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Consensus of Second Order Multi-Agent Systems with Exogenous Disturbance Generated by Unknown Exosystems

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Abstract: This paper is concerned with consensus problem of a class of second-order multi-agent systems subjecting to external disturbance generated from some unknown exosystems. In comparison with the case where the disturbance is generated from some known exosystems, we need to combine adaptive control and internal model design to deal with the external disturbance generated from the unknown exosystems. With the help of the internal model, an adaptive protocol is proposed for the consensus problem of the multi-agent systems. Finally, one numerical example is provided to demonstrate the effectiveness of the control design.

Keywords: consensus; multi-agent systems; internal model; disturbance; unknown exosystems

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1. Introduction

The consensus problem of multi-agent systems has received increasing attention in recent years due to its broad applications in such areas as cooperative control of unmanned aircrafts and underwater vehicles, flocking of mobile vehicles, communication among wireless sensor networks, rendezvous, formation control, and so on, see [1–15]. In the past years, many researches have been firstly concerned with consensus problems of first order multi-agent systems [16–20]. In [16], the authors proposed a systematic framework to study the consensus problem of first-order multi-agent systems and showed that the consensus can be achieved if the diagraph is strongly connected. In [17], the authors extended the results obtained in [16] and further presented some improved conditions for state agreement under dynamically changing directed topology. In [18], the authors discussed average consensus problem by using a linear matrix inequality method in undirected networks of dynamic agents with fixed and switching topologies as well as multiple time-varying communication delays.

Recently, the consensus problem of second order multi-agent systems has received increasing attention due to the fact that second order dynamics can be used to model more complicated processes in reality [21–26]. In reality, many practical individual systems, especially mechanical systems, can be presented as second-order multi-agent systems; for instance, networks of mass-spring systems[27], coupled pendulum systems[28], harmonic oscillators [29] and frequency control of power systems [30]. In [21], the authors pointed out that the existence of a directed spanning tree is a necessary rather than a sufficient condition to reach the second order consensus. In [22], the authors discussed the consensus problems for undirected networks of point mass dynamic agents with fixed or switching topology. In [23], the authors proposed a Lyapunov-based approach to consider multi-agent systems with switching jointly connected interconnection. In [24], the authors presented some necessary and sufficient conditions for second order consensus in multi-agent dynamical systems. In [25], the authors studied the exponential second order consensus problem of a network of inertial agents using passive decomposition approach with time-varying coupling delays and variable balanced topologies.

However, there are few results that have considered the second order consensus problem for multi-agent systems with exogenous disturbance [31,32]. In [31], by using linear matrix inequality method, the authors studied the consensus problem of second order multi-agent systems with exogenous disturbances generated from linear exogenous system under the assumption that the coefficient matrix of the exogenous system can be used for designing a disturbance observer, and a disturbance observer based protocol was proposed to achieve consensus for the second order multi-agent systems. In [32], by using the input-to-state stability and dynamic gain technique, Zhang et al. further investigated the consensus problem of second order multi-agent systems with exogenous disturbances generated from linear exogenous system and nonlinear exogenous system, respectively.

Nevertheless, the case when consensus problem of multi-agent systems with exogenous disturbance generated from linear unknown exogenous system seems more realistic and has greater practical significance [33–35]. In this paper, we will consider the consensus problem of second order multi-agent systems with exogenous disturbance generated from linear unknown exogenous system. It is worth noting that, unlike [31,32], since the disturbances are generated from some linear unknown exogenous systems and the information of the coefficient matrix of the exogenous system can not be used for designing of disturbance observer and feedback control, we cannot apply the approaches developed in [31,32] to solve the present problem. Meanwhile, the method that used in [34] to solve the problem of asymptotic rejection of unknown sinusoidal disturbances can not be used directly to tackle the consensus problem of multi-agent system, because the multi-agent system is multi-input and multi-output. Therefore, to overcome this difficulty, we need to develop a different technique.

The remainder of this paper is organized as follows. In Section 2, some preliminaries are briefly reviewed and the problem formulation is presented. Some internal models, which are used to deal with the disturbances generated from some linear unknown exosystems, are designed in Section 3. Based on the internal models proposed in Section 3, an adaptive consensus protocol is presented for the second order multi-agent systems in Section 4. In Section 5, an example will be given to illustrate our design. Finally, the conclusions are drawn in Section 6.

2. Preliminaries and Problem Formulation

Assuming that each agent can be viewed as a node, and the interaction topology of information exchange between n nodes can be described by a graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$, where $\mathscr{V} = \{1, \cdots, n\}$ be an index set of n nodes with i representing the ith node, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the set of edges of paired nodes and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ with non-negative adjacency elements a_{ij} is the weighted adjacency matrix of the graph $\mathscr{G}$. An edge of $\mathscr{G}$ is denoted by (i, j), representing that node i can get information from node j. The adjacency elements associated with the edges are positive, i.e., $(i, j) \in \mathscr{E}$ if and only if $a_{ij} > 0$. Moreover, it is assumed that $a_{ii} = 0$ for all $i \in \mathscr{V}$. A graph is called an undirected graph if the graph has the property that $a_{ij} = a_{ji}$ for any $i, j \in \mathscr{V}$. The neighborhood of node i is denoted by $\mathscr{N}_i = \{j \in \mathscr{V} : (j, i) \in \mathscr{E}\}$. A path on $\mathscr{G}$ from node i_1 to node i_n is a sequence of ordered edges of the form $(i_k, i_{k+1}) \in \mathscr{E}, k = 1, \cdots, n-1$, and i_k's are distinct. A graph $\mathscr{G}$ is said to be connected if there exists a path from node i to node j for any two nodes $i, j \in \mathscr{E}$. A diagonal matrix $\mathscr{D} = diag\{d_1, \cdots, d_n\}$ is a degree matrix of graph $\mathscr{G}$, whose diagonal matrix elements $d_i = \sum_{j \in \mathscr{N}_i} a_{ij}, i \in \mathscr{V}$. Then, the Laplacian matrix of a weighted graph can be defined as $\mathscr{L} = \mathscr{D} - \mathscr{A}$, which is a symmetric positive semi-definite matrix.

Considering a group of agents, the dynamics of the ith agent is given by

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= -\textstyle\sum_{j \in \mathscr{N}_i} a_{ij}[(x_i - x_j) + \gamma(v_i - v_j)] + u_i + g_i d_i, \quad i \in \mathscr{V}, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^m$ and $v_i \in \mathbb{R}^m$ are the position and velocity of agent i, respectively. a_{ij} is the (i, j)th entry of the adjacency matrix, and $\gamma > 0$ denotes a scaling factor. $u_i \in \mathbb{R}^m$ and $g_i \in \mathbb{R}^m$ denote the control input and a coefficient matrix, respectively. $d_i \in \mathbb{R}$ is the external disturbance, which is generated from the following unknown exosystem

$$\begin{aligned} \dot{\xi}_i &= A_i \xi_i, \\ d_i &= C_i \xi_i, \end{aligned} \tag{2}$$

where $\xi_i \in \mathbb{R}^{m_i}$, $A_i \in R^{m_i \times m_i}$ and $C_i \in R^{1 \times m_i}$ are the coefficient matrices.

As in [31] and [32], we assume that the desired state is described by

$$\dot{\bar{x}} = \bar{v}, \tag{3}$$

where $\bar{x} \in \mathbb{R}^m$ and $\bar{v} \in \mathbb{R}^m$ are the position and velocity of the leader agent, respectively.

Definition 1. *The consensus problem of the multi-agent systems* (1) *is formulated as follows: For the multi-agent systems* (1)*, design an adaptive consensus protocol such that the states of the close-loop system exist and are bounded, and the states of agents satisfy*

$$\lim_{t \to \infty} \|x_i - \bar{x}\| = 0, \lim_{t \to \infty} \|v_i - \bar{v}\| = 0, \tag{4}$$

for any initial values $x_i(0)$ and $v_i(0), i \in \mathscr{V}$.

Remark 1. *Note that, unlike the cases in [31,32], we allow that the disturbance $d_i, i \in \mathscr{V}$ is generated from different unknown exosystems, which makes our problem more challenging and realistic.*

3. Designing of Internal Models

In this section, in order to deal with the external disturbances, we will design some internal models. To this end, let

$$\begin{aligned} s_i(t) &= [x_i^T(t), v_i^T(t)]^T, \\ s(t) &= [x_1^T(t), \cdots, x_n^T(t), v_1^T(t), \cdots, v_n^T(t)]^T. \end{aligned}$$

Then, it follows from system (1) that

$$\dot{s}_i = L_i s(t) + H_i u_i + G_i d_i \tag{5}$$

where $H_i = [0_{m \times m}, I_m]^T \in \mathbb{R}^{2m \times m}, G_i = [0_{m \times 1}^T, g_i^T]^T \in \mathbb{R}^{2m \times 1}$, L_i is a matrix with its rows are chosen from rows $(i-1)m + 1$ to im and from rows $mn + (i-1)m + 1$ to $mn + im$ of the following matrix

$$\begin{bmatrix} 0_{mn \times mn} & I_n \otimes I_m \\ -\mathscr{L} \otimes I_m & -\gamma \mathscr{L} \otimes I_m \end{bmatrix}.$$

Before proceeding further, some standard assumptions are introduced as follows:

Assumption 1. *The matrix pair $(A_i, C_i), i \in \mathscr{V}$ is observable, and the eigenvalues of $A_i, i \in \mathscr{V}$ are with zero real parts and are distinct.*

Assumption 2. *There exists a function $h_i(s_i) : \mathbb{R}^{2m} \to \mathbb{R}^{m_i}, i \in \mathscr{V}$, such that $\frac{\partial h_i(s_i)}{\partial s_i} G_i = N_i, i \in \mathscr{V}$, where $N_i, i \in \mathscr{V}$ is a nonzero constant vector in $\mathbb{R}^{m_i}$.*

Under the Assumptions 1 and 2, for any nonzero vector $N_i, i \in \mathscr{V}$, there exists a Hurwitz matrix $M_i, i \in \mathscr{V}$ such that $(M_i, N_i), i \in \mathscr{V}$ is controllable.

Now, we can define a dynamic system of the following form

$$\dot{z}_i = M_i z_i + M_i h_i(s_i) - \frac{\partial h_i(s_i)}{\partial s_i}(L_i s + H_i u_i), i \in \mathscr{V}, \tag{6}$$

which is called an internal model and can be used to handle the disturbance d_i generated from (2).

Furthermore, there exists a nonsingular matrix $T_i, i \in \mathscr{V}$ satisfying the following Sylvester equation

$$T_i A_i - M_i T_i = N_i C_i, \;\; i \in \mathscr{V}, \tag{7}$$

because the pair $(M_i, N_i), i \in \mathscr{V}$, is controllable with M_i being Hurwitz, and the pair $(M_i, N_i), i \in \mathscr{V}$, is observable.

With the internal model (6) and Sylvester Equation (7) ready, the biased error can be defined by

$$e_i = T_i \xi_i - z_i - h_i(s_i), i \in \mathscr{V}. \tag{8}$$

Then, it can be verified that the internal model (6) and the biased error (8) have a nice property as described in the following lemma.

Lemma 1. *There exist some positive constants d_{e_i} and λ_{e_i} such that the biased error defined by (8) satisfies the following inequality*

$$\|e_i\| \leq d_{e_i} e^{-\lambda_{e_i} t}, i \in \mathscr{V}, \tag{9}$$

which implies that e_i is exponentially stable.

Proof. Firstly, by Equations (2), (5) and (6), a straightforward computation shows that

$$\begin{aligned}\dot{e}_i &= T_i \dot{\xi}_i - \dot{z}_i - \frac{\partial h_i(s_i)}{\partial s_i}\dot{s}_i \\ &= T_i A_i \xi_i - [M_i z_i + M_i h_i(s_i) - \frac{\partial h_i(s_i)}{\partial s_i}(L_i s + H_i u_i)] \\ &\quad - \frac{\partial h_i(s_i)}{\partial s_i}[L_i s(t) + H_i u_i + G_i d_i].\end{aligned}$$

Then, under the Assumption 2, by the Sylvester Equation (7), one has

$$\begin{aligned}\dot{e}_i &= (M_i T_i + N_i C_i)\xi_i - [M_i z_i + M_i h_i(s_i) - \tfrac{\partial h_i(s_i)}{\partial s_i}(L_i s + H_i u_i)] \\ &\quad - \tfrac{\partial h_i(s_i)}{\partial s_i}[L_i s(t) + H_i u_i + G_i d_i] \\ &= (M_i T_i + N_i C_i)\xi_i - [M_i z_i + M_i h_i(s_i)] - N_i d_i \\ &= M_i e_i.\end{aligned} \tag{10}$$

Next, using the Lyapunov stability theory of [36], it is easy to verify that the solution of e_i system (10) can be given as

$$e_i(t) = e_i(0) e^{M_i t}.$$

Furthermore, owing to M_i is a Hurwitz matrix, it follows that there exist some positive constants d_{i0} and λ_{i0} such that

$$\|e^{M_i t}\| \leq d_{i0} e^{-\lambda_{i0} t},$$

which implies that

$$\|e_i(t)\| \leq d_{ei} e^{-\lambda_{ei} t},$$

where $d_{ei} = d_{i0}\|e_i(0)\|$ and $\lambda_{ei} = \lambda_{i0}$. $\square$

Remark 2. *Based on Lemma 1, it can be shown that, for the following first-order system*

$$\dot{\bar{e}}_i = -\lambda_{ei}\bar{e}_i, \bar{e}_i(0) = d_{ei}, i \in \mathscr{V}, \tag{11}$$

where d_{ei} and λ_{ei} are the same positive constants given in Lemma 1, the following nice property

$$\|e_i(t)\| \leq \bar{e}_i(t), i \in \mathscr{V}, \tag{12}$$

is hold, which is very useful for managing the disturbances caused by the unknown exosystems (2).

4. Main Result

In this section, we will present an adaptive protocol for solving the consensus problem of the multi-agent systems (1). To do this, we further make one more standard assumption and recall one lemma which can be found in [31,32,37].

Assumption 3. *The graph $\mathscr{G}$ describing the interaction topology is connected.*

Lemma 2. *Under Assumption 3, suppose that $\gamma > 0$ is a positive real number, then the following matrix*

$$\begin{bmatrix} 0 & I_n \\ -(\mathscr{L}+\mathscr{B}) & -\gamma(\mathscr{L}+\mathscr{B}) \end{bmatrix}$$

is Hurwitz, where $\mathscr{B} = diag\{b_1, \cdots, b_n\}$ with b_i is the control gain of control law (16), and $b_i > 0$ if agent is pinned, otherwise, $b_i = 0$.

In order to make the problem more tractable, let

$$\eta_i = T_i\xi_i, \tag{13}$$

where T_i is the nonsingular matrix satisfying the Sylvester Equation (7). Then, we have

$$\dot{\eta}_i = T_iA_i\xi_i. \tag{14}$$

By (7) and (13), one can obtain that

$$\begin{aligned} \dot{\eta}_i &= M_i\eta_i + N_i\psi_i\eta_i, \\ d_i &= \psi_i\eta_i, i \in \mathscr{V}, \end{aligned} \tag{15}$$

where $\psi_i = C_iT_i^{-1}$ is unknown vector since C_i and T_i are unknown matrices.

Remark 3. *It is worth pointing out that, after the linear transformation (13), the external disturbance $d_i, i \in \mathscr{V}$, can be generated from the system (13), in which M_i is a known Hurwitz matrix and only the matrix $\psi_i, i \in \mathscr{V}$, is unknown. Thus, one can estimate the disturbance d_i through estimating the unknown constant vector $\psi_i, i \in \mathscr{V}$.*

Now, we are ready to state our main result.

Theorem 1. *Under Assumptions 1–3, with the help of the internal model presented in (6), the adaptive protocol given by*

$$\begin{aligned} u_i &= -b_i[(x_i - \bar{x}) + \gamma(v_i - \bar{v})] - g_i\hat{\psi}_i(z_i + h_i(s_i)), \\ \dot{\hat{\psi}}_i &= \varrho_i \omega^T P_{n+i} g_i (z_i + h_i(s_i))^T, i \in \mathscr{V}, \end{aligned} \tag{16}$$

where the control gain $b_i > 0$ if agent is pinned, otherwise, $b_i = 0$, $\hat{\psi}_i$ is the estimation of the unknown vector ψ_i, ϱ_i is a positive constant which is used to modify the update rate, ω and P_{n+i} are defined by (18) and (22), respectively, solves the consensus problem of second order multi-agent systems (1) with external disturbance generated from linear unknown exosystem (2).

Proof. let

$$\begin{aligned} \tilde{x}_i(t) &= x_i(t) - \bar{x}, \\ \tilde{v}_i(t) &= v_i(t) - \bar{v}, i \in \mathscr{V}, \end{aligned} \tag{17}$$

and

$$\begin{aligned} \tilde{x}(t) &= [\tilde{x}_1^T(t), \cdots, \tilde{x}_n^T(t)]^T, \\ \tilde{v}(t) &= [\tilde{v}_1^T(t), \cdots, \tilde{v}_n^T(t)]^T, \\ w &= [\tilde{x}^T, \tilde{v}^T]^T, \\ \eta &= [\eta_1^T, \cdots, \eta_n^T]^T, \\ z &= [z_1^T, \cdots, z_n^T]^T, \\ h &= [h_1^T, \cdots, h_n^T]^T, \\ \psi &= diag(\psi_1, \cdots, \psi_n), \\ \hat{\psi} &= diag(\hat{\psi}_1, \cdots, \hat{\psi}_n). \end{aligned} \tag{18}$$

Then, by combining Equations (1), (2), (15) and (16), the following system can be derived

$$\dot{\omega} = \bar{\mathscr{L}}\omega + \Psi[\psi\eta - \hat{\psi}(z + h)], \tag{19}$$

where

$$\begin{aligned} \bar{\mathscr{L}} &= \begin{bmatrix} 0 & I_n \otimes I_m \\ -(\mathscr{L} + \mathscr{B}) \otimes I_m & -\gamma(\mathscr{L} + \mathscr{B}) \otimes I_m \end{bmatrix}, \\ \Psi &= \begin{bmatrix} I_n \otimes 0_{m\times 1} \\ g \end{bmatrix}, \\ g &= \text{blockdiag}(g_1, \cdots, g_n). \end{aligned}$$

Furthermore, as noted in [32], according to the Lemma 2 and Theorem 4.2.12 of [38], it follows that the matrix $\bar{\mathscr{L}}$ is Hurwitz.

Next, consider the following Lyapunov function candidate

$$V = \omega^T P \omega + \frac{2}{\varrho_i} \sum_{i=1}^{n} \tilde{\psi}_i \tilde{\psi}_i^T + \frac{1}{2} \sum_{i=1}^{n} c_i \bar{e}_i^2, \tag{20}$$

where $\tilde{\psi}_i = \psi_i - \hat{\psi}_i$, $c_i, i \in \mathscr{V}$, is a positive real constant number which will be specified later, $\bar{e}_i$ is the state defined by (11), and P is a positive definite matrix satisfying the following Lyapunov equation

$$P\bar{\mathscr{L}} + \bar{\mathscr{L}}^T P = -I.$$

The existence of the matrix P is due to the Hurwitzness of $\bar{\mathscr{L}}$.

Then, taking the derivative of V along the system composed of (11), (16) and (19) gives

$$\begin{aligned}\dot{V} &= \dot{\omega}^T P\omega + \omega^T P\dot{\omega} + \tfrac{2}{\varrho_i}\textstyle\sum_{i=1}^n \tilde{\psi}_i\dot{\tilde{\psi}}_i^T + \sum_{i=1}^n c_i\bar{e}_i\dot{\bar{e}}_i \\ &= -\|\omega\|^2 + 2\omega^T P\Psi[\psi\eta - \hat{\psi}(z+h)] \\ &\quad - \tfrac{2}{\varrho_i}\textstyle\sum_{i=1}^n \tilde{\psi}_i\dot{\hat{\psi}}_i^T - \sum_{i=1}^n c_i\lambda_{e_i}\bar{e}_i^2.\end{aligned} \tag{21}$$

Now, in order to overcome the difficulties caused by the unknown vectors $\psi_i, i \in \mathscr{V}$, let us split the matrix P as

$$P = [P_1, \cdots, P_n, P_{n+1}, \cdots, P_{2n}], \tag{22}$$

where $P_i, i = 1, 2, \cdots, 2n$, are $2mn \times m$ blocks.

Then, in light of (8) and (22), it follows from (21) that

$$\begin{aligned}\dot{V} = &-\|\omega\|^2 + 2\textstyle\sum_{i=1}^n \omega^T P_{n+i} g_i \tilde{\psi}_i(z_i + h_i) \\ &+ 2\textstyle\sum_{i=1}^n \omega^T P_{n+i} g_i \psi_i e_i - \tfrac{2}{\varrho_i}\sum_{i=1}^n \tilde{\psi}_i\dot{\hat{\psi}}_i^T - \sum_{i=1}^n c_i\lambda_{e_i}\bar{e}_i^2.\end{aligned} \tag{23}$$

Furthermore, substituting the adaptive law $\dot{\hat{\psi}}_i$ proposed by (16) into (23) yields that

$$\begin{aligned}\dot{V} &= -\|\omega\|^2 + 2\textstyle\sum_{i=1}^n \omega^T P_{n+i} g_i \psi_i e_i - \sum_{i=1}^n c_i\lambda_{e_i}\bar{e}_i^2 \\ &\leq -\|\omega\|^2 + \textstyle\sum_{i=1}^n [\epsilon_i\|\omega\|^2 + \frac{1}{\epsilon_i}\|P_{n+i} g_i \psi_i\|^2\|e_i\|^2] - \sum_{i=1}^n c_i\lambda_{e_i}\bar{e}_i^2,\end{aligned} \tag{24}$$

where $\epsilon_i, i = 1, 2, \cdots, n$, are any positive real constants.

From the inequality (12), we obtain that

$$\dot{V} \leq -(1-\epsilon)\|\omega\|^2 + \sum_{i=1}^n (\frac{1}{\epsilon_i}\|P_{n+i} g_i \psi_i\|^2 - c_i\lambda_{e_i})\bar{e}_i^2, \tag{25}$$

where $\epsilon = \sum_{i=1}^n \epsilon_i$.

Next, choosing $\epsilon \leq \frac{1}{2}$, and $c_i = \frac{2}{\lambda_{e_i}\epsilon_i}\|P_{n+i} g_i \psi_i\|^2$, which will lead to

$$\dot{V} \leq -\frac{1}{2}\|w\|^2. \tag{26}$$

Hence, we can conclude that all the variables are bounded. Finally, by invoking the Barbalat's Lemma, one can obtain that

$$\lim_{t\to\infty} \tilde{x}(t) = 0,$$
$$\lim_{t\to\infty} \tilde{v}(t) = 0,$$

which complete this proof. $\square$

Remark 4. *It is worth pointing out that, distributed proportional-integral control law was also studied in [30] for second-order multi-agent systems with constant disturbances. Unlike the results in [30], the disturbances in this paper are assumed to be generated from some unknown exosystems, which include the constant disturbance as special case. In addition, the results in this paper are proved by combining Lyapunov-based method and adaptive control technique, which are totally different proof techniques from that used in [30].*

5. Illustrative Example

In this section, an example will be provided to illustrate our design. The model parameters are taken from [31,32] with some adjustments. We assume that there are ten agents with an undirected communication graph $\mathscr{G}$ shown in Figure 1. The gain γ is set to 1 and the coefficient matrix of system (1) is $g_i = 1$, respectively. The desired consensus state is described by $\dot{x} = 0.08$. However, unlike [31,32], we assume the disturbance d_i is generated from

$$\dot{\xi}_i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \xi_i,$$
$$d_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \xi, \ i \in \mathscr{V}.$$

Then, we have

$$A_i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix}, \ C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ G_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{27}$$

Let $h_i(s_i) = s_i$, one has

$$\frac{\partial h_i(s_i)}{s_i} G_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Therefore, Assumptions 1–3 are satisfied.

Furthermore, select

$$M_i = \begin{bmatrix} 0 & 1 \\ -9 & -8 \end{bmatrix}, \ i \in \mathscr{V},$$

such that (M_i, N_i) is controllable with M_i being Hurwitz.

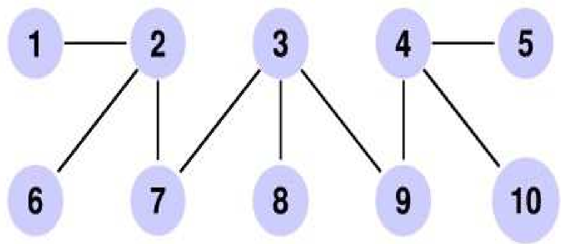

Figure 1. Communication graph $\mathscr{G}$.

Then, based on the proposed approach, the internal model (6) and adaptive protocol (16) can be designed. Numerical simulations are conducted to show the performance of the presented control law. Some of the results are depicted in Figures 2 and 3 with initial conditions of states and initial velocities of agents are chosen randomly from $[0, 4]$ and $[0, 5]$, respectively. The unknown parameters of the exosystems are set as $\sigma_1 = 0.1, \sigma_2 = 0.2, \sigma_3 = 0.3, \sigma_4 = 0.4, \sigma_5 = 0.5, \sigma_6 = 0.6, \sigma_7 = 0.7, \sigma_8 = 0.8$, $\sigma_9 = 0.9, \sigma_{10} = 1$, and the initial conditions of the exosystem are all set as $\xi_i(0) = [0.5 \sin 1, 0.5 \cos 1]^T$. The pining control gains are selected as $b_2 = b_4 = 1$. All the other initial conditions in the controller are set to zero. From Figures 2 and 3, it can be seen that the consensus protocol proposed in this paper allows the agents to reach consensus, in the presence of external disturbance generated from some unknown exosystems.

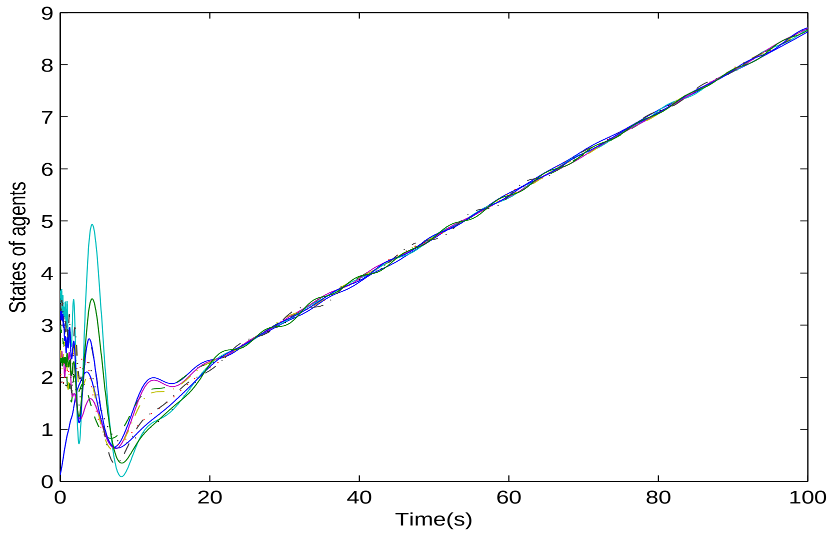

Figure 2. States of the agents.

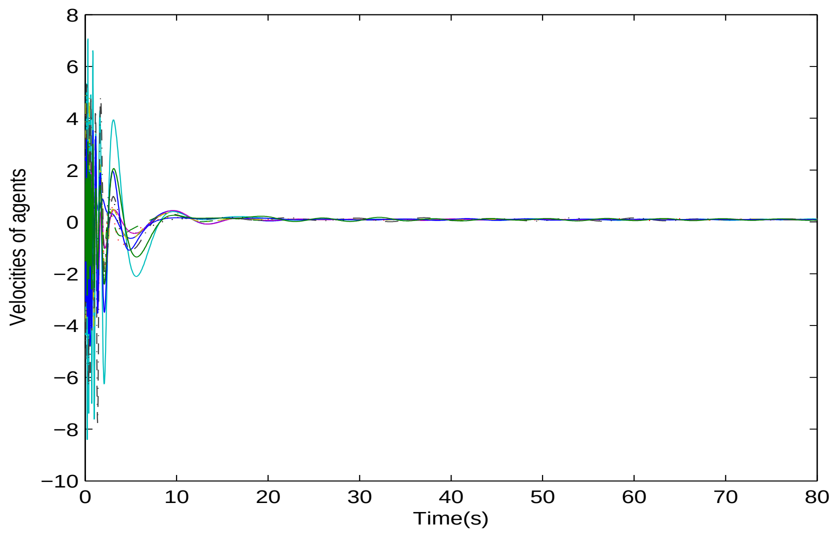

Figure 3. Velocities of the agents.

6. Conclusions

This paper address a consensus problem of second order multi-agent systems with exogenous disturbance generated by unknown exosystems. A class of internal model was proposed for deal with the disturbance caused by the unknown exosystems. Based on the internal model, an adaptive consensus protocol was presented for the second order multi-agent systems. Finally, the effectiveness of our results is validated by numerical simulations.

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