0} = \Delta(M_j, R_j)$, where

$$M_j = Q^*_{PLj0}\sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}; \quad (27)$$

$$R_j = Q^*_{PLj0}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + \frac{t_{0.005;n-1}}{\sqrt{n}}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}}. \quad (28)$$

According to Equations (27) and (28), the membership function of $\widetilde{Q}^*_{PLj0}$ is written as

$$\eta_j(x) = \begin{cases} 0 \;\; if \; x < M_j \\ 1 \;\; if \; x = M_j \\ \alpha \;\; if \; M_j < x < R_j \\ 0 \;\; if \; R_j \le x \end{cases}, \quad (29)$$

where α is designated by $UQ_{PLj0}(\alpha) = x$. Next, let $x_h = 0.01 \times (R_j - M_j) \times h$ we have $y_h = \eta_j(x_h) = \eta_h(0.01 \times (R_j - M_j) \times h)$, $h = 1, 2, 3, \ldots, 100$. Then draw all (x_h, y_h) points and vertical line $x = v'$ in Figure 1.

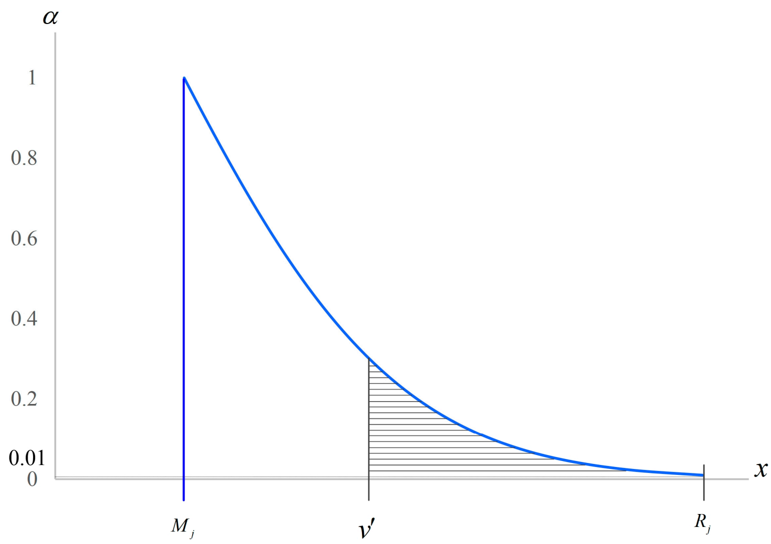

Figure 1. Membership function $\eta_j(x)$ with vertical line $x = v'$.

As suggested by Yu et al. [21], suppose set A_j is an area in Figure 1. Then

$$A_j = \{(x, \alpha) | M_j \le x \le UQ_{PLj0}(\alpha),\ 0 \le \alpha \le 1\}. \quad (30)$$

Likewise, suppose set B_j is an area, but to the right of vertical line $x = v'$ in Figure 1. Then

$$B_j = \{(x, \alpha) | v' \le x \le UQ_{PLj0}(\alpha),\ 0 \le \alpha \le b\}, \quad (31)$$

where $UQ_{PLj0}(b) = v'$. Let $d_{Tj} = 2(R_j - M_j)$ be twice the bottom length of set A_j. Then

$$d_{Tj} = 2Q^*_{PLj0}\left(\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} - \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}\right) + 2\frac{t_{0.005;n-1}}{\sqrt{n}}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}}. \quad (32)$$

Similarly, let $d_{Rj} = R_j - v'$ be the length of the bottom of set R_j. Then d_{Rj} is shown below:

$$d_{Rj} = Q^*_{PLj0}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + \frac{t_{0.005;n-1}}{\sqrt{n}}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} - v'. \quad (33)$$

Based on Equations (32) and (33), we have

$$\frac{d_{Rj}}{d_{Tj}} = \frac{R_j - v'}{2(R_j - M_j)} = \frac{Q^*_{PLj0}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} + \frac{t_{0.005;n-1}}{\sqrt{n}}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} - v'}{2Q^*_{PLj0}\left(\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}} - \sqrt{\frac{\chi^2_{0.5;n-1}}{n-1}}\right) + 2\frac{t_{0.005;n-1}}{\sqrt{n}}\sqrt{\frac{\chi^2_{0.995;n-1}}{n-1}}}. \quad (34)$$

As noted by Yu et al. [21], suppose $0 < \phi \leq 0.5$. Accordingly, the decision value of the jth quality evaluation index can be obtained by the following equation:

$$\frac{R_j - Ev_j}{2(R_j - M_j)} = \phi. \quad (35)$$

Therefore, we have

$$Ev_j = (1 - 2\phi)R_j + 2\phi M_j. \quad (36)$$

Based on Chen et al. [22], the fuzzy testing rules for evaluation are presented below:

(1) When $Ev_j \leq v'$, then reject H_0 and conclude that quality evaluation index is $Q_{PLj} < v'$.
(2) When $Ev_j > v'$, then do not reject H_0 and conclude that quality evaluation index is $Q_{PLj} \geq v'$.

4. A Case Study

In fact, strength, pulling force, and yellowing resistance are three quality characteristics of eco-friendly yarn, two of which are closely related to the functionality of sports shoes. Among them, pulling force and strength are functions with which a pair of high-quality sports shoes must equip. Although yellowing resistance has least relevance to functionality, it is related to the aesthetic appearance and brand image of the shoes. Consequently, all of them are essential quality characteristics. Subsequently, lower specification limits and quality evaluation indices of these three important quality characteristics are depicted in Table 3.

Table 3. Three important quality characteristics and the lower specification limits for eco-friendly yarn.

Quality Characteristics	*LSL*	Quality Evaluation Index
1. Pulling force	$LSL_1 = 0.27$	$Q_{PL1} = \frac{\mu_1 - 0.270}{\sigma_1}$
2. Strength	$LSL_2 = 1.70$	$Q_{PL2} = \frac{\mu_2 - 1.70}{\sigma_2}$
3.Yellowing resistance	$LSL_3 = 3$	$Q_{PL3} = \frac{\mu_3 - 3}{\sigma_3}$

Process engineers designate the required product quality index as $v = 6$. According to Equation (7), the required quality evaluation index for each quality characteristic is $v' = \Phi^{-1}\left(\sqrt[3]{\Phi(6)}\right) = 6.176$. Obviously, when the three individual quality evaluation indices exceed or equal 6.176, then the product quality index exceeds or equals 6. Therefore, to determine whether the value for each individual quality evaluation index exceeds or equals 6.176, the hypotheses for testing are described below:

H_0: $Q_{PLj} \geq$ *6.176 (The quality characteristic is able to reach the required quality level).*

H_1: $Q_{PLj} <$ *6.176 (The quality characteristic is unable to reach the required quality level).*

The observed values of $(X_{j,1}, \cdots, X_{j,i}, \cdots, X_{j,16})$ are $(x_{j,1}, \cdots, x_{j,i}, \cdots, x_{j,16})$ with n = 16 and j = 1, 2, 3. Based on Equations (22), (23) and (36), all the values of $\bar{x}_j$, s_j, Q^*_{PLj0}, UQ_{PLj0} and Ev_j of three quality characteristics are calculated, as summarized in Table 4.

Table 4. Values of $\bar{x}_j$, s_j, Q^*_{PLj0}, UQ_{PLj0} and Ev_j.

j	$\bar{x}_j$	s_j	Q^*_{PLj0}	UQ_{PLj0}	Ev_j
1	0.283	0.0035	3.753	6.639	5.451
2	1.765	0.0136	4.747	8.109	6.722
3	5.313	0.4787	4.831	8.233	6.829

According to Table 4, the value of Ev_1, equal to 5.451, is less than 6.176. Based on the decision rules of fuzzy testing, we reject H_0 and conclude that quality evaluation index is presented as $Q_{PLj} < 6.176$. Clearly, this quality characteristic does not reach the desired quality level, so it requires improvement. The value of Ev_2, equal to 6.722, and the value of Ev_3, equal to 6.829, are both greater than 6.176, so there is no need for improvement.

5. Conclusions

Since the awareness of green environmental protection and the idea of athletic health-care have been valued by lots of countries with top-notch industries, numerous world-renowned sports brands are actively developing new products with high functionality, environmental friendliness, and pleasing appearance. Accordingly, in this paper, we took the eco-friendly yarn as an example and proposed a fuzzy process quality evaluation model possessing multifunctional quality characteristics. Strength, pulling force, and yellowing resistance are three significant quality characteristics for eco-friendly yarn, two of which are closely related to the functionality of sports shoes. Therefore, we employed a Six Sigma quality index as an evaluation index for each individual quality characteristic and then integrated all the indexes for all individual quality characteristics into a product evaluation index for eco-friendly yarn. Apart from reflecting the process quality level, the Six Sigma quality index is also directly related to process yield with a one-to-one mathematical relationship. To assure the final product quality of eco-friendly yarn, we specified the required Six Sigma quality index for each quality characteristic based on the required product quality level. Because the index has unidentified parameters, we must estimate it using sample data. Building upon the sample data, we proposed three estimators for the LTB Six Sigma quality indicators. Meanwhile, these three estimators were integrated into an estimator of the product quality index. Subsequently, the $100(1-\alpha)\%$ upper confidence limits of these three LTB Six Sigma quality indicators were derived in Section 3, and the required Six Sigma quality index for each quality characteristic derived from Section 2 was employed as the required value for fuzzy testing. Additionally, since companies strive for quick response, the sample size of sample data is usually not large. As a result, big sampling errors may result in an increase in the risk of misjudgment. Moreover, considering the product quality index of eco-friendly yarn, we came up with a fuzzy evaluation model built on the $100(1-\alpha)\%$ upper confidence limits of the three LTB Six Sigma quality indices. This approach can incorporate past data and experience, thereby enhancing evaluation accuracy. Finally, a case study was employed to demonstrate how to apply the proposed fuzzy testing model, hoping to benefit the application and promotion of related industries. In the case study, the estimated index value of quality characteristic 1 is 3.753 (Q^*_{PL10} = 3.753), and its upper confidence limit is equal to 6.639 (UQ_{PL10} = 6.639). According to the statistical testing rule, when the upper confidence limit is greater than the required value (6.639 > 6.176), then H_0 is not rejected. However, the estimated index value of 3.753 is far smaller than the required value of 6.176. From a practical point of view, it is obviously unreasonable and will miss the immediacy of improvement. This is caused by a large sampling error due to a small sample size.

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