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# The Influence of Reverse Yielding on the Plastic Conditioning of Interference Fits in Power Transmission Engineering

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**Abstract:** Interference fits are very common shaft–hub connections due to their low manufacturing costs and excellent technical properties. The Plastic Conditioning of this machine element is a new and not very well-known method. During the development of this method, it was discovered that Reverse Yielding occurs in certain applications and has a negative impact on the result. This paper examines the effects of Reverse Yielding on the technology of Plastic Conditioning of interference fits in Power Transmission Engineering. Based on the Shear Stress Hypothesis (SH), the Plane Stress State (PSS), and the ideal plastic behavior of materials, established stress–mechanical relationships are used to find the influencing parameters of Reverse Yielding on the technology of Plastic Conditioning and their limits. As a result, a new computational concept is developed that allows the user to maximize Plastic Conditioning while avoiding Reverse Yielding. Analytical calculation suggestions and diagrams for practical application are provided. Furthermore, the deviations in the obtained results, taking into account other material models such as the Von Mises Yield Criterion (VMYC) and material hardening, as well as the Bauschinger effect, are examined in comparison with our own numerical results from the development of Plastic Conditioning, and the resulting need for further research is defined. In addition, the method of Plastic Conditioning of interference fits is introduced and its basic principles are briefly explained.

**Keywords:** reverse yielding; plastic conditioning; plastically joined interference fits; elastic stress limits; strain hardening; plastic behavior of materials; residual stresses; joint pressure

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## 1. Introduction

Frictional shaft–hub connections in the form of interference fits are commonly used in power transmission applications. In order to save material, space and weight, and to increase load capacity, the stress limits of these components have been extended beyond their elastic limits into the plastic region. Rees studied the elastic–plastic stress distribution within discs rotating at high speed [1]. He examined and compared the Tresca criterion, which provides a closed solution traditionally associated with this problem, with an alternative solution from the Von Mises criterion. Jang and Bi presented an elastic–plastic analysis of interference fit connections to improve the fatigue life of aircraft structures, established the finite element model, and performed a numerical simulation [2]. Laghzale and Bouzid described an analytical solution for the residual stresses of elastically–plastically deformed interference fits with hollow shafts under plane strain conditions [3]. The constitutive law governing their material strain hardening behavior was assumed to follow a general power law that also covers the special cases of elastic–perfect plastic and bilinear hardening. Lätzer and Leidich studied the transmissibility of clamp connections with special consideration of the plastic deformations [4]. Zhang et al. proposed a new phase method for predicting the crack propagation path and mechanical response of a fiber-reinforced composite laminate, and a new three-dimensional (3D) crack surface density function considering material anisotropy for composite laminates [5]. Hu et al. presented an overview of the implicit and explicit phase field models for quasi-static failure processes and explained their theories and implementation in the commercial software ABAQUS [6]. Methods to improve computational efficiency were introduced.

This paper focuses on the effect of Reverse Yielding on the Plastic Conditioning of interference fits (especially in the outer part/hub) in Power Transmission Engineering. Extensive work has been performed on Reverse Yielding in the context of autofrettage technology. Chen provided a closed-form solution for calculating residual stresses in autofrettaged thick-walled cylinders to better describe the actual load/unload behavior in a high-strength steel with regard to Reverse Yielding, Bauschinger, and hardening effects [7]. Parker et al. presented a complete analysis procedure that encompasses the representation of elastic–plastic uniaxial loading material behavior and reverse loading material behavior as a function of plastic strain in pressure vessels with respect to the Bauschinger effect [8]. Ghorbanpour et al. performed an elastoplastic and residual stress analysis of thick-walled SUS 304 stainless-steel cylinders with closed ends using the incremental theory of plasticity, and the loading and unloading properties of the material, including the Bauschinger Effect Factor, were obtained experimentally and represented mathematically as a continuous function of the equivalent plastic strain [9]. The critical pressures of direct and Reverse Yielding were investigated using the Von Mises Yield Criterion (VMYC) as well as the Bauschinger Effect Factor. Jahed and Dubey described an axisymmetric method of elastic–plastic analysis capable of predicting the residual stress field [10]. The residual stress calculation was based on the actual material curve, isotropic and kinematic hardening models, and a variable Bauschinger Effect Factor.

The main objective of autofrettage is to improve the strength properties and fatigue life (from a fracture mechanics point of view) of tubular components/pressure vessels under high internal pressure loading, such as those used in nuclear reactors, reciprocating compressors, or common rail applications. In addition to improving the strength properties, the use of Plastic Conditioning Technology for interference fits must take into account much more complex loading situations both in the components and in their contact zones. Important aspects are shape stability and functional reliability, even under dynamic loads.

When considering multiple interference fits, the complexity of the parameters increases with the number of joining partners. Such interference fits are very common in industrial applications in the form of clamp connections, such as clamping elements. An example of this can be found in wind turbines, where the connection between the gear shaft and the rotor shaft of the wind turbine is joined using clamping elements, and where material stress reserves are often sought in industrial practice because of the reduced strength limit due to the geometric size effect.

The phenomenon of Reverse Yielding of elastically–plastically loaded interference fits was investigated in detail by Kollmann and Önöz [11] on the basis of the Shear Stress Hypothesis (SH); the associated flow rule according to Melan, Prager, and Koiter [12]; the Plane Stress State (PSS); and ideal plastic material behavior. Corresponding calculation principles were derived analytically.

The question of the occurrence of Reverse Yielding during the deceleration of a rotating interference fit with a plastified hub and the calculation of residual stresses at standstill are addressed in the work of Gamer [13]. The occurrence of Reverse Yielding in rotating thick-walled hollow cylinders with elastic–ideal plastic material and free ends after a reduction in the angular velocity is described by Mack [14].

Initial studies on the effects of Reverse Yielding on Plastic Conditioning Technology have been conducted by Schierz [15].

## 2. Interference Fits under Plastic Stress Conditions

### *2.1. Consideration of Stresses in Interference Fits*

Interference fits are based on the transmission of forces and moments by frictional contact between the inner part (shaft) and the outer part (hub), which generates compressive stress in their contact zone. Figure 1 shows such a connection between a hub and a hollow shaft in a cross-section, where the hub represents a hollow cylinder under internal pressure and the hollow shaft represents one under external pressure. The distribution of radial and tangential stresses (principal stresses) over the cross-sections of the hub and hollow shaft is shown in Figure 1. For the hub subject to this study, these stresses are calculated according to Equation (1) ($p_i = p_F$), which is well known from the scientific literature.

$$\begin{aligned}
\sigma_{rA}(d) &= \frac{-p_i}{1-Q_A^2} \cdot \left( \left( \frac{D_{iA}}{d} \right)^2 - Q_A^2 \right) \\
\sigma_{tA}(d) &= \frac{p_i}{1-Q_A^2} \cdot \left( \left( \frac{D_{iA}}{d} \right)^2 + Q_A^2 \right) \\
Q_A &= \frac{D_{iA}}{D_{aA}}
\end{aligned} \tag{1}$$

where

d (mm): Diameter coordinate (control variable);
$D_{aA}$ (mm): Outer diameter of the outer part;
$D_{iA}$ (mm): Inner diameter of the outer part;
$p_i$ (MPa): Internal pressure of the hub;
$Q_A$: Diameter ratio of the outer part;
$\sigma_{rA}(d)$ (MPa): Radial stress at d of outer part;
$\sigma_{tA}(d)$ (MPa): Tangential stress at d of outer part.

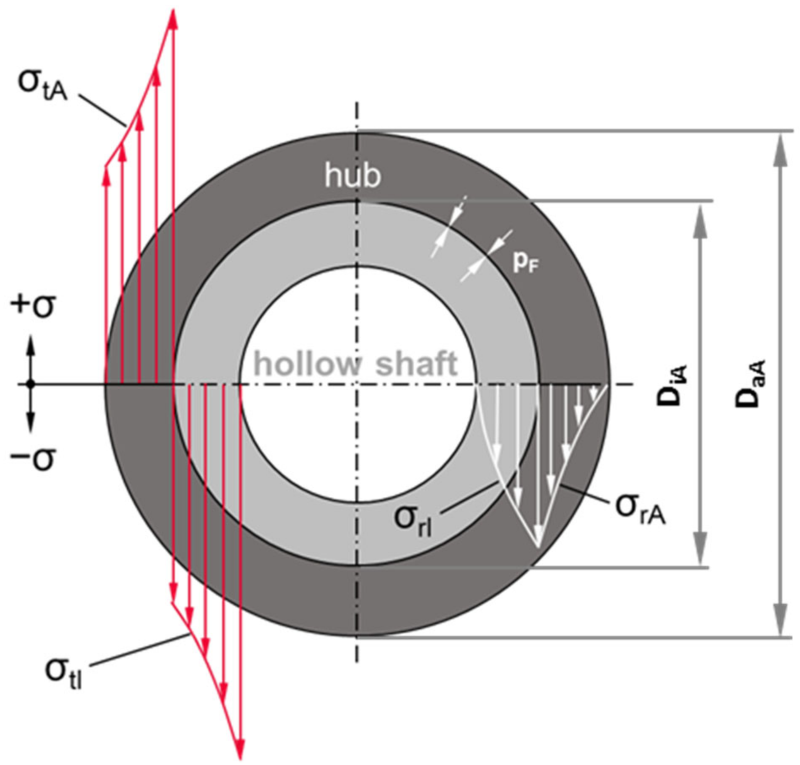


**Figure 1.** Cross-section of an interference fit (hub and hollow shaft) with principal stresses.

From the stress curves in the cross-section, it can be seen that for both components, the greatest stresses occur at the respective inner diameters and that the onset of plastic stresses is always found here. If the joint is designed purely elastically, the high stress gradients mean that the rest of the component cross-section remains purely elastic, which in many cases can lead to unfavorable material utilization. That is why partially plastic components with plastically stressed inner zones are permitted. However, when the outer part is subjected to plastic deformation, it can experience additional plastic deformation in the opposite direction in certain stress relief situations, known as Reverse Yielding.

The purpose of this study is to investigate how Reverse Yielding affects Plastic Conditioning Technology by examining analytical relationships based on the Shear Stress Hypothesis (SH) and the Plane Stress State (PSS) in the context of the physical principles of Plastic Conditioning. Of particular interest are the conditions under which Reverse Yielding can occur and how to calculate them. The main focus is on the diameter ratio $Q_A$ (Equation (2)) and the maximum achievable internal pressures $p_i$ or radial stresses $\sigma_r$. This study also examines how these conditions change under the Von Mises Yield Criterion (VMYC), with hardening of the material, and by considering the Bauschinger effect. Ultimately, this research aims to develop a reliable calculation system for practical use.

$$\sigma_t = -m \cdot \sigma_r \ with \ m = \frac{1+Q_A^2}{1-Q_A^2} = -\tan(\alpha) = \tan(180^\circ - \alpha) \ and \ Q_A = \frac{D_{iA}}{D_{aA}} \tag{2}$$

where

$D_{aA}$ (mm): Outer diameter of the outer part;
$D_{iA}$ (mm): Inner diameter of the outer part;
m: Factor for determining the angle of inclination;
$Q_A$: Diameter ratio of the outer part;
a (°): Inclination angle for load line and relief straight line;
$\sigma_r$ (MPa): Radial stress;
$\sigma_t$ (MPa): Tangential stress.

### *2.2. Plastic Conditioning of Interference Fits, Stress–Mechanical Principles*

The idea of the Plastic Conditioning Method developed by the author is based on the fact that an elastic–plastic pretreatment of the joining partners of an interference fit is carried out before or during the joining process, e.g., by applying a conditioning pressure $p_K$ to a hub ($p_K = p_i$, see also Section 4). During the subsequent elastic relief and, if necessary, reloading of these joining partners, a purely elastic stress state can be established in such a way that all additional stresses to be expected on the hub during operation, such as rotating bending moments, torsion, temperature changes, and centrifugal forces, result exclusively in purely elastic stress changes. This property is also required for other frictionally engaged joints, such as bolted joints, because plastic deformation due to operating loads usually results in a reduction in transmittable forces and moments after these loads are relieved.

Compared to conventionally joined interference fits, the load bearing capacity in the elastic range can be increased by almost 200% and a specifically defined additional safety against plastic deformation in the elastic–plastic range can be ensured. In addition, several articles have been published on the Plastic Conditioning method [15–17].

The stress curves during the conditioning process are shown in Figure 2. It shows the changes in stress on the inner diameter $D_{iA}$ of an outer part (hub) under internal pressure in the principal stress plane, assuming the Plane Stress State (PSS). Its axes are formed by the radial stress $\sigma_r$ on the abscissa and the tangential stress $\sigma_t$ on the ordinate. The closed dashed line in Figure 2 represents the ideal plastic yield strength of the material according to the Shear Stress Hypothesis (SH) used, and it is calculated according to the scientific literature with reference to Equation (1) as follows.

$$\sigma_v = \max(|\sigma_t - \sigma_r| \ ; \ |\sigma_r| \ ; \ |\sigma_t|) \tag{3}$$

where

$\sigma_r$ (MPa): Radial stress;
$\sigma_t$ (MPa): Tangential stress;
$\sigma_v$ (MPa): Equivalent stress.

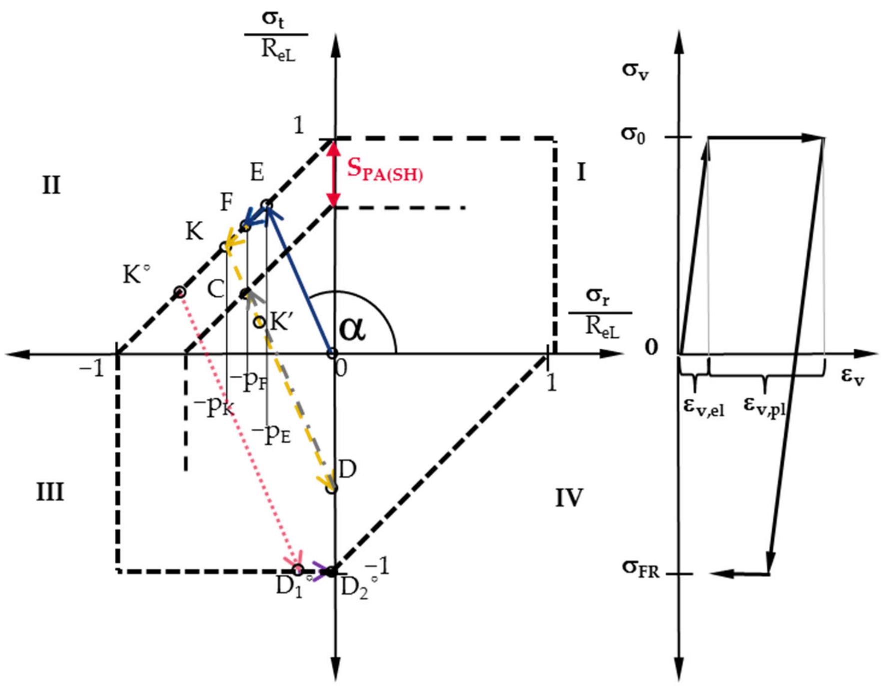


**Figure 2.** Principal stresses at the inner diameter $D_{iA}$ of the outer part of an interference fit with the ideal plastic yield strength according to the Shear Stress Hypothesis (SH) during Plastic Conditioning (**left**); stress–strain curve of the equivalent stress (**right**).

The production of elastically–plastically joined interference fits with an internal plastic zone, as described in Section 1, has so far been carried out in practice according to Figure 2 along the load path $\overline{0E} - \overline{EF}$ (blue arrow/solid line). According to Kollmann, the angle of inclination α of the straight line $\overline{0E}$ is determined exclusively by the diameter ratio $Q_A$ of the component according to Equation (2).

When point E in Figure 2 is reached, the stress state is at the yield point. As the negative radial stress continues to increase, the associated stress states can only develop along the yield curve (e.g., along the load path $\overline{EF}$).

Point F then marks the stress state at the inner diameter $D_{iA}$ of the outer part of the interference fit after completion of the joining process for conventional elastically–plastically joined hubs.

Since point F lies on the graph of the yield strength, increases in stress lead to changes in stress along the yield strength and thus inevitably to further plastic deformations. In this example, the stress state moves to point K by increasing the radial stress, for instance, due to additional stresses on the hub during operation. After this increase is relieved, elastic relief takes place along the relief path $\overline{KK'}$ whereby the inclination angle of the relief straight line in accordance with Kollmann is also defined by the diameter ratio $Q_A$ of the component according to Equation (2) and is thus always parallel to the load line. As a result of the plastic deformations, the stress state that develops thereafter must lie in a region to the right of $-p_F$ along the relief path (for example, at point K′). Thus, the previous joint pressure $p_F$ can no longer be reached due to the plastic interference loss, which may result in impairments of the operational safety or the service life of the components [15,16].

When producing a conditioned hub with the same joint pressure but with an additional safety $S_{PA(SH)}$ (see Figure 2, red double arrow) against plastic deformation, the radial stress is further increased up from point F to the conditioning pressure at point K in Figure 2. This virtually anticipates the additional operating load.

There is then the complete elastic relief ($\sigma_r$ = 0) to point D, which illustrates the remaining tangential residual stresses and where the end of the conditioning process is reached.

To calculate point D, Equation (4) was formulated by the author for the Shear Stress Hypothesis as follows.

$$D_{\sigma r(SH)} = 0\ ; D_{\sigma t(SH)} = -\frac{2 \cdot (p_K - p_E)}{1 - Q_A^2} \quad (4)$$

where

$D_{\sigma r(SH)}$ (MPa): Radial Stress Value at Point D (SH);
$D_{\sigma t(SH)}$ (MPa): Tangential Stress Value at Point D (SH);
$p_E$ (MPa): Elastic joint pressure at yield strength;
$p_K$ (MPa): Conditioning pressure.

From a technical point of view, the conditioning of a hub can be carried out in a simple manner, for example, by means of clamping elements or, in the case of hubs for conical interference fits, by means of a corresponding cone. In these cases, the interference in the hub bore generates a surface pressure corresponding to the radial stress in Figure 2.

The subsequent reloading along the straight line $\overline{DC}$ (gray arrow/dashed-dotted-line) leads to the final joined state of the interference fit at stress point C. Here, the joint pressure $p_F$ is identical to that of conventional elastically–plastically joined interference fits at point F, but with the above-mentioned safety $S_{PA(SH)}$. Additional stresses caused by operating loads can thus be absorbed by a conditioned interference fit in a purely elastic manner and, after they have subsided, it always returns unchanged to its original state.

To calculate the safety against plastic deformation $S_{PA(SH)}$ (at point C relative to point K in Figure 2), Equation (5) was developed by the author.

$$S_{PA(SH)} = \frac{1}{1 - \frac{\Delta p_K}{R_{eL,A}} \cdot \left(1 + \frac{1+Q_A^2}{1-Q_A^2}\right)} \text{with } \Delta p_K = p_K - p_F \quad (5)$$

where

$R_{eL,A}$ (MPa): Lower yield strength of the outer part;
$S_{PA(SH)}$: Safety against plastic deformation of the outer part achieved by conditioning;
$p_F$ (MPa): Joint pressure;
$\Delta p_K$ (MPa): pressure difference between $p_K$ and $p_F$.

A description of the items in Figure 2 is as follows:
E: purely elastic state at the yield strength;
F: elastic–plastic state (conventionally joined according to DIN 7190-1 [18]);
K: stress state at yield curve due to conditioning;
C: joined final state of the conditioned interference fit;
D: stress state after complete relief (tangential residual stresses).

## 3. Reverse Yielding

### *3.1. Occurrence of Reverse Yielding in Plastic Conditioning*

Reverse Yielding is a mechanical process that occurs when plastically stressed parts (hubs) in interference fits undergo elastic relief, resulting in renewed plastic stress in the opposite direction. For one-dimensional loading, as can be seen, for example, in the diagram of equivalent stress $\sigma_v$ and equivalent strain $\varepsilon_v$ in Figure 2 (right side), tensile stresses above $\sigma_0$ lead to equivalent plastic strains $\varepsilon_{v,pl}$. After complete unloading at $\sigma_v = 0$, the equivalent elastic strains $\varepsilon_{v,el}$ disappear. If the material is then subjected to compressive stresses, the yield point is reached again in the opposite direction at $\sigma_{FR}$ and further stresses again lead to equivalent plastic strains in the opposite direction (Reverse Yielding).

In the Plane Stress State (PSS), the situation is more complex. Here, however, Reverse Yielding is always characterized by a transition of the stress state to another quadrant in the stress plane (in the example in Figure 2, left side, from II to III).

In the example of Figure 2, the stress point D at the end of the conditioning process is located in the elastic region within the yield curve. Under certain conditions, which will be explained in more detail below, the relief straight line $\overline{K^{\circ}D_1{}^{\circ}}$ (pink dotted line) may intersect the yield curve at point $D_1{}^{\circ}$. In this case, the material will return to a plastic state and the stress will inevitably continue to follow the yield curve until it is completely relieved at point $D_2{}^{\circ}$. This process is known as Reverse Yielding. This shows that the area between $D_1{}^{\circ}$ and $D_2{}^{\circ}$ can no longer contribute to conditioning. For the maximum usable conditioning pressure $p_{Kond,max}$, therefore, only those values whose elastic relief straight lines end at point $D_2{}^{\circ}$ are meaningful. If Reverse Yielding is induced beyond this point, this will only result in unnecessary plastic deformation and compromise the desired residual stress state.

It should be mentioned at this point that Reverse Yielding cannot occur in hollow cylinders under external pressure, in principle (see Figure 1). In the case of interference fits, for example, this applies to hollow shafts which, in principle, can also be subjected to elastic–plastic stresses. Solid shafts can only be subjected to purely elastic stresses due to the stress–mechanical principles that apply. Since hollow shafts are subject to compressive stresses only, their stress states are generally in the IIIrd quadrant (see Figure 2) and their relief straight lines always end in the elastic region of the ordinate. More detailed information about this can be found in Kollmann and Önöz [11].

### *3.2. Influencing Parameters for Reverse Yielding*

This section deals with the conditions under which Reverse Yielding can occur. Assuming the Shear Stress Hypothesis (SH); the associated flow rule according to Melan, Prager, and Koiter [12]; the Plane Stress State (PSS); and an ideal plastic material behavior, Kollmann and Önöz [11] found the following equation as a necessary condition for the critical pressure $p_{krit}$ that must be generated at the inner diameter $D_{iA}$ of the hub in order for Reverse Yielding to occur with subsequent elastic relief.

$$p_{krit,\,SH} = R_{eL,A} \cdot \left(1 - Q_A^2\right) \tag{6}$$

where

$p_{krit,SH}$ (MPa): Critical pressure for Reverse Yielding (SH).

It is also shown that this pressure can only occur in hubs with a diameter ratio $Q_A < 0.45076$. For larger diameter ratios, the maximum possible fully plastic state of the hub according to Equation (7) [11], which should be avoided for ideal plastic material, is reached before the critical pressure $p_{krit}$.

$$p_{\max,\,SH} = -R_{eL,A} \cdot \ln Q_A \tag{7}$$

where

$p_{max,SH}$ (MPa): Maximum possible pressure before the fully plastic state of the hub for ideal plastic material (SH).

For information only, it should be noted that for diameter ratios $Q_A < 0.368$ (so-called "thick-walled hubs"), the fully plastic state cannot be achieved and the maximum achievable pressure is equal to the yield strength.

Figure 3 illustrates these relationships using the example of relief straight lines with maximum possible pressures $p_{max,MSH}$ (Equation (8)) of three characteristic diameter ratios $Q_A$ at the inner diameters of the respective hubs in the principal stress plane.

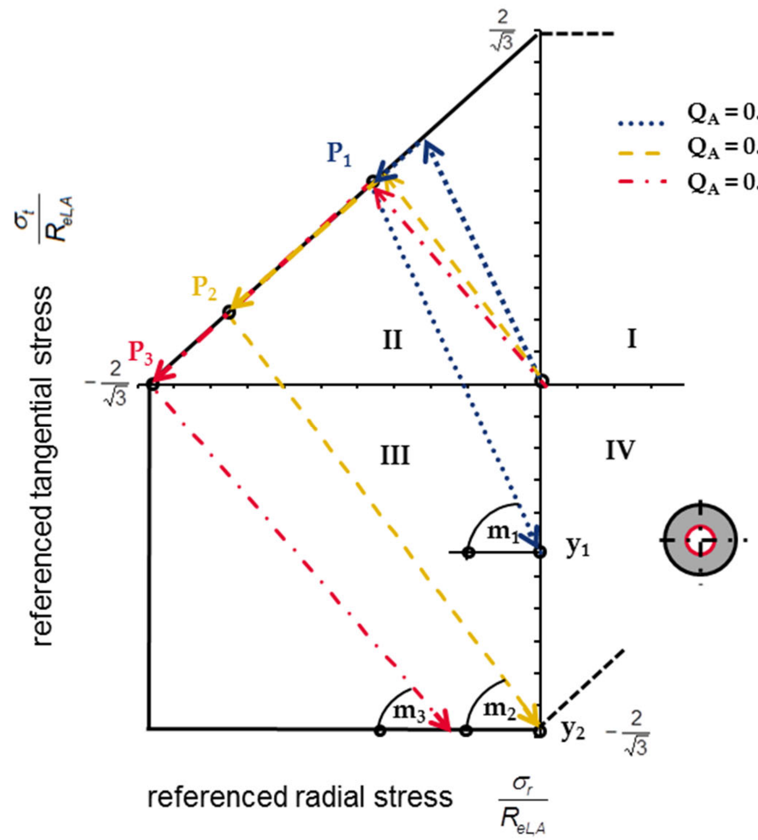


**Figure 3.** Stress states at any point of the inner diameter $D_{iA}$ (marked in red) of three characteristic diameter ratios $Q_A$ in the principal stress plane (ideal plastic material) during conditioning with maximum conditioning joint pressure $p_{Kond,max}$ and subsequent stress relief.

The yield curve here corresponds to the Modified Shear Stress Hypothesis (MSH), for which Equations (6) and (7) of Kollmann have been adapted as follows.

$$p_{\max,\, MSH} = -R_{eLA} \cdot \frac{2}{\sqrt{3}} \cdot \ln Q_A \tag{8}$$

where

$p_{max,MSH}$ (MPa): Maximum possible pressure before the fully plastic state of the hub for ideal plastic material (MSH).

$$p_{krit,\, MSH} = \frac{2}{\sqrt{3}} \cdot R_{eL,A} \cdot \left(1 - Q_A^2\right) \tag{9}$$

where

$p_{krit,MSH}$ (MPa): Critical pressure for Reverse Yielding (MSH).

Equation (10) was designed by the author to determine the relief straight lines in the elastic region of quadrants II and III of Figure 3. Their gradients $m_n$ are calculated from Equation (2) using $Q_A$ and y determines the points of intersection of the relief straight lines with the ordinate and thus the tangential residual stresses at the inner diameter after complete relief (Figure 3).

$$\frac{\sigma_t}{R_{eL,A}} = -\, m \cdot \frac{\sigma_r}{R_{eL,A}} - \frac{y}{R_{eL,A}} \tag{10}$$

where

$y$ (MPa): Intersection of the relief straight line with the ordinate (tangential residual stress after complete relief).

The points of intersection ($P_1$ to $P_3$) of the relief straight lines with the yield curve in the IInd quadrant mark the stress states at the inner diameter at the maximum plastic loading of the component according to Kollmann and are defined analytically only for the SH and MSH (see also Equations (7) and (8)).

It can clearly be seen that the relief straight lines for $Q_A > 0.45076$ (and $\sigma_r = 0$) always end in the elastic region of the principal stress plane (dotted line for $Q_A = 0.65$). Only when $Q_A < 0.45076$ do the relief straight lines intersect the yield curves again and Reverse Yielding occurs (dashed line for $Q_A = 0.45$ and dashed dotted line for $Q_A = 0.368$).

In the diagram in Figure 4, the referenced internal pressures $p_F/R_{eL,A}$ are plotted against the diameter ratios $Q_A$ of the outer parts (hubs) of an interference fit. The black shaded area marks the range of possible pressures and diameter ratios at which Reverse Yielding can occur. It can be seen that, under the above conditions, Reverse Yielding occurs mainly with so-called thick-walled hubs. These in turn are of particular interest for Plastic Conditioning as they offer the greatest potential for this technology [15].

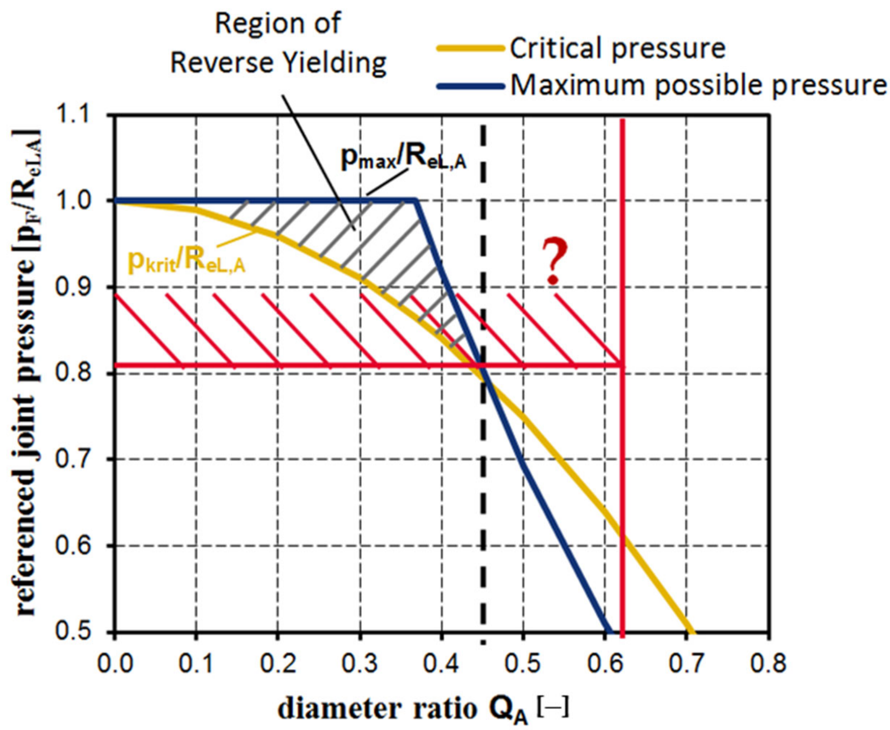


**Figure 4.** Region of reverse yielding in an outer part for ideal plastic material (SH) according to Kollmann and regions of risk (red question mark) with respect to VMYC and hardening material on the basis of this work (red lines).

To prevent Reverse Yielding after complete stress relief during conditioning, the condition given in Equation (11) must be satisfied for all relief straight lines with $Q_A < 0.45$. The maximum conditioning joint pressure $p_{Kond,max}$, at which the residual stress state at the inner diameter $D_{iA}$ after complete stress relief is just not at the yield curve, is identical to the negative radial stress $-\sigma_r$ according to

$$y = -m \cdot \sigma_r - \sigma_t < R_{eL,A} \tag{11}$$

## 4. Calculation Results

### *4.1. Results under MSH and Ideal Plastic Material Behavior Assumptions*

Figure 5 illustrates the maximum joint pressures (calculated using Equations (3) and (10) when $y = 1.1547 \cdot R_{eL,A}$) that can be achieved by Plastic Conditioning, assuming the MSH and ideal plastic material behavior, when Reverse Yielding is avoided and the specified safety factors $S_{PA}$ are met. The maximum achievable joint pressures are compared with those of conventional, purely elastically joined interference fits according to DIN 7190 [18].

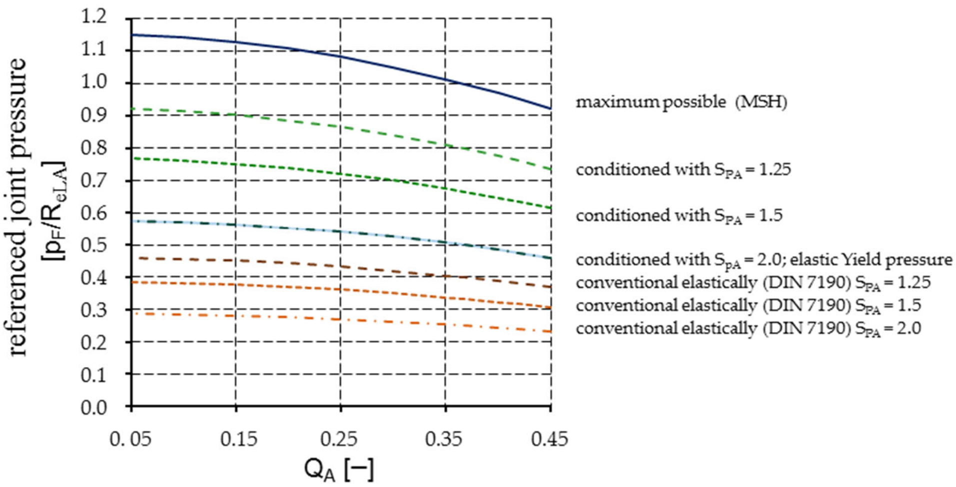


**Figure 5.** Maximum achievable joint pressures (MSH) for plastic conditioned interference fits ($0.05 \leq Q_A \leq 0.45$) compared with those of conventional, purely elastically joined fits according to DIN 7190 with specified safety against plastic deformation $S_{PA}$ and avoidance of Reverse Yielding.

In Figure 6, we compare the highest possible pressures that can be achieved in joints that are not affected by Reverse Yielding and have a $Q_A$ value above 0.45. These joints have safety factors built in to prevent plastic deformation during assembly and in service. We compare the pressure achieved by purely elastic joints according to DIN 7190 with the pressure achieved by interference fits with prior conditioning.

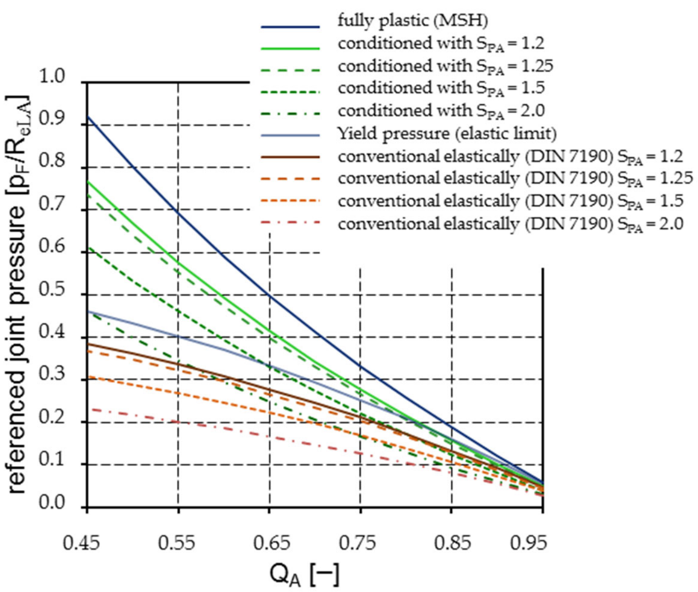


**Figure 6.** Maximum achievable joint pressures (MSH) for plastic conditioned interference fits ($0.45 \leq Q_A \leq 0.95$) compared with those of conventional, purely elastically joined fits according to DIN 7190 with specified safety against plastic deformation $S_{PA}$ and avoidance of Reverse Yielding.

The maximum pressure that can be tolerated during conditioning based on the MSH is called the fully plastic joint pressure (see Equation (8)) and is the reference point for this comparison. The previous explanations were based on the Shear Stress Hypothesis (SH/MSH) and ideal plastic material behavior. Our own numerical investigations based on the Von Mises Yield Criterion (VMYC) and an isotropically hardening material have shown that there is a potential risk of Reverse Yielding beyond the limits described so far (red shaded area in Figure 4).

*4.2. Results Regarding VMYC and Hardening Material Behavior*

Contrary to ideal plastic material behavior, real hardening materials do not exhibit unrestricted flow in the region of uniform elongation of the tensile test stress–strain curve that is of interest for interference fits. Plastic deformation during strengthening is limited by the inhibition of lattice dislocations and the resulting increase in yield strength. This means that the maximum joint pressure is not determined by the yield strength as in the ideal plastic approach, because the inner diameter of a hub or an outer part can now theoretically be loaded at least up to the limit of uniform elongation of the tensile test stress–strain curve. As a result, fully plastic states are theoretically possible even for "thick-walled hubs", and for hubs with $Q_A > 0.45076$, the fully plastic state is only reached at higher pressures than for ideal plastic material according to Equations (7) and (8).

In addition to a significant increase in the joint pressure and the maximum plasticity diameter for "thick-walled hubs", the range of Reverse Yielding as a function of the diameter ratio according to Kollmann and Önöz [11] is also extended by taking into account material hardening and the VMYC. A comparison of the stress curves obtained from the FE calculations for the study of plastically conditioned interference fits showed this to be the case. This involved a two-dimensional FE model corresponding to Figure 7 with a diameter ratio of $Q_A = 0.625$ and a pressure of $p_i = 300$ MPa (corresponding to point K in Figure 2) applied to the inner diameter $D_{iA}$. The pressure was then completely released (corresponding to point D in Figure 2). The hardening behavior was modeled on the basis of tensile tests for material C45 according to Table 1.

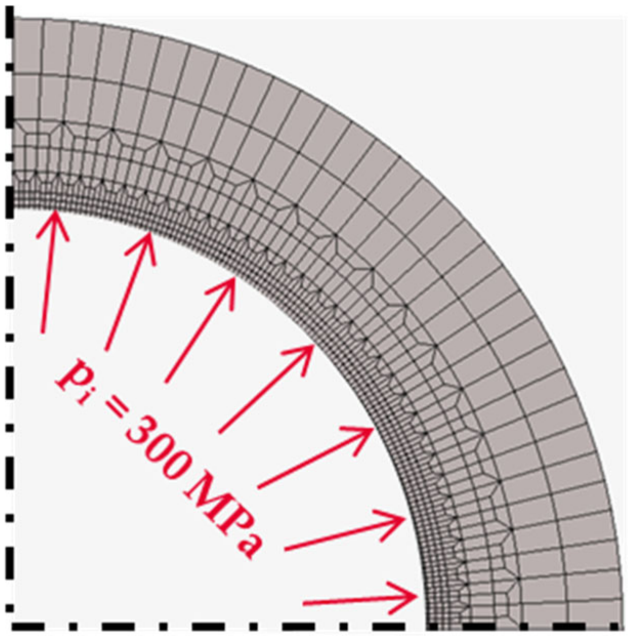


**Figure 7.** Quarter section of the meshed FE model of a hub with $Q_A = 0.625$.

**Table 1.** Data from the tensile test for C45 (IKAT).

| $\sigma_v$ (MPa) | $\varepsilon_v$ (−) |
|---|---|
| 370.00 | 0.00171 |
| 390.00 | 0.00182 |
| 400.00 | 0.00189 |
| 410.00 | 0.01303 |
| 420.00 | 0.01417 |
| 430.00 | 0.01549 |
| 440.00 | 0.01665 |
| 470.00 | 0.02075 |
| 500.00 | 0.02543 |
| 530.00 | 0.03097 |
| 560.00 | 0.03775 |
| 590.00 | 0.04612 |
| 620.00 | 0.05967 |

When the component is completely relieved by reducing the internal pressure, the plastic deformation will produce a residual stress state as shown in Figure 8. The blue solid line and the yellow dashed line in Figure 8 show the stress profile in the cross-section of the FE model (see red arrow in sketch) at full load. The red solid line and the gray dashed line show the residual stress state after complete unloading.

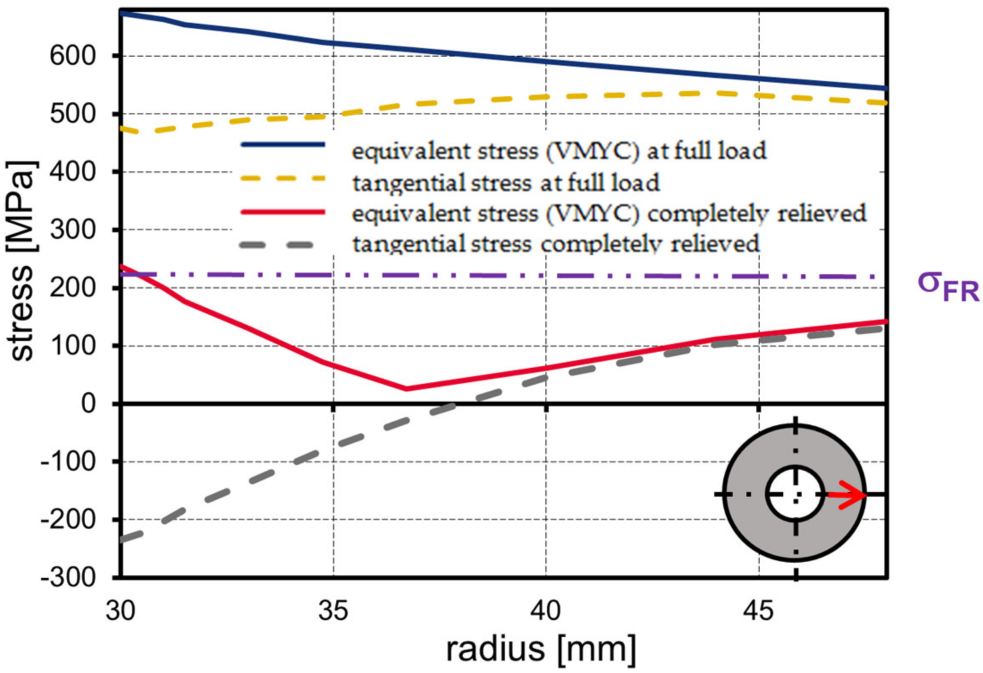


**Figure 8.** Stress states of a C45 hub with $Q_A$ = 0.625 loaded with an internal pressure of $p_i$ = 300 MPa and relieved by Reverse Yielding.

The equivalent stress curve after complete unloading indicates in the area of the inner diameter that the relief will produce compressive residual stresses of an order of magnitude that will again exceed the yield strength and cause Reverse Yielding if the load history is in the opposite direction. To verify this, the residual stresses were compared to the reverse yield strength, shown in Figure 8 as a purple dash-dot-dot line.

According to Section 3.2, this means that, depending on the maximum load and the hardening behavior of the material, Reverse Yielding can also occur in hubs with hardening material if they have a larger diameter ratio (see red line at $Q_A$ = 0.625 in Figure 4) than that marked in Figure 4 by the dashed line at $Q_A$ = 0.45 [11] for ideal plastic material.

*4.3. Results with Respect to the Bauschinger Effect*

The reverse yield strength in Figure 8 shows a significant reduction from the original yield strength before work hardening due to the Bauschinger effect. The influence of the Bauschinger effect on the reduction in yield strength is highly dependent on the material and technological factors and can be defined in a simplified way by the Bauschinger Stress Parameter (Equation (12)) [19].

$$\beta_{BS} = \frac{\sigma_{\max}}{\sigma_{FR}} \tag{12}$$

where

$\beta_{BS}$: Bauschinger Stress Parameter;
$\sigma_{FR}$ (MPa): Yield strength that is reached when the material is subsequently subjected to stress in the opposite direction;
$\sigma_{max}$ (MPa): Maximum stress achieved by plastic deformation as a result of hardening.

In this equation, $\sigma_{max}$ is the maximum stress achieved by plastic deformation as a result of hardening and $\sigma_{FR}$ is the reverse yield strength that is reached when the material is subsequently stressed in the reverse direction (Figure 9). The yield point is defined as $\sigma_0$.

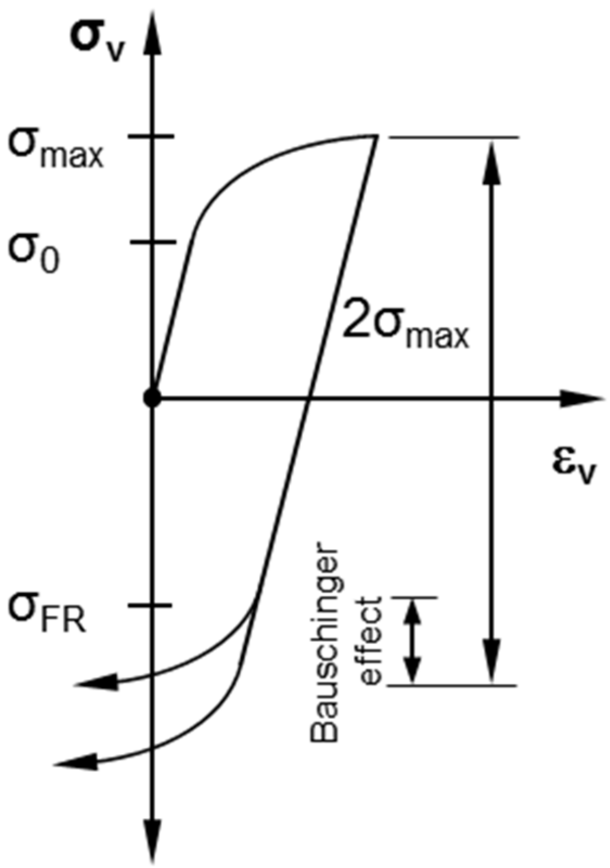


**Figure 9.** Bauschinger effect.

The C45 steel considered here has a large Bauschinger effect. Therefore, the reverse yield strength in this calculation example was determined to be $\sigma_{FR}$ = 227 MPa, assuming a Bauschinger Stress Parameter of $\beta_{BS}$ = 2.95. As can be seen in Figure 8, the equivalent stress at the inner diameter $D_{iA}$ of the hub (red line at radius 30 mm) exceeds this new yield point and plastic stresses reappear.

Therefore, the only way to prevent the Reverse Yielding of at-risk components is either to relieve the stress only until it reaches $\sigma_{FR}$, or to select $\sigma_{max}$ so that the stresses do not reach $\sigma_{FR}$ after complete relief.

It should be noted that Equation (12) is primarily considered from a fracture mechanics point of view [19]. It does not take into account the dependence of the reverse yield strength on the magnitude of the plastic strains. For this purpose, a number of investigations on autofrettaged cylinders made of higher-strength steels using different approaches have also focused on the influence of the Bauschinger effect [20–23]. The deviations in the respective results clearly show the decisive influence of the material models used. For larger plastic deformations, a nonlinear unloading behavior was found for the higher-strength steels investigated. Other interesting calculation models from current research have great potential for the numerical investigation of such complex FE models with regard to their plastic material behavior and also for the optimization of the computational processes [24].

The results of this work highlight the need for research regarding the Plastic Conditioning of interference fits and the materials commonly used in this process.

### *4.4. Technological Avoidance of Reverse Yielding*

Reverse Yielding can also be avoided technologically by integrating Plastic Conditioning into the joining process, which does not require relieving the components in the Reverse Yielding area. Such an approach is conceivable, for example, in the case of conical interference fits [25] or the use of clamping elements, as shown in Figure 10 (RING-SPANN GmbH).

In these cases, however, it should be borne in mind that Reverse Yielding may also occur in outer (tensile stressed) parts during the disassembly of joints subjected to such plastic stresses. When reassembling such components, there is a risk that the original joint pressures will not be achieved.

**Figure 10.** Shaft–hub connection with a cone clamping element (hub clamped on the inside) from RINGSPANN GmbH (image source: RINGSPANN GmbH).

## 5. Conclusions

This paper examines in detail the phenomenon of Reverse Yielding of elastically–plastically loaded interference fits and its influencing parameters. The limits of Plastic Conditioning effectiveness set by Reverse Yielding were presented under the conditions of SH, PSS, and ideal plastic material behavior, and analytical calculation options for engineering practice were derived. In addition, it was recognized that these limits change significantly under the assumption of the VMYC, material hardening, and the Bauschinger effect, and the research needs derived from this finding were defined. It has been shown that it is important to avoid Reverse Yielding during Plastic Conditioning as it limits the intended residual stress states and the extension of the elastic stress range. We defined the associated parameters, such as diameter ratio $Q_A$ and maximum pressure $\mathbf{p_{max}}$, as well as their limits of influence, and identified the components at risk. We also analytically presented possible methods to prevent Reverse Yielding, assuming an ideal plastic material and the Shear Stress Hypothesis.

Since our own numerical investigations for hardening materials based on the Von Mises Yield Criterion (VMYC) have shown an extension of the hazard potential with respect to larger diameter ratios $Q_A$ and lower pressures $p_i$ (or lower amounts of radial stress $|\sigma_r|$), there is a need for further numerical and experimental research on these prerequisites. In this context, material models that represent the Bauschinger effect as realistically as possible should also be used. This is important because previous results have shown that the Bauschinger effect significantly increases the risk of Reverse Yielding, and thus ignoring it can lead to severe miscalculations. The main task here is to determine the extent to which nonlinear unloading behavior [10,21] must be taken into account for the materials used and the common plastic strains.

The research results in this publication have been collected analytically and numerically. Therefore, another important concern of this work was to identify the need for further research activities in order to be able to plan the necessary resources. This mainly concerns the experimental verification of the analytical and numerical results and the determination of appropriate material models as well as data acquisition and calculation methods. This also includes investigations of the fatigue strength of plastically conditioned interference fits.

For components with larger diameter ratios made of hardening material, investigations beyond the fully plastic state are also of interest. Due to material hardening, for diameter ratios $Q_A > 0.45076$, stress states can be achieved before reaching the fully plastic state, resulting in Reverse Yielding after complete unloading, which is not possible with ideal plastic material. Especially for such diameter ratios, it is interesting to investigate conditioning pressures that exceed the fully plastic loading of the component cross-section, since this could significantly intensify the performance increase by conditioning for these components. Special attention is paid to the load limits as a function of the yield strength and stress–strain curve of the material, as well as the increasing risk of Reverse Yielding.

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## Abbreviations

| Abbreviation | Unit | Meaning |
|---|---|---|
| d | mm | Diameter coordinate (control variable) |
| $D_{aA}$ | mm | Outer diameter of the outer part |
| $D_F$ | mm | Joint diameter (nominal) |
| $D_{iA}$ | mm | Inner diameter of the outer part |
| $D_{PA}$ | mm | Plasticity diameter of the outer part |

| | | |
|---|---|---|
| $D_{\sigma r(SH)}$, $D_{\sigma t(SH)}$ | MPa | Stress values at point D in the principal stress plane for Shear Stress Hypothesis (SH) |
| m | - | Factor for determining the angle of inclination |
| $p_i$ | MPa | Internal pressure of the hub |
| $p_E$ | MPa | Elastic joint pressure at the yield strength |
| $p_F$ | MPa | Joint pressure |
| $p_K$ | MPa | Conditioning pressure |
| $p_{Kond,max}$ | MPa | Maximum joint pressure when conditioning |
| $p_{krit,SH}$ | MPa | Critical pressure for Reverse Yielding according to Shear Stress Hypothesis (SH) |
| $p_{krit,MSH}$ | MPa | Critical pressure for Reverse Yielding according to Modified Shear Stress Hypothesis (MSH) |
| $p_{max,SH}$ | MPa | Maximum possible pressure before the fully plastic state of the hub for ideal plastic material according to Shear Stress Hypothesis (SH) |
| $p_{max,MSH}$ | MPa | Maximum possible pressure before the fully plastic state of the hub for ideal plastic material according to Modified Shear Stress Hypothesis (MSH) |
| $Q_A$ | - | Diameter ratio of the outer part |
| $R_{eL,A}$ | MPa | Lower yield strength of the outer part |
| $S_{PA}$ | - | Safety against plastic deformation of the outer part |
| $S_{PA(SH)}$ | - | Safety against plastic deformation of the outer part achieved by conditioning |
| y | MPa | Intersection of the relief straight line with the ordinate (tangential residual stress after complete relief) |
| $\alpha$ | ° | Inclination angle for load line and relief straight line |
| $\beta_{BS}$ | - | Bauschinger Stress Parameter |
| $\varepsilon_v$ | - | Equivalent strain |
| $\varepsilon_{v,el}$ | - | Equivalent elastic strain |
| $\varepsilon_{v,pl}$ | - | Equivalent plastic strain |
| $\sigma_0$ | MPa | Yield strength |
| $\sigma_{FR}$ | MPa | Yield strength that is reached when the material is subsequently subjected to stress in the opposite direction |
| $\sigma_{max}$ | MPa | Maximum stress achieved by plastic deformation as a result of hardening |
| $\sigma_r$ | MPa | Radial stress |
| $\sigma_{rA}(d)$ | MPa | Radial stress at d of outer part |
| $\sigma_t$ | MPa | Tangential stress |
| $\sigma_{tA}(d)$ | MPa | Tangential stress at d of outer part |
| $\sigma_v$ | MPa | Equivalent stress |

| | |
|---|---|
| AT | Outer part of the Interference fit |
| PSS | Plane Stress State |

| | |
|---|---|
| FE | Finite elements |
| FEM | Finite element method |
| MSH | Modified Shear Stress Hypothesis according to Kollmann |
| VMYC | Von Mises Yield Criterion |
| SH | Shear Stress Hypothesis according to Tresca |
| IKAT | Institute of Construction and Drive Technology (TU Chemnitz) |

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