Consensus Tracking Control of Multiple Unmanned Aerial Vehicles Subject to Distinct Unknown Delays

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Abstract: This article deals with the consensus tracking problem for multi-agent systems (MAS) under the influence of unknown time-varying delays. Each agent of the MAS is a quadrotor unmanned aerial vehicle (UAV) represented as a linear continuous-time system. The main objective of this paper is the stabilization of multi-agent systems where the control input is affected by unknown time-varying delays, which are assumed to be upper-bounded, and where these bounds are not required to be known. The proposed observer-based control scheme guarantees the consensus tracking of multi-UAV systems with the desired H_∞ performance, which adds a further level of mitigation of unknown delays present in MAS systems by minimizing the H_∞ norm, which measures the maximum gain from the disturbance to the controlled output of the system. For each UAV agent, an unknown input observer is employed to isolate the unknown time-varying delays in the state estimation process. With the use of an unknown input observer-based consensus tracking control, sufficient conditions are derived to ensure that all follower UAVs can reach a consensus with the leader, despite the presence of distinct unknown time-varying delays. The stability of the proposed scheme is proven using Lyapunov theory for the leader and follower agents. Finally, numerical examples are provided to illustrate the effectiveness of the proposed method.

Keywords: multi-agent systems; unknown time-varying delay; consensus control; multi-quadrotors

1. Introduction

In recent years, the study of multiple unmanned aerial vehicles (UAVs) in formation has attracted attention, because of their diverse applications in target enclosure and tracking, search and rescue, surveillance, and heavy payload transportation, among others [1–4]. Compared with a single UAV, as in [5], using multiple UAVs working cooperatively has advantages, such as greater thrust force, high efficiency, wide coverage area, and increased versatility.

A multiple UAV system can be conceptualized as a multi-agent system (MAS). MAS formation and tracking control is considered a fundamental problem, in which agents are required to produce a desired trajectory. This topic has been extensively studied in the literature [6–8]. One of the most commonly proposed approaches for formation control in MASs is consensus protocols [9–12]. Consensus means that a team of agents reaches an agreement about a particular variable of interest by interacting with each other via a communication network. Some studies [13–22] suggested that consensus protocols in multi-agent systems provide benefits such as achieving agreement in various conditions

(antagonistic interactions, noisy environments, and heterogeneous systems), enabling distributed and adaptive solutions, and ensuring stability and finite settling times.

Consensus protocols can be affected by the presence of time-varying delays, which can lead to reduced performance, instability, or prevent the desired agreement. Certain protocol designs can compensate for these delays, ensuring proper system performance and achieving consensus under specific conditions such as network topologies [23,24] and design parameters [25]. Other authors have explored different approaches, such as consensus protocols based on observation. For example, the authors in [26,27] addressed this problem by designing a leader–follower consensus control strategy based on predictive control, where an observer is used for estimation of the state of the agents when communication constraints exist. The authors in [28] presented a predictive extended state observer (ESO) to estimate the consensus tracking error when having time-varying input/output delays and mismatched disturbances in a linear MAS, and the ESO was used for designing the controller. In addition, ref. [29] developed a distributed extended state observer and a leader–follower consensus control based on the relative estimated states between each agent and its neighbors in the presence of unknown external disturbances.

Moreover, the authors in [30,31] focused on the problem of fault detection for a MAS in continuous time, where they used consensus protocols and unknown input observer (UIO) schemes to deal with disturbances, failures, and time-varying delays, a where numerical example demonstrated the effectiveness and feasibility of the proposed approach. The authors in [32] implemented a fault diagnosis strategy based on the stages of detection, isolation, and estimation for satellites in formation. In the isolation stage, a bank of robust and non-linear UIOs were designed to isolate the faulty actuator, allowing the unknown input disturbances to be decoupled and their effects attenuated. In addition, UIOs can effectively track state errors in multi-agent systems with directed switching topologies, enabling consensus tracking, even with external inputs affecting the leader [33]. For example, the authors in [34] used a UIO to estimate an unknown state affected by simultaneous connectivity-mixed attacks, actuator/sensor faults, and disturbances, and developed a leader–follower consensus control for a MAS. In summary, research indicates that consensus in multi-agent systems can be achieved despite unknown input delays or faults. Protocols that utilize UIOs to detect, isolate, or estimate these elements have been developed, ensuring that the systems can reach consensus.

In general, time-varying delays are unavoidable in complex systems such as UAVs. Time-varying delays in UAVs are primarily caused by factors related to communication and computation, such as wireless communication issues, satellite communication transport delays, or problems with vision-based navigation systems. In UAV systems, time-delays can reduce the effectiveness of control performance, affecting the stability and making the system vulnerable to an actuator, sensor, or process fault. These problems become more critical in multiple-UAV systems in formation schemes. Therefore, developing formation control schemes for multiple UAVs subject to delays has attracted the attention of academia and industry. For example, the authors of [35] investigated the robust control problem of time-varying formation flight for multiple-UAV systems with external disturbances and time delay. However, ref. [35] only considered a model based on fixed-wing vehicles. Moreover, the authors in [36] proposed an algorithm for leader–following consensus control of multiple fixed-wing UAVs with time-varying delays and unknown external disturbances but limited their study to the dynamic model of the attitude (aircraft orientation).

The time-varying delay problem for a single quadrotor was addressed in [37], where a Lyapunov barrier was used to ensure that the tracking error was limited within a range, and a Pade approximation was applied to compensate for the effect of input delay.

In the literature, many works have addressed the problem of observer-based consensus formation control for different cases of multi-agent systems with time-varying delay. However, few studies have addressed this problem in the specific case of multi-UAV systems. Therefore, the main contribution of this work is a new methodology to obtain mitigation of an unknown input delay in a tracking control in the formation of multiple-UAV systems

under consensus protocols, where the stability of the proposed scheme is proved in the leader and followers, despite unknown time-varying delays. It is essential to mention that unknown delays are assumed to be upper-bounded, and these bounds are not required to be known. In addition, the proposed observer-based control scheme guarantees the consensus tracking of multi-UAV systems with the desired H_∞ performance, which adds a further level of mitigation to unknown delays present in an MAS system by minimizing the H_∞ norm, which measures the maximum gain from the disturbance to the controlled output of the system. In other words, in the proposed method, we address the problem related to a distinct unknown delay of the input of cooperative control, where consensus tracking control for the translation dynamics based on an unknown input observer is proposed. Thus, the designed consensus control between the leader and followers is guaranteed, and this scheme can be applied to solve the cooperative control problem of multiple UAVs in the presence of distinct unknown delays.

The paper is organized as follows. Section 2 is related to the dynamic model of the considered quadrotors and the problem formulation. Section 3 is dedicated to the control strategy design, which involves the inner attitude control and the consensus tracking control for translation. Section 3 analyzes the observer-based control strategy for the tracking control of the leader and follower agents, which involves observers used to isolate the unknown time-varying delays in the state estimation process; in addition, in this section, consensus tracking control under unknown time-varying delays is presented. Section 4 presents the results of numerical simulations, to show the effectiveness of the observer-based consensus control applied to multiple UAVs subject to distinct unknown time-varying delays. Finally, Section 5 ends the document with the conclusions.

2. System Dynamic Model

In this article, the MAS consists of a group of quadrotors under a leader—follower scheme. Each of the quadrotors in the MAS is a system with six degrees of freedom that consists of the translation coordinates $r_i(t) = (x_i(t), y_i(t), z_i(t))$; and three rotations, one in each of the axes $\eta_i(t) = (\phi_i(t), \theta_i(t), \psi_i(t))$ called roll, pitch, and yaw, respectively. The MAS can be described with respect to an inertial reference I, where the UAVs are considered mass points. The translational and rotational dynamics of the vehicles are defined by the second law of Newton and the external torque equation as follows [38]:

$$m\ddot{r}_i(t) = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R(\eta_i) \begin{bmatrix} 0 \\ 0 \\ F_i(t) \end{bmatrix} \tag{1}$$

$$I_r(\eta_i)\ddot{\eta}_i = \tau_i - \dot{\eta}_i \times I_r(\eta_i)\dot{\eta}_i = \begin{bmatrix} \tau_{i2}(t) \\ \tau_{i3}(t) \\ \tau_{i4}(t) \end{bmatrix} + \begin{bmatrix} C_{i1} \\ C_{i2} \\ C_{i3} \end{bmatrix} \tag{2}$$

where g, m, and $\ddot{r}_i(t)$ are the gravitational acceleration, mass, and lineal acceleration; $\dot{\eta}_i(t)$ and $\ddot{\eta}_i(t)$ are the angular velocity and angular acceleration of the vehicle, respectively. In addition, $F_i(t) = F_{i1}(t) + F_{i2}(t) + F_{i3}(t) + F_{i4}(t)$ is the total thrust of the vehicle and $F_{ip}(t) = k_F \omega_{ip}^2(t)$, $p = 1, 2, 3, 4$. The term k_F is a constant, and $\omega_{ip}(t)$ is the angular velocity of each rotor. The terms $R(\eta_i)$ represent the orientation relative to I, and $I_r(\eta_i)$ is the inertia tensor of the body:

$$R(\eta_i) = \begin{bmatrix} s_{\theta_i}c_{\psi_i} + s_{\phi_i}c_{\theta_i}s_{\psi_i} \\ s_{\theta_i}s_{\psi_i} - s_{\phi_i}c_{\theta_i}c_{\psi_i} \\ c_{\phi_i}c_{\theta_i} \end{bmatrix} \qquad I_r(\eta_i) = \begin{bmatrix} I_{xx}c_{\theta_i} & 0 & -I_{xx}c_{\phi_i}s_{\theta_i} \\ 0 & I_{yy} & I_{yy}s_{\phi_i} \\ I_{zz}s_{\theta_i} & 0 & I_{zz}c_{\phi_i}c_{\theta_i} \end{bmatrix}$$

where the shorthand notations s_{α_i} and c_{α_i} represent the sine and cosine of the angle $\alpha_i(t)$, respectively; τ_i are the torques defined as $\tau_{i2}(t) = L(F_{i2}(t) - F_{i4}(t))$, $\tau_{i3}(t) = L(F_{i3}(t) - F_{i1}(t))$, $\tau_{i4}(t) = M_{i1}(t) - M_{i2}(t) + M_{i3}(t) - M_{i4}(t)$; L is the distance between the rotors and the

center of mass, $M_{ip}(t) = k_M \omega_{ip}^2(t)$ is the reaction torque and k_M is a constant. In addition, C_{i1}, C_{i2}, and C_{i3} have the following definitions:

$$
\begin{aligned}
C_{i1} =& \left[I_{xx} + I_{yy} - I_{zz}\right]\dot{\phi}_i(t)\dot{\theta}_i(t)s_{\theta_i} + \left[-I_{xx} + I_{yy} - I_{zz}\right]\dot{\phi}_i(t)\dot{\psi}_i(t)s_{\phi_i}s_{\theta_i} \\
&+ \left[I_{xx} + I_{yy} - I_{zz}\right]\dot{\theta}_i(t)\dot{\psi}_i(t)c_{\phi_i}c_{\theta_i} + \left[I_{yy} - I_{zz}\right]\dot{\psi}_i^2(t)s_{\phi_i}c_{\phi_i}c_{\theta_i} \\
C_{i2} =& \left[-I_{yy} + \left[-I_{xx} + I_{zz}\right]c_\theta^2\right]\dot{\phi}_i(t)\dot{\psi}_i(t)c_{\theta_i} + \left[-I_{xx} + I_{zz}\right]\left[\dot{\phi}_i^2(t) - \dot{\psi}_i^2(t)c_{\theta_i}^2\right]s_{\theta_i}c_{\theta_i} \\
C_{i3} =& \left[I_{xx} - I_{yy} - I_{zz}\right]\dot{\phi}_i(t)\dot{\theta}_i(t)c_{\theta_i} + \left[I_{xx} - I_{yy} + I_{zz}\right]\dot{\phi}_i(t)\dot{\psi}_i(t)s_{\phi_i}c_{\theta_i} \\
&+ \left[-I_{xx} + I_{yy} + I_{zz}\right]\dot{\theta}_i(t)\dot{\psi}_i(t)c_{\phi_i}s_{\theta_i} + \left[-I_{xx} + I_{yy}\right]\dot{\psi}_i^2(t)s_{\phi_i}c_{\phi_i}s_{\theta_i}
\end{aligned}
$$

The Equations (1) and (2) are the nonlinear dynamic model of the quadrotor multi-agent system. We use the Taylor series and obtain the following expressions:

$$
m\ddot{r}_i(t) = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \begin{bmatrix} \theta_i(t)c_{\psi_{di}} + \phi_i(t)s_{\psi_{di}} \\ \theta_i(t)s_{\psi_{di}} - \phi_i(t)c_{\psi_{di}} \\ 1 \end{bmatrix} F_i(t) \tag{3}
$$

$$
\ddot{\eta}_i(t) = \begin{bmatrix} \frac{1}{I_{xx}}\tau_{i2}(t) \\ \frac{1}{I_{yy}}\tau_{i3}(t) \\ \frac{1}{I_{zz}}\tau_{i4}(t) \end{bmatrix} \tag{4}
$$

Thus, the linear representation of the quadrotor multi-agent system is defined as:

$$
\begin{aligned}
\ddot{x}_i(t) &= g\left[\theta_i(t)c_{\psi_{di}} + \phi_i(t)s_{\psi_{di}}\right] & \ddot{\phi}_i(t) &= \frac{1}{I_{xx}}\tau_{i2}(t) \\
\ddot{y}_i(t) &= g\left[\theta_i(t)s_{\psi_{di}} - \phi_i(t)c_{\psi_{di}}\right] & \ddot{\theta}_i(t) &= \frac{1}{I_{yy}}\tau_{i3}(t) \\
\ddot{z}_i(t) &= \frac{1}{m}F_i(t) - g & \ddot{\psi}_i(t) &= \frac{1}{I_{zz}}\tau_{i4}(t)
\end{aligned} \tag{5}
$$

where ψ_{di} is the desired yaw angle of each UAV.

3. Control Strategy

We consider a leader–follower strategy for the formation control of the MAS. The system has the scheme shown in Figure 1, in which it is considered that the quadrotors are underactuated systems. For all agents, the coordinates $z_i(t)$, $\phi_i(t)$, $\theta_i(t)$, and $\psi_i(t)$ can be controlled directly and individually, without sharing information between the vehicles. At the same time, the translational movement is conducted in a coordinated way.

For the translational subsystem, it is considered that the $x_i(t)$ coordinate can be controlled indirectly by the θ_i coordinate, and the $y_i(t)$ coordinate is controlled indirectly using the coordinate $\phi_i(t)$. In this work, we address the problem of a possible actuator time-varying delay affecting the motion in the $x - y$ plane. Therefore, the coordinates θ_i and ϕ_i are controlled by schemes based on state estimation to isolate the time-varying delay. Specifically, a coordinated control of the translational movement based on a leader–follower scheme is considered. Thus, a consensus tracking control based on an unknown input observation is proposed, for the follower agents to maintain formation flight, with the leader agent's estimated translational position as the reference. In contrast, a disturbance observer-based control is formulated for the leader agent, to follow a desired trajectory despite the presence of time-varying delays.
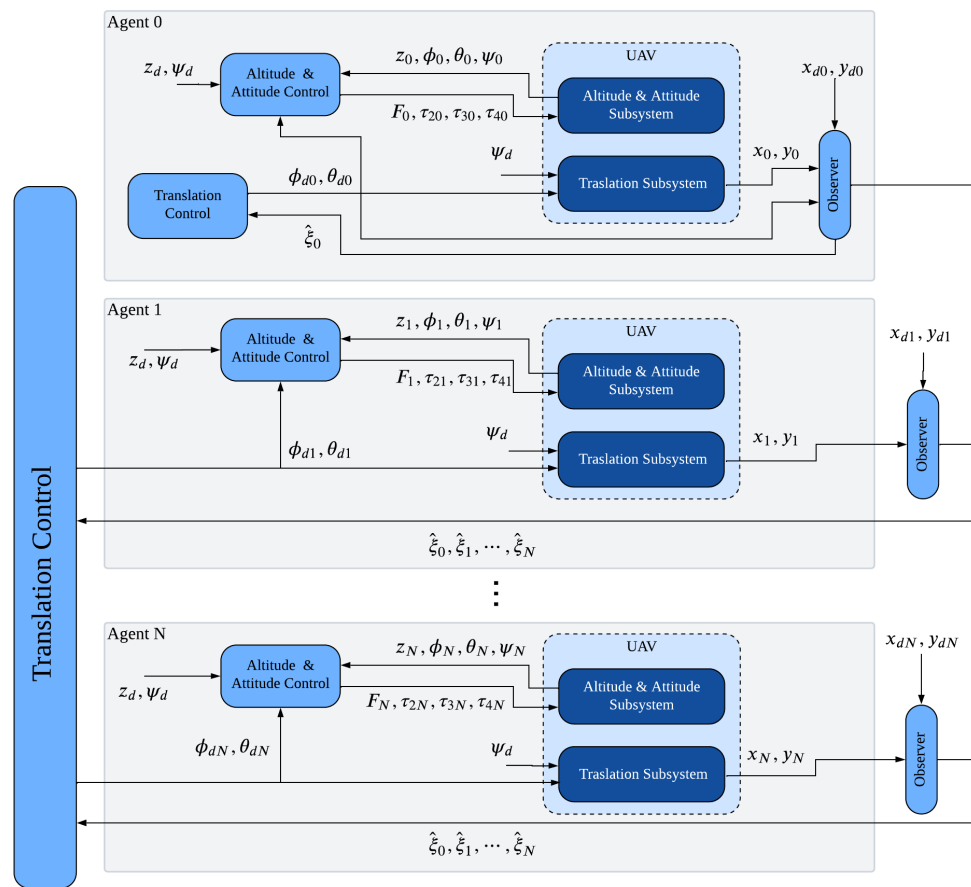
Figure 1. Control scheme for the multi-UAV system.

3.1. Altitude and Attitude Control

This section proposes using simple linear control laws for the altitude and attitude subsystems, to ensure the stability of the flying attitude of the quadrotor multi-agent system. Thus, the control inputs $F_i(t)$, $\tau_{i2}(t)$, $\tau_{i3}(t)$, $\tau_{i4}(t)$ for each of the agents are denoted as

$$F_i(t) = mg + k_{pz}e_{zi}(t) + k_{iz}\int e_{zi}(t)dt + k_{dz}\dot{e}_{zi}(t) \tag{6}$$

$$\tau_{i2}(t) = k_{p\phi}e_{\phi_i}(t) + k_{d\phi}\dot{e}_{\phi_i}(t) \tag{7}$$

$$\tau_{i3}(t) = k_{p\theta}e_{\theta_i}(t) + k_{d\theta}\dot{e}_{\theta_i}(t) \tag{8}$$

$$\tau_{i4}(t) = k_{p\psi}e_{\psi_i}(t) + k_{d\psi}\dot{e}_{\psi_i}(t) \tag{9}$$

where

$$e_{zi}(t) = z_d(t) - z_i(t) \quad e_{\phi_i}(t) = \phi_{di}(t) - \phi_i(t) \quad e_{\theta_i}(t) = \theta_{di}(t) - \theta_i(t) \quad e_{\psi_i}(t) = \psi_{di}(t) - \psi_i(t)$$
$$\dot{e}_{zi}(t) = \dot{z}_d(t) - \dot{z}_i(t) \quad \dot{e}_{\phi_i}(t) = \dot{\phi}_{di}(t) - \dot{\phi}_i(t) \quad \dot{e}_{\theta_i}(t) = \dot{\theta}_{di}(t) - \dot{\theta}_i(t) \quad \dot{e}_{\psi_i}(t) = \dot{\psi}_{di}(t) - \dot{\psi}_i(t)$$

The control inputs are substituted in the equations of the linear multi-agent system (5); thus, the following closed-loop expressions are obtained:

$$\ddot{\phi}_i(t) = \frac{1}{I_{xx}}\left[k_{p\phi}e_{\phi_i}(t) + k_{d\phi}\dot{e}_{\phi_i}(t)\right]$$

$$\ddot{z}_i(t) = \frac{1}{m}\left[k_{pz}e_{zi}(t) + k_{iz}\int e_{zi}(t)dt + k_{dz}\dot{e}_{zi}(t)\right] \qquad \ddot{\theta}_i(t) = \frac{1}{I_{yy}}\left[k_{p\theta}e_{\theta_i}(t) + k_{d\theta}\dot{e}_{\theta_i}(t)\right]$$

$$\ddot{\psi}_i(t) = \frac{1}{I_{zz}}\left[k_{p\psi}e_{\psi_i}(t) + k_{d\psi}\dot{e}_{\psi_i}(t)\right]$$

3.2. Translation Subsystem

From the linear quadrotor multi-agent system (5), the translation movement of the quadcopter can be obtained:

$$\ddot{x}_i(t) = g\left[\theta_i(t)c_{\psi_{di}} + \phi_i(t)s_{\psi_{di}}\right]$$
$$\ddot{y}_i(t) = g\left[\theta_i(t)s_{\psi_{di}} - \phi_i(t)c_{\psi_{di}}\right] \tag{10}$$

Now, since all orientation errors of the quadrotor multi-agent system tend to be zero, it can be assumed that $\theta_i \approx \theta_{d_i}$, $\phi_i \approx \phi_{d_i}$. Moreover, if ψ_{d_i} is considered a constant equal to zero, the dynamics corresponding to the x_i and y_i coordinates are described in the following form:

$$\ddot{x}_i(t) = gc_{\psi_d}\theta_{di}(t)$$
$$\ddot{y}_i(t) = -gc_{\psi_d}\phi_{di}(t) \tag{11}$$

Then, one can define the following state vector $\xi_i(t) = \begin{bmatrix} \xi_{ik}^1(t) & \xi_{ik}^2(t) \end{bmatrix}^T \in \mathbb{R}^6$ for each agent, where

$$\xi_{ik}^1(t) = \left[\int e_{xi}(t)dt \quad e_{xi}(t) \quad \dot{e}_{xi}(t)\right]^T \in \mathbb{R}^3, \quad k = 1, 2, 3$$

$$\xi_{ik}^2(t) = \left[\int e_{yi}(t)dt \quad e_{yi}(t) \quad \dot{e}_{yi}(t)\right]^T \in \mathbb{R}^3, \quad k = 1, 2, 3 \tag{12}$$

Thus, the x and y coordinate errors are defined as follows:

$$e_{xi}(t) = x_{d_i}(t) - x_i(t) \qquad e_{yi}(t) = y_{d_i}(t) - y_i(t)$$
$$\dot{e}_{xi}(t) = \dot{x}_{d_i}(t) - \dot{x}_i(t) \qquad \dot{e}_{yi}(t) = \dot{y}_{d_i}(t) - \dot{y}_i(t)$$

Considering x_{d_i} and y_{d_i} as constants, the elements of the derivative $\dot{\xi}_i(t) = \begin{bmatrix} \dot{\xi}_{ik}^1(t) & \dot{\xi}_{ik}^2(t) \end{bmatrix}^T \in \mathbb{R}^6$ are represented as follows:

$$\dot{\xi}_{i1}^1 = \xi_{i2}^1$$
$$\dot{\xi}_{i2}^1 = \xi_{i3}^1$$
$$\dot{\xi}_{i3}^1 = -gc_{\psi_d}\theta_{di}$$
$$\dot{\xi}_{i1}^2 = \xi_{i2}^2$$
$$\dot{\xi}_{i2}^2 = \xi_{i3}^2$$
$$\dot{\xi}_{i3}^2 = gc_{\psi_d}\phi_{di}$$

Therefore, the translation movement x-y of the quadcopter multi-agent dynamics can be expressed in the following state-space form:

$$\dot{\xi}_i(t) = A\xi_i(t) + Bu_i(t)$$
$$\bar{y}_i(t) = C\xi_i(t) \tag{13}$$

where matrix $C = I_{6\times6}$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -gc_{\psi_d} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & gc_{\psi_d} \end{bmatrix} \quad u_i(t) = \begin{bmatrix} \theta_{di}(t) \\ \phi_{di}(t) \end{bmatrix}$$

A practical problem in the position tracking and formation control of quadrotor multi-agent systems is related to the time difference between the translation sensor as a global

positioning system (GPS) that detects a position measurement and the force input that applies torque to each motor (input signal) of the aerial vehicle. This time difference is due to the communication or synchronization time between each servo controller, which entails UAVs not receiving the proper real-time input, meaning the system has an inherent input delay. Therefore, it is necessary to address the effect of variable time-varying delay in the input application on the UAV tracking control design. Thus, an observer-based consensus control is proposed, to ensure that all UAV agents can reach consensus with the leader, despite the presence of distinct unknown time-varying delays.

Considering the previous problem, the lineal multi-agent system of the position tracking error dynamics (13) has inherent input delays. Therefore, the following delayed lineal multi-agent system is formulated:

$$
\begin{aligned}
\dot{\xi}_i(t) &= A\xi_i(t) + Bu_i(t) + Bu_i(t - \tau_i(t)) \\
\bar{y}_i(t) &= C\xi_i(t)
\end{aligned}
\tag{14}
$$

where $u_i(t - \tau_i(t))$ is the unknown control input under the effect of unknown time-varying delay $\tau_i(t)$, which is present in every agent i, with $i \in \{0, \ldots, N\}$. The delay problem in multi-agent systems is focused on the unknown time-varying delay $\tau_i(t)$, where each unknown delay is assumed to be continuous and bounded; i.e., $0 \leq \tau_i(t) \leq \tau_{i,\max}$, $\forall t \geq 0$; $\tau_{i,\max}$ is the maximum delay of the i-th agent, which is assumed to be upper bounded, and this bound is not required to be known; the assumptions about the control input and the unknown time-varying delay are given in [39,40]. Thus, it is essential to note that each agent has its own inherent unknown delay, unlike other approaches proposed in the literature; e.g., in [31], a single random time-varying delay is present in the whole multi-agent system.

Thus, considering the unknown delayed input in the translation movement of each agent, the delayed linear multi-agent system in (14) is modified as follows:

$$
\begin{aligned}
\dot{\xi}_i(t) &= A\xi_i(t) + Bu_i(t) + B\bar{u}_i(t) \\
\bar{y}_i(t) &= C\xi_i(t)
\end{aligned}
\tag{15}
$$

where $\bar{u}_i(t)$ is the unknown input signal of each agent, which is given by $\bar{u}_i(t) = u_i(t - \tau_i(t)) - u_i(t)$, as in [39].

3.3. Translation Control

According to Equation (5), the control inputs are computed to determine the desired roll and pitch angles for all agents. These angles are functions of the tracking errors in the y_i and x_i coordinates, respectively. Specifically, in the leader–follower scheme, a disturbance observer-based control is implemented on the leader agent to enable it to track the desired trajectory within the plane. Consequently, an observer-based consensus control is formulated for the follower agents to maintain formation flight, with the leader agent as the reference. Addressing the challenge of an unknown delayed input in Equation (15), control schemes are developed based on estimating the state vectors for all agents.

Leader agent control

Thus, for the leader agent, the control inputs are as follows:

$$
\theta_{d_0}(t) = K_0^1 \hat{\xi}_{0k}^1(t) \qquad \phi_{d_0}(t) = -K_0^2 \hat{\xi}_{0k}^2(t)
\tag{16}
$$

where $\hat{\xi}_{0k}^1$ and $\hat{\xi}_{0k}^2$ are the estimations of ξ_{0k}^1 and ξ_{0k}^2. The terms $K_0^1 \in \mathbb{R}^3$ and $K_0^2 \in \mathbb{R}^3$ are the gain feedback matrices. Then, the signal u_0 can be expressed as

$$
u_0 = K_0 \hat{\bar{\xi}}_0(t)
\tag{17}
$$

where

$$K_0 = \begin{bmatrix} K_0^1 & 0 \\ 0 & -K_0^2 \end{bmatrix} \quad \hat{\xi}_0(t) = \begin{bmatrix} \hat{\xi}_{0k}^1 & \hat{\xi}_{0k}^2 \end{bmatrix}^T$$

Therefore, the dynamics of the leader agent are given by

$$\begin{aligned} \dot{\xi}_0(t) &= A\xi_0(t) + Bu_0(t) + B\bar{u}_0(t) \\ \bar{y}_0(t) &= C\xi_0(t) \end{aligned} \tag{18}$$

where $\bar{u}_0(t)$ is the unknown input of the leader agent, which is affected by the time-varying delay $\tau_0(t)$.

Now, the following unknown input observer is proposed for the translation movement of the leader agent under an unknown time-varying delay:

$$\begin{cases} \dot{w}_0(t) = Nw_0(t) + Hu_0(t) + J\bar{y}_0(t) \\ \hat{\xi}_0(t) = w_0(t) + E\bar{y}_0(t) \end{cases} \tag{19}$$

where $w_0(t) \in \mathbb{R}^{Nn}$ and $\hat{\xi}_0(t) \in \mathbb{R}^{Nn}$ are the states of the observer, and the estimations of the system states $\xi_0(t)$. The constant matrices N, H, J and E are chosen to make $\hat{\xi}_0(t)$ converge to $\xi_0(t)$ in the delay-free case.

Now, the following lemma is used in the design of the unknown input observer.

Lemma 1. *Given any matrices C and B, equation $ECB = B$ has a solution if and only if $\mathrm{rank}(CB) = \mathrm{rank}(B)$.*

Then, the following theorem provides sufficient conditions that guarantee the convergence of the observation error $\bar{e}_0(t)$.

Theorem 1. *The unknown input $\bar{u}_0(t)$ of the leader agent system (18) can be decoupled form the state estimation under the unknown input observer (19) if the following conditions are satisfied:*

$$(A - ECA + NEC - JC - N) = 0 \tag{20}$$
$$((I - EC)B - H) = 0 \tag{21}$$
$$(I - EC)B = 0 \tag{22}$$

where N is a Hurwitz matrix used to achieve exponential convergence of the observation error $\bar{e}_0(t)$. This means the observer is not subject to an unknown time-varying delay $\tau_0(t)$.

Proof. Now, the observation error is $\bar{e}_0(t) = \xi_0(t) - \hat{\xi}_0(t)$; then, the dynamics of the estimation error is

$$\begin{aligned} \dot{\bar{e}}_0(t) &= \dot{\xi}_0(t) - \dot{\hat{\xi}}_0(t) \\ &= (A - ECA)\xi_0(t) - Nw_0(t) - JC\xi_0(t) \\ &\quad + ((I - EC)B - H)u_0(t) + (I - EC)B\bar{u}_0(t) \\ &= N\bar{e}_0(t) + (A - ECA + NEC - JC - N)\xi_0(t) \\ &\quad + ((I - EC)B - H)u_0(t) + (I - EC)B\bar{u}_0(t) \\ &= N\bar{e}_i(t) + (A - ECA + NEC - JC - N)\xi_0(t) \\ &\quad + ((I - EC)B - H)u_0(t) + (I - EC)B\bar{u}_0(t) \end{aligned} \tag{23}$$

$$\tag{24}$$

Now, in order to obtain E and H matrices, from Lemma 1, one can deduce that the general solution of E is as follows:

$$E = B(CB)^\dagger + X\left(I - (CB)(CB)^\dagger\right), \tag{25}$$

where X is an arbitrary matrix. Thus, $X = 0$ is selected for ease of calculation, and one obtains $E = B(CB)^\dagger$.

From the matrix E, H is obtained as follows: $H = (I - EC)B$. Now, this denotes that $M = (I - EC)$; thus, Equation (20) can be rewritten as

$$MA + NEC - JC = N$$

Then,

$$N = MA - LC \tag{26}$$

where $L = (J - NE)$. Now, $N = MA - LC$ is a Hurwitz matrix used to ensure convergence of the observation error. Thus, the matrix $(MA - LC)$ should be placed anywhere in the complex s-plane by a suitable selection of matrix L, where (C, YA) must be observable. Subsequently, it is possible to find J, computed as $J = (L + NE)$. Then, the unknown input observer provides an easy-to-implement algorithm that eliminates the influence of the unknown delayed input in the state estimation process.

Based on the above, if these conditions are fulfilled, the following dynamics of the error of observation can be obtained:

$$\dot{e}_0(t) = N\bar{e}_0(t) \tag{27}$$

where N should be a Hurwitz matrix for achieving exponential convergence of the observation error. This means the observer is not subject to an unknown delayed input $\tau_0(t)$. This ends the proof. $\square$

Now, it is possible to design an observer-based control strategy to stabilize the leader agent system (18), where the following observer-based control for the leader agent is considered:

$$
\begin{aligned}
u_0 &= K_0 \hat{\xi}_0(t) \tag{28}\\
&= K_0(w_0(t) + E\bar{y}_0(t)) = K_0(w_0(t) + EC\xi_0(t))\\
&= K_0((I_n - EC)\xi_0(t) - e_0(t) + EC\xi_0(t))\\
&\leq K_0\xi_0(t)
\end{aligned}
$$

Then, the closed-loop dynamics of the leader are

$$\dot{\xi}_0(t) = [A + BK_0]\xi_0(t) + B\bar{u}_0(t) \tag{29}$$

Now, it is evident that there is an influence of the unknown signal $\bar{u}_0(t)$ on the closed-loop system (29). Therefore, the stability of the closed-loop system must be guaranteed.

Theorem 2. *Consider the disturbance observer-based control (28) of the leader agent system (18), which is exponentially stable if there exists a $P_c > 0$ such that the following inequation holds for $\mu_1 > \mu_2 > 0$:*

$$
\begin{bmatrix} P_c A^T + WB^T + AP_c + BW + \mu_1 P_c & B \\ B^T & -\mu_2 I_n \end{bmatrix} < 0, \tag{30}
$$

where the control gain is $K_o = WP_c^{-1}$. Thus, the system's stability is guaranteed, even in the presence of the unknown time-varying delay $\tau_0(t)$.

Proof. Considering the quadratic function $V(\xi_0(t)) = \xi_0^T(t)P\xi_0(t)$, one obtains the following dynamics:

$$\dot{V}(\xi_0(t)) = \dot{\xi}_0^T(t)P\xi_0(t) + \xi_0^T(t)P\dot{\xi}_0(t)$$

$$= ((A + BK_0)\xi_0(t) + B\bar{u}_0(t))^T P\xi_0(t) + \xi_0(t)^T P((A + BK_0)\xi_0(t) + B\bar{u}_0(t))$$

$$= \xi_0(t)^T((A + BK_0)^T P + P(A + BK_0))\xi_0(t) + \bar{u}_0(t)^T B^T P\xi_0(t) + \xi_0(t)^T PB\bar{u}_0(t))$$

$$= \begin{bmatrix} \xi_0(t) \\ \bar{u}_0(t) \end{bmatrix}^T \begin{bmatrix} ((A + BK_0)^T P + P(A + BK_0)) & PB \\ B^T P & 0 \end{bmatrix} \begin{bmatrix} \xi_0(t) \\ \bar{u}_0(t) \end{bmatrix} \tag{31}$$

The following proposition provides a criterion that guarantees the exponential stability of the disturbance system (18).

Proposition 1. *Taking into account the system (29). If there exist matrix $P > 0$ and positive scalars $\mu_1 > \mu_2$ such that the following inequality holds:*

$$\dot{V}(\xi_0(t)) + \mu_1 V(\xi_0(t)) - \mu_2 \bar{u}_0(t)^T P\bar{u}_0(t) \leq 0 \tag{32}$$

where $V(\xi_0(t))$ is set $V(\xi_0(t)) = \xi_0^T(t)P\xi_0(t)$ and $\dot{V}(\xi_0(t))$ is its derivative, then the closed-loop system (29) is exponentially stable.

From Proposition 1, one can rewrite (31) as follows:

$$\dot{V}(\xi_0(t)) + \mu_1 V(\xi_0(t)) - \mu_2 \bar{u}_0(t)^T P\bar{u}_0(t) =$$

$$\begin{bmatrix} \xi_0(t) \\ \bar{u}_0(t) \end{bmatrix}^T \begin{bmatrix} ((A + BK_0)^T P + P(A + BK_0)) + \mu_1 P & PB \\ B^T P & -\mu_2 P \end{bmatrix} \begin{bmatrix} \xi_0(t) \\ \bar{u}_0(t) \end{bmatrix}$$

Therefore, the following inequality must hold

$$\begin{bmatrix} (A + BK_0)^T P + P(A + BK_0) + \mu_1 P & PB \\ B^T P & -\mu_2 P \end{bmatrix} \leq 0 \tag{33}$$

Now, the above equation is a bilinear matrix inequality; thus, a non-singular transformation, such as $T = \begin{bmatrix} P^{-1} & 0 \\ 0 & I_n \end{bmatrix} > 0$, must be introduced:

$$T\begin{bmatrix} (A + BK_0)^T P + P(A + BK_0) + \mu_1 P & PB \\ B^T P & -\mu_2 P \end{bmatrix} T = \tag{34}$$

$$\begin{bmatrix} P^{-1}(A + BK_0)^T + (A + BK_0)P^{-1} + \mu_1 P^{-1} & B \\ B^T & -\mu_2 I_n \end{bmatrix} \leq 0 \tag{35}$$

Defining $P^{-1} = P_c$, this yields

$$\begin{bmatrix} P_c(A + BK_0)^T + (A + BK_0)P_c + \mu_1 P_c & BP_c \\ P_c B^T & -\mu_2 I_n \end{bmatrix} < 0. \tag{36}$$

Thus, the following LMI is obtained,

$$\begin{bmatrix} P_c A^T + WB^T + AP_c + BW + \mu_1 P_c & B \\ B^T & -\mu_2 I_n \end{bmatrix} < 0, \tag{37}$$

where $K_o = WP_c^{-1}$.

Using Proposition 1, the closed-loop dynamics of the leader (29) is exponentially stable under (37). It is obvious that the effect of the unknown input $\bar{u}_0(t)$ is reduced, which depends on the influence of the unknown time-varying delay $\tau_0(t)$. This ends the proof. $\square$

Follower agent control

For the follower agents, the consensus control for formation is based on the estimation of the states $\xi_i(t)$ of each agent, which are error states in the x_i and y_i coordinates. In the consensus protocols, the communication between agents in a MAS can be described by the graph $\mathcal{G}$. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty finite node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set. The edge (v_i, v_j) in the edge set $\mathcal{E}$ denotes that agent v_j can obtain information from agent v_i, but not vice versa. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ associated to the graph $\mathcal{G}$ is defined such that $a_{ii} = 0$ and $a_{ij} = 1 \Leftrightarrow (j, i) \in \mathcal{E}$ for $i, j = 0, 1, \ldots, N$. The Laplacian matrix $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbb{R}^{N \times N}$ of the graph $\mathcal{G}$ is defined as $\mathcal{L}_{ii} = \sum_{j \neq i} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}, i \neq j$. In leader–follower schemes, $\mathcal{L}$ can be partitioned as $\mathcal{L} = \begin{bmatrix} 0 & 0_{1 \times N} \\ \mathcal{L}_2 & \mathcal{L}_1 \end{bmatrix}$, where $\mathcal{L}_2 \in \mathbb{R}^{N \times 1}$ and $\mathcal{L}_1 \in \mathbb{R}^{N \times N}$. In addition, the constant formation structure of the agents in the graph $\mathcal{G}$ in a reference coordinate frame is denoted by $\widetilde{H} = (h_0, h_1, \ldots, h_N) \in \mathbb{R}^{n \times N}$, where $h_i \in \mathbb{R}^n$ is the formation variable corresponding to agent i.

Assuming that each agent has access to its own state and the states of its neighbors, the agents achieve consensus in the sense of $\lim_{t \to \infty} \|(\xi_i(t) - h_i) - (\xi_j(t) - h_j)\| = 0, \forall i, j = 0, \ldots, N$. Thus, the following law of observer-based consensus control for the coordinates x and y is considered:

$$\theta_{di}(t) = K_1 \sum_{j=0}^{N} a_{ij}(\hat{\xi}_{ik}^1(t) - \hat{\xi}_{jk}^1(t) - h_i^1 + h_j^1), \quad i = 1, \ldots, N. \tag{38}$$

$$\phi_{di}(t) = K_2 \sum_{j=0}^{N} a_{ij}(\hat{\xi}_{ik}^2(t) - \hat{\xi}_{jk}^2(t) - h_i^2 + h_j^2), \quad i = 1, \ldots, N. \tag{39}$$

The terms $K_1 \in \mathbb{R}^{1 \times 3}$ and $K_2 \in \mathbb{R}^{1 \times 3}$ are the feedback gain matrices, which will be defined later; a_{ij} is the (i, j)-th element of the adjacency matrix associated with $\mathcal{G}$, and $\hat{\xi}_{ik}^1(t)$, $\hat{\xi}_{ik}^2(t)$, $\hat{\xi}_{jk}^1(t)$, and $\hat{\xi}_{jk}^2(t)$ are the state estimations of $\xi_{ik}^1(t)$, $\xi_{ik}^2(t)$, $\xi_{jk}^1(t)$, and $\xi_{jk}^2(t)$. Moreover, h_i^1, h_i^2, h_j^1 and h_j^2 are constant vectors that describe the formation structure in coordinates x and y.

Considering the vectors $h_i = \begin{bmatrix} h_i^1 & h_i^2 \end{bmatrix}^T$, $\hat{\xi}_i(t) = \begin{bmatrix} \hat{\xi}_{ik}^1(t) & \hat{\xi}_{ik}^2(t) \end{bmatrix}^T$ and $\hat{\xi}_j(t) = \begin{bmatrix} \hat{\xi}_{jk}^1(t) & \hat{\xi}_{jk}^2(t) \end{bmatrix}^T$, the consensus control law based on state estimation can be formulated as a single control input for each agent:

$$u_i(t) = K \sum_{j=0}^{N} a_{ij}(\hat{\xi}_i(t) - \hat{\xi}_j(t) - h_i + h_j), \quad i = 1, \ldots, N. \tag{40}$$

where

$$K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}$$

Then, the follower agent dynamics are given:

$$\begin{cases} \dot{\xi}_i(t) = A\xi_i(t) + Bu_i(t) + B\bar{u}_i(t) \\ \bar{y}_i(t) = C\xi_i(t) \end{cases} \tag{41}$$

where $\bar{u}_i(t)$ is the unknown input of the follower agent, which is affected by time-varying delay $\tau_i(t)$.

Substitute the observer-based control law (40) into the follower agent system (41) of agent i. By making use of the properties of the Kronecker product, one can rewrite this in the following form:

$$\begin{cases} \dot{\xi}(t) = (I_N \otimes A)\xi(t) + (\mathcal{L}_1 \otimes BK)\hat{\xi}(t) + (I_N \otimes B)\bar{u}(t) \\ \bar{y}(t) = (I_N \otimes C)\xi(t) \end{cases} \tag{42}$$

where $\xi(t) = \left[\xi_1^T(t),\dots,\xi_N^T(t)\right]^T, \bar{u}(t) = \left[\bar{u}_1^T(t),\dots,\bar{u}_N^T(t)\right]^T, \bar{y}(t) = \left[\bar{y}_1^T(t),\dots,\bar{y}_N^T(t)\right]^T$ and $\hat{\xi}(t) = \left[\hat{\xi}_1^T(t),\dots,\hat{\xi}_N^T(t)\right]^T$ denotes the follower agent's state estimation. This yields the following follower agent system:

$$\begin{cases} \dot{\xi}(t) = \tilde{A}\xi(t) + \tilde{K}\hat{\xi}(t) + \tilde{B}\bar{u}(t) \\ \bar{y}(t) = \tilde{C}\xi(t) \end{cases} \tag{43}$$

with $\tilde{A} = (I_N \otimes A), \tilde{K} = (\mathcal{L}_1 \otimes BK), \tilde{B} = (I_N \otimes B), \tilde{C} = (I_N \otimes C)$.

Now, the same estimation approach is used in the follower agent system; thus, unknown input observers are applied to isolate the external disturbances with the unknown boundary $\bar{u}_i(t)$, where the unknown delayed input of each agent is present $u_i(t - \tau_i(t))$. For isolation of each delayed control input, an observer like a nonlinear unknown input observer on agent i is constructed as follows:

$$\begin{cases} \dot{w}_i(t) = \mathbb{N}w_i(t) + \mathbb{J}\bar{y}_i(t) \\ \hat{\xi}_i(t) = w_i(t) + \mathbb{E}\bar{y}_i(t) \end{cases} \tag{44}$$

where $w_i(t) \in \mathbb{R}^N$ and $\hat{\xi}_i(t) \in \mathbb{R}^N$ are the states of the observer, and the estimations of the system states $\xi_i(t)$ for each agent i. The constant matrices $\mathbb{N}, , \mathbb{J}$ and $\mathbb{E}$ are chosen to make $\hat{\xi}_i(t)$ converge to $\xi_i(t)$ in a delay-free case.

Now, based on the properties of the Kronecker product, one can formulate the following unknown input observer on the agent:

$$\begin{cases} \dot{w}(t) = \mathbf{N}w(t) + \mathbf{J}\bar{y}(t) \\ \hat{\xi}(t) = w(t) + \mathbf{E}\bar{y}(t) \end{cases} \tag{45}$$

where $w(t) = \left[w_1^T(t),\dots,w_N^T(t)\right]^T, \mathbf{N} = (I_N \otimes \mathbb{N}), \mathbf{J} = (I_N \otimes \mathbb{J}), \mathbf{E} = (I_N \otimes \mathbb{E})$.

Now, the following theorem is provided to ensure convergence of the proposed observer:

Theorem 3. *The unknown inputs $\bar{u}_i(t)$ of the follower agent system* (41) *can be decoupled from the state estimation under the unknown input observer* (45) *if the following conditions are satisfied:*

$$\left(\tilde{A} - \mathbf{E}\tilde{C}\tilde{A} + \mathbf{N}\mathbf{E}\tilde{C} - \mathbf{J}\tilde{C}\right) = \mathbf{N} - \left(I - \mathbf{E}\tilde{C}\right)\tilde{K} \tag{46}$$

$$\left(I - \mathbf{E}\tilde{C}\right)\tilde{B} = 0 \tag{47}$$

where $\left[\mathbf{N} - \left(I - \mathbf{E}\tilde{C}\right)\tilde{K}\right]$ is a Hurwitz matrix to achieve an exponential convergence of the observation error $\bar{e}(t)$. This means the follower observers are not subject to the unknown time-varying delay $\tau_i(t)$.

Proof. Now, the observation error is $\bar{e}(t) = \xi(t) - \hat{\xi}(t)$, and using Equations (43) and (45), one obtains the following dynamics:

$$\begin{aligned} \dot{\bar{e}}(t) = &\dot{\xi}(t) - \dot{\hat{\xi}}(t) \\ = &\left(\mathbf{N} - \left(I - \mathbf{E}\tilde{C}\right)\tilde{K}\right)\bar{e}(t) + \left(\tilde{A} - \mathbf{E}\tilde{C}\tilde{A} + \mathbf{N}\mathbf{E}\tilde{C} - \mathbf{J}\tilde{C} - \mathbf{N} + \left(I - \mathbf{E}\tilde{C}\right)\tilde{K}\right)\xi(t) \\ &+ \left(I - \mathbf{E}\tilde{C}\right)\tilde{B}\bar{u}(t) \end{aligned} \tag{48}$$

Then,

$$\dot{\bar{e}}(t) = \left(\mathbf{N} - \mathbf{M}\tilde{K}\right)\bar{e}(t) + \left(\mathbf{M}\tilde{A} + \mathbf{N}\mathbf{E}\tilde{C} - \mathbf{J}\tilde{C} - \mathbf{N} + \mathbf{M}\tilde{K}\right)\xi(t) + \mathbf{M}\tilde{B}\bar{u}(t) \tag{49}$$

where $\mathbf{M} = \left(I - \mathbf{E}\tilde{C}\right)$. Since $\mathbf{M}\tilde{B} = 0$, using Lemma 1, one can easily obtain the matrix $\mathbf{E}$. In addition, the following assumption must be satisfied:

$$\mathbf{M}\tilde{A} + \mathbf{N}\mathbf{E}\tilde{C} - \mathbf{J}\tilde{C} + \mathbf{M}\tilde{K} = \mathbf{N}$$
$$\mathbf{M}\tilde{A} - \mathbf{L}\tilde{C} = \mathbf{N} - \mathbf{M}\tilde{K}$$
$$\mathbf{M}\tilde{A} - \mathbf{L}\tilde{C} = \tilde{\mathbf{N}} \tag{50}$$

where $\mathbf{L} = (\mathbf{J} - \mathbf{N}\mathbf{E})$ and the matrix $\tilde{\mathbf{N}} = \mathbf{N} - \mathbf{M}\tilde{K}$. It is worth recalling that the matrix $\tilde{K}$ depends on the gain matrix of the consensus control, which will be further detailed below. Therefore, to guarantee the convergence of the observation error $\bar{e}$, one should satisfy that $\tilde{\mathbf{N}} = \mathbf{M}\tilde{A} - \mathbf{L}\tilde{C}$ is a Hurwitz matrix, to ensure convergence of the observation error. Therefore, $\tilde{\mathbf{N}}$ should be placed anywhere in the complex s-plane by a suitable selection of matrix $\mathbf{L}$.

Based on the above, the dynamics of the observation error is given by $\dot{\bar{e}}(t) = \tilde{\mathbf{N}}\bar{e}(t)$, where the Hurwitz matrix $\tilde{\mathbf{N}}$ achieves exponential convergence of the observation error. This means that the followers' observers will be not subject to the unknown time-varying delays $\tau_i(t)$. $\quad\square$

Now, it is possible to design an observer-based control strategy to stabilize the follower agents' system, where the following observer-based control is considered:

$$u_i(t) = K \sum_{j=0}^{N} a_{ij}(\hat{\xi}_i(t) - \hat{\xi}_j(t) - h_i + h_j) \tag{51}$$

$$= K \sum_{j=0}^{N} a_{ij}(w_i(t) - w_j(t) + \mathbb{E}C(\xi_i(t) - \xi_j(t)) - h_i + h_j)$$

$$= K \sum_{j=0}^{N} a_{ij}((I_n - \mathbb{E}C)(\xi_i(t) - \xi_j(t)) + \mathbb{E}C(\xi_i(t) - \xi_j(t)) - \bar{e}_i(t) + \bar{e}_j(t) - h_i + h_j)$$

$$= K \sum_{j=0}^{N} a_{ij}(\xi_i(t) - \xi_j(t) - \bar{e}_i(t) + \bar{e}_j(t) - h_i + h_j)$$

Since all estimation errors $\bar{e}_i(t)$ and $\bar{e}_j(t)$ tend to zero, the control law can be simplified as

$$u_i(t) = K \sum_{j=0}^{N} a_{ij}(\xi_i(t) - \xi_j(t) - h_i + h_j), \quad i = 1, \ldots, N. \tag{52}$$

For the follower agents, let

$$\bar{\xi}_i(t) = \xi_i(t) - h_i$$
$$\tilde{\xi}_i(t) = \bar{\xi}_i(t) - \xi_0(t)$$

According to Equations (29), (41) and (51), for $i = 1, \ldots, N$, we have

$$\dot{\tilde{\xi}}_i(t) = \dot{\bar{\xi}}_i(t) - \dot{\xi}_0(t)$$

$$= A\xi_i(t) + BK \sum_{j=0}^{N} a_{ij}(\xi_i(t) - \xi_j(t) - h_i + h_j) + B\bar{u}_i(t) - [A + BK_0]\xi_0 - B\bar{u}_0(t)$$

$$= A\bar{\xi}_i(t) + Ah_i + BK \sum_{j=0}^{N} a_{ij}(\bar{\xi}_i(t) - \bar{\xi}_j(t)) + B\bar{u}_i(t) - [A + BK_0]\xi_0 - B\bar{u}_0(t)$$

Considering the new variables $\tilde{u}_i(t) = \bar{u}_i(t) - \bar{u}_0(t)$ and $\bar{A} = A - A_0$, where $A_0 = [A + BK_0]$ is a stable matrix according to the solution of the LMI in Theorem 2. Then, $\dot{\tilde{\xi}}_i(t)$ is now defined as follows:

$$\dot{\tilde{\xi}}_i(t) = \bar{A}\tilde{\xi}_i(t) + Ah_i + BK \sum_{j=0}^{N} a_{ij}(\bar{\xi}_i(t) - \bar{\xi}_j(t)) + B\tilde{u}_i(t) \tag{53}$$

It is important to note that the achievable formation structures must satisfy the constraints $Ah_i = 0$. Therefore, the closed-loop dynamic of the followers is

$$\dot{\tilde{\xi}}(t) = (I_N \otimes \bar{A} + \mathcal{L}_1 \otimes BK)\tilde{\xi}(t) + (I_N \otimes B)\tilde{u}(t) \tag{54}$$

where $\tilde{\xi}(t) = \left[\tilde{\xi}_1(t), \dots, \tilde{\xi}_N(t)\right]^T$ and $\tilde{u}(t) = [\tilde{u}_1(t), \dots, \tilde{u}_N(t)]^T$. Now, as happened with the leader agent, it is evident that the dynamics of the followers are influenced by the unknown signal $\tilde{u}(t)$. Therefore, the stability of the closed-loop system must also be guaranteed.

Theorem 4. *For a direct communications graph $\mathcal{G}$, the agents (41) reach the formation $\tilde{H}$ under the protocol (52) if there exists a $P_c > 0$ such that the following inequation holds for $\alpha_1 > \alpha_2 > 0$:*

$$\begin{bmatrix} I_N \otimes P_c\bar{A}^T + \mathcal{L}_1 \otimes W^T B^T + I_N \otimes \bar{A}P_c + \mathcal{L}_1 \otimes BW + I_N \otimes \alpha_1 P_c & I_N \otimes B \\ I_N \otimes B^T & I_N \otimes -\alpha_2 I_N \end{bmatrix} < 0, \tag{55}$$

where the control gain is $K = WP_c^{-1}$, and it satisfies that $Ah_i = 0, \forall i = 1, \dots, N$. Thus, the observer-based control (40) of the follower agent system (41) is exponentially stable, even in the presence of unknown time-varying delays τ_i. Then, the formation described by $\tilde{H}$ can be accomplished for the leader–follower system (41) via the protocol (40).

Proof. Considering the quadratic function $V(\tilde{\xi}) = \tilde{\xi}^T(t)(I_n \otimes P)\tilde{\xi}(t)$, one obtains the following dynamics:

$$\begin{aligned} \dot{V}(\tilde{\xi}) &= \dot{\tilde{\xi}}^T(t)(I_N \otimes P)\tilde{\xi}(t) + \tilde{\xi}^T(t)(I_N \otimes P)\dot{\tilde{\xi}}(t) \\ &= ((I_N \otimes \bar{A} + \mathcal{L}_1 \otimes BK)\tilde{\xi}(t) + (I_N \otimes B)\tilde{u}(t))^T(I_N \otimes P)\xi(t) \\ &\quad + \tilde{\xi}(t)^T(I_N \otimes P)((I_N \otimes \bar{A} + \mathcal{L}_1 \otimes BK)\tilde{\xi}(t) + (I_N \otimes B)\tilde{u}(t)) \\ &= \tilde{\xi}(t)^T((I_N \otimes \bar{A} + \mathcal{L}_1 \otimes BK)^T(I_N \otimes P) + (I_N \otimes P)(I_N \otimes \bar{A} + \mathcal{L}_1 \otimes BK))\tilde{\xi}(t) \\ &\quad + \tilde{u}(t)^T B^T(I_N \otimes P)\tilde{\xi}(t) + \tilde{\xi}(t)^T(I_N \otimes P)B\tilde{u}(t)) \end{aligned} \tag{56}$$

Then, Equation (56) can be expressed in matrix form as

$$\dot{V}(\tilde{\xi}) = \begin{bmatrix} \tilde{\xi}(t) \\ \tilde{u}(t) \end{bmatrix}^T \begin{bmatrix} I_N \otimes \bar{A}^T P + \mathcal{L}_1^T \otimes K^T B^T P + I_N \otimes P\bar{A} + \mathcal{L}_1 \otimes PBK & I_N \otimes PB \\ I_N \otimes B^T P & 0 \end{bmatrix} \begin{bmatrix} \tilde{\xi}(t) \\ \tilde{u}(t) \end{bmatrix} \tag{57}$$

Proceeding as in the stability analysis of the leader system, one can use Proposition 1, and making use of a non-singular transformation, one obtains the following LMI:

$$\begin{bmatrix} I_N \otimes P_c\bar{A}^T + \mathcal{L}_1 \otimes W^T B^T + I_N \otimes \bar{A}P_c + \mathcal{L}_1 \otimes BW + I_N \otimes \alpha_1 P_c & I_N \otimes B \\ I_N \otimes B^T & I_N \otimes -\alpha_2 I_N \end{bmatrix} < 0, \tag{58}$$

where the control gain is $K = WP_c^{-1}$ and the positive constants $\alpha_1 > \alpha_2 > 0$ guarantee the above LMI, which can ensure the exponential stability and the $\mathcal{H}_\infty$ performance of the follower system in the presence of the unknown inputs, being affected by the unknown time-varying delays τ_i. $\square$

4. Simulation Results

A numerical simulation was performed to validate the performance of the control scheme based on an unknown input observer for the formation flight of three UAVs. For this formation, consider the communication graph $\mathcal{G}$ in Figure 2, where node 0 is associated with the leader agent and the other nodes are associated with the follower agents. Then, the Laplacian matrix of the graph $\mathcal{G}$ is defined as follows:

$$\mathcal{L} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \tag{59}$$

Figure 2. Communication graph $\mathcal{G}$.

The formation structure is shown in Figure 3, where the followers (gray) have a distance of $d = 1\,\mathrm{m}$ in the y coordinate from the leader (red). Therefore, because the leader is the origin of the formation, the vector $h_0 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]^T$. For the follower agents, constant vectors that describe the formation structure are $h^1 = -h^2 = [0 \;\; 0 \;\; 0 \;\; 0 \;\; d \;\; 0]^T$.

Figure 3. Formation structure of the UAVs.

Furthermore, the linear dynamic model of altitude and attitude is given by (5), and the control scheme was implemented according to (6)–(9). The parameters used in the simulation are presented in Table 1. The translation movement is described by Equation (15) with input delays as shown in Table 2. The observer-based consensus control for the coordinates x and y was designed according to Equations (40) and (44), the feedback gain matrices $K_1 = -K_2 = [2.5775 \;\; 3.3450 \;\; 1.1890]$ were obtained through Theorem 4 with $\alpha_1 = 6$ and $\alpha_2 = 5$. In addition, the observer gain selected was as follows:

$$\mathbf{L} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2.7 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.81 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2.7 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0.81 \end{bmatrix} \tag{60}$$

Table 1. Parameters employed in the simulations for each agent.

Model		Control			
Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
m [kg]	0.4	k_{iz}	0.09	$k_{p\theta}$	50.88
g [m/s^2]	9.807	k_{pz}	4.29	$k_{d\theta}$	3.97
I_{xxx} [kg·m^2]	0.177	k_{dz}	2.69	$k_{p\psi}$	98.33
I_{yyy} [kg·m^2]	0.177	$k_{p\phi}$	65.18	$k_{d\psi}$	8.27
I_{zzz} [kg·m^2]	0.354	$k_{d\phi}$	3.99		

Table 2. Signal of input delays created for each agent.

Time-Varying Delays	
t	[10 s, 80 s)
$\tau_0(t)$	0.6 + 0.6 sin(0.6 t)
$\tau_1(t)$	0.8 + 0.8 sin(0.6 t)
$\tau_2(t)$	1 + sin(0.6 t)

Moreover, the control law based on an unknown input observer for the leader agent was designed using Equations (17) and (19), the gain matrices $K_0^1 = -K_0^2 = [1.6108 \quad 2.0904 \quad 0.8038]$ were selected according to Theorem 2 with $\mu_1 = 1.5$ and $\mu_2 = 1$. In this agent, the observer gain is the same as in Equation (60).

The desired trajectory r_d, the initial conditions of the agents, and the unknown input observer were as follows:

$$r_0 = [0 \quad 0 \quad 0]^T$$
$$r_1 = [0.15 \quad -0.5 \quad 0]^T$$
$$r_2 = [0.3 \quad 0.8 \quad 0]^T$$
$$\eta_0 = [0 \quad 0 \quad 3\pi/2]^T$$
$$\eta_1 = [0 \quad 0 \quad 3\pi/4]^T$$
$$\eta_2 = [0 \quad 0 \quad 2\pi/5]^T$$
$$w_0 = [0 \quad 0.3 \quad 0.3 \quad 0 \quad 0.2 \quad 0.2]^T$$
$$w_1 = [0 \quad 0.4 \quad 0.1 \quad 0 \quad 0.6 \quad 0.2]^T$$
$$w_2 = [0 \quad 0.3 \quad 0.1 \quad 0 \quad 0.1 \quad 0.3]^T$$

$$r_d(t) = \begin{cases} [0 \quad 0 \quad 1]^T & if & t \leq 10\,\text{s} \\ [0 \quad 1 \quad 1]^T & if & 10\,\text{s} < t \leq 25\,\text{s} \\ [1 \quad 1 \quad 1]^T & if & 25\,\text{s} < t \leq 40\,\text{s} \\ [1 \quad 0 \quad 1]^T & if & 40\,\text{s} < t \leq 55\,\text{s} \\ [0 \quad 0 \quad 1]^T & if & 55\,\text{s} < t \leq 80\,\text{s} \end{cases}$$

Figure 4 shows the trajectories of all agents under the formation structure and Figure 5 shows the time-varying delays trajectories affecting the system. Note that, in this simulation, the linear control law provided good tracking in altitude of all UAVs, as shown in Figure 6, where the errors in z and ψ coordinates are close to zero. These results were because the control scheme was independent of the translational movements.

Tracking errors in x, y, ϕ and θ coordinates are shown in Figure 7. It is appreciated that there are peaks in the errors due to the reference changes and the presence of time-varying delays. However, the proposed scheme caused the error to converge to zero, despite the presence of distinct unknown delays.

Figure 8 illustrates the evolution of the x, y, ϕ, and θ coordinates of the leader vehicle, where it is shown that the tracking performance is quite satisfactory. We can note that when the desired x position changed at $t = 25\,\text{s}$ and $t = 55\,\text{s}$, and the desired y position at $t = 10\,\text{s}$ and $t = 40\,\text{s}$, the effects of the input time-varying delay were visible and caused a small tracking error in both the translation and rotation of the leader vehicle.
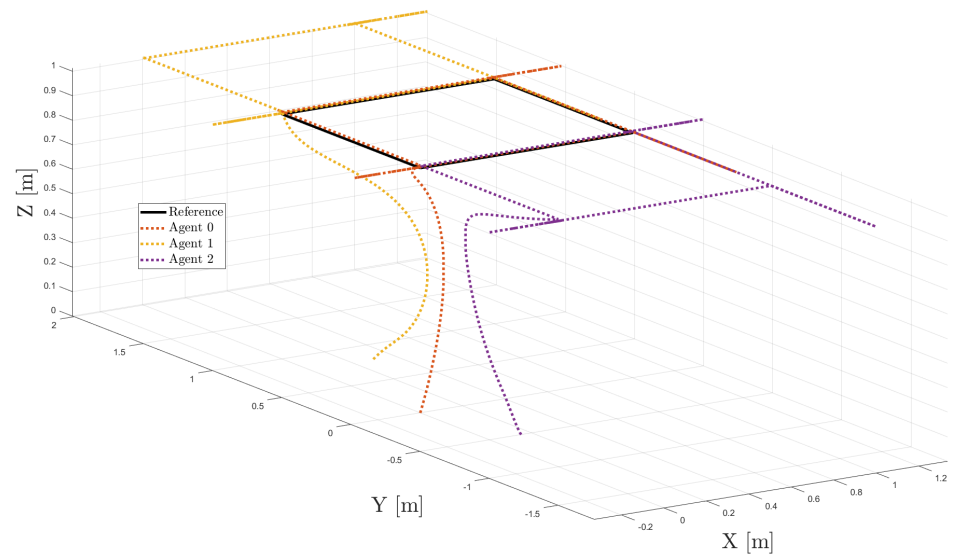
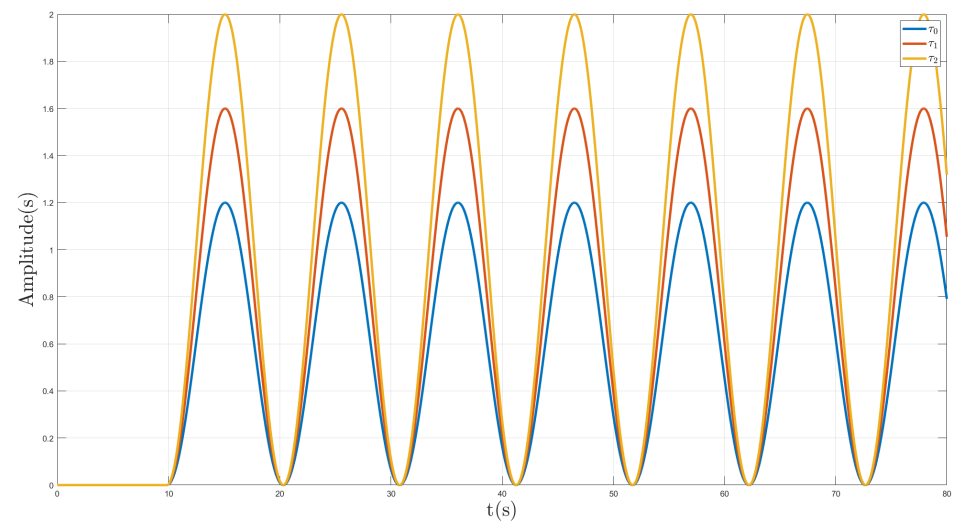**Figure 4.** Trajectories of all agents in formation.
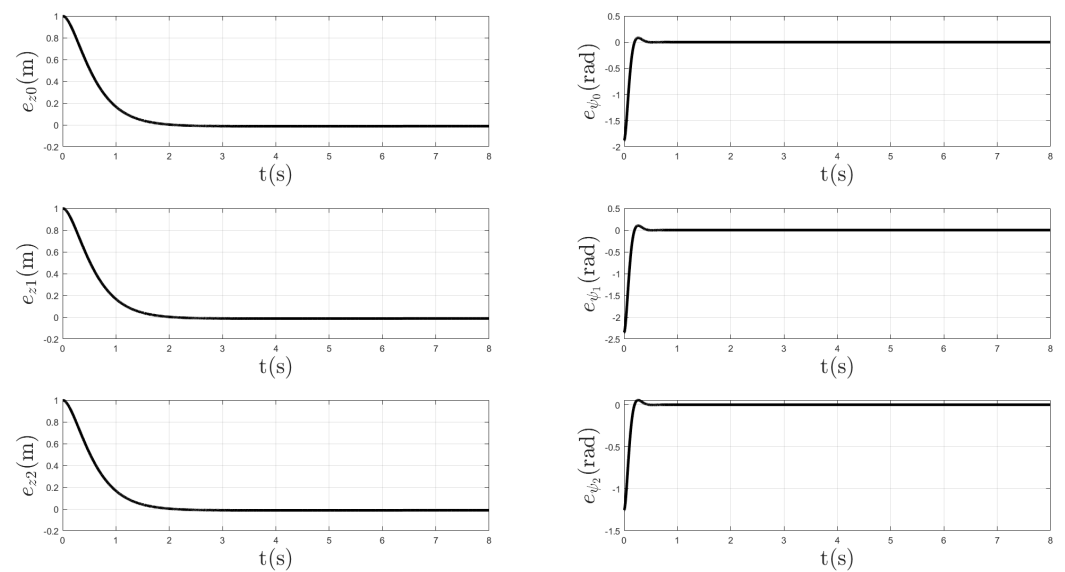

Figure 5. Time-varying delay trajectories.
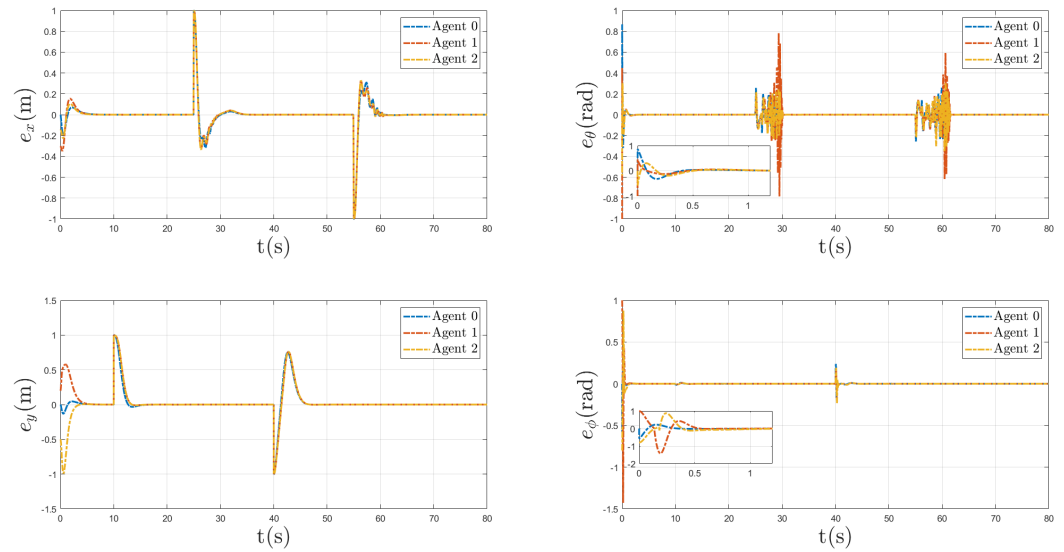

Figure 6. Tracking error of all agents in z and ψ coordinates.

Figure 7. Tracking error of all agents in x, y, ϕ, and θ coordinates.
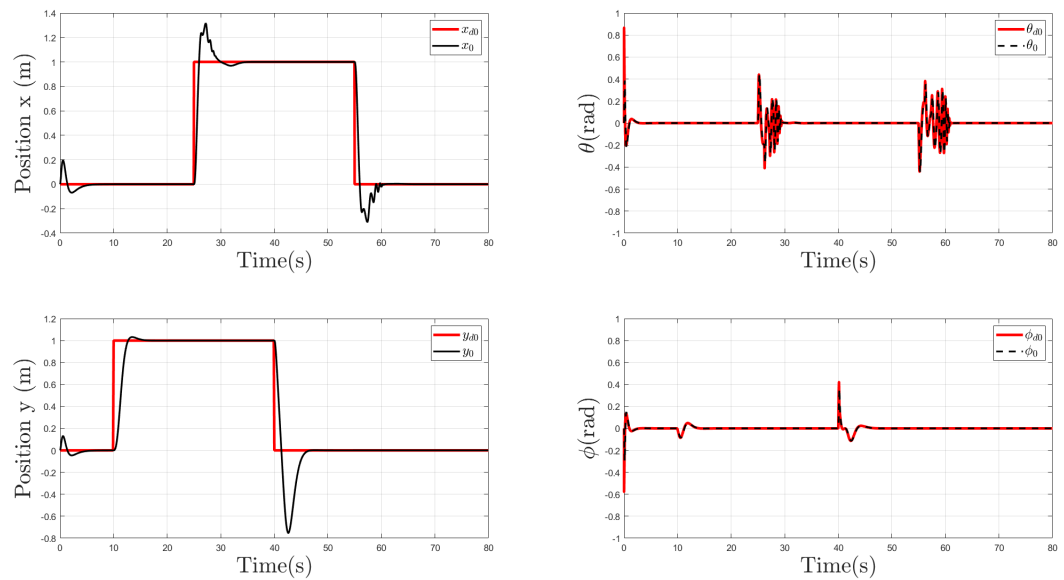

Figure 8. Trajectories of Agent 0 (Leader) in x, y, ϕ and θ coordinates.

Figure 9 shows the behavior of follower Agent 1. We can observe that it is evident that the input time-varying delay impacted the follower more significantly than the leader. The trajectories of x, y, ϕ, and θ remained unaffected, except when the desired position changed. Moreover, the x coordinate was more affected by time-varying delays than the y coordinate; the reason for this behavior was that, during the time intervals when the x coordinate changed position, the functions $\tau_0(t)$, $\tau_1(t)$ and $\tau_2(t)$ had periods of greater amplitude, as shown in Figure 5, thus resulting in a more significant delay in the control input.

Figure 10 illustrates the results of follower Agent 2. Notably, the behavior closely mirrored that of Agent 1, except for the initial part of the simulation, due to the initial conditions. This is characteristic of such control schemes, wherein any signal delay in the leader agent's state inevitably ripples through the entire agent system.

In Figure 11, the estimation errors between $\hat{\xi}^1$ and ξ^1 of all agents are depicted. It is evident that the error approached zero, indicating the convergence of $\hat{\xi}_1$ to ξ_1. Finally, Figure 12 shows the estimation errors between $\hat{\xi}^2$ and ξ^2 of all agents, we can see that the proposed algorithm caused the error to converge to zero; thus, the effectiveness of the designed control schemes based on state estimation was verified by isolating these errors in all agents, despite the presence of distinct unknown delays.
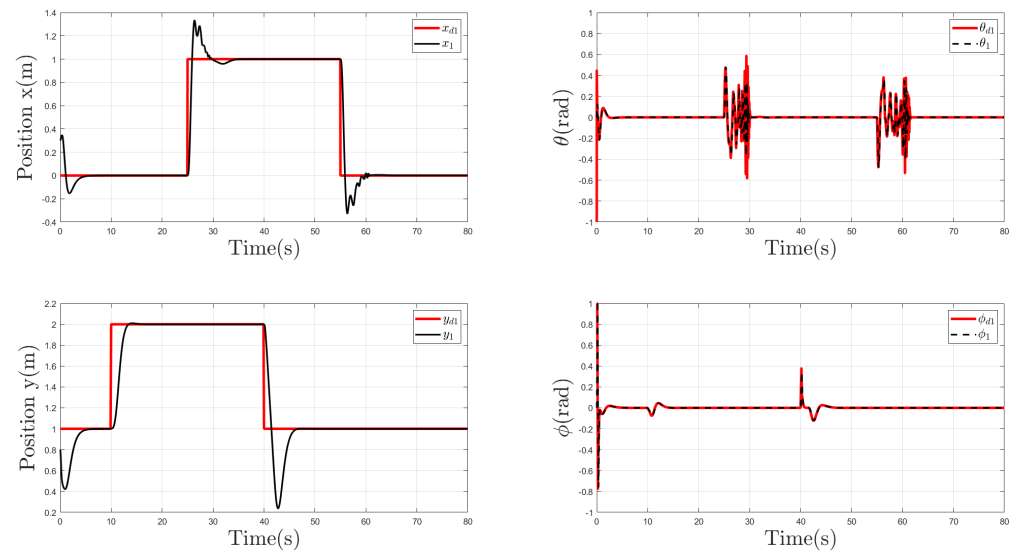
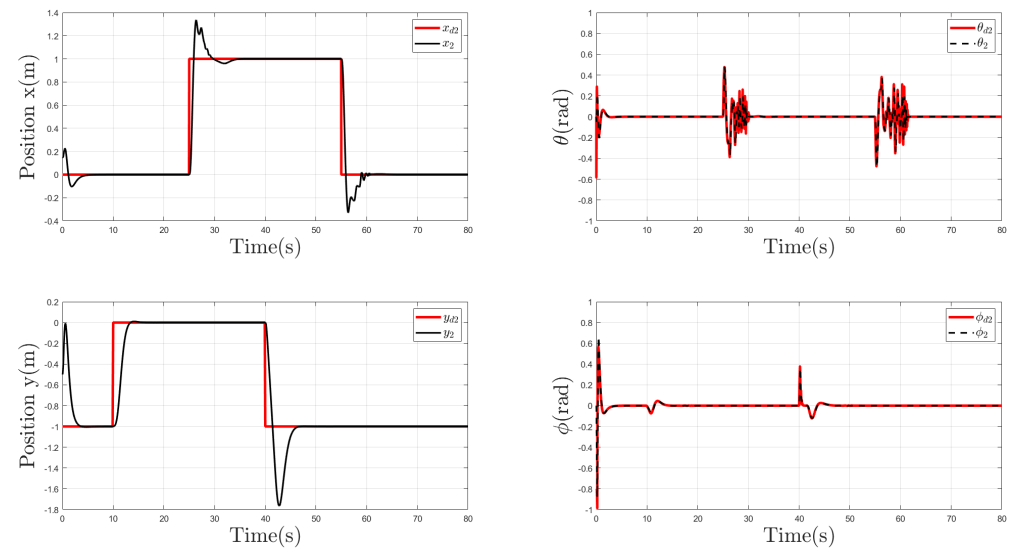**Figure 9.** Trajectories of Agent 1 in x, y, ϕ and θ coordinates.

Figure 10. Trajectories of Agent 2 in x, y, ϕ and θ coordinates.
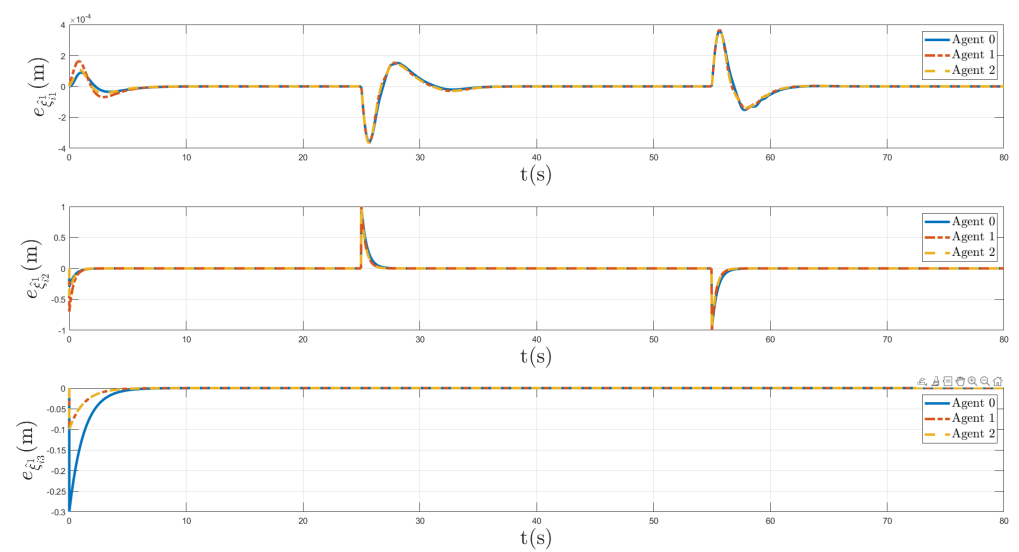

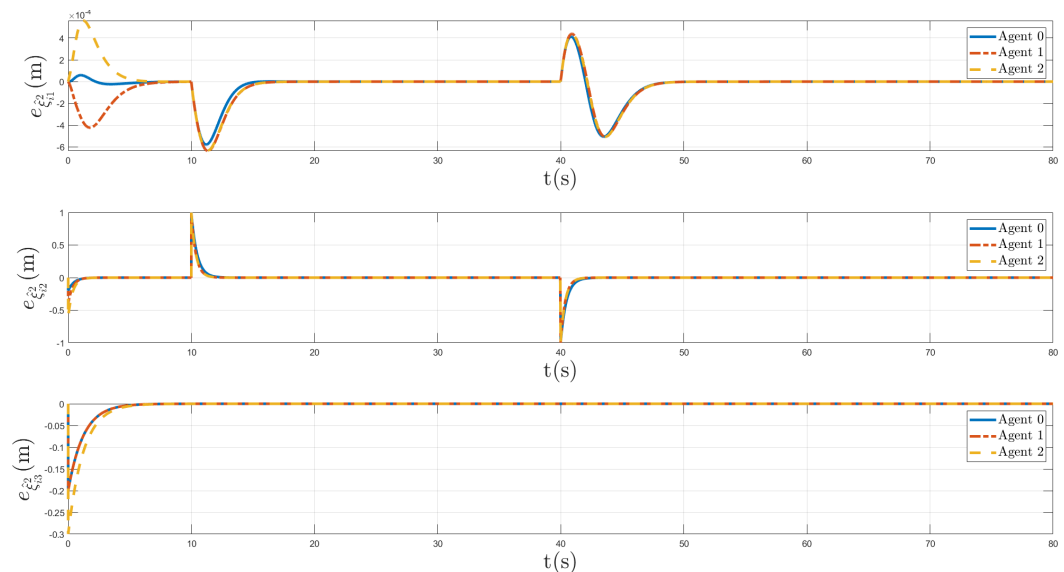**Figure 11.** Error estimation between $\hat{\zeta}^1$ and ζ^1 of all agents.

Figure 12. Error estimation between $\hat{\xi}^2$ and ξ^2 of all agents.

5. Conclusions

This paper addressed an observer-based consensus tracking control algorithm for multiple quadrotor unmanned aerial vehicles under the presence of unknown time-varying delays. The state estimation, consensus, and tracking errors of the closed-loop system converged to small values, and the stability of the proposed strategy was guaranteed using Lyapunov theory for the leader and followers, despite a distinct unknown time-varying delay. Future work includes extending the scheme to a multi-quadrotor nonlinear dynamic model and control, and considering external disturbances, measurement noise, and parametric uncertainty, as well as the case of fault isolation.

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