

Editorial for Special Issue “Various Approaches for Generalized Integral Transforms”

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The Laplace transform can be interpreted as a method of converting a function from the time domain to the complex domain. This is primarily effective in investigating and analyzing initial value problems and the dynamic characteristics of linear systems. When it is difficult to find a solution to a given differential equation in a certain space, we think of transforming it to another space and then finding the solution. Currently, integral transform uses integration. If you transform \mathcal{L}_1 to equation A and then inversely transform it, you obtain A . Similarly, transforming \mathcal{L}_2 to this equation and then inversely transforming it also yields A . Therefore, the choice of transform makes no significant difference; only the transformed space varies. To ensure convergence in these transforms, it is necessary to bind the kernel. Therefore, it is common to set the kernel to e^{kt} ($k < 0$) on a positive interval. Achieving a bound kernel using differential transform or logarithm function is challenging. For instance, expressing the generalized Laplace transform as a logarithm function results in

$$s^{-\alpha} \int_1^{\infty} f(\ln x) x^{-s-1} dx$$

Binding this function is not easy. On the other hand, as observed with Fourier transform or Laplace transform, integral transform calculations are not inherently straightforward. Considering computational simplicity, differential transforms with appropriately limited domains are also worth studying. We must study, in greater depth, the theory of the study of $\mathcal{L}(fg)$, which is closely related to integral theory. The calculation of $\mathcal{L}(f) \mathcal{L}(g)$ can be conducted through convolution, where it is commonly known that $\mathcal{L}(fg) \neq \mathcal{L}(f) \mathcal{L}(g)$. Although researching this is challenging due to the nature of integration, finding a space where $\int fg = \int f \int g$ can lead to significant advancements. In traditional theory, further research should be conducted on integral transforms in ODEs with variable coefficients and PDEs.

Now, let us look at the aspects related to AI. The concept of convolution in integral transform is connected to the convolution concept in convolutional neural networks (CNN) used in image processing or analysis. The connected tool is the trace of AB^T , where A and B are input images and B^T is the transpose of B . This concept of convolution can be interpreted as a weight for a given input. AI updates these weights to find an optimal solution that approximates the desired solution. Since CNN's convolution operation requires a massive amount of computation, heat generation becomes a significant issue. Similarly, AI semiconductor devices perform enormous computations, leading to high electricity consumption and heat generation problems. Consequently, an immense amount of coolant is used for cooling purposes. For instance, the estimated water usage as a coolant for Google and Microsoft in 2022 amounted to 2.9 billion liters [1].

This presents a compelling research idea: developing a new algorithm to accelerate processes can help mitigate heating issues, ultimately reducing coolant usage. The contributions are listed in List of Contributions.

Contribution 1 introduces the concept of (p, q) -calculus to establish the (p, q) -analog of Laplace-type integral transforms. They delve into its unique characteristics and apply it to solve some (p, q) -differential equations. Contribution 2 employs weighting functions



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and real analysis techniques to establish equivalence conditions for Hardy-type integral inequalities with inhomogeneous kernels. Contribution 3 presents generalizations of the three classical summation formulas ${}_2F_1$.

Studies related to the Sumudu transform are Contribution 4 and Contribution 5. Contribution 4 introduces the Sumudu-generalized Laplace transform decomposition method for solving linear and nonlinear non-homogeneous dispersive Korteweg–de Vries-type equations. Contribution 5 establishes a technique using the double Sumudu transform in combination with a new generalized Laplace transform decomposition method. This technique, called the double Sumudu-generalized Laplace transform decomposition method, is applied to solve general two-dimensional singular pseudo-hyperbolic equations subject to the initial conditions. Contributions 6 and 7 utilize the Laplace transform. Contribution 6 investigates stability in a Volterra integrodifferential equation using the Laplace transform. Contribution 7 develops a numerical method based on the Laplace transform for modeling fractional-order differential equations. Contribution 8 uses G-transform to provide precise solutions for both homogeneous and non-homogeneous coupled Burgers' equations. Contributions 9 and 10 focus on the application of integral transform. Contribution 9 uses the Laplace transform method to analyze soft soil foundation deformations and proposes an improved quantum genetic algorithm. Contribution 10 explores the relationship between the unified Mittag–Leffler function and known special functions, deriving integral transforms of the unified Mittag–Leffler function in terms of Wright generalized functions.

A transformative new approach that simplifies complex calculations would be a welcome advancement. Many thanks to all the authors for their contributions to this Special Issue and to the editorial team for their hard work.

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List of Contributions:

1. Jirakulchaiwong, S.; Nonlaopon, K.; Tariboon, J.; Ntouyas, S.; Kim, H. On (p,q) -Analogues of Laplace-Typed Integral Transforms and Applications. *Symmetry* 2021, 13(4), 631; <https://doi.org/10.3390/sym13040631>. <https://www.mdpi.com/2073-8994/13/4/631>
2. Rassias, M.; Yang, B.; Raigorodskii, A. Equivalent Properties of Two Kinds of Hardy-Type Integral Inequalities. *Symmetry* 2021, 13(6), 1006; <https://doi.org/10.3390/sym13061006>. <https://www.mdpi.com/2073-8994/13/6/1006>
3. Choi, J. Certain Applications of Generalized Kummer's Summation Formulas for ${}_2F_1$. *Symmetry* 2021, 13(8), 1538; <https://doi.org/10.3390/sym13081538>. <https://www.mdpi.com/2073-8994/13/8/1538>
4. Eltayeb, H.; Alhefthi, R. Solution of Fractional Third-Order Dispersive Partial Differential Equations and Symmetric KdV via Sumudu-Generalized Laplace Transform Decomposition. *Symmetry* 2023, 15(8), 1540; <https://doi.org/10.3390/sym15081540>. <https://www.mdpi.com/2073-8994/15/8/1540>
5. Eltayeb, H. Application of the Double Sumudu-Generalized Laplace Transform Decomposition Method to Solve Singular Pseudo-Hyperbolic Equations. *Symmetry* 2023, 15(9), 1706; <https://doi.org/10.3390/sym15091706>. <https://www.mdpi.com/2073-8994/15/9/1706>
6. Inoan, D.; Marian, D. Semi-Hyers–Ulam–Rassias Stability of a Volterra Integro-Differential Equation of Order I with a Convolution Type Kernel via Laplace Transform. *Symmetry* 2021, 13(11), 2181; <https://doi.org/10.3390/sym13112181>. <https://www.mdpi.com/2073-8994/13/11/2181>
7. Kamran; Khan, S.; Haque, S.; Mlaiki, N. On the Approximation of Fractional-Order Differential Equations Using Laplace Transform and Weeks Method. *Symmetry* 2023, 15(6), 1214; <https://doi.org/10.3390/sym15061214>. <https://www.mdpi.com/2073-8994/15/6/1214>
8. Alhefthi, R.; Eltayeb, H. The Solution of Coupled Burgers's Equation by G-Laplace Transform *Symmetry* 2023, 15(9), 1764; <https://doi.org/10.3390/sym15091764>. <https://www.mdpi.com/2073-8994/15/9/1764>

9. Zhang, H.; Su, C.; Bai, J.; Yuan, R.; Ma, Y.; Wang, W. The Rheological Analytical Solution and Parameter Inversion of Soft Soil Foundation. *Symmetry* 2021, 13(7), 1228; <https://doi.org/10.3390/sym13071228>. <https://www.mdpi.com/2073-8994/13/7/1228>
10. Jung, C.; Farid, G.; Yasmeen, H.; Nonlaopon, K. More on the Unified Mittag-Leffler Function. *Symmetry* 2022, 14(3), 523; <https://doi.org/10.3390/sym14030523>. <https://www.mdpi.com/2073-8994/14/3/523>

Reference

1. Firstpost. Available online: <https://www.firstpost.com/tech/big-tech-water-thirsty-for-ai-microsoft-used-22-22-billion-litres-in-2022-google-7-billion-13741850.html> (accessed on 26 February 2024).

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