

# Complex and Fractional Dynamics

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Complex systems (CS) are pervasive in many areas, namely financial markets; highway transportation; telecommunication networks; world and country economies; social networks; immunological systems; living organisms; computational systems; and electrical and mechanical structures. CS are often composed of a large number of interconnected and interacting entities exhibiting much richer global scale dynamics than could be inferred from the properties and behavior of individual elements.

This Special Issue focuses on the topics of complex dynamical systems, nonlinearity, chaos and fractional dynamics, from the perspectives of both thermodynamics and information processing. This selection of 21 papers addressing advanced issues and specific topics illustrates the broad impact of entropy-based techniques in complexity, nonlinearity and fractionality.

In the paper A Memristor-Based Complex Lorenz System and Its Modified Projective Synchronization, Shibing Wang, Xingyuan Wang and Yufei Zhou introduce a novel complex Lorenz system with a flux-controlled memristor, investigating its synchronization [1].

The manuscript A Novel Image Encryption Algorithm Based on DNA Encoding and Spatiotemporal Chaos, by Chunyan Song and Yulong Qiao, proposes a novel image encryption scheme based on DNA encoding and spatiotemporal chaos [2].

In the work A Novel Weak Fuzzy Solution for Fuzzy Linear System, Soheil Salahshour, Ali Ahmadian, Fudziah Ismail and Dumitru Baleanu propose a novel weak fuzzy solution for the fuzzy linear system. Two complex and non-complex linear systems under uncertainty are tested to validate the effectiveness and correctness of the method [3].

In the article Adaptive Synchronization for a Class of Uncertain Fractional-Order Neural Networks, Heng Liu, Shenggang Li, Hongxing Wang, Yuhong Huo and Junhai Luo study the synchronization of a class of uncertain fractional-order neural networks subject to external disturbances and disturbed system parameters [4].

In Approximate Analytical Solutions of Time Fractional Whitham–Broer–Kaup Equations by a Residual Power Series Method, Linjun Wang and Xumei Chen apply a new analytic iterative technique, called the residual power series method, to time fractional Whitham–Broer–Kaup equations [5].

In Characterization of Complex Fractionated Atrial Electrograms by Sample Entropy: An International Multi-Center Study, Eva Cirugeda-Roldán, Daniel Novak, Vaclav Kremen, David Cuesta-Frau, Matthias Keller, Armin Luik and Martina Srutova propose the use of sample entropy for discerning between non-fractionated and fractionated atrial electrograms, and for characterizing the degree of trial electrograms regularity [6].

The paper Chaos on the Vallis Model for El Niño with Fractional Operators, by Badr Saad T. Alkahtani and Abdon Atangana, investigates the Vallis model for El Niño. The authors study and compare both integer and non-integer order versions of the model [7].

The manuscript Cloud Entropy Management System Involving a Fractional Power, by Rabha W. Ibrahim, Hamid A. Jalab and Abdullah Gani, develops a new algorithm for a multi-agent system based on fractional power. The existence of solutions for the system as well as the stability, utilizing the Hadamard well-posed problem, is discussed. Experimental results show stability and performance [8].

In Complexity Analysis and DSP Implementation of the Fractional-Order Lorenz Hyperchaotic System, Shaobo He, Kehui Sun and Huihai Wang solve the fractional-order hyperchaotic Lorenz system as a discrete map by applying the Adomian decomposition method [9].

The paper Existence of Ulam Stability for Iterative Fractional Differential Equations Based on Fractional Entropy, by Rabha W. Ibrahim and Hamid A. Jalab, introduces conditions for the existence of solutions for an iterative functional differential equation of fractional order. The authors prove that the solutions of that class of fractional differential equations are bounded by Tsallis entropy [10].

In Fractional Differential Texture Descriptors Based on the Machado Entropy for Image Splicing Detection, Rabha W. Ibrahim, Zahra Moghaddasi, Hamid A. Jalab and Rafidah Md Noor introduce a texture enhancement technique involving the use of fractional differential masks based on the Machado entropy [11].

The article Fractional State Space Analysis of Economic Systems, by J. A. Tenreiro Machado, Maria Eugénia Mata and António M. Lopes, examines modern economic growth according to the multidimensional scaling method and state space portrait analysis. Electing GDP per capita as the main indicator for economic growth and prosperity, the long-run perspective from 1870 to 2010 identifies the main similarities among 34 world partners' modern economic growth and exemplifies the historical waving mechanics of the largest world economy, the USA [12].

The manuscript Generalized Boundary Conditions for the Time-Fractional Advection Diffusion Equation, by Yuriy Povstenko, analyzes the different kinds of boundary conditions for standard and fractional diffusion and advection diffusion equations [13].

The work  $H_\infty$  Control for Markov Jump Systems with Nonlinear Noise Intensity Function and Uncertain Transition Rate, by Xiaonian Wang and Yafeng Guo, investigates robust  $H_\infty$  control for Markov jump systems with nonlinear noise intensity function and uncertain transition rates [14].

The work Identify the Rotating Stall in Centrifugal Compressors by Fractal Dimension in Reconstructed Phase Space, by Le Wang, Jiazhong Zhang and Wenfan Zhang, proposes a method to extract and analyze the dynamics of the rotating stall in the impeller of a centrifugal compressor. The approach is based on phase space reconstruction and fractal dynamics [15].

The manuscript Increment Entropy as a Measure of Complexity for Time Series, by Xiaofeng Liu, Aimin Jiang, Ning Xu and Jianru Xue, introduces the increment entropy to measure the complexity of time series in which each increment is mapped onto a word of two letters, one corresponding to the sign and the other corresponding to the magnitude [16].

In the work Modeling of a Mass-Spring-Damper System by Fractional Derivatives with and without a Singular Kernel, by José Francisco Gómez-Aguilar, Huitzilin Yépez-Martínez, Celia Calderón-Ramón, Ines Cruz-Orduña, Ricardo Fabricio Escobar-Jiménez and Victor Hugo Olivares-Peregrino, the authors present the fractional equations of the mass-spring-damper system with Caputo and Caputo-Fabrizio derivatives [17].

The paper New Derivatives on the Fractal Subset of Real-Line, by Alireza Khalili Golmankhaneh and Dumitru Baleanu, introduces the generalized fractional Riemann–Liouville and Caputo-like derivative for functions defined on fractal sets. The advantage of using these new nonlocal derivatives on the fractals subset of real-line lies in the fact that they are better at modeling processes with memory effect [18].

In the work Predicting Traffic Flow in Local Area Networks by the Largest Lyapunov Exponent, Yan Liu and Jiazhong Zhang investigate the dynamics of network traffic by means of the largest Lyapunov exponent. They show that the presented method is feasible and efficient for predicting the complex dynamic behaviors in local area network traffic [19].

In Particular Solutions of the Confluent Hypergeometric Differential Equation by Using the Nabla Fractional Calculus Operator, Resat Yilmazer, Mustafa Inc, Fairouz Tchier and Dumitru Baleanu formulate a method for solving the second-order linear ordinary differential equation of hypergeometric type. Four different new discrete complex fractional solutions are presented [20].

The manuscript Stability Analysis and Synchronization for a Class of Fractional-Order Neural Networks, by GuanJun Li and Heng Liu addresses the stability of a class of fractional-order neural networks. Several numerical examples justify the feasibility of the proposed methods [21].

The guest editors hope that the selected papers will motivate researchers to pursue further advances in the emerging areas of complex systems and fractional dynamics.

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## References

1. Wang, S.; Wang, X.; Zhou, Y. A Memristor-based Complex Lorenz System and its Modified Projective Synchronization. *Entropy* **2015**, *17*, 7628–7644.
2. Song, C.; Qiao, Y. A Novel Image Encryption Algorithm Based on DNA Encoding and Spatiotemporal Chaos. *Entropy* **2015**, *17*, 6954–6968.
3. Salahshour, S.; Ahmadian, A.; Ismail, F.; Baleanu, D. A Novel Weak Fuzzy Solution for Fuzzy Linear System. *Entropy* **2016**, *18*, 68.
4. Liu, H.; Li, S.; Wang, H.; Huo, Y.; Luo, J. Adaptive Synchronization for a Class of Uncertain Fractional-Order Neural Networks. *Entropy* **2015**, *17*, 7185–7200.
5. Wang, L.; Chen, X. Approximate Analytical Solutions of Time Fractional Whitham–Broer–Kaup Equations by a Residual Power Series Method. *Entropy* **2015**, *17*, 6519–6533.
6. Cirugeda-Roldán, E.; Novak, D.; Kremen, V.; Cuesta-Frau, D.; Keller, M.; Luik, A.; Srutova, M. Characterization of Complex Fractionated Atrial Electrograms by Sample Entropy: An International Multi-Center Study. *Entropy* **2015**, *17*, 7493–7509.
7. Alkahtani, B.S.T.; Atangana, A. Chaos on the Vallis Model for El Niño with Fractional Operators. *Entropy* **2016**, *18*, 100.
8. Ibrahim, R.W.; Jalab, H.A.; Gani, A. Cloud Entropy Management System Involving a Fractional Power. *Entropy* **2016**, *18*, 14.
9. He, S.; Sun, K.; Wang, H. Complexity Analysis and DSP Implementation of the Fractional-Order Lorenz Hyperchaotic System. *Entropy* **2015**, *17*, 8299–8311.
10. Ibrahim, R.W.; Jalab, H.A. Existence of Ulam Stability for Iterative Fractional Differential Equations Based on Fractional Entropy. *Entropy* **2015**, *17*, 3172–3181.
11. Ibrahim, R.W.; Moghaddasi, Z.; Jalab, H.A.; Noor, R.M. Fractional Differential Texture Descriptors based on the Machado Entropy for Image Splicing Detection. *Entropy* **2015**, *17*, 4775–4785.
12. Machado, J.; Mata, M.E.; Lopes, A.M. Fractional State Space Analysis of Economic Systems. *Entropy* **2015**, *17*, 5402–5421.
13. Povstenko, Y. Generalized Boundary Conditions for the Time-Fractional Advection Diffusion Equation. *Entropy* **2015**, *17*, 4028–4039.
14. Wang, X.; Guo, Y.  $H_\infty$  Control for Markov Jump Systems with Nonlinear Noise Intensity Function and Uncertain Transition Rates. *Entropy* **2015**, *17*, 4762–4774.
15. Wang, L.; Zhang, J.; Zhang, W. Identify the Rotating Stall in Centrifugal Compressors by Fractal Dimension in Reconstructed Phase Space. *Entropy* **2015**, *17*, 7888–7899.
16. Liu, X.; Jiang, A.; Xu, N.; Xue, J. Increment Entropy as a Measure of Complexity for Time Series. *Entropy* **2016**, *18*, 22.
17. Gómez-Aguilar, J.F.; Yépez-Martínez, H.; Calderón-Ramón, C.; Cruz-Orduña, I.; Escobar-Jiménez, R.F.; Olivares-Peregrino, V.H. Modeling of a Mass-Spring-Damper System by Fractional Derivatives with and without a Singular Kernel. *Entropy* **2015**, *17*, 6289–6303.

18. Khalili Golmankhaneh, A.; Baleanu, D. New Derivatives on the Fractal Subset of Real-Line. *Entropy* **2016**, *18*, 1.
19. Liu, Y.; Zhang, J. Predicting Traffic Flow in Local Area Networks by the Largest Lyapunov Exponent. *Entropy* **2016**, *18*, 32.
20. Yilmazer, R.; Inc, M.; Tchier, F.; Baleanu, D. Particular Solutions of the Confluent Hypergeometric Differential Equation by Using the Nabla Fractional Calculus Operator. *Entropy* **2016**, *18*, 49.
21. Li, G.; Liu, H. Stability Analysis and Synchronization for a Class of Fractional-Order Neural Networks. *Entropy* **2016**, *18*, 55.



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