Entropy **2009**, *11*, 1121-1122; doi:10.3390/e11041121

OPEN ACCESS

entropy

ISSN 1099-4300

www.mdpi.com/journal/entropy

Discussion

Comment on "Equiprobability, Entropy, Gamma Distributions and Other Geometrical Questions in Multi-Agent Systems", Entropy 2009, 11, 959-971

Raúl Toral

IFISC, Instituto de Física Interdisciplinar y Sistemas Complejos, CSIC-UIB, Campus UIB, E-07122 Palma de Mallorca, Spain; E-Mail: raul@ifisc.uib-csic.es; Tel.: +34-971173235

Received: 11 December 2009 / Accepted: 18 December 2009 / Published: 22 December 2009

Abstract: The volume of the body enclosed by the n-dimensional Lamé curve defined by $\sum_{i=1}^{n} x_i^b = E$ is computed.

Keywords: Lamé curves

A recent paper [1] derives asymptotic expressions for the volume of the n-dimensional body defined by $0 \leq \sum_{i=1}^{n} x_i^b \leq E$ for $b > 0$, $x_i \geq 0$. This is the body enclosed by a Lamé curve in n dimensions. Here I compute exactly this volume by using a straightforward modification of the calculation that gives the volume of the n-dimensional sphere, the case $b = 2$, see [2].

Writing $E = R^b$, the volume $V_n(R)$ is

$$V_n(R) = \underset{0\leq\sum_{i=1}^{n} x_i^b \leq R^b}{\int dx_1 \cdots \int dx_n} \tag{1}$$

By dimensional analysis $V_n(R) = C_n R^n$. Let us now compute the integral

$$\int_0^\infty dx_1 \cdots \int_0^\infty dx_n \exp[-(x_1^b + \cdots + x_n^b)] = \left[\int_0^\infty dx \exp[-x^b]\right]^n = \left[\Gamma\left(1+\frac{1}{b}\right)\right]^n \tag{2}$$

by using the change of variables $r = (x_1^b + \cdots + x_n^b)^{1/b}$ and the volume element $dV_n(r) = nC_n r^{n-1} dr$ as

$$\int_0^\infty dV_n(r) \exp[-r^b] = C_n \Gamma\left(1+\frac{n}{b}\right) \tag{3}$$

Equaling these two expressions one gets:

$$V_n(R) = \frac{\left[\Gamma\left(1+\frac{1}{b}\right)\right]^n}{\Gamma\left(1+\frac{n}{b}\right)} R^n \tag{4}$$

which is the desired formula. This validates the results in [1], since it coincides with the approximate calculation of that paper in the asymptotic limit $n \to \infty$ although, as proven here, it turns out to be valid for any value of n.

Acknowledgements

I acknowledge financial support by the MEC (Spain) and FEDER (EU) through project FIS2007-60327.

References

1. López-Ruiz, R.; Sañudo, J.; Calbet X. *Entropy* **2009**, *11*, 959–971.
2. Pathria, R.K. Appendix C. *Statistical Mechanics*, 2nd ed.; Butterwort-Heinemann: Stoneham, MA, USA, 1996.