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A Supply Chain Coordination Optimization Model with Revenue Sharing and Carbon Awareness

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Abstract: The present study explores the impact of carbon emissions on supply chain coordination, where the supply chain entities are a retailer and a distributor. The study also involves two types of systems, namely centralized and decentralized. A centralized system computes the profit of the entire supply chain, including the profit of a retailer and a distributor, using the traditional optimization technique. In contrast, a decentralized system computes the profit of both a retailer and a distributor independently and uses the Stackelberg sequence for profit optimization. According to the Stackelberg sequence, one entity is considered a leader and the other a follower. When the profit in both systems is compared, it is found to be higher in the centralized system. So, to coordinate the system, a revenue-sharing contract is applied to coordinate the supply chain under a stock–time–price-sensitive demand rate. Finally, a carbon emission cost is implemented to the profits of both systems to make the model more sustainable. The main objective of the research is to optimize the profit of the supply chain by considering the concept of revenue-sharing contracts and making the system more sustainable through the implementation of carbon emission cost. The overall study concludes that the revenue-sharing fraction 'δ' helps in coordinating the system and 0.4 is the value of the revenue-sharing fraction 'δ' that perfectly coordinates the system. Due to this coordination, both the parties will gain profit, i.e., retailer and distributor, and this whole phenomenon increases the profit of the supply chain. A sensitivity analysis is also performed to check the stability of the model, and the model is found to be quite stable. A numerical example is illustrated, providing the result of the model.

Keywords: supply chain coordination; centralized and decentralized system; revenue sharing; carbon emission; time-dependent demand; optimization

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1. Introduction

The supply chain has a crucial role in the production of carbon emissions across the whole product lifespan, from origin to disposal. During the disposal phase, this covers sourcing raw materials, manufacturing methods, storage, and distribution, and even the product's end-of-life cycle. The need to reduce carbon emissions is not only essential from an environmental perspective, but with increasing awareness of climate change and ESG (environmental, social, and governance) among the global population, organizations that neglect to track and reduce their carbon footprint face the prospect of reputational damage due to the negative perceptions of the market. The implication of carbon emissions in supply chain coordination with revenue-sharing contracts also leads to profit for both the supply chain's entities, such as the distributor and the retailer. Researchers in the field of supply chain coordination have conducted numerous studies over the past few decades. Coordination is a strategic issue for a supply chain since it can maximize revenues across the entire chain. For decades, researchers have conducted extensive research on supply chain coordination using various contractual forms, gaining valuable insights into coordinating supply chains with a single supplier and retailer. Many businesses, such as fashion clothes, publishing, and cosmetics, use buyback contracts. The main objective of the coordination mechanism is to motivate supply chain participants to make the best choices possible on a global scale, resulting in a win-win scenario where supply chain performance is comparable to centralized decision making. For example, Arani et al. (2016) [1] developed a novel hybrid revenue-sharing contract to connect the supply chain, consisting of a producer and a retailer. This contract combines the features of the standard revenue-sharing contract with those of the European call option contract, and they claimed that the revenue-sharing contract could lessen the supply chain's dual critique effect. This option has the potential to improve the performance of the supply chain. De Giovammi (2020) [2] created a supply chain game model in which the retailer and supplier perform a Stackelberg game, with the supplier leading. They also assessed the significance of the supplier selling goods to the retailer and looked into the consequences of the wholesale price contract and revenue-sharing contract between the two entities.

The supply chain can be coordinated by executing a revenue-sharing contract mechanism. Current research shows that revenue-sharing contracts have proven to be favorable, which can reduce buyers' risk abomination, demand can be matched more desirably to supply, and many sorts of supply chains can attain coordination [3,4]. Numerous industries are broadly using revenue-sharing contracts [5]. According to Giannoccaro and Pontrandolfo (2004) [6], a revenue-sharing contract proved to be efficacious for the supply chain when it ensured coordination, but it is preferred by chain members only when they both make a profit that is at least equal to what they would make if they did not use a revenue-sharing contract. Employing a revenue-sharing contract simplifies the coordination challenge of a disrupted supply chain involving one supplier and numerous retailers with demand disturbances.

A supermarket's display of items often leads to frequent sales due to their wide variety. For these prompts, the demand is stock-dependent on the inventory model. The actual market demand also relies on the stock in the inventory systems along with the price of the item. Pal et al. (2014) [7] studied an inventory for deteriorating items with inflation and trade credit policy. They observed that the most optimal solution for the total cost of inventory may be found when the deterioration rate is constant, and the demand is dependent on stock and pricing.

It is critical to raise carbon consciousness while also growing earnings in today's world. Carbon emissions, the greenhouse gas with the highest amount of emissions in the atmosphere, have a massive impact on the world. Of course, this leads to global warming and, eventually, climate change. Some studies have investigated the expense of transportation as well as its carbon dioxide emissions. Mallidis et al. (2012) [8] studied the environmental impact of freight transportation, focusing on particulate matter (PMs) and CO_2 emissions. They examined emissions from several means of transportation for warehouses that were either dedicated or shared. In recent years, a few studies have investigated the sustainability of EOQ (economic order quantity) models. Inman (2002) [9] proposed a few key ideas to help researchers in the field of environmentally conscious operations management develop research guidelines. Turkay (2008) [10] is working on a lot-sizing model that takes into consideration a company's carbon footprint and compares the following five potential approaches: carbon tax, carbon offsets, direct accounting, cap and trade, and direct cap and trade.

For practitioners and researchers, green inventory management and carbon emission reduction have become global concerns. To compete in this competitive green trade market, producing a high-quality product at a low cost with low carbon emissions is critical. Inventory management that is both effective and environmentally friendly helps to save the environment from harmful carbon emissions while also increasing total profit. The increased global warming due to carbon emissions from industrial activities poses a significant problem for scholars and organizations. Incorporating the cost of carbon emissions and a trade credit policy into modern inventory management models creates a new paradigm for inventory system success. In recent years, it has been observed that academia and industry need to place more emphasis on approaches that promote environmental protection.

In this paper, revenue-sharing contract mechanisms are used to coordinate a decentralized system, and this mechanism is defined by two parameters, namely the percentage of revenue shared and the wholesale price shared among supply chain members. The carbon emission cost is also implemented on both centralized and decentralized systems to know how it affects the profit of a distributor and a retailer. The study's remaining sections are organized as follows: In Sections 4.1 and 4.2, results from centralized and decentralized systems, including carbon emissions costs, are obtained. These results are then used to implement a revenue-sharing system. Section 6 presents a numerical example of the model's improvement, following the description of the optimal solution criteria in Section 5. Sensitivity analysis is completed in Section 7, and Section 8 summarizes the results of the entire investigation, including the future scope and limitations.

Research objectives:

- To develop a model containing centralized and decentralized scenarios in which supply chain coordination takes place through revenue-sharing contracts, aiming to increase the profit of the entire supply chain.
- To develop a more sustainable model through the implementation of carbon emission costs in both scenarios.

2. Literature Review

A comprehensive literature review on the various categories of supply chain coordination is presented in this section. The objective is to conduct an exhaustive investigation into the present state of research, identify any voids in knowledge, and elucidate areas of consensus or divergence of opinion.

2.1. Review of Supply Chain Coordination

To investigate whether the quantity discount schedule is linear or if it is an all-unit quantity discount schedule, Xiao et al. (2007) [11] investigated supply chain coordination with demand disruptions. They also analyzed several problem scenarios, such as whether the expense of production deviation was paid by retailers or by the manufacturer. Following the disruption of the manufacturer's production cost, Xiao and Qi (2008) [12] examined the coordination of a supply chain between a single manufacturer and two rival retailers, employing either an all-unit quantity discount or an incremental quantity discount. According to Yan et al. (2016) [13], in the scenario of a dominating retailer having more knowledge, negotiating strength, and influence than the supplier, the retailer acts as the leader in the Stackelberg game. According to Zhao et al. (2019) [14], both risk-neutral suppliers and retailers who are risk-averse may gain profit from the call option. Their investigation focused on how supply chain earnings are impacted by stochastic yield volatility, option contract parameters, and uncertain demand. Under the standard vendor issue model framework, a revenue-sharing contract was created by Yao et al. (2008) [15] to set up a supply chain that included two rival retailers and a manufacturer. To investigate channel coordination with risk limitations for the two-echelon supply chain in the call option contract examined by Zhuo et al. (2018) [16], suppliers would need to provide both contract and wholesale price alternatives to identify the supply chain price and optimal ordering, where retailers must then deal with demand unpredictability and consumer refunds. For their research, Wang et al. (2020) [17] created a newsvendor model, while Fan et al. (2020) [18] considered a two-tier supply chain in which an upstream manufacturer and a downstream retailer share the costs of product liability due to quality defects. They made the assumptions that the manufacturer and the retailer share the expected liability cost for each unit of the poor-quality product. Xie et al. (2021) [19] investigated the supply chain buyback contract and looked at how to yield relative bargaining strength as well as determined the uncertainties that impact the performance of the buyback contract. In order to investigate and evaluate the difference in decision-making between centralized and decentralized settings, Bangjun et al. (2023) [20] examined the coordination game problem between the supply chains of coal power companies and renewable energy businesses. To represent risk-averse, risk-neutral, and risk-taking decision makers, Jammernegg et al. (2024) [21] used risk measurements based on the traditional normative definition of risk preferences by certainty equivalents. They proposed the mean conditional value-at-risk model, which is based on the popular risk metrics—value-at-risk and conditional value-at-risk. Linze et al. (2024) [22] researched the Stackelberg game model to explore the conditions for developing environmental, social and governance related cost-sharing contracts and their subsequent implications for supply chain coordination. Zhan et al. (2023) [23] proposed a model that includes the following three scenarios: no return freight insurance, customer purchase return freight insurance, and free e-retail return freight insurance. A Stackelberg game between e-retailers and manufacturers is also modelled for study. In various contexts, they examined the optimal price decisions and RI premium distribution plans for manufacturers and e-retailers.

2.2. Review of Revenue-Sharing Contract

Sappington (2024) [24] studied the best design of regulatory policy in situations where the regulatory company faces potentially huge variations in the demand for its service (e.g., electricity supply). He investigated a class of regulatory approaches that comprised two common kinds of incentive regulation, namely price cap regulation and revenue cap regulation. Nerja and Sanchez (2023) [25] explored the effectiveness of revenue-sharing contracts between ports and shipping lines in mitigating the negative consequences of rival shipping alliances. They considered a vertical structural method established by port–shipping line chains, where ports are regarded as the upstream market and shipping lines represent the downstream market. Liu et al. (2024) [26] discussed a three-tier supply chain finance system that includes a supplier, an electronic business platform, and a financially troubled retailer. In this system, the electronic business platform serves as both a seller and a lender. They used a revenue-sharing contract to establish a multi-tier game model and looked at each member's operating plans. If production is unpredictable, and demand is uncertain and determined, Tang and Kouvelis (2014) [27] investigated whether the supply chain could then be regulated by the cashback contract. They demonstrated how a particularly constructed contract, such as the contract for purchasing revenue sharing, may coordinate the supply chain and discuss how it might be used in an agricultural context.

Choi and Guo (2019) [28] presented the revenue-sharing plus side-payment agreement for the zero wholesale price, which improved on the zero wholesale price contract. They argued that the zero wholesale price contracts may lead to coordination that would benefit both producers and retailers, with the supply chain being maximized because of the contract's adoption. In a three-level supply chain, they investigate how to establish supply chain coordination utilizing bilateral tariff contracts and zero wholesale price contracts vs. wholesale pricing contracts. Giri et al. (2021) [29] investigated a single-product news vendor model, highlighting the significance of coordination in the face of supply and demand uncertainty, a scenario where raw materials are purchased without access to emergency resources from two unreliable sources (the main supplier and the backup supplier). To alleviate demand and supply concerns in the decentralized model, they recommended a price-only agreement and a new revenue-sharing agreement, noting that the latter may fully coordinate the supply chain and result in a win-win situation for all parties concerned. The revenue contract was utilized by Lu et al. (2010) [30] to examine the supply chain for mobile carriers. However, concerning supply chain coordination, almost all the models discussed above are based on deterministic, price-sensitive, or stochastic news vendor situations. Wang et al. (2008) [31] examined the effects of a revenue-sharing mechanism in a supply chain with uncertain demand. Avinadav et al. (2015) [32] used a consignment contract with revenue sharing and quality investment to investigate how risk sensitivity affected the supply chain for mobile apps. They found that developers who take more risks may anticipate larger profits than developers who take no risks, and that revenue-sharing agreements may help avoid double marginalization. Bellantuono et al. (2009) [33] concentrated on two types of contract schemes (a revenue-sharing contract and an advance booking discount program) that aimed to coordinate a decentralized supply chain while also increasing sales. They demonstrated that each solution would boost the supply chain's predicted profit.

2.3. Review of Stock–Price–Time-Dependent Demand

Yang et al. (2023) [34] analyzed the delivery investment choice in the context of trade credit, in which the retailer postponed payments from its supplier. In addition to ordering procurement, the store must use limited cash to shorten delivery times, which encourages time-sensitive demand. They created a Stackelberg game in which the supplier sets the wholesale price, and the retailer sets the delivery investment and order volume. Bahrami et al. (2024) [35] developed a dynamic model that evaluates market demand, price choices, and advertising costs using the features of the product life cycle. Their contributions go beyond traditional analysis. They provided dynamic mathematical models that reflect market demand, price choices, and advertising expenditures while meticulously matching them with the distinct characteristics of each product's life cycle phase. Parthasarathi et al. (2010) [36] considered the stock- and price-dependent demand patterns to cope with the pricing consequences of stock-dependent demands. Lee et al. (2017) [37] investigated forward and backward stocking techniques under consignment stock and stock-dependent demand. A coordinated model with a single vendor and a single buyer for a declining item in a price- and stock-dependent demand environment with a vendor management inventory and consignment stock agreement was established by Hemmati et al. (2017) [38]. Giri and Bardhan (2015) [39] were the first to use a joint economic lot size model to incorporate the concept of stock-dependent demand. Chang (2013) [40] developed an inventory model with a price-dependent demand rate that determines the optimum buying and selling prices. Moreover, an inventory holding the cost that accounts for all unit price deductions and is proportional to the unit cost was formed based on the method used by Burwell et al. (1997) [41]. Because selling price, freshness, and displayed stocks influence the rate of demand for perishable commodities, Feng et al. (2017) [42] estimated the best-selling price, cycle time, and non-zero finishing stock levels. They demonstrated that the profit in the dependent variables is pseudo-concave.

2.4. Review of Carbon Emission

Zhang et al. (2024) [43] examined, empirically, the influence of digital transformation on carbon emissions in the manufacturing business, using data from publicly traded firms in China's manufacturing sector from 2011 to 2019. Their results show that business digital transformation has a significant impact on reducing carbon emissions in the industrial sector. Significant carbon emissions from building sites demand better management by combining several data sources. Although ontologies are commonly used for data integration, there are no domain-specific ontologies for managing carbon emissions during construction. Lu et al. (2024) [44] created a carbon emission management ontology using a hybrid development methodology. According to Fleischmann et al. (2003) [45], using improved green supply chain management, International Business Machines remanufactured products can save up to 80% in cost when compared to newly built products. Hua et al. (2011) [46] added a carbon cap and trade mechanism to the basic EOQ model, comparing the optimal order amounts to reduce economic and environmental costs. A model was developed by Hovelaque and Bironneau (2015) [47], who considered the carbon price to be an environmental regulation and demand to be environmentally dependent. They compared endogenous and exogenous prices in the EOQ model to find optimal amounts that maximize production profitability while minimizing carbon emissions. The outcomes showed that the improved EOQ model would substantially lower carbon emissions.

Rani et al. (2017) [48] developed an inventory model in the green supply chain for deteriorating items, considering recycling and reverse logistics and taking inflation into account. A green supply chain fuzzy inventory model that considered the effects of learning and remanufacturing was also created by Rani et al. (2019) [49]. Chai et al. (2021) [50] proposed policies for a controlled low-strength material model for remanufactured green items. A model for the deterioration of low-quality items was created by Sepehri et al. (2021) [51], in which the degree of deterioration remained constant and could be reduced by spending money on preservation techniques. Carbon emissions from manufacturing operations may be minimized by investing in tax reform. Singh et al. (2021) [52] developed a technique for degrading items that integrated the effects of carbon emissions costs and trade credit schemes. A three-tiered sustainable supply chain model with a single supplier, a single manufacturer, and multiple retailers was developed by Sarkar et al. (2021) [53]. Sarkar et al. (2022) [54] analyzed the environmental hazards connected to the production of novel green products, given that both the demand and the return rate are arbitrary and lacking appropriate data. Kugele et al. (2022) [55] presented a smart manufacturing system based on product reliability. The optimal solution was found using geometric programming. Arora et al. (2022) [56] investigated the fuzzy model under the cap-and-trade system to limit the carbon emissions caused by various modes of transportation. This model includes both recycling and remanufacturing, with uncertain cost parameters. Poswal et al. (2022) [57] developed a carbon emissions sustainable policy for both the producer and the remanufacturer under uncertain environments. Singh et al. (2023) [58] proposed a methodology to reduce supply chain costs while both decreasing environmental impact and optimizing social impact. Table 1 identifies research gaps based on earlier work. which are outlined below:

Table 1. Research gaps shown within previous studies.

References	Supply Chain Coordination	Revenue-Sharing Contract	Stock-Dependent Demand	Price-Dependent Demand	Time-Dependent Demand	Carbon Emission
[1]	consider	consider	Not consider	Not consider	Not consider	Not consider
[6]	consider	consider	Not consider	Not consider	Not consider	Not consider
[30]	consider	consider	Not consider	Not consider	Not consider	Not consider
[14]	consider	consider	Not consider	Not consider	Not consider	Not consider
[33]	Not consider	consider	Not consider	Not consider	Not consider	Not consider
[37]	Not consider	Not consider	consider	Not consider	Not consider	Not consider
[38]	Not consider	Not consider	consider	consider	Not consider	Not consider
[41]	Not consider	Not consider	Not consider	consider	Not consider	Not consider
[42]	Not consider	Not consider	consider	consider	consider	Not consider
[47]	Not consider	Not consider	Not consider	Not consider	Not consider	consider
[40]	Not consider	Not consider	Not consider	consider	Not consider	Not consider
[51]	Not consider	Not consider	Not consider	Not consider	Not consider	consider
[52]	Not consider	Not consider	Not consider	Not consider	Not consider	consider
[55]	Not consider	Not consider	Not consider	Not consider	Not consider	consider
This paper	consider	consider	consider	consider	consider	consider

3. Notations and Assumptions

3.1. Notations

$B(>0)$ = initial demand rate
$a(>0)$ = time-responsive parameters of demand
$\theta(>0)$ = reflects the elasticity of the demand rate with respect to the inventory level
$\varepsilon(>0)$ = price sensitive perimeter
P = the product's end selling price per unit

$D(I(T), T, P)$ = demand rate, which is a function of time selling price and on-hand inventory level
$I(T)$ = on-hand stock level
B_1, B_2 = the distributor's and the retailer's fixed ordering costs
u_d, u_r = the distributor's and the retailer's respective unit costs
H = holding cost per unit quantity per unit time for the retailer
q = the quantity of stock that the retailer orders from the distributor at the beginning of the replenishment cycle.
e = carbon emission unit related to the order's beginning.
g = inventory level per unit of time related to the warehouse carbon emission unit
k = quantities of carbon emissions related to each purchased or generated unit.
μ_r = retailer's profit
μ_d = distributor's profit
δ = revenue-sharing fraction
l = replenishment cycle
HC = total holding cost
μ_r = total profit for the supply chain
q_c = optimal order quantity in centralized system
μ_{rdc} = optimal profit of retailer for decentralized system
μ_{ddc} = optimal profit of distributor for decentralized system
P_c = optimal selling price in centralized system
P_d = optimal selling price in decentralized system
μ_{rrs} = optimal profit of retailer in revenue-sharing system
μ_{drs} = optimal profit of distributor in revenue-sharing system
$P_r{}^*$ = optimal selling price of retailer in revenue-sharing system
$u_r{}^*$ = unit price of retailer in revenue-sharing system

3.2. Assumptions

- The level of stock on hand determines the demand rate, pricing, and time, i.e., $D(I(T), T, P) = Be^{-aT} + \theta I(T) - \varepsilon P$ and this assumption is taken from Hseih et al. (2010) [59], in which they look at the effectiveness of the price discount strategy as a way for the distributor to manage the ordering and pricing choices made by the retailers in two typical scenarios, namely linear demand in price and constant elasticity demand in price.
- Visual merchandising is considered, i.e., the practice in the retail industry of optimizing the presentation of products and services to better highlight their features and benefits. Its purpose is to attract, engage, and motivate the customer towards making a purchase.
- Demand for fashion apparel depends on time and price. As time passes, the display of the product attracts more demand but with time, it decreases.

4. Mathematical Model

4.1. Model for Stock–Price–Time-Dependent Demand

To develop a model, we contemplate a two-stage supply chain that includes a single distributor and a retailer. Based on the assumption that demand rate is dependent on price, time, and on-hand stock level. The demand will be in the form:

$$D(I(T), T, P) = Be^{-aT} + \theta I(T) - \varepsilon P \tag{1}$$

The retailer differential equations are as follows:

$$\frac{dI(T)}{dT} = -Be^{-aT} - \theta I(T) + \varepsilon P \tag{2}$$

On integrating and applying the condition $I(l) = 0$ in Equation (2):

$$I(T) = \frac{B}{\theta - a}\left(e^{(\theta-a)l}e^{-\theta T} - e^{-aT}\right) - \frac{\varepsilon P}{\theta}\left(e^{\theta(l-T)} - 1\right) \tag{3}$$

Applying condition $I(0) = q$, the total order quantity for the retailer will be obtained as follows:

$$q = \frac{B}{\theta - a}\left(e^{(\theta-a)l} - 1\right) - \frac{\varepsilon P}{\theta}\left(e^{\theta l} - 1\right) \tag{4}$$

Now, the total holding cost for the time interval [0, l]. So, the holding cost will be:

$$HC = H\left[\left(\frac{Be^{(\theta-a)l}}{\theta - a} - \frac{\varepsilon P e^{\theta l}}{\theta}\right)\left(\frac{1 - e^{-\theta l}}{\theta}\right) - \left\{\frac{B}{\theta - a}\left(\frac{1 - e^{-al}}{a}\right) - \frac{\varepsilon P}{\theta} l\right\}\right] \tag{5}$$

The retailer's total profit for planning horizons will be:

$$\begin{gathered}\mu_r(P) = Pq - u_r q - HC - B_1 \\ \mu_r(P) = (P - u_r)\left\{\tfrac{B}{\theta-a}\left(e^{(\theta-a)l} - 1\right) - \tfrac{\varepsilon P}{\theta}\left(e^{\theta l} - 1\right)\right\} - H\left[\tfrac{B}{\theta-a}\left\{\tfrac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right) - \tfrac{1-e^{-al}}{a}\right\} - \right. \\ \left.\tfrac{\varepsilon P}{\theta}\left(\tfrac{e^{\theta l}-1}{\theta} - l\right)\right] - B_1\end{gathered} \tag{6}$$

Distributor's total profit for planning horizon will be:

$$\mu_d(u_r) = (u_r - u_d)\left\{\frac{B}{\theta - a}\left(e^{(\theta-a)l} - 1\right) - \frac{\varepsilon P}{\theta}\left(e^{\theta l} - 1\right)\right\} - B_2 \tag{7}$$

Now, we will obtain total profit for the supply chain by adding Equations (6) and (7), i.e., $\mu_c = \mu_r(P) + \mu_d(u_r)$.

Without carbon emission:

$$\begin{gathered}\mu_c = (P - u_d)\left\{\tfrac{B}{\theta-a}\left(e^{(\theta-a)l} - 1\right) - \tfrac{\varepsilon P}{\theta}\left(e^{\theta l} - 1\right)\right\} - H\left[\tfrac{B}{\theta-a}\left\{\tfrac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right) - \tfrac{1-e^{-al}}{a}\right\}\right. \\ \left. - \tfrac{\varepsilon P}{\theta}\left(\tfrac{e^{\theta l}-1}{\theta} - l\right)\right] - B_1 - B_2\end{gathered} \tag{8}$$

With carbon emission:

$$\begin{gathered}\mu_c = \left(P - u_d - \tfrac{g}{2}\right)\left\{\tfrac{B}{\theta-a}\left(e^{(\theta-a)l} - 1\right) - \tfrac{\varepsilon P}{\theta}\left(e^{\theta l} - 1\right)\right\} - H\left[\tfrac{B}{\theta-a}\left\{\tfrac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right) - \tfrac{1-e^{-al}}{a}\right\}\right. \\ \left. - \tfrac{\varepsilon P}{\theta}\left(\tfrac{e^{\theta l}-1}{\theta} - l\right)\right] - K\left\{\tfrac{`B}{2(\theta-a)}\left(e^{(\theta-a)l} - 1\right) - \tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta} - l\right) - \tfrac{u_d \varepsilon}{2\theta}\left(e^{\theta l} - 1\right)\right\} - B_1 - B_2 - \tfrac{e}{l}\end{gathered} \tag{9}$$

4.1.1. For the Centralized System

All decisions are taken by considering a single entity, so we need to optimize the total supply chain profit we obtained from above equation. The condition for optimization is as follows:

$$\frac{d\mu_c}{dP} = 0 \tag{10}$$

We will ascertain the value of P as:

The product's unit end selling price without carbon emission:

$$P = \frac{(B/(\theta - a))\left(e^{(\theta-a)l} - 1\right)}{(2\varepsilon/\theta)\left(e^{\theta l} - 1\right)} + \frac{H(((e^{\theta l} - 1)/\theta) - l)}{2(e^{\theta l} - 1)} + \frac{u_d}{2} = P_c \tag{11}$$

The product's unit end selling price with carbon emission:

$$P = \frac{(B/(\theta-a))\left(e^{(\theta-a)l}-1\right)}{(2\varepsilon/\theta)\left(e^{\theta l}-1\right)} + \frac{H(((e^{\theta l}-1)/\theta)-l)}{2(e^{\theta l}-1)} + \frac{u_d}{2} + \frac{g}{4} = P_c \tag{12}$$

Now, $\frac{d^2\mu_c}{dP^2} = -2\varepsilon l < 0$, thus we reach the observation that for the centralized system, the total supply chain profit function is concave.

Putting the value of P from Equations (11) and (12) in Equations (8) and (9), respectively, then the value of the total supply chain profit will be:

The total supply chain profit without carbon emissions:

$$\begin{gathered}\mu_c = \frac{1}{(4\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 + \frac{u_d H\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) \\ -H\left\{\frac{B}{\theta-a}\left(\frac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right) - \frac{1-e^{-al}}{a}\right\} - B_1 - B_2\end{gathered} \tag{13}$$

The total supply chain profit with carbon emissions:

$$\begin{gathered}\mu_c = \frac{1}{\left(4\varepsilon/\theta\right)\left(e^{\theta l}-1\right)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 \\ +\frac{u_d H\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - H\left\{\frac{B}{\theta-a}\left(e^{(\theta-a)l}-1\right) - \frac{\varepsilon P}{\theta}\left(e^{\theta l}-1\right)\right\} \\ -\frac{g}{4}\left\{\frac{B}{(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)\right\} \\ +\frac{g^2}{16}\frac{\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}\right) - K\left\{\frac{`B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\} \\ -B_1 - B_2 - \frac{e}{l}\end{gathered} \tag{14}$$

Now, using the value of P from Equation (12) in Equation (4), the optimal order quantity in the centralized system is obtained.

$$\begin{gathered}q_c = \frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right) \\ -\frac{g\varepsilon}{4\theta}\left(e^{\theta l}-1\right)\end{gathered} \tag{15}$$

4.1.2. For a Decentralized System

It is assumed that all members involved in the system will make liberal decisions to maximize their respective profits. Since there are many methods to analyze the present condition, in our paper, we conduct optimization by considering the Stackelberg sequence. This paper involves a supply chain of two stages, that involve the retailer and the distributor. Thus, according to the Stackelberg sequence, we take the distributor as a leader and the retailer as a follower. The main aim of the distributor is to maximize his profit after assuming the ways that retailer may apply to maximize their profit. Firstly, the retailer determines his optimal decision, and the optimal condition is as follows:

$$\frac{d\mu_r}{dP} = 0 \tag{16}$$

After solving above equation, the value of P will be given as:

$$P = \frac{(B/(\theta-a))\left(e^{(\theta-a)l}-1\right)}{(2\varepsilon/\theta)\left(e^{\theta l}-1\right)} + \frac{H(((e^{\theta l}-1)/\theta)-l)}{2(e^{\theta l}-1)} + \frac{u_r}{2} \tag{17}$$

Also, the second order differentiation of $_r$ with respect to P will be $-2\varepsilon l < 0$, i.e., $\frac{d^2\mu_r}{dP^2} = -2\varepsilon l < 0$.

From the above equation, it is observed that the retailer's total profit function is concave.

Using Equation (17) in Equation (7), we obtain the distributor profit function as:

$$\mu_d(u_r) = (u_r - u_d)\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{\varepsilon u_r}{2\theta}\left(e^{\theta l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)\right\} - B_2 \tag{18}$$

The maximization condition for the $_d(u_r)$ is as follows:

$$\frac{d\mu_d(u_r)}{du_r} = 0 \tag{19}$$

The above equation will give the value of u_r.

$$u_r = \frac{(B/(\theta-a))\left(e^{(\theta-a)l}-1\right)}{(2\varepsilon/\theta)\left(e^{\theta l}-1\right)} - \frac{H(((e^{\theta l}-1)/\theta)-l)}{2(e^{\theta l}-1)} + \frac{u_d}{2} \tag{20}$$

Also, $\frac{d^2\mu_d}{du_r{}^2} = -\varepsilon l < 0$. From the above equation, the total profit function for the distributor is found to be concave.

Putting the optimal value of u_r from Equation (20) in Equation (17), we obtain the optimal value of selling price in decentralized system.

$$P = \frac{(3B/(\theta-a))\left(e^{(\theta-a)l}-1\right)}{(4\varepsilon/\theta)\left(e^{\theta l}-1\right)} + \frac{H(((e^{\theta l}-1)/\theta)-l)}{4(e^{\theta l}-1)} + \frac{u_d}{4} = P_d. \tag{21}$$

Now, the optimal profit of the retailer for the decentralized system without carbon emission is as follows:

$$\begin{aligned}\mu_{rdc} = {} & \frac{1}{(4\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 \\ & + \frac{BH(((e^{\theta l}-1)/\theta)-l)(e^{(\theta-a)l}-1)}{(\theta-a)\,(e^{\theta l}-1)} - H[\frac{B}{\theta-a}\left\{\frac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right) - \frac{1-e^{-al}}{a}\right\}] - B_1\end{aligned} \tag{22}$$

Optimal profit of the retailer for the decentralized system with carbon emission:

$$\begin{aligned}\mu_{rdc} = {} & \frac{1}{(4\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 \\ & + \frac{BH(((e^{\theta l}-1)/\theta)-l)(e^{(\theta-a)l}-1)}{(\theta-a)(e^{\theta l}-1)} - H[\frac{B}{\theta-a}\left\{\frac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right) - \frac{1-e^{-al}}{a}\right\}] - B_1 - \frac{e}{l} \\ & -K\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\} \\ & -\frac{g}{8}\left\{\frac{B}{(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)\right\}\end{aligned} \tag{23}$$

Optimal profit of the distributor for decentralized system without carbon emission:

$$\mu_{ddc} = \frac{1}{(2\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_2 \tag{24}$$

Optimal profit of the distributor for decentralized system with carbon emission:

$$\begin{aligned}\mu_{ddc} = {} & \frac{1}{(2\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_2 \\ & -K\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{2\,\theta}\left(e^{\theta l}-1\right)\right\} \\ & -\frac{g}{8}\left\{\frac{B}{(\theta-a)}\left(e^{(\theta-a)l}-1\right) - \frac{H\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right) - \frac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)\right\} - \frac{e}{l}\end{aligned} \tag{25}$$

Now, the difference in profit within the centralized and decentralized situations for the entire supply chain:

$$\mu_c - (\mu_{rdc} + \mu_{ddc}) = (P_c - P_d)\Big\{ \tfrac{B}{(\theta-a)}\Big(e^{(\theta-a)l} - 1\Big) - \tfrac{\varepsilon}{\theta}\Big(e^{\theta l} - 1\Big)(P_c + P_d) + \tfrac{u_d\varepsilon}{\theta}\Big(e^{\theta l} - 1\Big) + \tfrac{H\varepsilon}{\theta}\Big(\tfrac{e^{\theta l}-1}{\theta} - l\Big)\Big\} \tag{26}$$

Since

$$(P_c + P_d)\frac{\varepsilon}{\theta}\Big(e^{\theta l} - 1\Big) = \frac{5B}{4(\theta - a)}\Big(e^{(\theta-a)l} - 1\Big) + \frac{3}{4}\frac{H\varepsilon}{\theta}\left(\frac{e^{\theta l} - 1}{\theta} - l\right) + \frac{3}{4}\frac{u_d\varepsilon}{\theta}\Big(e^{\theta l} - 1\Big) + \frac{g\varepsilon}{4\theta}\Big(e^{\theta l} - 1\Big) \tag{27}$$

upon simplifying, we attain the following:

$$\begin{aligned} &\mu_c - (\mu_{rdc} + \mu_{rdc}) \\ &= \tfrac{1}{(\varepsilon/\theta)(e^{\theta l}-1)}\Big\{\tfrac{B}{4(\theta-a)}\Big(e^{(\theta-a)l} - 1\Big) - \tfrac{H\varepsilon}{4\theta}\Big(\tfrac{e^{\theta l}-1}{\theta} - l\Big) - \tfrac{u_d\varepsilon}{4\theta}\Big(e^{\theta l} - 1\Big)\Big\}^2 \\ &+K\Big\{\tfrac{B}{2(\theta-a)}\Big(e^{(\theta-a)l} - 1\Big) - \tfrac{H\varepsilon}{2\theta}\Big(\tfrac{e^{\theta l}-1}{\theta} - l\Big) - \tfrac{u_d\varepsilon}{2\theta}\Big(e^{\theta l} - 1\Big)\Big\} + \tfrac{e}{l} \\ &+\tfrac{g^2}{16}\tfrac{\varepsilon}{\theta}\Big(\tfrac{e^{\theta l}-1}{\theta}\Big) > 0 \end{aligned} \tag{28}$$

From Equation (28), it is observed that for the price–stock–time-dependent demand rate, the supply chain's profit in a centralized system is higher than the profit in the decentralized system.

4.1.3. Revenue-Sharing System

In a centralized system, optimal supply chain profit is found to be greater than the total profit in a decentralized system, which means we gain the suboptimal solution. There are many coordination methods that are suggested by supporters to escape the condition of suboptimality. In the present paper, we implement the revenue-sharing contract methods to be sure if they can coordinate the system. In this contract, both the wholesale price u_r and δ (where 0 < δ < 1 and δ denotes the percentage) are addressed, and δ will provide the profit sharing between the retailer and the distributor.

Now,

$$\begin{aligned}\mu_{rrs} = (\delta P - u_r)\Big\{\tfrac{B}{2(\theta-a)}\Big(e^{(\theta-a)l} - 1\Big) - \tfrac{\varepsilon u_r}{2\theta}\Big(e^{\theta l} - 1\Big) - \tfrac{H\varepsilon}{2\theta}\Big(\tfrac{e^{\theta l}-1}{\theta} - l\Big)\Big\} - H\Big[\tfrac{1-e^{-al}}{a}\Big) - \\ -\Big[\tfrac{B}{\theta-a}\Big(\tfrac{1-e^{-\theta l}}{\theta}\Big(e^{(\theta-a)l}\Big) - \tfrac{\varepsilon P}{\theta}\Big(\tfrac{e^{\theta l}-1}{\theta} - l\Big)\Big] - B_1\end{aligned} \tag{29}$$

Retailer profit function: After applying the revenue-sharing contract mechanisms without carbon emissions, the aim is to optimize the $_{rrs}$, which is a function of P. The maximization condition is as follows:

$$\frac{d\mu_{rrs}(P)}{dP} = 0 \tag{30}$$

The above equation gives the value of P as:

$$P = \frac{(B/(\theta - a))\Big(e^{(\theta-a)l} - 1\Big)}{(2\varepsilon/\theta)\big(e^{\theta l} - 1\big)} + \frac{H(((e^{\theta l} - 1)/\theta) - l)}{2\delta\big(e^{\theta l} - 1\big)} + \frac{u_r}{2\delta} = P_r^* \tag{31}$$

To perfectly coordinate the system, it is necessary to achieve the optimal profit of the supply chain in a centralized system, which must be higher than the total profit obtained in a decentralized system.

Comparing Equation (20) with Equation (32), the unit price of the retailer can be obtained as follows:

$$u_r = \frac{(\delta-1)H(((e^{\theta l}-1)/\theta)-l)}{\left(e^{\theta l}-1\right)} + \frac{\delta g}{2} + \delta u_d = u_r^* \tag{32}$$

Putting the values of P_r^* and u_r^* in Equations (6) and (7), we can attain the following:

Retailer's and distributor's optimal profit under the revenue-sharing contract without carbon emissions:

$$\begin{aligned}\mu_{rrs} = {} & \frac{\delta}{\left(\varepsilon/\theta\right)\left(e^{\theta l}-1\right)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 \\ & +\frac{BH\left(\left(\left(e^{\theta l}-1\right)/\theta\right)-l\right)\left(e^{(\theta-a)l}-1\right)}{(\theta-a)\left(e^{\theta l}-1\right)} - H\left[\frac{B}{\theta-a}\left\{\frac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right)-\frac{1-e^{-al}}{a}\right\}\right]-B_1\end{aligned} \tag{33}$$

$$\mu_{drs} = \frac{(1-\delta)}{(\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_2 \tag{34}$$

Retailer's and distributor's optimal profit under the revenue-sharing contract with carbon emissions:

$$\begin{aligned}\mu_{rrs} = {} & \frac{\delta}{(\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 + \\ & \frac{BH\left(\left(\left(e^{\theta l}-1\right)/\theta\right)-l\right)\left(e^{(\theta-a)l}-1\right)}{(\theta-a)\left(e^{\theta l}-1\right)} - H\left[\frac{B}{\theta-a}\left(\left\{\frac{1-e^{-\theta l}}{\theta}\left(e^{(\theta-a)l}\right)-\frac{1-e^{-al}}{a}\right\}\right]-K\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)-\right. \\ & \left.\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}-\frac{g}{2}\delta\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)-\frac{H\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)-\right. \\ & \left.\frac{g\varepsilon}{4\theta}\left(e^{\theta l}-1\right)\right\}-\frac{g}{4}\frac{B}{(\theta-a)}\left(e^{(\theta-a)l}-1\right)+\frac{g^2\varepsilon}{16\theta}\left(e^{\theta l}-1\right)-B_1-\frac{e}{l}\end{aligned} \tag{35}$$

$$\begin{aligned}\mu_{drs} = {} & \frac{(1-\delta)}{(\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2-\frac{g(\delta-1)}{2}\left\{\frac{H\varepsilon}{\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)+\right. \\ & \left.\frac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)-\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)\right\}-\frac{g^2(\delta-1)\varepsilon}{8\theta}\left(e^{\theta l}-1\right)-K\left\{\frac{B}{2(\theta-a)}\left(e^{(\theta-a)l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)\right. \\ & \left.-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}-B_2-\frac{e}{l}\end{aligned} \tag{36}$$

It is clearly seen that the value of the revenue-sharing component that coordinates the system is $\delta \in [1/4,1/2]$, which follows the Stackelberg game.

4.2. Model for Stock- and Price-Sensitive Demand

Since demand is stock–price sensitive, the retailer's and the distributor's optimal profit in a decentralized system can be obtained by putting $a \to 0$ in Equations (22)–(25) as follows:

4.2.1. For a Decentralized System

Retailer and distributor optimal profit in a decentralized system without carbon emissions:

$$\mu_{rdc} = \frac{1}{(4\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2\theta}\left(e^{\theta l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_1 \tag{37}$$

$$\mu_{ddc} = \frac{1}{(2\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2\theta}\left(e^{\theta l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_2 \tag{38}$$

Retailer and distributor optimal profit in a decentralized system with carbon emissions:

$$\begin{aligned}\mu_{rdc} = \tfrac{1}{(4\varepsilon/\theta)(e^{\theta l}-1)}\left\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_1 - K\Big\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)\\ -\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\\ -\tfrac{g}{8}\left\{\tfrac{B}{\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)\right\}-\tfrac{e}{l}\end{aligned} \tag{39}$$

$$\begin{aligned}\mu_{ddc} = \tfrac{1}{(2\varepsilon/\theta)(e^{\theta l}-1)}\left\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_2 - K\Big\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)\\ -\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\\ -\tfrac{g}{8}\left\{\tfrac{B}{\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)\right\}-\tfrac{e}{l}\end{aligned} \tag{40}$$

4.2.2. For a Revenue-Sharing System

Now, we calculate the optimal profits for the revenue-sharing system from Equation (33) to Equation (36) using $a \to 0$.

Retailer's and distributor's optimal profit in the revenue-sharing system without carbon emissions:

$$\mu_{rrs} = \frac{\delta}{(\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2\theta}\left(e^{\theta l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_1 \tag{41}$$

$$\mu_{drs} = \frac{(1-\delta)}{(\varepsilon/\theta)(e^{\theta l}-1)}\left\{\frac{B}{2\theta}\left(e^{\theta l}-1\right)-\frac{H\varepsilon}{2\theta}\left(\frac{e^{\theta l}-1}{\theta}-l\right)-\frac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_2 \tag{42}$$

Retailer's and distributor's optimal profit in the revenue-sharing system with carbon emissions:

$$\begin{aligned}\mu_{rrs} = \tfrac{\delta}{(\varepsilon/\theta)(e^{\theta l}-1)}\left\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2 - B_1\\ -K\Big\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\Big\}\\ -\tfrac{g}{2}\delta\Big\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)\Big\}-\tfrac{g}{4}\tfrac{B}{\theta}\left(e^{\theta l}-1\right)\\ +\tfrac{g^2\varepsilon}{16\theta}\left(e^{\theta l}-1\right)-\tfrac{e}{l}\end{aligned} \tag{43}$$

$$\begin{aligned}\mu_{drs} = \tfrac{(1-\delta)}{(\varepsilon/\theta)(e^{\theta l}-1)}\left\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\right\}^2\\ -\tfrac{g(\delta-1)}{2}\left\{\tfrac{H\varepsilon}{\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)+\tfrac{u_d\varepsilon}{\theta}\left(e^{\theta l}-1\right)-\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)\right\}\\ -\tfrac{g^2(\delta-1)\varepsilon}{8\theta}\left(e^{\theta l}-1\right)\\ -K\Big\{\tfrac{B}{2\theta}\left(e^{\theta l}-1\right)-\tfrac{H\varepsilon}{2\theta}\left(\tfrac{e^{\theta l}-1}{\theta}-l\right)-\tfrac{u_d\varepsilon}{2\theta}\left(e^{\theta l}-1\right)\Big\}-B_2-\tfrac{e}{l}\end{aligned} \tag{44}$$

It is observed that the value of the revenue-sharing component, i.e., ϵ [1/4,1/2], will coordinate the system for both retailers and distributors, where the system follows the Stackelberg sequence.

4.3. Model for Time- and Price-Sensitive Demand

When the demand is time- and price-sensitive, then the optimal profits of the retailer and the distributor under a decentralized system are obtained by putting $\theta \to 0$ in Equations (22)–(25).

4.3.1. For a Decentralized System

Retailer's and distributor's optimal profit in a decentralized system without carbon emissions:

$$\mu_{rdc} = \frac{1}{4\varepsilon l}\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{H\varepsilon}{4}l^2 - \frac{u_d \varepsilon l}{2}\right\}^2 - \frac{HB}{a}\left(\frac{1-e^{-al}}{a} - e^{-al}l\right) - B_1 \tag{45}$$

$$\mu_{ddc} = \frac{1}{2\varepsilon l}\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{H\varepsilon}{4}l^2 - \frac{u_d \varepsilon l}{2}\right\}^2 - B_2 \tag{46}$$

Retailer's and distributor's optimal profit in a decentralized system with carbon emissions:

$$\begin{gathered}\mu_{rdc} = \frac{1}{4\varepsilon l}\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{H\varepsilon}{4}l^2 - \frac{u_d \varepsilon l}{2}\right\}^2 - \frac{HB}{a}\left(\frac{1-e^{-al}}{a} - e^{-al}l\right) \\ -B_1 - \frac{e}{l} \\ -K\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{H\varepsilon l^2}{4} - \frac{u_d \varepsilon l}{2}\right\} - \frac{g}{8}\left\{\frac{B}{a}\left(1-e^{-al}\right) - \frac{H\varepsilon l^2}{2} - u_d \varepsilon l\right\}\end{gathered} \tag{47}$$

$$\begin{gathered}\mu_{ddc} = \frac{1}{2\varepsilon l}\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{H\varepsilon}{4}l^2 - \frac{u_d \varepsilon l}{2}\right\}^2 - B_2 - \frac{e}{l} \\ -K\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{H\varepsilon l^2}{4} - \frac{u_d \varepsilon l}{2}\right\} \\ -\frac{g}{8}\left\{\frac{B}{a}\left(1-e^{-al}\right) - \frac{H\varepsilon l^2}{2} - u_d \varepsilon l\right\}\end{gathered} \tag{48}$$

4.3.2. For Revenue-Sharing System

Similarly, the optimal profit for retailers and distributors in the revenue-sharing system will be obtained by putting $\theta \to 0$ in Equation (33) to Equation (36).

Retailer's and distributor's optimal profit in revenue-sharing system without carbon emissions:

$$\mu_{rrs} = \frac{\delta}{\varepsilon l}\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{u_d \varepsilon l}{2}\right\}^2 - \frac{HB}{a}\left(\frac{1-e^{-al}}{a} - e^{-al}l\right) - B_1 \tag{49}$$

$$\mu_{rdc} = \frac{(1-\delta)}{\varepsilon l}\left\{\frac{B}{2a}\left(1-e^{-al}\right) - \frac{H\varepsilon}{4}l^2 - \frac{u_d \varepsilon l}{2}\right\}^2 - B_2 \tag{50}$$

Retailer's and distributor's optimal profit in revenue-sharing system without carbon emissions:

$$\begin{gathered}\mu_{rrs} = \frac{\delta}{\varepsilon l}\left\{\frac{B\,l}{2} - \frac{u_d \varepsilon l}{2}\right\}^2 - \frac{HB}{a}\left(\frac{1-e^{-al}}{a} - e^{-al}l\right) - K\left\{\frac{Bl}{2} - \frac{u_d \varepsilon l}{2}\right\} - \frac{g}{2}\delta\left\{\frac{B\,l}{2} - u_d\,\varepsilon\,l - \frac{g\,l\,\varepsilon}{4}\right\} - \frac{g\,B\,l}{4} \\ +\frac{g^2 \varepsilon l}{16} - B_1 - \frac{e}{l}\end{gathered} \tag{51}$$

$$\mu_{drs} = \frac{(1-\delta)}{\varepsilon l}\left\{\frac{B\,l}{2} - \frac{u_d \varepsilon l}{2}\right\}^2 - \frac{g}{2}(\delta-1)\left\{u_d\,\varepsilon\,l - \frac{B\,l}{2}\right\} - \frac{g^2(\delta-1)\varepsilon l}{8} - K\left\{\frac{Bl}{2} - \frac{u_d \varepsilon l}{2}\right\}B_2 - \frac{e}{l} \tag{52}$$

It is observed that for the above models, the only value of the revenue-sharing fraction that coordinates the system is 0.4 for the retailer and distributor, where the distributor is a Stackelberg leader, and the retailer is a follower.

5. Optimality Solution Criteria

To obtain the optimal profit for the decentralized scenario, i.e., $\mu_r(P)$ for retailers and $\mu_d(u_r)$ for distributors with respect to P and u_r, respectively, the steps given below will be followed:

Step 1: In this step, the derivatives of the first order $\mu_r(P)$ with respect to P and $\mu_d(u_r)$ with respect to u_r are found as follows:

$$\frac{d\mu_r(P)}{dP} = 0 \text{ and } \frac{d\mu_d(u_r)}{du_r} = 0$$

Step 2: For optimality, the equation obtained in Step 1 will be solved simultaneously.

Step 3: Again, differentiating the equations obtained in Step 1 and substituting the values of P and u_r from Step 2, we obtain the following:

$$\frac{d^2\mu_r(P)}{dP^2} < 0, \frac{d^2\mu_d(u_r)}{du_r{}^2} < 0$$
$$\frac{2\left(-a\varepsilon+ae^{\theta l}+\varepsilon\theta-e^{\theta l}\right)}{\theta(-a+\theta)} = -2\varepsilon l < 0 \; and \; \frac{\left(1-e^{\theta l}\right)\varepsilon}{2\theta} = -\varepsilon l < 0$$

This condition holds for a, ε, θ, l, δ, $u_r, u_d, P, \mu_r, \mu_d, g, k, B, H, B_1, B_2, e$.

6. Empirical Verification through Illustration

To further clarify the ideas above, numerical examples are given in this section. Consider the parameter values as follows:

$l = 7.00$ weeks, where l is the replenishment cycle

$B = 80.0$, where B is the initial demand rate

$a = 0.01$ unit, where a is the time-responsive parameter of demand

$\varepsilon = 0.2$, where ε is the price-sensitive parameter

$u_d = \$30/\text{unit}$, where u_d is the distributor's unit cost

$H = \$0.01/\text{unit time}$, where H is the holding cost per unit quantity per unit time

$\theta = 0.5$, where θ reflects the elasticity of the demand rate with respect to the inventory level

$B_1 = \$1000$, where B_1 is the distributor's fixed ordering cost

$B_2 = \$3000$, where B_2 is the retailer's fixed ordering cost

$k = 2$, where k is the quantity of carbon emissions related to each purchased or generated unit

$g = 1$, where g is the inventory level per unit of time related to the warehouse carbon emission unit

$e = 1$, where e is the carbon emission unit related to the order's beginning

It is assumed that both parties (retailer and distributor) will execute a revenue-sharing contract with $\delta = 0.4$, along with the cost of carbon emissions with any value of $\delta \epsilon \left[\frac{1}{4}, \frac{1}{2}\right]$. Table 2 displays the equivalent ideal solutions for the carbon emission effect. Table 2 also shows that both the retailer and the distributor make money in both the Stackelberg game and the centralized system, but after applying the revenue-sharing contract, the retailer's profit rises while the distributor's profit falls. the following values of retailer and distributor are obtained in different systems: In case 1, when demand is stock–price–time sensitive, the optimal values in the centralized system are $q = 2242.89$ and the combined value of the retailer and distributor's profit, i.e., $\mu_r + \mu_d = 383{,}107$, in the Stackelberg games are $q = 998.301$, $\mu_r = 91{,}847.7$ and $\mu_d = 188{,}307$, whereas in the revenue-sharing system, they are $q = 2243.04$, $\mu_r = 150{,}028$, $\mu_d = 51{,}639.1$. In case 2, when demand is stock–price sensitive, then the Stackelberg game optimal values are $q = 1186.61$, $\mu_r = 103{,}262$, $\mu_d = 211{,}462$ and those of the revenue-sharing system are $q = 2373.37$, $\mu_r = 168{,}435$, $\mu_d = 59{,}551.5$. As for case 3, when demand is time–price sensitive, then the values in the Stackelberg game are $q = 129.325$, $\mu_r = 8665.37$, $\mu_d = 18{,}657.5$ and in revenue-sharing system are $q = 270.425$, $\mu_r = 16{,}083.2$, $\mu_d = 1253.92$.

Table 2. The optimal order quantity, unit end selling price, and profit of the retailer and distributor under the effect of carbon emission costs.

When Demand Is Stock–Time–Price Sensitive						
Distributor	q	P	u_r	μ_r	μ_d	$\mu_r + \mu_d$
Stackelberg game in decentralized system	999.301	292.412	205.096	91,847.7	188,307	28,054.7
centralized system	2242.89	205.114				383,107
Revenue-sharing system	2243.04	205.114	12.1893	150,028	51,639.1	201,667.1
When demand is stock–price sensitive:						
Distributor	q	P	u_r	μ_r	μ_d	$\mu_r + \mu_d$
Stackelberg game in decentralized system	1186.61	307.629	215.241	103,262	211,462	314,724
Revenue-sharing system	2373.37	215.259	12.1893	168,435	59,551.5	227,986.5
When demand is price–time sensitive:						
Distributor	q	P	u_r	μ_r	μ_d	$\mu_r + \mu_d$
Stackelberg game in decentralized system	129.325	307.625	215.25	8665.37	18,657.5	27,322.87
Revenue-sharing system	270.425	215.25	12.2	16,083.2	1253.92	17,337.12

7. Results and Discussion

By considering the above parametric values, the following results and discussion are obtained:

1. When demand is stock–price–time sensitive, then the optimal values obtained in the Stackelberg game are as follows: 998.301 for q (ordered quantity of stock by the retailer), 91,847.7 for μ_r (retailer's profit), 188,307 for μ_d (distributor's profit), 205.096 for u_r, 280,154.7 for $\mu_r + \mu_d$ (total profit of the supply chain), and 292.412 for P (unit end selling price). The values obtained in the centralized system are as follows: 2242.89 for q, 383,107 for $\mu_r + \mu_d$ (total profit of the supply chain), and 205.114 for P. Finally, the values in the revenue-sharing system are as follows: 2243.04 for q, 150,028 for μ_r, 51,639.1 for μ_d, 12.1893 for u_r, 205.114 for P, and 201,667.1 for $\mu_r + \mu_d$.
2. When demand is stock–price sensitive, then the optimal values obtained in the Stackelberg game for q (ordered quantity of stock by the retailer) is 1186.61, for μ_r (retailer's profit) is 103,262, for μ_d (distributor's profit) is 211,462, for u_r is 215.241, for P (unit end selling price) is 307.629, for $\mu_r + \mu_d$ (total profit of the supply chain) is 314,724, whereas the values in the revenue-sharing system for q is 2373.37, for μ_r is 168,435, for μ_d is 59,551.5, for u_r is 12.1893, for P is 215.25, and for $\mu_r + \mu_d$ is 227,986.5.
3. When demand is price-time sensitive, then the optimal values obtained in the Stackelberg game for q (ordered quantity of stock by the retailer) is 129.325, for μ_r (retailer's profit) is 8665.37, for μ_d (distributor's profit) is 18,657.5, for u_r is 215.25, for P (unit end selling price) is 307.625, and for $\mu_r + \mu_d$ (total profit of the supply chain) is 27,322.87, whereas the values in the revenue-sharing system for q is 270.425, for μ_r is 16,083.2, for μ_d is 1253.92, for u_r is 12.2, for P is 215.25, and for $\mu_r + \mu_d$ is 17,337.12.

From the above results, it is clear that the profit of the total supply chain is highest in the centralized system when demand is stock–time–price sensitive as compared to the profits determined in the other systems when demand is stock–price sensitive and price–time sensitive.

8. Sensitivity Analysis

This section describes the sensitivity of the models in different systems and discusses how the cost of carbon emissions impacts the profit of both entities in different systems (centralized, decentralized, and revenue-sharing contracts) by altering the percentage of certain pertinent parameters.

From Table 3, the following observations are observed:

- In a centralized system, when the value of the parameter "B" changes, the value of $\mu_r + \mu_d$ and its corresponding value 'P' likewise change, and vice versa. The values of μ_r, μ_d, P, and u_r directly depend on the change in parameter B in the Stackelberg game and the revenue-sharing system, respectively. In other words, the values of all these variables decrease as the value of B falls, and vice versa.
- When the value of the parameter 'a' declines in a centralized system, the value of $\mu_r + \mu_d$ and the related value of 'P' also somewhat increase, and vice versa. This indicates that $\mu_r + \mu_d$ and P show an indirect variation to the change in parameter 'a'. The values of μ_r, μ_d, P, and u_r in the Stackelberg game and revenue-sharing system increase somewhat when the value of parameter 'a' declines and decrease as the value of parameter 'a' increases, suggesting that all these variables show indirect variation to the change in parameter 'a'.
- When the value of a parameter 'θ' is changed in a centralized system, the value $\mu_r + \mu_d$ shows a direct variation, whereas the value 'P' shows an inverse variation. As a result, when the value of 'θ' decreases, the value of $\mu_r + \mu_d$ shows a significant decrease, although 'P' increases slightly, and vice versa. A change in the value of the parameter 'θ' in the Stackelberg game and revenue-sharing system significantly affects the values of P, u_r and μ_r, μ_d. The values of P and u_r exhibit very little change, i.e., they both increase while $_r$, $_d$, exhibit substantial decrement when the value of the parameter 'θ' falls, suggesting that both P and u_r increase as 'θ' decreases and vice versa.
- The values of P and $\mu_r + \mu_d$ exhibit an inverse variation to the parameter 'ε' in the centralized system because their values increase when the value of parameter 'ε' decreases. The values of μ_r, μ_d, P, and u_r exhibit a large increase when the value of 'ε' drops, and vice versa, in both the Stackelberg game and the revenue-sharing system. This illustrates how each variable changes when the value of parameter 'ε' changes.
- In a centralized system, when the value of the parameter 'u_d' reduces, the value of $\mu_r + \mu_d$ raises slightly, whereas the value of 'P' somewhat lowers, and vice versa. This shows that 'P' directly varies with parameter 'u_d', whereas $\mu_r + \mu_d$ shows an indirect variation with 'u_d'. In the Stackelberg game and revenue-sharing system, the values of μ_r and u_r fall slightly when the value of parameter 'u_d' decreases, and they both increase when the value of 'u_d' grows, but the value of 'P' remains constant whether the value of 'u_d' increases or decreases. Furthermore, the value of 'μ_d' shows an indirect variation with change in parameter, 'u_d', i.e., on decreasing the value of 'u_d', its value increases and vice versa.
- In a centralized system, the value of $\mu_r + \mu_d$ changes very little in response to changes in parameter 'H', i.e., marginally increases when 'H' falls and slightly decreases when it rises, i.e., $\mu_r + \mu_d$ indirectly varies with parameter 'H', whereas 'P' directly affects 'H' changes. The values of μ_r, μ_d and u_r in the Stackelberg game and revenue-sharing system somewhat increase when parameter 'H' drops and vice versa, and the value of 'P' reduces as parameter 'H' falls, i.e., μ_r, μ_d and u_r impart an inverse variation with parameter 'H', whereas 'P' imparts a direct variation with 'H'.

Table 3. Effect of changes in parameters when demand is stock–price–time sensitive under carbon emissions.

Parameters	%Change	Integrated System		Stackelberg Game				Revenue-Sharing System			
		μ_r+μ_d in %	P in %	μ_r in %	P in %	μ_d in %	u_r in %	μ_r in %	P in %	μ_d in %	u_r in %
B	−30%	−55.43	−27.76	−91.72	−19.46	−55.99	−27.77	−55.78	−27.68	−67.0076	−24.98
	−20%	−39.37	−18.51	−71.23	−12.97	−39.79	−18.51	−39.66	−18.45	−48.25	−16.65
	−10%	−20.89	−9.2563	−40.65	−6.48	−21.12	−9.25	−21.06	−9.22	−25.92	−8.32
	10%	23.31	9.25	50.73	6.48	23.58	9.25	23.53	9.22	29.5	8.32
	20%	49.04	18.51	111.56	12.97	49.62	18.51	49.53	18.45	62.6	16.65
	30%	77.18	27.76	182.46	19.46	78.12	27.77	78.01	27.68	99.28	24.98
a	−30%	3.5	1.45	7.3	1.01	3.54	1.46	3.53	1.44	4.39	1.3
	−20%	2.32	0.96	4.82	0.67	2.34	0.97	2.34	0.96	2.91	0.87
	−10%	1.15	0.48	2.39	0.33	1.16	0.48	1.16	0.48	1.44	0.43
	10%	−1.14	−0.47	−2.35	−0.33	−1.15	−0.47	−1.15	−0.47	−1.43	−0.43
	20%	−2.26	−0.95	−4.66	−0.67	−2.29	−0.94	−2.28	−0.95	−2.84	−0.86
	30%	−3.38	−1.42	−6.93	−1.0021	−3.42	−1.42	−3.41	−1.42	−4.23	−1.28
θ	−30%	−53.01	0.38	−52.57	0.27	−53.3	0.38	−52.81	0.38	−55.43	0.34
	−20%	−39.94	0.24	−39.58	0.17	−40.16	0.24	−39.79	0.24	−41.76	0.22
	−10%	−22.74	0.11	−22.51	0.08	−22.86	0.11	−22.65	0.11	−23.77	0.1
	10%	30.17	−0.1	29.82	−0.07	30.33	−0.1	30.05	−0.1	31.52	−0.09
	20%	70.33	−0.2	69.47	−0.14	70.71	−0.2	70.06	−0.2	73.48	−0.18
	30%	123.98	−0.29	122.35	−0.2	124.64	−0.2959	123.5	−0.29	129.51	−0.26
ε	−30%	51.53	39.66	171.58	26.43	52.39	39.67	52.57	39.54	74.46	35.69
	−20%	30.03	23.13	96.38	16.21	30.53	23.14	30.63	23.06	43.23	20.82
	−10%	13.34	10.28	41.19	7.2	13.56	10.28	13.6	10.25	19.12	9.25
	10%	−10.89	−8.41	−31.006	−5.89	−11.07	−8.41	−11.11	−8.38	−15.5	−7.57
	20%	−19.96	−15.42	−54.37	−10.81	−20.29	−15.42	−20.35	−15.38	−28.29	−13.88
	30%	−27.62	−21.36	−71.86	−14.97	−92.8	−21.36	−28.15	−21.29	−38.98	−19.21
u_d	−30%	5.3	−2.19	−0.12	NC	5.36	−2.19	−0.07	NC	17.38	−4.93
	−20%	3.52	−1.46	−0.08	NC	3.56	−1.46	−0.05	NC	11.45	−3.29
	−10%	1.75	−0.73	−0.04	NC	1.77	−0.73	−0.02	NC	5.65	−1.645
	10%	−1.73	0.73	0.04	NC	−1.75	0.73	0.02	NC	−5.51	1.64
	20%	−3.46	1.46	0.08	NC	−3.5	1.46	0.05	NC	−10.89	3.29
	30%	−5.17	2.19	0.12	NC	−5.22	2.19	0.07	NC	−16.12	4.93
H	−30%	0.003	−0.0014	0.0064	−0.001	0.003	0.0014	0.0079	−0.0034	0.0098	0.0029
	−20%	0.002	−0.0009	0.0042	−0.0006	0.002	0.0009	0.0053	−0.0024	0.0065	0.0019
	−10%	0.001	−0.0004	0.0021	−0.0003	0.001	0.0004	0.0026	−0.0009	0.0032	0.0009
	10%	−0.0007	0.0004	−0.002	0.0003	−0.001	−0.0004	−0.0026	0.0009	−0.0032	−0.0009
	20%	−0.0018	0.0009	−0.0042	0.0006	−0.0015	−0.0009	−0.0053	0.0019	−0.0065	−0.0019
	30%	−0.0028	0.0009	−0.0063	0.0006	−0.0026	−0.0014	−0.008	0.0029	−0.0098	−0.0028

NC = No Change.

From Table 4, the following observations are found:

- The values of μ_r, μ_d in the Stackelberg game and revenue-sharing system are greatly influenced by the change in parameter 'B'; therefore, the values of μ_r and μ_d exhibit a considerable decrease when decreasing the value of 'B' and vice versa. However, the values of P and u_r, likewise, fall with lowering parameter 'B' values and increase with increasing parameter 'B' values, revealing that P and u_r also exhibit direct fluctuations with parameter 'B' in both the Stackelberg game and revenue-sharing system, but they are not greatly influenced by the parameter 'B'.
- In both the Stackelberg game and the revenue-sharing system, the values of μ_r, μ_d, and u_r show an inverse variation with the change in parameter 'H', i.e., they rise upon reducing the value of 'H' and vice versa, while the value of 'P' decreases on decreasing

'H' and increases on increasing it. However, the parameter 'H' causes a very slight fluctuation in each of the variables μ_r, μ_d, P, and u_r.

- The values of μ_r, μ_d and u_r in the Stackelberg game and revenue-sharing system exhibit substantial decrement when the value of 'θ' drops and a high increment when increasing the value of 'θ'. In contrast, the values of 'P' are only slightly impacted by the change in parameter 'θ'. The variables μ_r, μ_d and u_r exhibit a direct variation with the parameter 'θ', however, the value of 'P' increases upon reducing the value of 'θ' and vice versa. This implies that variable 'P' shows an inverse variation with parameter 'θ'.
- The values of variables μ_r, μ_d, in both the Stackelberg game and revenue-sharing system are very highly influenced by the change in parameter 'ε'. On the other hand, u_r and P are moderately influenced by the change in parameter 'ε', but the values of μ_r, μ_d, u_r and P all are increasing with the decreasing value of 'ε' and vice versa, which indicates that all these variables show inverse variation with parameter 'ε'.
- In the Stackelberg game and revenue-sharing system, when decreasing the value of parameter 'u_d', the value of μ_r, u_r decreases slightly, and vice versa. The value of μ_d increases on decreasing the value of 'u_d' and vice versa, whereas the value of 'P', will remain constant while either the value of 'u_d' increases or decreases, and all this indicates that μ_r, u_r shows a direct variation with the parameter 'u_d' and μ_d shows an inverse variation with parameter 'u_d'.

Table 4. Effect of change in parameters when demand is stock–price sensitive under carbon emissions.

Parameters	% Change	Stackelberg Game				Revenue-Sharing System			
		μ_r in %	P in %	μ_d in %	u_r in %	μ_r in %	P in %	μ_d in %	u_r in %
B	−30%	−91.24	−19.48	−55.67	−27.87	−55.5	−27.79	−65.67	−25.19
	−20%	−70.78	−12.99	−39.54	−18.58	−39.44	−18.52	−47.23	−16.79
	−10%	−40.36	−6.49	−20.98	−9.29	−20.94	−9.26	−25.34	8.39
	10%	50.31	6.49	23.41	9.2919	23.38	9.26	28.79	8.39
	20%	110.59	12.99	49.26	18.58	49.2	18.52	61.03	16.79
	30%	180.81	19.48	77.54	27.87	77.46	27.79	96.73	119.24
H	−30%	0.0067	−0.0006	0.0033	0.0013	0.0071	−0.0032	0.0092	0.0028
	−20%	0.0038	−0.0003	0.0023	0.0009	0.0047	−0.0023	0.0062	0.0018
	−10%	0.0019	307.879	0.0009	0.0004	0.0023	−0.0009	0.003	0.0009
	0	103,262	307.879	211,462	215.241	168,435	215.884	59,551.5	95.2411
	10%	−0.0019	0.0003	−0.0009	−0.0004	−0.0023	0.0009	−0.003	−0.0009
	20%	−0.0038	0.0006	−0.0018	−0.0009	−0.0053	0.0018	−0.0062	−0.0018
	30%	−0.0058	0.0009	−0.0028	−0.0013	0.0077	0.0032	−0.0092	−0.0028
θ	−30%	−53.41	0.0006	−53.65	−0.0009	−53.21	0.0023	−55.56	−0.0022
	−20%	−40.27	0.0006	−40.45	−0.0004	−40.12	0.0013	−41.89	−0.0013
	−10%	−22.94	0.0003	−23.04	−0.0004	−22.85	0.0004	−23.86	−0.0006
	0	103,262	307.879	211,462	215.241	168,435	215.884	59,551.5	95.2411
	10%	30.48	NC	30.62	0.0004	30.37	−0.0009	31.71	0.0006
	20%	71.13	−0.0003	71.44	0.0004	70.86	−0.0013	71.54	0.0011
	30%	125.48	−0.0003	126.03	0.0009	125.01	−0.0018	130.54	0.0016

Table 4. *Cont.*

Parameters	% Change	Stackelberg Game				Revenue-Sharing System			
ε	−30%	169.71	27.84	51.8	39.82	51.99	39.7	71.88	35.99
	−20%	95.37	16.24	30.19	23.22	30.3	23.16	41.75	20.99
	−10%	40.77	7.21	13.41	10.32	13.45	10.29	18.42	9.33
	0.2	103,262	307.879	211,462	215.241	168,435	215.884	59,551.5	95.2411
	10%	−30.72	−5.9	−10.95	−8.44	−10.99	−8.42	−14.99	−7.63
	20%	−53.9	−10.82	−20.07	−15.48	−20.13	−15.44	−27.37	−13.99
	30%	−71.28	−14.99	−27.77	−21.44	−27.86	−21.37	−37.74	−19.38
u_d	−30%	−0.11	NC	5.05	−2.09	−0.06	NC	16.06	−4.72
	−20%	−0.07	NC	3.49	−1.39	−0.04	NC	10.58	−3.14
	−10%	−0.03	NC	1.67	−0.69	−0.02	NC	5.23	−1.57
	0	103,262	307.879	211,462	215.241	168,435	215.884	59,551.5	95.2411
	10%	0.03	NC	−1.65	0.69	0.02	NC	−5.11	1.57
	20%	0.07	NC	−3.3	1.39	0.04	NC	−10.1	3.14
	30%	0.11	NC	−4.93	2.09	0.06	NC	−14.97	4.72

NC = No Change.

From Table 5, the following observations are found:

- The values of μ_r, μ_d, P, and u_r exhibit a direct variation with change in parameter 'B' in both the Stackelberg game and revenue-sharing system, i.e., μ_r, μ_d have high decrement when the value of parameter 'B' decreases and vice versa, whereas P and u_r have less significant drops when the value of parameter 'B' decreases.
- In the Stackelberg game as well as in the revenue-sharing system, the variables μ_r, μ_d, P, and u_r exhibit an inverse relation with changes in parameter 'a', which implies that when the value of parameter 'a' decreases, the values of μ_r, μ_d, P, and u_r increases to some degree, and vice versa.
- The values of all variables μ_r, μ_d, P, and u_r in the Stackelberg game and revenue-sharing system have inverse variations with the change in parameter 'ε'. But in the revenue-sharing system, the value of $_r$ displays a high increment when the value of 'ε' decreases, and the other variables show a modest increment when 'ε' decreases, and vice versa. In the Stackelberg game, the value of μ_r shows a very high increment as compared to other variables when the parameter 'ε' drops significantly.
- Variables μ_r and u_r directly variate with parameter 'u_d', while the variable μ_d variate inversely with parameter 'u_d' in both the Stackelberg game and revenue-sharing system. When the value of parameter 'u_d' decreases, then the values of variables μ_r and u_r also decrease whereas when the value of μ_d increases, the value of 'P' will remain constant, whether the value of 'u_d' increases or decreases in both Stackelberg and revenue-sharing system.

Observation 1: From Table 6, it is clearly seen that the change in the revenue-sharing fraction 'δ' greatly affects μ_r, μ_d, u_r, while P is less affected by the change in the revenue-sharing fraction 'δ'. When the value of the revenue-sharing fraction 'δ' is decreased, the value of μ_r, μ_d, u_r significantly decreases and vice versa, whereas the value of P, increases slightly on decreasing the value of the revenue-sharing fraction 'δ' and vice versa.

Table 5. Effect of change in parameters when demand is price–time sensitive.

Parameters	% Change	Stackelberg Game				Revenue-Sharing System			
		μ_r in %	P in %	μ_d in %	u_r in %	μ_r in %	P in %	μ_d in%	u_r in%
B	−30%	−103.5	−19.24	−63.89	−27.8	−58.8	−27.72	−121.58	−25.05
	−20%	−81.05	−12.83	−45.39	−18.53	−41.8	−18.48	−87.52	−16.7
	−10%	−46.55	−6.41	−24.09	−9.260	−22.2	−9.24	−46.99	−8.35
	10%	58.61	6.41	26.89	9.26	24.79	9.24	53.46	8.35
	20%	129.28	12.83	56.59	18.53	52.19	18.48	113.39	16.7
	30%	212	19.24	89.08	27.8	82.19	27.72	179.78	25.05
a	−30%	5.56	0.58	2.67	0.96	2.82	0.96	5.28	0.87
	−20%	3.68	0.35	1.77	0.64	1.87	0.64	3.5	0.58
	−10%	1.83	0.13	0.88	0.321	0.93	0.32	1.74	0.28
	10%	−1.8	−0.31	−0.87	−0.32	−0.92	−0.31	−1.72	−0.28
	20%	−3.59	−0.53	−1.74	−0.63	−1.84	−0.63	−3.44	−0.57
	30%	−5.35	−0.75	−2.61	−0.95	−2.74	−0.95	−5.14	−0.86
ε	−30%	201.79	27.49	59.66	39.72	55.33	39.6	134.4	35.79
	−20%	113.00	16.04	34.77	23.17	32.25	23.1	78.05	20.87
	−10%	48.12	7.02	15.44	10.29	14.32	10.26	34.53	9.27
	10%	−35.95	−5.83	−12.61	−8.42	−11.69	−8.4	−28.00	−7.59
	20%	−62.76	−10.68	−23.11	−15.44	−21.42	−15.4	−51.1	−13.91
	30%	−82.56	−14.8	−31.97	−21.39	−29.63	−21.32	−70.44	−19.27
u_d	−30%	−0.14	NC	6.01	−2.15	−0.07	NC	30.93	−4.86
	−20%	−0.09	NC	3.99	−1.43	−0.05	NC	20.38	−3.24
	−10%	−0.04	NC	1.98	−0.71	−0.02	NC	10.06	−1.62
	10%	0.04	NC	−1.97	0.71	0.02	NC	−9.82	1.62
	20%	0.09	NC	−3.92	1.43	0.05	NC	−19.4	3.24
	30%	0.14	NC	−5.86	2.15	0.07	NC	−28.74	4.86

NC = No Change.

Table 6. Effect of the changes in revenue-sharing fraction 'δ' on selling price, retailer's profit, unit cost of retailer and distributor's profit when demand is stock–price–time dependent.

	For Retailer		For Distributor	
δ (Revenue-Sharing Fraction)	μ_r (Retailer's Profit) in %	P (Unit End Selling Price) in %	μ_d (Distributor's Profit) in %	u_r (Unit Cost of Retailer) in %
−30%	−36.66	3.3	−50.69	−24.98
−20%	−24.47	1.93	−34.05	−16.65
−10%	−12.24	0.85	−17.12	−8.32
10%	12.26	−0.7	17.27	8.32
20%	24.54	−1.28	34.66	16.65
30%	36.83	−1.78	52.13	24.98

Observation 2: From Figures 1 and 2, it is clearly seen that when the percentage of the revenue-sharing fraction 'δ' is decreased, the percentage of μ_r, μ_d, u_r also decreases, and vice versa, whereas the percentage of 'P' increases when the percentage of the revenue-sharing fraction 'δ' decreases and vice versa.

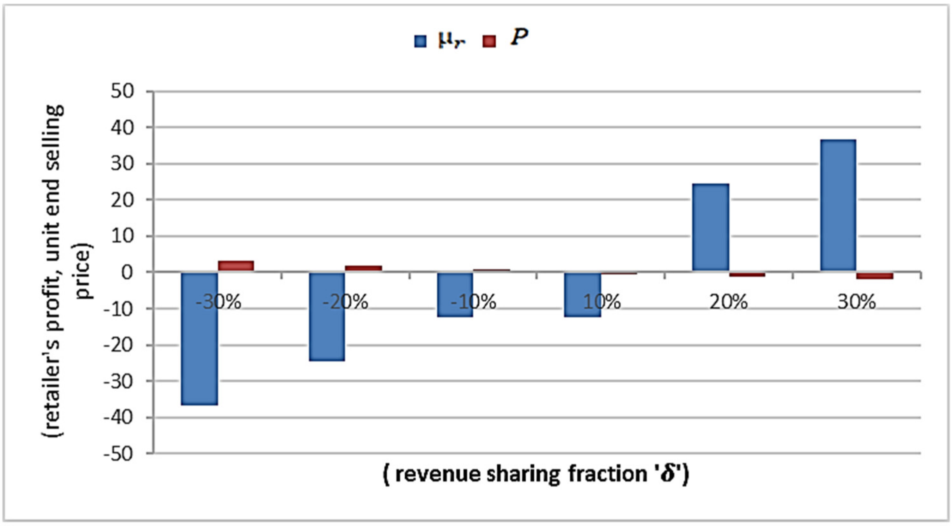

Figure 1. The behavior of the retailer's profit and unit end selling price when the percentage of revenue fraction 'δ' fluctuates.

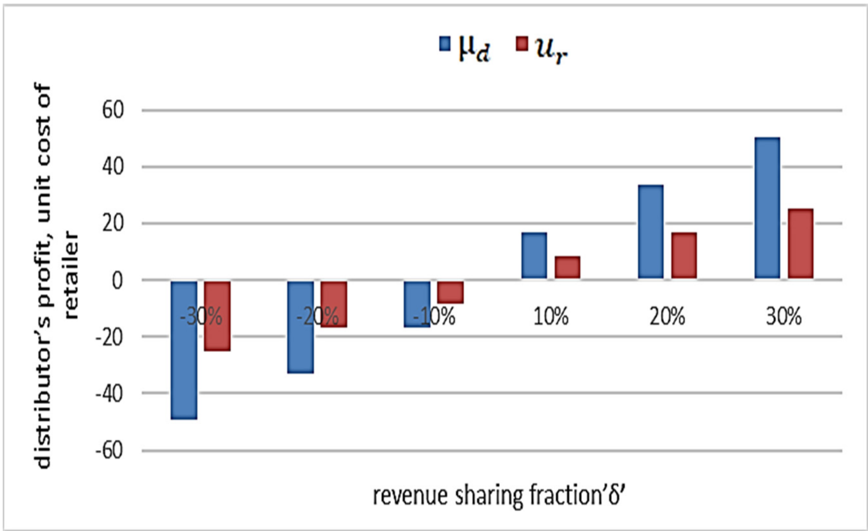

Figure 2. The behavior of the distributor's profit and unit cost of retailer when the percentage of revenue-sharing fraction 'δ' fluctuates.

Observation 3: Table 7 clearly indicates that on changing the value of the revenue-sharing fraction, 'δ' the values of μ_r, μ_d, show high sensitivity, u_r show moderate sensitivity, while 'P' shows very little sensitivity, i.e., on decreasing the value of the revenue-sharing fraction, 'δ' the values of μ_r, μ_d, and u_r decreases and vice versa, but the value of 'P' increases on decreasing the revenue-sharing fraction 'δ' and vice versa.

Observation 4: Figures 3 and 4 clearly indicate that on increasing the percentage of the revenue-sharing fraction 'δ'', the percentage of μ_r, μ_d, and u_r also increases while the percentage of P decreases, and vice-versa.

Table 7. The effect of change in revenue-sharing fraction 'δ' on retailer's profit, selling price, distributor's profit, and unit cost of retailer when demand is stock–price sensitive.

	For Retailer		For Distributor	
δ (Revenue Sharing Fraction)	**μ_r (Retailer's Profit) in %**	**P (Unit End Selling Price) in %**	**μ_d (Distributor's Profit) in %**	**u_r (Unit Cost of Retailer) in %**
−30%	−36.27	3.15	−49.08	−25.19
−20%	−24.21	1.83	−32.94	−16.79
−10%	−12.11	0.81	−16.55	−8.39
10%	12.13	−0.66	16.68	8.39
20%	24.27	−1.22	33.46	16.79
30%	36.43	−1.69	50.32	25.19

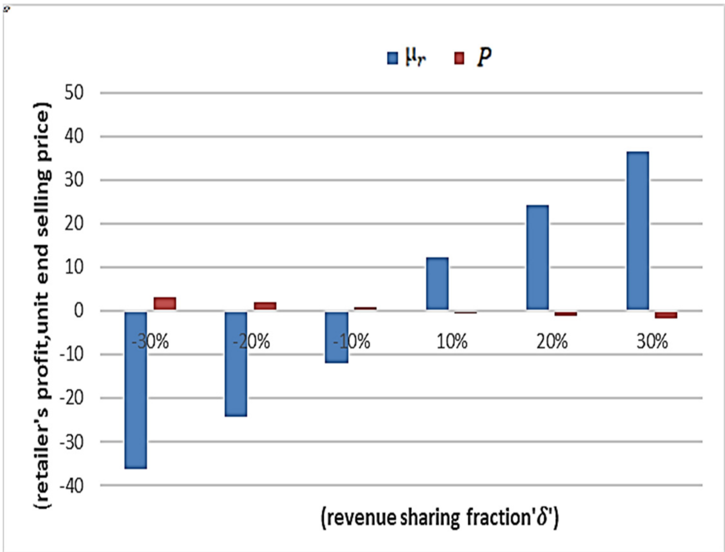

Figure 3. The behaviour of the retailer's profit and unit end selling price when value of 'δ' changes.

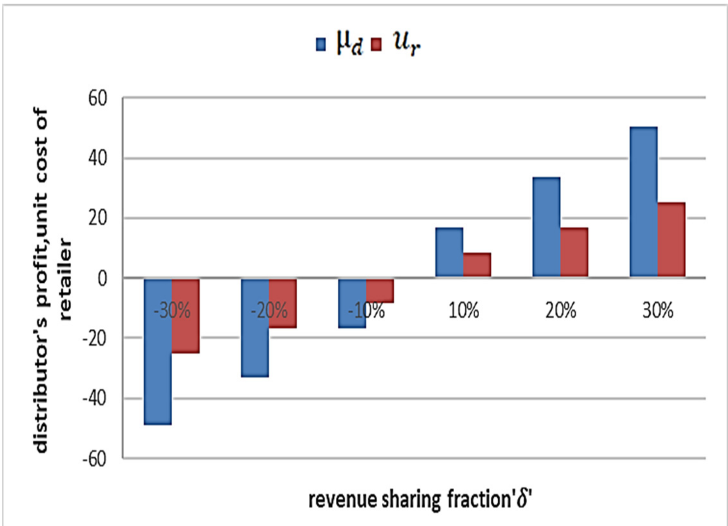

Figure 4. The behavior of unit cost of the retailer's profit, selling price, and distributor's profit when the percentage of revenue-sharing fraction 'δ' fluctuates.

Observation 5: From Table 8, it is clearly seen that μ_r, μ_d, shows high sensitivity, u_r shows moderate sensitivity, and 'δ' shows the least sensitivity to the change in revenue-sharing fraction 'δ', i.e., when the value of the revenue-sharing fraction is decreased, the value of μ_r, μ_d and u_r decreases but the value of 'δ' increases slightly, and vice versa in Figures 5 and 6.

Table 8. Effect of change of revenue-sharing fraction 'δ' on retailer and distributor's profit when demand is price–time sensitive.

	For Retailer		For Distributor	
δ (Revenue Sharing Fraction)	**μ_r (Retailer's Profit) in %**	***P* (Unit End Selling Price) in %**	**μ_d (Distributor's Profit) in %**	**u_r (Unit Cost of Retailer) in %**
−30%	−38.59	3.25	−91.6	−25.05
−20%	−25.76	1.89	−61.51	−16.7
−10%	−12.89	0.84	−30.93	−8.35
10%	12.91	−0.69	32.18	8.25
20%	26.11	−1.26	62.56	16.7
30%	38.77	−1.75	94.09	25.05

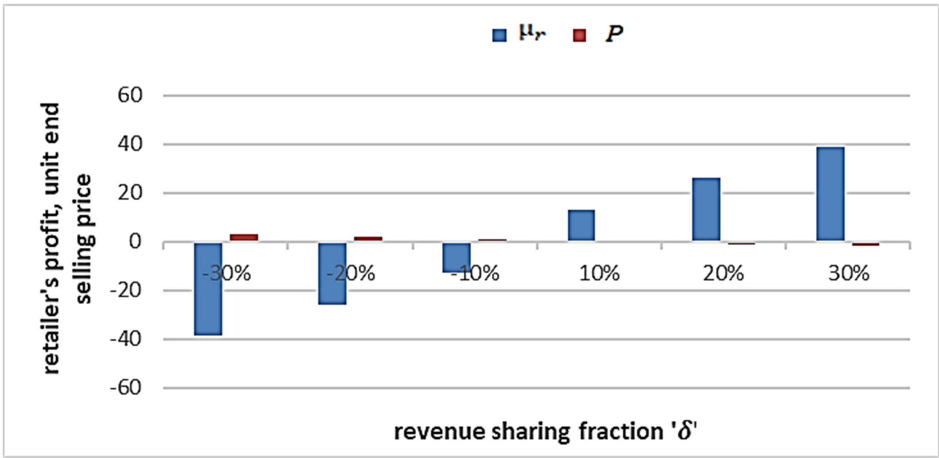

Figure 5. The behavior of the retailer's profit and unit end selling price when the percentage of revenue-sharing fraction 'δ' changes.

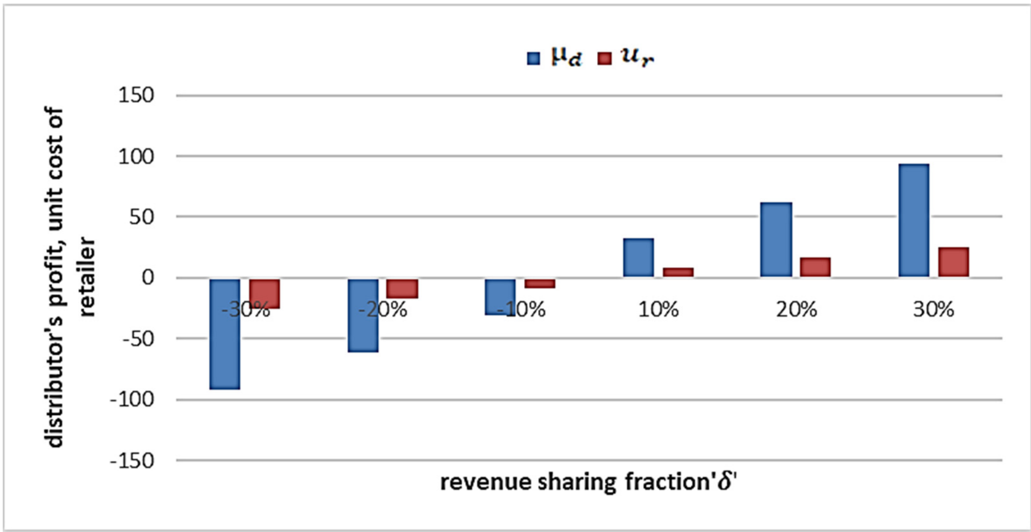

Figure 6. The behavior of the distributor's profit and unit cost of retailer when the percentage of revenue-sharing fraction 'δ' changes.

8.1. Conclusions

This article studied a two-tier supply chain with retailers and distributors as its members. Profit is determined in two scenarios, namely centralized and decentralized. Decentralized optimization uses the Stackelberg sequence, whereas centralized optimization involves the traditional optimization technique. Calculations showed that centralized systems create more profit than decentralized systems. To maximize the supply chain's profit, a revenue-sharing system is applied as a way to coordinate the process. Carbon emissions are regulated in both the centralized and decentralized systems to encourage awareness. The figures and the analytical analyses show that when the revenue-sharing fraction 'δ' increases, the percentage of 'P' decreases, while the percentage of ${}_r$, ${}_d$, u_r increases and vice versa. The main conclusion is that 0.4 is the only value of the revenue-sharing function 'δ' that coordinates the system, which implies that both parties will gain profit, i.e., retailer and distributor, and tends to raise the profit of the entire supply chain.

8.2. Future Scope

Future developments in this area could include several potential extensions of the current concept as follows:

- Online and offline price discount contracts must be suggested to coordinate the supply chain and set up the conditions for the contracts to enhance the decentralized system's performance.
- The current model can also prolong stockout cost through partial backlogging. Thus, another point to consider is the way that implementing a multi-recovery policy could potentially improve supply chain efficiency and reduce costs over time.
- Another extension of the present model could be the development of the present model under an uncertain environment.

8.3. Limitation

Since the demand is considered as a function of stock, price, and time, and it is assumed that the retailer replenishes the stock at the beginning of the season, the main limitation is that the holding cost of the retailer will also increase.

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