Supplementary file

Table S1. The relative RMSEs and biases of entropy estimators with MLE and approximate MLE.

Sch.	n	l	r	$\hat{H}$	$\hat{H}_A$	Sch.	n	l	r	$\hat{H}$	$\hat{H}_A$
I	10	1	8	.1779(-.0040)	.1780(-.0089)		40	1	36	.0857(-.0030)	.0875(-.0075)
		2	9	.1795(-.0029)	.1797(-.0078)			3	38	.0854(-.0028)	.0871(-.0072)
		3	10	.1772(.0006)	.1773(-.0036)			5	40	.0849(-.0026)	.0868(-.0067)
		1	6	.1825(.0034)	.1828(-.0018)			1	32	.0870(-.0052)	.0892(-.0100)
		3	8	.1780(-.0034)	.1781(-.0079)			5	36	.0857(-.0030)	.0876(-.0072)
		5	10	.1793(.0015)	.1794(-.0025)			9	40	.0849(-.0026)	.0863(-.0043)
	20	1	18	.1207(-.0072)	.1209(-.0105)		100	1	92	.0553(-.0009)	.0567(-.0066)
		2	19	.1195(-.0059)	.1195(-.0061)			5	96	.0551(-.0008)	.0565(-.0064)
		3	20	.1182(-.0053)	.1182(-.0056)			9	100	.0551(-.0008)	.0565(-.0064)
		1	16	.1208(-.0082)	.1209(-.0083)			1	84	.0561(-.0020)	.0579(-.0078)
		3	18	.1207(-.0072)	.1207(-.0074)			9	92	.0553(-.0009)	.0567(-.0066)
		5	20	.1181(-.0052)	.1181(-.0057)			17	100	.0551(-.0007)	.0569(-.0041)
	40	1	36	.0857(-.0031)	.0858(-.0033)	III	10	1	8	.1733(-.0081)	.1762(-.0104)
		3	38	.0854(-.0028)	.0854(-.0030)			2	9	.1735(-.0097)	.1764(-.0122)
		5	40	.0850(-.0027)	.0850(-.0028)			3	10	.1715(-.0059)	.1744(-.0086)
		1	32	.0870(-.0052)	.0872(-.0055)			1	6	.1825(.0034)	.1853(.0011)
		5	36	.0857(-.0031)	.0857(-.0032)			3	8	.1732(-.0075)	.1761(-.0104)
		9	40	.0850(-.0026)	.0849(-.0028)			5	10	.1733(-.0053)	.1762(-.0081)
	100	1	92	.0552(-.0009)	.0553(-.0013)		20	1	18	.1177(-.0100)	.1178(-.0102)
		5	96	.0550(-.0008)	.0550(-.0010)			2	19	.1163(-.0083)	.1164(-.0085)
		9	100	.0550(-.0008)	.0550(-.0009)			3	20	.1136(-.0066)	.1136(-.0068)
		1	84	.0560(-.0020)	.0561(-.0023)			1	16	.1186(-.0097)	.1187(-.0099)
		9	92	.0552(-.0009)	.0552(-.0010)			3	18	.1176(-.0101)	.1176(-.0103)
		17	100	.0551(-.0007)	.0551(-.0008)			5	20	.1135(-.0065)	.1135(-.0069)
II	10	1	8	.1779(-.0040)	.1831(-.0087)		40	1	36	.0832(-.0058)	.0833(-.0060)
		2	9	.1796(-.0029)	.1845(-.0073)			3	38	.0824(-.0045)	.0825(-.0046)
		3	10	.1772(.0006)	.1815(-.0032)			5	40	.0808(-.0035)	.0808(-.0036)
		1	6	.1825(.0034)	.1875(-.0013)			1	32	.0864(-.0057)	.0865(-.0060)
		3	8	.1780(-.0034)	.1824(-.0074)			5	36	.0832(-.0058)	.0832(-.0060)
		5	10	.1793(.0015)	.1824(-.0016)			9	40	.0808(-.0034)	.0808(-.0036)
	20	1	18	.1206(-.0071)	.1228(-.0105)		100	1	92	.0541(-.0021)	.0542(-.0024)
		2	19	.1194(-.0058)	.1215(-.0092)			5	96	.0529(-.0010)	.0530(-.0013)
		3	20	.1182(-.0053)	.1202(-.0083)			9	100	.0526(-.0009)	.0526(-.0009)
		1	16	.1207(-.0081)	.1231(-.0117)			1	84	.0555(-.0024)	.0556(-.0027)
		3	18	.1206(-.0071)	.1228(-.0103)			9	92	.0541(-.0021)	.0542(-.0021)
		5	20	.1181(-.0052)	.1194(-.0069)			17	100	.0527(-.0010)	.0527(-.0009)

Table S2. The relative RMSEs and biases of entropy estimators with Bayes estimator based on non-informative prior under squared error loss and general entropy loss function.

Sch.	n	l	r	$\tilde{\tilde{H}}_{S_1}$	$\tilde{\tilde{H}}_{E_1}$	Sch.	n	l	r	$\tilde{\tilde{H}}_{S_1}$	$\tilde{\tilde{H}}_{E_1}$
I	10	1	8	.1804(-.0219)	.1870(-.0650)		40	1	36	.0858(-.0065)	.0868(-.0173)
		2	9	.1817(-.0202)	.1872(-.0620)			3	38	.0854(-.0063)	.0864(-.0170)
		3	10	.1789(-.0166)	.1831(-.0579)			5	40	.0850(-.0061)	.0859(-.0168)
		1	6	.1834(-.0156)	.1872(-.0617)			1	32	.0881(-.0087)	.0889(-.0205)
		3	8	.1804(-.0214)	.1869(-.0646)			5	36	.0858(-.0065)	.0868(-.0173)
		5	10	.1808(-.0168)	.1850(-.0586)			9	40	.0850(-.0061)	.0859(-.0167)
	20	1	18	.1210(-.0145)	.1241(-.0358)		100	1	92	.0610(-.0008)	.0629(-.0062)
		2	19	.1195(-.0132)	.1223(-.0343)			5	96	.0609(-.0004)	.0628(-.0060)
		3	20	.1182(-.0126)	.1208(-.0337)			9	100	.0610(-.0004)	.0630(-.0057)
		1	16	.1212(-.0159)	.1251(-.0382)			1	84	.0612(-.0006)	.0632(-.0065)
		3	18	.1209(-.0146)	.1240(-.0359)			9	92	.0610(-.0009)	.0631(-.0062)
		5	20	.1181(-.0126)	.1207(-.0337)			17	100	.0611(-.0007)	.0632(-.0062)
	40	1	36	.0857(-.0067)	.0868(-.0175)	III	10	1	8	.1768(-.0255)	.1854(-.0676)
		3	38	.0854(-.0065)	.0863(-.0172)			2	9	.1772(-.0261)	.1853(-.0657)
		5	40	.0850(-.0063)	.0859(-.0170)			3	10	.1745(-.0217)	.1807(-.0599)
		1	32	.0873(-.0091)	.0889(-.0205)			1	6	.1835(-.0156)	.1874(-.0617)
		5	36	.0857(-.0067)	.0868(-.0175)			3	8	.1767(-.0250)	.1852(-.0673)
		9	40	.0849(-.0062)	.0858(-.0169)			5	10	.1763(-.0220)	.1825(-.0607)
	100	1	92	.0611(-.0007)	.0630(-.0060)		20	1	18	.1183(-.0171)	.1225(-.0374)
		5	96	.0608(-.0005)	.0627(-.0059)			2	19	.1168(-.0152)	.1203(-.0350)
		9	100	.0609(-.0005)	.0629(-.0058)			3	20	.1139(-.0134)	.1170(-.0330)
		1	84	.0612(-.0005)	.0631(-.0063)			1	16	.1192(-.0174)	.1237(-.0394)
		9	92	.0610(-.0007)	.0632(-.0060)			3	18	.1182(-.0171)	.1224(-.0374)
		17	100	.0611(-.0005)	.0631(-.0058)			5	20	.1138(-.0133)	.1169(-.0329)
II	10	1	8	.1805(-.0219)	.1871(-.0651)		40	1	36	.0835(-.0093)	.0852(-.0196)
		2	9	.1819(-.0202)	.1873(-.0620)			3	38	.0826(-.0079)	.0839(-.0179)
		3	10	.1790(-.0166)	.1831(-.0579)			5	40	.0809(-.0068)	.0821(-.0168)
		1	6	.1835(-.0156)	.1873(-.0618)			1	32	.0866(-.0096)	.0884(-.0209)
		3	8	.1805(-.0214)	.1870(-.0646)			5	36	.0835(-.0093)	.0852(-.0196)
		5	10	.1808(-.0168)	.1850(-.0586)			9	40	.0809(-.0068)	.0820(-.0167)
	20	1	18	.1209(-.0145)	.1240(-.0358)		100	1	92	.0552(-.0003)	.0584(-.0068)
		2	19	.1195(-.0132)	.1222(-.0343)			5	96	.0551(-.0014)	.0561(-.0051)
		3	20	.1182(-.0126)	.1208(-.0337)			9	100	.0550(-.0012)	.0560(-.0049)
		1	16	.1211(-.0159)	.1250(-.0382)			1	84	.0561(-.0015)	.0571(-.0068)
		3	18	.1209(-.0145)	.1240(-.0358)			9	92	.0561(-.0015)	.0571(-.0068)
		5	20	.1181(-.0126)	.1207(-.0336)			17	100	.0548(-.0012)	.0559(-.0049)

Table S3. The relative RMSEs and biases of entropy estimators with Bayes estimator based on non-informative prior under balanced loss function.

Sch.	n	l	r	$\tilde{H}_{BS_1}$			$\tilde{H}_{BE_1}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
I	10	1	8	.1781(-.0163)	.1775(-.0127)	.1774(-.0092)	.1820(-.0477)	.1795(-.0358)	.1779(-.0235)
		2	9	.1795(-.0148)	.1790(-.0113)	.1790(-.0079)	.1827(-.0452)	.1805(-.0337)	.1792(-.0217)
		3	10	.1769(-.0112)	.1765(-.0078)	.1765(-.0044)	.1791(-.0413)	.1773(-.0299)	.1763(-.0181)
		1	6	.1817(-.0097)	.1814(-.0059)	.1816(-.0021)	.1832(-.0433)	.1815(-.0306)	.1808(-.0174)
		3	8	.1781(-.0158)	.1776(-.0122)	.1775(-.0086)	.1820(-.0473)	.1795(-.0353)	.1779(-.0230)
		5	10	.1788(-.0110)	.1784(-.0074)	.1785(-.0038)	.1710(-.0416)	.1791(-.0299)	.1782(-.0177)
	20	1	18	.1208(-.0123)	.1208(-.0108)	.1208(-.0094)	.1224(-.0274)	.1216(-.0217)	.1210(-.0159)
		2	19	.1195(-.0110)	.1194(-.0095)	.1194(-.0081)	.1207(-.0259)	.1200(-.0203)	.1196(-.0146)
		3	20	.1182(-.0104)	.1181(-.0090)	.1181(-.0075)	.1193(-.0254)	.1187(-.0197)	.1183(-.0140)
		1	16	.1210(-.0136)	.1210(-.0120)	.1209(-.0105)	.1231(-.0294)	.1221(-.0234)	.1213(-.0174)
		3	18	.1208(-.0124)	.1207(-.0109)	.1207(-.0094)	.1223(-.0274)	.1215(-.0217)	.1210(-.0160)
		5	20	.1181(-.0104)	.1181(-.0089)	.1181(-.0074)	.1193(-.0253)	.1186(-.0196)	.1182(-.0139)
	40	1	36	.0857(-.0056)	.0857(-.0049)	.0857(-.0042)	.0862(-.0132)	.0859(-.0103)	.0858(-.0074)
		3	38	.0854(-.0054)	.0854(-.0047)	.0854(-.0039)	.0858(-.0129)	.0856(-.0101)	.0854(-.0072)
		5	40	.0850(-.0052)	.0850(-.0045)	.0850(-.0038)	.0854(-.0127)	.0851(-.0099)	.0850(-.0070)
		1	32	.0872(-.0080)	.0871(-.0072)	.0871(-.0064)	.0881(-.0160)	.0876(-.0129)	.0873(-.0099)
		5	36	.0857(-.0056)	.0857(-.0049)	.0857(-.0042)	.0862(-.0132)	.0859(-.0103)	.0858(-.0074)
		9	40	.0849(-.0051)	.0849(-.0044)	.0849(-.0037)	.0853(-.0127)	.0851(-.0098)	.0850(-.0069)
	100	1	92	.0600(-.0006)	.0592(-.0006)	.0590(-.0007)	.0625(-.0047)	.0621(-.0037)	.0615(-.0027)
		5	96	.0602(-.0005)	.0600(-.0005)	.0591(-.0006)	.0624(-.0045)	.0620(-.0035)	.0614(-.0025)
		9	100	.0605(-.0005)	.0602(-.0005)	.0600(-.0006)	.0626(-.0045)	.0623(-.0035)	.0620(-.0025)
		1	84	.0608(-.0007)	.0600(-.0010)	.0595(-.0013)	.0627(-.0051)	.0624(-.0042)	.0620(-.0034)
		9	92	.0610(-.0006)	.0608(-.0006)	.0600(-.0007)	.0627(-.0047)	.0625(-.0037)	.0621(-.0026)
		17	100	.0607(-.0005)	.0604(-.0005)	.0602(-.0006)	.0628(-.0045)	.0625(-.0035)	.0622(-.0025)
II	10	1	8	.1782(-.0163)	.1776(-.0127)	.1775(-.0092)	.1822(-.0478)	.1796(-.0359)	.1779(-.0235)
		2	9	.1797(-.0148)	.1792(-.0113)	.1791(-.0079)	.1829(-.0452)	.1807(-.0337)	.1793(-.0217)
		3	10	.1770(-.0112)	.1765(-.0077)	.1766(-.0043)	.1792(-.0413)	.1774(-.0299)	.1763(-.0180)
		1	6	.1817(-.0097)	.1814(-.0059)	.1816(-.0021)	.1832(-.0434)	.1815(-.0306)	.1808(-.0175)
		3	8	.1782(-.0158)	.1776(-.0121)	.1776(-.0086)	.1821(-.0473)	.1796(-.0353)	.1779(-.0230)
		5	10	.1788(-.0110)	.1784(-.0074)	.1785(-.0038)	.1810(-.0416)	.1792(-.0299)	.1782(-.0177)
	20	1	18	.1208(-.0123)	.1207(-.0108)	.1207(-.0093)	.1223(-.0273)	.1215(-.0216)	.1210(-.0159)
		2	19	.1194(-.0110)	.1194(-.0095)	.1194(-.0080)	.1207(-.0259)	.1200(-.0203)	.1196(-.0145)
		3	20	.1182(-.0104)	.1181(-.0089)	.1181(-.0075)	.1193(-.0253)	.1187(-.0197)	.1183(-.0140)
		1	16	.1210(-.0135)	.1209(-.0120)	.1208(-.0104)	.1230(-.0293)	.1220(-.0234)	.1213(-.0173)
		3	18	.1207(-.0123)	.1207(-.0108)	.1207(-.0093)	.1223(-.0274)	.1215(-.0217)	.1209(-.0159)
		5	20	.1180(-.0104)	.1180(-.0089)	.1180(-.0074)	.1192(-.0253)	.1185(-.0196)	.1181(-.0139)

Table S3. Cont.

Sch.	n	l	r	$\tilde{H}_{BS_1}$			$\tilde{H}_{BE_1}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
	40	1	36	.0857(-.0055)	.0857(-.0048)	.0857(-.0041)	.0862(-.0130)	.0859(-.0102)	.0857(-.0073)
		3	38	.0854(-.0053)	.0854(-.0046)	.0853(-.0039)	.0858(-.0128)	.0855(-.0099)	.0854(-.0071)
		5	40	.0850(-.0051)	.0849(-.0044)	.0849(-.0037)	.0853(-.0126)	.0851(-.0098)	.0850(-.0069)
		1	32	.0876(-.0076)	.0873(-.0067)	.0871(-.0062)	.0881(-.0159)	.0876(-.0129)	.0873(-.0098)
		5	36	.0857(-.0055)	.0857(-.0048)	.0857(-.0041)	.0862(-.0130)	.0859(-.0102)	.0857(-.0073)
		9	40	.0850(-.0050)	.0849(-.0043)	.0849(-.0036)	.0853(-.0125)	.0851(-.0097)	.0849(-.0069)
	100	1	92	.0599(-.0007)	.0592(-.0007)	.0588(-.0009)	.0626(-.0050)	.0623(-.0040)	.0614(-.0030)
		5	96	.0603(-.0008)	.0601(-.0006)	.0592(-.0008)	.0623(-.0047)	.0621(-.0039)	.0615(-.0035)
		9	100	.0606(-.0009)	.0603(-.0008)	.0601(-.0007)	.0627(-.0049)	.0624(-.0037)	.0621(-.0028)
		1	84	.0609(-.0010)	.0601(-.0012)	.0596(-.0013)	.0628(-.0055)	.0625(-.0045)	.0621(-.0040)
		9	92	.0611(-.0007)	.0609(-.0008)	.0601(-.0009)	.0626(-.0049)	.0624(-.0039)	.0622(-.0029)
		17	100	.0608(-.0008)	.0605(-.0007)	.0603(-.0007)	.0627(-.0050)	.0624(-.0039)	.0621(-.0035)
III	10	1	8	.1742(-.0200)	.1734(-.0165)	.1731(-.0131)	.1797(-.0507)	.1767(-.0391)	.1743(-.0271)
		2	9	.1746(-.0209)	.1738(-.0176)	.1735(-.0144)	.1800(-.0498)	.1771(-.0388)	.1748(-.0275)
		3	10	.1722(-.0168)	.1715(-.0136)	.1713(-.0105)	.1761(-.0445)	.1738(-.0339)	.1720(-.0230)
		1	6	.1817(-.0097)	.1814(-.0059)	.1816(-.0021)	.1833(-.0433)	.1816(-.0306)	.1808(-.0174)
		3	8	.1741(-.0195)	.1734(-.0160)	.1731(-.0126)	.1796(-.0503)	.1765(-.0386)	.1743(-.0266)
		5	10	.1739(-.0168)	.1733(-.0134)	.1730(-.0101)	.1779(-.0450)	.1755(-.0341)	.1737(-.0229)
	20	1	18	.1181(-.0149)	.1180(-.0135)	.1178(-.0121)	.1205(-.0293)	.1194(-.0239)	.1185(-.0184)
		2	19	.1166(-.0131)	.1165(-.0118)	.1164(-.0104)	.1185(-.0271)	.1176(-.0218)	.1169(-.0165)
		3	20	.1137(-.0113)	.1137(-.0100)	.1136(-.0086)	.1153(-.0252)	.1145(-.0119)	.1140(-.0146)
		1	16	.1190(-.0151)	.1188(-.0135)	.1187(-.0120)	.1215(-.0307)	.1203(-.0248)	.1194(-.0188)
		3	18	.1180(-.0150)	.1179(-.0136)	.1177(-.0122)	.1204(-.0294)	.1193(-.0239)	.1184(-.0184)
		5	20	.1137(-.0112)	.1136(-.0099)	.1136(-.0085)	.1153(-.0251)	.1145(-.0199)	.1139(-.0146)
	40	1	36	.0834(-.0083)	.0833(-.0076)	.0833(-.0069)	.0843(-.0155)	.0839(-.0128)	.0836(-.0100)
		3	38	.0825(-.0069)	.0825(-.0062)	.0825(-.0055)	.0832(-.0139)	.0829(-.0112)	.0826(-.0085)
		5	40	.0809(-.0058)	.0809(-.0051)	.0809(-.0045)	.0815(-.0128)	.0812(-.0102)	.0810(-.0075)
		1	32	.0865(-.0084)	.0865(-.0077)	.0864(-.0069)	.0875(-.0164)	.0871(-.0134)	.0867(-.0103)
		5	36	.0834(-.0083)	.0833(-.0076)	.0833(-.0069)	.0844(-.0155)	.0839(-.0128)	.0836(-.0100)
		9	40	.0808(-.0057)	.0808(-.0051)	.0808(-.0044)	.0814(-.0127)	.0811(-.0101)	.0809(-.0074)
	100	1	92	.0550(-.0007)	.0550(-.0010)	.0548(-.0015)	.0567(-.0055)	.0561(-.0046)	.0557(-.0037)
		5	96	.0550(-.0013)	.0550(-.0012)	.0549(-.0011)	.0543(-.0039)	.0536(-.0031)	.0532(-.0023)
		9	100	.0550(-.0011)	.0548(-.0010)	.0547(-.0009)	.0543(-.0037)	.0535(-.0030)	.0530(-.0021)
		1	84	.0553(-.0017)	.0550(-.0017)	.0545(-.0019)	.0560(-.0054)	.0553(-.0045)	.0547(-.0035)
		9	92	.0560(-.0016)	.0558(-.0017)	.0558(-.0018)	.0560(-.0054)	.0553(-.0045)	.0547(-.0035)
		17	100	.0547(-.0011)	.0545(-.0010)	.0543(-.0009)	.0541(-.0037)	.0533(-.0030)	.0529(-.0021)

Table S4. The relative RMSEs and biases of entropy estimators with Bayes estimator based on natural conjugate prior under squared error loss and general entropy loss function.

Sch.	n	l	r	$\tilde{H}_{S_2}$	$\tilde{H}_{E_2}$	Sch.	n	l	r	$\tilde{H}_{S_2}$	$\tilde{H}_{E_2}$
I	10	1	8	.1594(-.0178)	.1653(-.0564)		40	1	36	.0833(-.0063)	.0843(-.0168)
		2	9	.1608(-.0165)	.1658(-.0540)			3	38	.0830(-.0061)	.0839(-.0166)
		3	10	.1588(-.0136)	.1627(-.0507)			5	40	.0826(-.0059)	.0835(-.0164)
		1	6	.1614(-.0129)	.1651(-.0539)			1	32	.0846(-.0086)	.0862(-.0198)
		3	8	.1593(-.0174)	.1652(-.0561)			5	36	.0833(-.0063)	.0843(-.0168)
		5	10	.1602(-.0136)	.1640(-.0512)			9	40	.0826(-.0059)	.0834(-.0163)
	20	1	18	.1138(-.0133)	.1168(-.0334)		100	1	92	.0560(-.0021)	.0569(-.0057)
		2	19	.1126(-.0121)	.1153(-.0321)			5	96	.0559(-.0019)	.0566(-.0060)
		3	20	.1115(-.0117)	.1140(-.0316)			9	100	.0560(-.0015)	.0569(-.0058)
		1	16	.1139(-.0145)	.1175(-.0355)			1	84	.0557(-.0021)	.0561(-.0038)
		3	18	.1138(-.0134)	.1168(-.0335)			9	92	.0561(-.0022)	.0564(-.0060)
		5	20	.1114(-.0116)	.1139(-.0315)			17	100	.0560(-.0017)	.0570(-.0058)
	40	1	36	.0832(-.0064)	.0842(-.0169)	III	10	1	8	.1566(-.0207)	.1642(-.0585)
		3	38	.0829(-.0062)	.0838(-.0166)			2	9	.1573(-.0212)	.1645(-.0570)
		5	40	.0825(-.0060)	.0834(-.0164)			3	10	.1557(-.0179)	.1613(-.0525)
		1	32	.0846(-.0087)	.0862(-.0197)			1	6	.1616(-.0128)	.1654(-.0538)
		5	36	.0832(-.0064)	.0842(-.0169)			3	8	.1565(-.0203)	.1640(-.0582)
		9	40	.0825(-.0060)	.0834(-.0164)			5	10	.1570(-.0180)	.1626(-.0531)
	100	1	92	.0561(-.0018)	.0569(-.0055)		20	1	18	.1116(-.0155)	.1155(-.0348)
		5	96	.0558(-.0018)	.0565(-.0055)			2	19	.1103(-.0139)	.1136(-.0327)
		9	100	.0559(-.0018)	.0568(-.0055)			3	20	.1078(-.0123)	.1107(-.0309)
		1	84	.0557(-.0020)	.0560(-.0041)			1	16	.1121(-.0158)	.1163(-.0366)
		9	92	.0562(-.0018)	.0565(-.0055)			3	18	.1115(-.0156)	.1154(-.0349)
		17	100	.0561(-.0018)	.0570(-.0055)			5	20	.1077(-.0122)	.1106(-.0309)
II	10	1	8	.1594(-.0178)	.1654(-.0565)		40	1	36	.0811(-.0089)	.0827(-.0189)
		2	9	.1609(-.0164)	.1659(-.0540)			3	38	.0803(-.0075)	.0816(-.0173)
		3	10	.1588(-.0136)	.1627(-.0507)			5	40	.0788(-.0066)	.0799(-.0162)
		1	6	.1614(-.0129)	.1652(-.0539)			1	32	.0840(-.0091)	.0857(-.0201)
		3	8	.1594(-.0173)	.0165(-.0561)			5	36	.0811(-.0089)	.0827(-.0189)
		5	10	.1602(-.0136)	.1640(-.0512)			9	40	.0787(-.0065)	.0798(-.0162)
	20	1	18	.1138(-.0133)	.1168(-.0334)		100	1	92	.0543(-.0020)	.0545(-.0068)
		2	19	.1126(-.0121)	.1152(-.0321)			5	96	.0539(-.0020)	.0541(-.0052)
		3	20	.1115(-.0116)	.1140(-.0315)			9	100	.0539(-.0020)	.0541(-.0048)
		1	16	.1138(-.0145)	.1175(-.0355)			1	84	.0542(-.0020)	.0553(-.0041)
		3	18	.1137(-.0133)	.1167(-.0335)			9	92	.0540(-.0020)	.0543(-.0068)
		5	20	.1113(-.0116)	.1138(-.0315)			17	100	.0537(-.0007)	.0539(-.0048)

Table S5. The relative RMSEs and biases of entropy estimators with Bayes estimator based on natural conjugate prior under balanced loss function.

Sch.	n	l	r	$\tilde{H}_{BS_2}$			$\tilde{H}_{BE_2}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
I	10	1	8	.1637(-.0135)	.1673(-.0107)	.1714(-.0080)	.1665(-.0416)	.1684(-.0314)	.1712(-.0208)
		2	9	.1652(-.0122)	.1689(-.0094)	.1729(-.0068)	.1674(-.0395)	.1695(-.0295)	.1725(-.0192)
		3	10	.1631(-.0091)	.1667(-.0063)	.1707(-.0035)	.1645(-.0362)	.1668(-.0262)	.1700(-.0158)
		1	6	.1665(-.0078)	.1707(-.0045)	.1752(-.0013)	.1673(-.0377)	.1701(-.0265)	.1739(-.0150)
		3	8	.1637(-.0130)	.1674(-.0101)	.1714(-.0074)	.1664(-.0412)	.1683(-.0309)	.1712(-.0202)
		5	10	.1646(-.0089)	.1684(-.0058)	.1726(-.0028)	.1660(-.0363)	.1683(-.0260)	.1717(-.0154)
	20	1	18	.1159(-.0114)	.1173(-.0102)	.1186(-.0090)	.1173(-.0257)	.1179(-.0205)	.1188(-.0152)
		2	19	.1147(-.0103)	.1160(-.0090)	.1174(-.0078)	.1158(-.0244)	.1165(-.0192)	.1175(-.0139)
		3	20	.1134(-.0098)	.1148(-.0085)	.1161(-.0072)	.1145(-.0239)	.1152(-.0187)	.1162(-.0134)
		1	16	.1159(-.0126)	.1173(-.0113)	.1187(-.0101)	.1177(-.0275)	.1183(-.0221)	.1191(-.0166)
		3	18	.1158(-.0115)	.1172(-.0103)	.1186(-.0090)	.1172(-.0258)	.1178(-.0205)	.1188(-.0152)
		5	20	.1134(-.0097)	.1147(-.0084)	.1161(-.0071)	.1144(-.0238)	.1151(-.0186)	.1161(-.0133)
	40	1	36	.0839(-.0054)	.0844(-.0047)	.0849(-.0041)	.0844(-.0128)	.0846(-.0100)	.0850(-.0072)
		3	38	.0836(-.0052)	.0841(-.0045)	.0846(-.0039)	.0840(-.0125)	.0843(-.0098)	.0847(-.0070)
		5	40	.0832(-.0050)	.0837(-.0044)	.0842(-.0037)	.0836(-.0124)	.0839(-.0096)	.0842(-.0068)
		1	32	.0853(-.0077)	.0858(-.0070)	.0863(-.0063)	.0861(-.0154)	.0863(-.0125)	.0865(-.0096)
		5	36	.0840(-.0054)	.0844(-.0047)	.0849(-.0041)	.0844(-.0128)	.0847(-.0100)	.0850(-.0073)
		9	40	.0832(-.0049)	.0837(-.0043)	.0842(-.0036)	.0836(-.0123)	.0838(-.0095)	.0842(-.0068)
	100	1	92	.0562(-.0012)	.0562(-.0009)	.0563(-.0008)	.0570(-.0042)	.0570(-.0033)	.0571(-.0024)
		5	96	.0559(-.0011)	.0560(-.0009)	.0562(-.0007)	.0566(-.0041)	.0567(-.0032)	.0568(-.0023)
		9	100	.0560(-.0012)	.0561(-.0009)	.0561(-.0007)	.0569(-.0041)	.0570(-.0032)	.0572(-.002)
		1	84	.0558(-.0020)	.0559(-.0019)	.0560(-.0019)	.0562(-.0044)	.0564(-.0038)	.0565(-.0031)
		9	92	.0564(-.0012)	.0565(-.0009)	.0566(-.0008)	.0566(-.0042)	.0567(-.0033)	.0568(-.0024)
		17	100	.0562(-.0012)	.0563(-.0009)	.0563(-.0007)	.0570(-.0041)	.0572(-.0032)	.0574(-.002)
II	10	1	8	.1637(-.0135)	.1674(-.0107)	.1714(-.0080)	.1666(-.0416)	.1684(-.0314)	.1712(-.0208)
		2	9	.1653(-.0122)	.1690(-.0094)	.1731(-.0067)	.1675(-.0395)	.1696(-.0295)	.1727(-.0192)
		3	10	.1631(-.0091)	.1668(-.0062)	.1708(-.0034)	.1645(-.0362)	.1668(-.0261)	.1700(-.0158)
		1	6	.1666(-.0078)	.1707(-.0045)	.1753(-.0013)	.1674(-.0377)	.1701(-.0265)	.1739(-.0150)
		3	8	.1637(-.0129)	.1674(-.0101)	.1715(-.0074)	.1665(-.0412)	.1684(-.0309)	.1712(-.0202)
		5	10	.1646(-.0089)	.1684(-.0058)	.1726(-.0028)	.1660(-.0363)	.1683(-.0260)	.1717(-.0154)
	20	1	18	.1158(-.0114)	.1172(-.0102)	.1185(-.0089)	.1172(-.0257)	.1178(-.0204)	.1187(-.0151)
		2	19	.1146(-.0102)	.1160(-.0090)	.1173(-.0077)	.1157(-.0244)	.1164(-.0192)	.1174(-.0139)
		3	20	.1134(-.0097)	.1148(-.0084)	.1161(-.0072)	.1145(-.0238)	.1152(-.0186)	.1162(-.0133)
		1	16	.1159(-.0126)	.1172(-.0113)	.1186(-.0100)	.1177(-.0275)	.1182(-.0220)	.1190(-.0165)
		3	18	.1158(-.0114)	.1171(-.0102)	.1185(-.0090)	.1172(-.0257)	.1178(-.0205)	.1187(-.0152)
		5	20	.1133(-.0097)	.1147(-.0084)	.1160(-.0071)	.1144(-.0238)	.1151(-.0186)	.1161(-.0133)

Table S5. Cont.

Sch.	n	l	r	$\tilde{H}_{BS_2}$			$\tilde{H}_{BE_2}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
	40	1	36	.0840(-.0053)	.0844(-.0046)	.0849(-.0040)	.0844(-.0127)	.0846(-.0100)	.0850(-.0072)
		3	38	.0837(-.0051)	.0841(-.0044)	.0846(-.0038)	.0840(-.0125)	.0843(-.0097)	.0846(-.0070)
		5	40	.0833(-.0049)	.0837(-.0043)	.0842(-.0036)	.0836(-.0123)	.0839(-.0096)	.0842(-.0068)
		1	32	.0853(-.0076)	.0858(-.0069)	.0863(-.0062)	.0862(-.0155)	.0863(-.0126)	.0865(-.0096)
		5	36	.0840(-.0053)	.0844(-.0046)	.0849(-.0040)	.0844(-.0127)	.0847(-.0100)	.0850(-.0072)
		9	40	.0833(-.0049)	.0837(-.0042)	.0842(-.0035)	.0836(-.0122)	.0839(-.0095)	.0842(-.0067)
	100	1	92	.0561(-.0014)	.0560(-.0012)	.0561(-.0010)	.0571(-.0049)	.0571(-.0040)	.0572(-.0034)
		5	96	.0560(-.0015)	.0561(-.0011)	.0563(-.0011)	.0567(-.0047)	.0568(-.0042)	.0569(-.0028)
		9	100	.0560(-.0017)	.0562(-.0011)	.0562(-.0012)	.0569(-.0048)	.0571(-.0030)	.0573(-.003)
		1	84	.0559(-.0022)	.0560(-.0019)	.0561(-.0019)	.0563(-.0050)	.0564(-.0032)	.0566(-.0038)
		9	92	.0563(-.0018)	.0564(-.0010)	.0565(-.0020)	.0565(-.0038)	.0568(-.0037)	.0569(-.0030)
		17	100	.0561(-.0015)	.0564(-.0012)	.0564(-.0025)	.0571(-.0045)	.0571(-.0039)	.0573(-.002)
III	10	1	8	.1604(-.0167)	.1637(-.0142)	.1673(-.0117)	.1646(-.0442)	.1658(-.0343)	.1678(-.0242)
		2	9	.1610(-.0176)	.1642(-.0152)	.1678(-.0130)	.1651(-.0436)	.1664(-.0343)	.1684(-.0247)
		3	10	.1594(-.0141)	.1625(-.0117)	.1659(-.0093)	.1623(-.0392)	.1638(-.0301)	.1660(-.0207)
		1	6	.1667(-.0078)	.1708(-.0045)	.1753(-.0013)	.1675(-.0377)	.1702(-.0265)	.1740(-.0149)
		3	8	.1603(-.0162)	.1636(-.0136)	.1673(-.0111)	.1644(-.0438)	.1656(-.0339)	.1678(-.0236)
		5	10	.1608(-.0140)	.1640(-.0114)	.1675(-.0089)	.1636(-.0395)	.1652(-.0301)	.1676(-.0205)
	20	1	18	.1134(-.0139)	.1146(-.0128)	.1158(-.0117)	.1155(-.0275)	.1158(-.0226)	.1164(-.0176)
		2	19	.1121(-.1220)	.1133(-.0111)	.1145(-.0100)	.1138(-.0255)	.1142(-.0207)	.1149(-.0158)
		3	20	.1095(-.0106)	.1106(-.0094)	.1118(-.0083)	.1109(-.0237)	.1114(-.0189)	.1121(-.0140)
		1	16	.1140(-.0140)	.1153(-.0127)	.1166(-.0115)	.1163(-.0287)	.1166(-.0234)	.1171(-.0179)
		3	18	.1133(-.0139)	.1145(-.0128)	.1157(-.0117)	.1154(-.0276)	.1157(-.0226)	.1163(-.0176)
		5	20	.1094(-.0105)	.1106(-.0093)	.1118(-.0082)	.1108(-.0237)	.1113(-.0188)	.1120(-.0139)
	40	1	36	.0817(-.0080)	.0822(-.0074)	.0826(-.0067)	.0826(-.0150)	.0827(-.0124)	.0828(-.0098)
		3	38	.0809(-.0066)	.0813(-.0060)	.0818(-.0054)	.0816(-.0135)	.0817(-.0109)	.0819(-.0084)
		5	40	.0794(-.0056)	.0798(-.0050)	.0802(-.0044)	.0799(-.0124)	.0801(-.0099)	.0803(-.0073)
		1	32	.0847(-.0081)	.0852(-.0074)	.0856(-.0068)	.0856(-.0159)	.0857(-.0130)	.0859(-.0101)
		5	36	.0817(-.0088)	.0822(-.0074)	.0826(-.0067)	.0826(-.0150)	.0827(-.0124)	.0828(-.0098)
		9	40	.0793(-.0055)	.0797(-.0049)	.0801(-.0043)	.0799(-.0124)	.0800(-.0098)	.0802(-.0072)
	100	1	92	.0544(-.0020)	.0545(-.0021)	.0547(-.0022)	.0554(-.0058)	.0556(-.0050)	.0560(-.0041)
		5	96	.0540(-.0017)	.0543(-.0015)	.0545(-.0013)	.0542(-.0047)	.0543(-.0040)	.0545(-.0031)
		9	100	.0543(-.0022)	.0544(-.0023)	.0546(-.0024)	.0539(-.0044)	.0540(-.0037)	.0542(-.0029)
		1	84	.0542(-.0020)	.0543(-.0020)	.0544(-.0020)	.0546(-.0046)	.0546(-.0040)	.0547(-.0034)
		9	92	.0542(-.0020)	.0545(-.0020)	.0550(-.0020)	.0544(-.0057)	.0544(-.0050)	.0543(-.0041)
		17	100	.0537(-.0006)	.0539(-.0006)	.0540(-.0006)	.0544(-.0044)	.0545(-.0037)	.0547(-.0029)

Table S6. The relative Kullback-Leibler divergence of entropy estimators with MLE and approximate MLE

Sch.	n	l	r	$\hat{H}$	$\hat{H}_A$	Sch.	n	l	r	$\hat{H}$	$\hat{H}_A$
I	10	1	8	.034957	.036723		40	1	36	.007931	.008348
		2	9	.035545	.037277			3	38	.007867	.008269
		3	10	.034444	.036194			5	40	.007776	.008182
		1	6	.036492	.038270			1	32	.008210	.008712
		3	8	.034940	.036692			5	36	.007932	.008365
		5	10	.035220	.037011			9	40	.007774	.008065
	20	1	18	.016001	.016039		100	1	92	.003267	.003473
		2	19	.015619	.015643			5	96	.003236	.003438
		3	20	.015202	.015208			9	100	.003236	.003439
		1	16	.016049	.016090			1	84	.003369	.003623
		3	18	.015977	.015986			9	92	.003264	.003472
		5	20	.015187	.015194			17	100	.003248	.003471
	40	1	36	.007933	.007955	III	10	1	8	.033479	.035258
		3	38	.007872	.007887			2	9	.033603	.035362
		5	40	.007780	.007782			3	10	.032618	.034399
		1	32	.008217	.008243			1	6	.036493	.038270
		5	36	.007937	.007939			3	8	.033433	.035211
		9	40	.007776	.007775			5	10	.033261	.035080
	100	1	92	.003257	.003266		20	1	18	.015333	.015371
		5	96	.003225	.003233			2	19	.014934	.014959
		9	100	.003228	.003230			3	20	.014084	.014092
		1	84	.003360	.003372			1	16	.015556	.015596
		9	92	.003259	.003260			3	18	.015305	.015318
		17	100	.003242	.003242			5	20	.014071	.014081
II	10	1	8	.034991	.038061		40	1	36	.007530	.007552
		2	9	.035623	.038620			3	38	.007375	.007390
		3	10	.034484	.037093			5	40	.007054	.007056
		1	6	.036516	.039579			1	32	.008102	.008127
		3	8	.034992	.037705			5	36	.007531	.007535
		5	10	.035217	.037241			9	40	.007042	.007043
	20	1	18	.015968	.016689		100	1	92	.003138	.003149
		2	19	.015601	.016276			5	96	.002989	.002997
		3	20	.015198	.015828			9	100	.002952	.002953
		1	16	.016022	.016808			1	84	.003303	.003316
		3	18	.015964	.016682			9	92	.003140	.003142
		5	20	.015170	.015606			17	100	.002964	.002964

Table S7. The relative Kullback-Leibler divergence of entropy estimators with Bayes estimator based on non-informative prior under squared error loss and general entropy loss function.

Sch.	n	l	r	$\tilde{H}_{S_1}$	$\tilde{H}_{E_1}$	Sch.	n	l	r	$\tilde{H}_{S_1}$	$\tilde{H}_{E_1}$
I	10	1	8	.037551	.043082		40	1	36	.007998	.008331
		2	9	.038049	.043124			3	38	.007929	.008234
		3	10	.036703	.041047			5	40	.007835	.008132
		1	6	.038585	.043122			1	32	.008442	.008766
		3	8	.037513	.043014			5	36	.008000	.008332
		5	10	.037521	.041990			9	40	.007832	.008128
	20	1	18	.016252	.017671		100	1	92	.004644	.005645
		2	19	.015824	.017097			5	96	.004599	.005684
		3	20	.015388	.016601			9	100	.004605	.005687
		1	16	.016352	.017982			1	84	.004844	.005843
		3	18	.016229	.017649			9	92	.004647	.005660
		5	20	.015372	.016583			17	100	.004624	.005696
	40	1	36	.007989	.008325	III	10	1	8	.036355	.042494
		3	38	.007920	.008229			2	9	.036529	.042453
		5	40	.007826	.008127			3	10	.035237	.040107
		1	32	.008308	.008766			1	6	.038601	.043170
		5	36	.007993	.008328			3	8	.036293	.042408
		9	40	.007820	.008120			5	10	.035969	.040997
	100	1	92	.004632	.005632		20	1	18	.015678	.017310
		5	96	.004589	.005679			2	19	.015216	.016628
		9	100	.004596	.005678			3	20	.014310	.015550
		1	84	.004836	.005830			1	16	.015907	.017663
		9	92	.004639	.005630			3	18	.015651	.017285
		17	100	.004617	.005689			5	20	.014297	.015535
II	10	1	8	.037614	.043182		40	1	36	.007623	.008053
		2	9	.038145	.043250			3	38	.007446	.007800
		3	10	.036751	.041113			5	40	.007112	.007425
		1	6	.038637	.043210			1	32	.008201	.008682
		3	8	.037575	.043096			5	36	.007624	.008054
		5	10	.037548	.042022			9	40	.007099	.007412
	20	1	18	.016224	.017644		100	1	92	.003532	.003608
		2	19	.015810	.017085			5	96	.003273	.003381
		3	20	.015387	.016602			9	100	.003235	.003344
		1	16	.016329	.017963			1	84	.004889	.004662
		3	18	.016220	.017642			9	92	.003532	.003610
		5	20	.015356	.016568			17	100	.003242	.003351

Table S8. The relative Kullback-Leibler divergence of entropy estimators with Bayes estimator based on non-informative prior under balanced loss function.

Sch.	n	l	r	$\tilde{H}_{BS_1}$			$\tilde{H}_{BE_1}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
I	10	1	8	.035742	.035284	.035060	.039457	.037543	.036064
		2	9	.036294	.035850	.035636	.039715	.037924	.036549
		3	10	.035027	.034632	.034465	.037943	.036349	.035167
		1	6	.036942	.036579	.036449	.039826	.038186	.037030
		3	8	.035715	.035260	.035038	.039397	.037492	.036026
		5	10	.035780	.035384	.035224	.038774	.037131	.035922
	20	1	18	.016164	.016111	.016064	.016986	.016615	.016314
		2	19	.015750	.015707	.015668	.016472	.016140	.015878
		3	20	.015320	.015280	.015245	.016001	.015684	.015438
		1	16	.016247	.016184	.016126	.017201	.016774	.016425
		3	18	.016140	.016088	.016040	.016963	.016592	.016291
		5	20	.015304	.015265	.015230	.015982	.015666	.015421
	40	1	36	.007970	.007958	.007947	.008161	.008073	.008004
		3	38	.007903	.007893	.007884	.008076	.007996	.007933
		5	40	.007809	.007800	.007791	.007977	.007899	.007838
		1	32	.008277	.008258	.008241	.008549	.008429	.008329
		5	36	.007973	.007961	.007951	.008164	.008077	.008007
		9	40	.007804	.007794	.007786	.007970	.007893	.007832
	100	1	92	.003932	.003617	.003389	.004964	.004133	.003621
		5	96	.003894	.003581	.003355	.004978	.004129	.003604
		9	100	.003900	.003586	.003359	.004978	.004130	.003606
		1	84	.004312	.003970	.003647	.004524	.004412	.003754
		9	92	.003937	.003621	.003392	.004964	.004133	.003622
		17	100	.003923	.003597	.003367	.004996	.004141	.003625
II	10	1	8	.035800	.035335	.035104	.039535	.037608	.036117
		2	9	.036389	.035941	.035722	.039825	.038025	.036641
		3	10	.035077	.034680	.034510	.038001	.036402	.035216
		1	6	.036988	.036619	.036483	.039892	.038240	.037072
		3	8	.035777	.035320	.035095	.039470	.037559	.036086
		5	10	.035781	.035382	.035221	.038791	.037140	.035924
	20	1	18	.016134	.016081	.016032	.016956	.016584	.016283
		2	19	.015735	.015691	.015651	.016458	.016125	.015861
		3	20	.015318	.015278	.015242	.015999	.015682	.015435
		1	16	.016223	.016159	.016100	.017179	.016750	.016399
		3	18	.016130	.016077	.016028	.016954	.016581	.016279
		5	20	.015288	.015248	.015213	.015967	.015650	.015404

Table S8. Cont.

Sch.	n	l	r	$\tilde{H}_{BS_1}$			$\tilde{H}_{BE_1}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
	40	1	36	.007971	.007956	.007943	.008159	.008070	.007999
		3	38	.007903	.007889	.007878	.008073	.007990	.007926
		5	40	.007810	.007796	.007786	.007974	.007893	.007832
		1	32	.008345	.008289	.008244	.008549	.008429	.008329
		5	36	.007972	.007957	.007945	.008161	.008071	.008001
		9	40	.007808	.007794	.007784	.007971	.007891	.007829
	100	1	92	.003943	.003625	.003399	.004973	.004148	.003632
		5	96	.003907	.003594	.003369	.004988	.004137	.003618
		9	100	.003914	.003597	.003367	.004983	.004140	.003614
		1	84	.004327	.003989	.003658	.004531	.004425	.003764
		9	92	.003941	.003633	.003404	.004977	.004140	.003639
		17	100	.003930	.003604	.003374	.005003	.004158	.003632
III	10	1	8	.034486	.033967	.033679	.038658	.036581	.034920
		2	9	.034688	.034154	.033843	.038766	.036748	.035107
		3	10	.033531	.033052	.032790	.036884	.035153	.033780
		1	6	.036954	.036588	.036455	.039862	.038214	.037049
		3	8	.034432	.033915	.033629	.038577	.036507	.034855
		5	10	.034192	.033701	.033434	.037650	.035856	.034438
	20	1	18	.015563	.015491	.015424	.016550	.016121	.015757
		2	19	.015120	.015062	.015008	.015962	.015592	.015282
		3	20	.014232	.014184	.014141	.014954	.014630	.014366
		1	16	.015787	.015715	.015647	.016835	.016374	.015988
		3	18	.015536	.015464	.015397	.016524	.016095	.015730
		5	20	.014218	.014171	.014128	.014939	.014615	.014351
	40	1	36	.007592	.007573	.007555	.007854	.007741	.007644
		3	38	.007422	.007407	.007394	.007633	.007540	.007462
		5	40	.007092	.007080	.007069	.007274	.007193	.007126
		1	32	.008167	.008147	.008128	.008456	.008330	.008224
		5	36	.007594	.007574	.007556	.007855	.007742	.007646
		9	40	.007080	.007068	.007057	.007261	.007180	.007113
	100	1	92	.003356	.003256	.003179	.003415	.003308	.003222
		5	96	.003126	.003061	.003018	.003173	.003085	.003029
		9	100	.003088	.003023	.002980	.003137	.003049	.002992
		1	84	.004326	.003958	.003612	.004467	.004386	.004334
		9	92	.003357	.003257	.003180	.003417	.003311	.003224
		17	100	.003095	.003030	.002997	.003144	.003056	.003009

Table S9. The relative Kullback-Leibler divergence of entropy estimators with Bayes estimator based on natural conjugate prior under squared error loss and general entropy loss function.

Sch.	n	l	r	$\tilde{H}_{S_2}$	$\tilde{H}_{E_2}$	Sch.	n	l	r	$\tilde{H}_{S_2}$	$\tilde{H}_{E_2}$
I	10	1	8	.028587	.032605		40	1	36	.007527	.007838
		2	9	.029063	.032763			3	38	.007468	.007754
		3	10	.028239	.031433			5	40	.007383	.007663
		1	6	.029211	.032526			1	32	.007796	.008229
		3	8	.028550	.032537			5	36	.007528	.007840
		5	10	.028736	.031982			9	40	.007381	.007659
	20	1	18	.014312	.015540		100	1	92	.003559	.004019
		2	19	.013974	.015082			5	96	.003554	.003975
		3	20	.013618	.014677			9	100	.003557	.003977
		1	16	.014352	.015755			1	84	.003981	.003695
		3	18	.014291	.015521			9	92	.003552	.004020
		5	20	.013602	.014659			17	100	.003570	.003996
	40	1	36	.007514	.007826	III	10	1	8	.027763	.032219
		3	38	.007457	.007746			2	9	.028018	.032341
		5	40	.007371	.007653			3	10	.027346	.030946
		1	32	.007793	.008217			1	6	.029249	.032583
		5	36	.007517	.007830			3	8	.027706	.032137
		9	40	.007366	.007646			5	10	.027791	.031469
	100	1	92	.003549	.004007		20	1	18	.013840	.015250
		5	96	.003549	.003961			2	19	.013490	.014717
		9	100	.003546	.003968			3	20	.012751	.013834
		1	84	.003976	.003686			1	16	.013976	.015485
		9	92	.003545	.004013			3	18	.013817	.015228
		17	100	.003563	.003989			5	20	.012738	.013819
II	10	1	8	.028604	.032639		40	1	36	.007180	.007580
		2	9	.029106	.032817			3	38	.007028	.007359
		3	10	.028253	.031453			5	40	.006723	.007016
		1	6	.029219	.032550			1	32	.007695	.008141
		3	8	.028573	.032568			5	36	.007181	.007581
		5	10	.028746	.031994			9	40	.006711	.007033
	20	1	18	.014286	.015515		100	1	92	.003392	.003439
		2	19	.013961	.015069			5	96	.003046	.003243
		3	20	.013616	.014676			9	100	.003005	.003206
		1	16	.014331	.015735			1	84	.003549	.004070
		3	18	.014282	.015513			9	92	.003397	.003444
		5	20	.013588	.014645			17	100	.003012	.003213

Table S10. The relative Kullback-Leibler divergence of entropy estimators with Bayes estimator based on natural conjugate prior under balanced loss function.

Sch.	n	l	r	$\tilde{H}_{BS_2}$			$\tilde{H}_{BE_2}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
I	10	1	8	.029756	.031031	.032506	.032304	.032522	.033119
		2	9	.030286	.031581	.033071	.032622	.032935	.033615
		3	10	.029387	.030627	.032060	.031356	.031713	.032439
		1	6	.030649	.032100	.033755	.032550	.033056	.034004
		3	8	.029729	.031008	.032485	.032245	.032472	.033081
		5	10	.029933	.031232	.032731	.031920	.032307	.033084
	20	1	18	.014800	.015135	.015476	.015505	.015562	.015686
		2	19	.014450	.014775	.015108	.015070	.015142	.015281
		3	20	.014076	.014390	.014710	.014661	.014731	.014868
		1	16	.014843	.015178	.015522	.015658	.015679	.015772
		3	18	.014779	.015112	.015453	.015484	.015540	.015663
		5	20	.014060	.014374	.014694	.014643	.014714	.014851
	40	1	36	.007637	.007720	.007804	.007813	.007826	.007856
		3	38	.007578	.007661	.007744	.007739	.007755	.007789
		5	40	.007490	.007572	.007654	.007646	.007663	.007697
		1	32	.007916	.008000	.008086	.008168	.008158	.008167
		5	36	.007640	.007723	.007807	.007816	.007829	.007859
		9	40	.007485	.007567	.007649	.007639	.007657	.007691
	100	1	92	.003398	.003328	.003282	.003624	.003461	.003347
		5	96	.003387	.003310	.003258	.003583	.003423	.003312
		9	100	.003386	.003310	.003259	.003589	.003428	.003316
		1	84	.003587	.003411	.003351	.003503	.003428	.003383
		9	92	.003397	.003328	.003282	.003629	.003465	.003350
		17	100	.003399	.003331	.003265	.003599	.003445	.003338
II	10	1	8	.029784	.031061	.032537	.032339	.032558	.033155
		2	9	.030343	.031644	.033140	.032684	.033002	.033687
		3	10	.029412	.030657	.032094	.031383	.031744	.032475
		1	6	.030666	.032119	.033777	.032575	.033081	.034030
		3	8	.029764	.031047	.032529	.032283	.032514	.033127
		5	10	.029932	.031229	.032728	.031925	.032309	.033083
	20	1	18	.014772	.015106	.015446	.015477	.015532	.015655
		2	19	.014435	.014760	.015091	.015055	.015127	.015265
		3	20	.014073	.014387	.014706	.014659	.014728	.014865
		1	16	.014819	.015154	.015496	.015635	.015655	.015746
		3	18	.014768	.015102	.015441	.015474	.015529	.015651
		5	20	.014045	.014358	.014678	.014628	.014698	.014835

Table S10. Cont.

Sch.	n	l	r	$\tilde{H}_{BS_2}$			$\tilde{H}_{BE_2}$		
				$w=0.3$	$w=0.5$	$w=0.7$	$w=0.3$	$w=0.5$	$w=0.7$
	40	1	36	.007641	.007720	.007802	.007817	.007826	.007854
		3	38	.007580	.007658	.007740	.007739	.007752	.007784
		5	40	.007494	.007571	.007650	.007648	.007661	.007693
		1	32	.007913	.007995	.008079	.008172	.008158	.008164
		5	36	.007642	.007721	.007803	.007818	.007827	.007855
		9	40	.007492	.007569	.007649	.007645	.007658	.007690
	100	1	92	.003402	.003337	.003295	.003637	.003473	.003352
		5	96	.003396	.003321	.003267	.003596	.003434	.003327
		9	100	.003399	.003324	.003269	.003599	.003437	.003327
		1	84	.003597	.003423	.003362	.003514	.003435	.003393
		9	92	.003404	.003335	.003299	.003636	.003472	.003420
		17	100	.003406	.003348	.003272	.003606	.003452	.003345
III	10	1	8	.028763	.029906	.031242	.031677	.031699	.032067
		2	9	.029015	.030129	.031428	.031884	.031927	.032286
		3	10	.028287	.029337	.030563	.030637	.030771	.031210
		1	6	.030676	.032120	.033768	.032596	.033092	.034029
		3	8	.028713	.029857	.031194	.031599	.031628	.032004
		5	10	.028764	.029855	.031129	.031155	.031302	.031769
	20	1	18	.014272	.014568	.014869	.015124	.015110	.015154
		2	19	.013908	.014194	.014486	.014636	.014650	.014721
		3	20	.013137	.013401	.013670	.013764	.013785	.013861
		1	16	.014432	.014745	.015064	.015329	.015308	.015354
		3	18	.014248	.014542	.014843	.015100	.015085	.015128
		5	20	.013124	.013388	.013657	.013749	.013770	.013847
	40	1	36	.007282	.007352	.007422	.007525	.007507	.007505
		3	38	.007129	.007198	.007268	.007325	.007321	.007332
		5	40	.006819	.006885	.006952	.006989	.006990	.007004
		1	32	.007814	.007894	.007976	.008081	.008063	.008064
		5	36	.007283	.007353	.007423	.007526	.007509	.007506
		9	40	.006807	.006873	.006940	.006976	.006977	.006992
	100	1	92	.003242	.003177	.003140	.003238	.003170	.003138
		5	96	.003083	.002983	.002924	.003187	.003102	.003018
		9	100	.002997	.002996	.002995	.003201	.003056	.003023
		1	84	.003406	.003337	.003296	.003995	.003885	.003705
		9	92	.003246	.003180	.003143	.003341	.003273	.003170
		17	100	.003004	.003003	.003002	.003218	.003063	.003030