Proof of Lemma 1: Consumer's indifferent point of buying with mobile coupons or not is $x' = \dfrac{V - p + m}{t}$, so the targeted segment is between x and x', and has a size of $\dfrac{m}{t}$. By maximizing the firm's profit function $px + (p - m - c)(x' - x)$, we obtain $p^* = \dfrac{2V - c}{3}$, $m^* = \dfrac{V - 2c}{3}$, $x = \dfrac{V + c}{3t}$, $x' = \dfrac{2V - c}{3t}$, and $\pi^* = \dfrac{V^2 - Vc + c^2}{3t}$, where $c < \dfrac{V}{2}$ and $\dfrac{2V - c}{3t} < 1$. For $c < \dfrac{V}{2}$ and $t > \dfrac{2V - c}{3}$, we have

(a) $\pi^*\big|_{target} = \dfrac{V^2 - Vc + c^2}{3t} > \pi^*\big|_{non-target} = \dfrac{V^2}{4t}$;

(b) $p^*\big|_{target} = \dfrac{2V - c}{3} > p^*\big|_{non-target} = \dfrac{V}{2}$ and the non-targeted area shrinks for $\dfrac{V + c}{3t} < \dfrac{V}{2t}$;

(c) $d^*\big|_{target} = \dfrac{2V - c}{3t} > d^*\big|_{non-target} = \dfrac{V}{2t}$.

Proof of Lemma 2: Comparing the equilibrium profits between subgames (*NC*, *NC*) and (*C*, *NC*), which are given in Tables 2 and 3, respectively, we have

(a) $\pi_1^*\big|_{(C,N)} > \pi_1^*\big|_{(N,N)}$ always holds if the conditions on the degree of competition and the cost structure are satisfied in Table 3.

(b) For $\dfrac{V}{t} \in (0, \dfrac{6}{7})$ and $c \in (0, \dfrac{V}{2})$, and $\dfrac{V}{t} \in [\dfrac{6}{7}, \dfrac{6}{5}]$ $c \in (0, \dfrac{t}{2})$, we have $\pi_2^*\big|_{(C,N)} = \pi_2^*\big|_{(N,N)}$.

(c) For $\dfrac{V}{t} \in (\dfrac{6}{5}, \dfrac{3}{2})$ and $c \in (0, 3t - 2V)$, we have $\pi_2^*\big|_{(C,N)} < \pi_2^*\big|_{(N,N)}$.

Proof of Corollary 1: Comparing the prices of firm 2 that does not adopt LBMC promotion between subgames (*N*, *N*) and (*NC*, *N*) under the same degree of competition, for $\dfrac{V}{t} \in (\dfrac{6}{7}, 1)$ and $\dfrac{V}{t} \in (\dfrac{6}{5}, \dfrac{3}{2})$, we have $p_2^*\big|_{(C,N)} < p_2^*\big|_{(N,N)}$. This implies that unilateral adoption of LBMC promotion, e.g., only firm 1 adopts mobile coupons leads to firm 2 having a lower price, which results in a fiercer competition in the market.

Proof of Lemma 3: Comparing the equilibrium profits of subgames (*NC*, *NC*) and (*C*, *C*), which are given in Tables 2 and 4, respectively, we have $\pi_i^*\big|_{(C,C)} > \pi_i^*\big|_{(N,N)}$. Comparing the profits between subgames (*C*, *C*) and (*C*, *NC*), we have $\pi_2^*\big|_{(C,C)} > \pi_2^*\big|_{(C,N)}$.

Proof of Proposition 1: Comparing the equilibrium profits among subgames (*NC*, *NC*), (*C*, *NC*), (*NC*, *C*), and (*C*, *C*), we have $\pi_1^*\big|_{(C,N)} > \pi_1^*\big|_{(N,N)}$ and $\pi_2^*\big|_{(N,C)} > \pi_2^*\big|_{(N,N)}$, which motive both firms to adopt LBMC promotion. Furthermore, we have $\pi_1^*\big|_{(C,C)} > \pi_1^*\big|_{(C,N)}$ when $\frac{V}{t} \in (0, \frac{6}{5}]$ and $\pi_1^*\big|_{(C,C)} = \pi_1^*\big|_{(C,N)}$ when $\frac{V}{t} \in (\frac{6}{5}, +\infty)$, i.e., if firm 1's profit is no worse off while firm 2 adopts LBMC promotion. In conclusion, strategy (*C*, *C*) is an SPNE as long as the properties of the cost structure are fulfilled under each scenario. Otherwise, strategy (*NC*, *NC*) is an SPNE.