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On the Definition of Diversity Order Based on Renyi Entropy for Frequency Selective Fading Channels

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Abstract: Outage probabilities are important measures of the performance of wireless communication systems, but to obtain outage probabilities it is necessary to first determine detailed system parameters, followed by complicated calculations. When there are multiple candidates of diversity techniques applicable for a system, the diversity order can be used to roughly but quickly compare the techniques for a wide range of operating environments. For a system transmitting over frequency selective fading channels, the diversity order can be defined as the number of multi-paths if multi-paths have all equal energy. However, diversity order may not be adequately defined when the energy values are different. In order to obtain a rough value of diversity order, one may use the number of multi-paths or the reciprocal value of the multi-path energy variance. Such definitions are not very useful for evaluating the performance of diversity techniques since the former is meaningful only when the target outage probability is extremely small, while the latter is reasonable when the target outage probability is very large. In this paper, we propose a new definition of diversity order for frequency selective fading channels. The proposed scheme is based on Renyi entropy, which is widely used in biology and many other fields. We provide various simulation results to show that the diversity order using the proposed definition is tightly correlated with the corresponding outage probability, and thus the proposed scheme can be used for quickly selecting the best diversity technique among multiple candidates.

Keywords: diversity; diversity order; Renyi entropy; frequency selective fading; outage probability

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1. Introduction

The outage probability is a critical measurement for evaluating the performance of wireless communication systems over fading channels. In order to obtain an outage probability, however, it is necessary to determine detailed system parameters, followed by complicated high-precision calculations. On the other hand, a diversity order can be used to roughly but quickly compare multiple diversity techniques for a wide range of system parameters and operating environments [1–4].

5G cellular communication systems consider various usage scenarios such as enhanced mobile broadband, massive machine type communications, and ultra-reliable and low latency communications [5]. Especially in order to achieve reliable communications over fading channels, various diversity techniques need to be considered including antenna, time, frequency, polarity, angle, and site diversity techniques [6–8], and the best scheme or the best combination of techniques should be selected within a short time to satisfy the latency requirement.

If the average energy values of multiple paths are all equal in a frequency selective fading channel, the diversity order can be defined as the number of multi-paths and the outage performance can be reduced as the diversity order becomes larger. However, if the average energy values of multiple paths are different, there is no adequate definition to represent the diversity order. In order to roughly represent the diversity achievable in a frequency selective fading channel, one may use the number of multi-paths regardless of the average energy values of the multi-paths, or some others use the reciprocal value of the multi-path energy variance [9–12]. Diversity order values produced by these schemes, however, are not tightly correlated with the corresponding outage probabilities, and thus they are not very suitable for evaluating the performance of diversity techniques. In this paper, we propose a new definition of a diversity order for a frequency selective fading channel. The proposed scheme is based on Renyi entropy, which is widely used in biology and many other fields. We provide various simulation results showing that diversity orders using the proposed definition are very useful for comparing diversity techniques. The diversity order obtained from the proposed definition is tightly correlated with the corresponding outage probability, and the proposed scheme can be used for quickly selecting the best diversity technique among multiple candidates.

This paper is organized as follow: Section 2 describes the relationships between the outage probabilities and the diversity orders in frequency selective fading channel environments. Section 3 proposes a new definition of a diversity order based on Renyi entropy for frequency selective fading channels and Section 4 shows the effectiveness of the proposed definition of diversity order through various simulation results. Section 5 provides an application example of the proposed scheme and finally, conclusions are drawn in Section 6.

2. Conventional Definitions of Diversity Order

The average power information of multi-paths in a frequency selective fading channel is called the power delay profile (PDP) and a PDP with L multi-paths can be presented as:

$$\mathbf{p} = P_{total}[p_0, p_1, \cdots, p_{L-1}] \tag{1}$$

where P_{total} is the total energy of the multi-paths, and the vector $[p_0, p_1, \cdots, p_{L-1}]$ is assumed to satisfy the following equation:

$$\sum_{l=0}^{L-1} p_l = 1 \tag{2}$$

2.1. Outage Probability

The outage probability for a communication system is the probability that the channel capacity will fail to satisfy its target data transmission rate. When instantaneous channel information is not available but the PDP is given, the outage probability can be calculated as:

$$\begin{aligned} p_{out} &= \Pr\left\{ W\log_2\left(1 + \tfrac{P_{total}}{\sigma^2}\sum_{l=0}^{L-1}|h_l|^2 p_l\right) < R \right\} \\ &= \Pr\left\{ W\log_2\left(1 + SNR\sum_{l=0}^{L-1}|h_l|^2 p_l\right) < R \right\} \\ &= \Pr\left\{ \sum_{l=0}^{L-1}|h_l|^2 p_l < \xi \right\} \end{aligned} \tag{3}$$

where h_l is an independent complex Gaussian random variable with zero mean and unit variance, R is the target data transmission rate, W is the bandwidth; σ^2 is the noise variance, SNR is the signal-to-noise ratio defined as P_{total}/σ^2, and $\xi \equiv \left(2^{R/W} - 1\right)/SNR$. The outage probability in Equation (3) can be calculated with the equations described in [12,13].

2.2. Equal Energy Values of Multi-Paths

If the average energy values of multi-paths in a frequency selective fading channel are all equal, the diversity order can be precisely defined. There are L paths with the same energy, the outage probability can be written as Equation (4) [12] and thus the diversity order can be written as L:

$$p_{out} = 1 - \left(\sum_{l=0}^{L-1} \frac{L\xi}{\Gamma(l+1)} \right) e^{-L\xi} \tag{4}$$

2.3. Different Energy Values of Multi-Paths

If the average energy values of multi-paths are different, there is no adequate definition of the diversity order. One scheme uses the number of multi-paths as the diversity order as shown in Equation (5) assuming that the operating SNR is very large, in other words, the outage probability is very small [12,14]:

$$D_0 = L \tag{5}$$

In Equation (5), the effect of different average energy values of multi-paths are not taken into account and thus it may not be tightly correlated with the outage probability except when the operating SNR is very high and thus all energy values of multi-paths are meaningfully large. Another scheme, called effective frequency diversity, uses the reciprocal value of the multi-path energy variance [10,14–16], written as follows:

$$D_2 = \left(\sum_{l=0}^{L-1} p_l^2 \right)^{-1} \tag{6}$$

In Equation (6), if the average energy values of multi-paths are all equal, in other words, $p_l = 1/L$ with L multi-paths, the diversity order becomes L, which coincides with the result in Section 2.2. The scheme described in Equation (6) mainly takes into account paths with large average energy values and undervalues the effect of small-energy paths. Therefore, this diversity order is not tightly correlated with the outage probability when the operating SNR is large, where a path with a relatively small energy value can contribute to the reduction of outage probabilities.

3. New Definition of Diversity Order

In this paper, we propose a new definition of a diversity order, which is a superset of the two conventional definitions described in Section 2 and applicable to a wide range of target outage probabilities. In order to define a diversity order, we borrow the definition of Renyi entropy, which is widely used to define a diversity order in biology and many other fields [17–22]. Renyi entropy is written as:

$$H_\alpha = \frac{1}{1-\alpha} \log \left(\sum_{i=1}^{n} \rho_i^\alpha \right) \; (0 \leq \alpha,\, \alpha \neq 1) \tag{7}$$

where ρ_i is a probability of an event. The diversity order can be represented as two to the power of Renyi Entropy:

$$D_\alpha = 2^{H_\alpha} = \left(\sum_{l=0}^{L-1} \rho_l^\alpha \right)^{\frac{1}{1-\alpha}} \; (0 \leq \alpha \leq 2,\, \alpha \neq 1) \tag{8}$$

Although PDP values are not probabilities but energy values, we can use a similar definition as Renyi entropy for a frequency selective fading channel, written as:

$$H_\alpha = \frac{1}{1-\alpha} \log \left(\sum_{i=1}^{n} p_i^\alpha \right) \; (0 \leq \alpha,\, \alpha \neq 1) \tag{9}$$

and the diversity order can be expressed as follows:

$$D_\alpha = 2^{H_\alpha} = \left(\sum_{l=0}^{L-1} p_l^\alpha \right)^{\frac{1}{1-\alpha}} \; (0 \leq \alpha \leq 2,\, \alpha \neq 1) \tag{10}$$

If the average energy values of multi-paths are all equal, in other words, $p_l = 1/L$ with L paths, the diversity order in Equation (10) becomes L, which also coincides with the result in Section 2.2. Note that in the case of $\alpha = 0$, Equation (10) is the same to Equation (5), and in the case of $\alpha = 2$, it becomes Equation (6). In Equation (10), α is a value which needs to be determined according to the target outage probability. If a target outage probability is very small, a small α can be used. On the other hand, if it is very large, a large α will be preferred. Equation (10) includes the definitions of Equation (5) (with $\alpha = 0$) and Equation (6) (with $\alpha = 2$), and can be used in a wide range of target outage probabilities assuming that a proper value of α can be chosen.

In order to make diversity orders in Equation (10) highly correlated with the corresponding outage probabilities, it is important to choose a proper value of α. We define mean square error (MSE) as the expected value of the square of the difference between a linear interpolation function of an outage probability and a diversity order obtained from Equation (10), written as:

$$MSE = E\left\{ \left(f(P_{out}(\mathbf{p}_k)) - D_\alpha(\mathbf{p}_k)\right)^2 \right\} \tag{11}$$

where f is a linear interpolation function created from the cases in which the diversity order is precisely defined, in other words, all paths have equal energy values. To express the equation more clearly, the inverse function of f, denoted as g, is defined as:

$$\begin{aligned} g\left(\widetilde{D}\right) &= f^{-1}\left(\widetilde{D}\right) \\ &= \log_{10}\left\{ \frac{P_{out}\left(\widetilde{\mathbf{p}}_{[\widetilde{D}]+1}\right)}{P_{out}\left(\widetilde{\mathbf{p}}_{[\widetilde{D}]}\right)} \right\} \left(\widetilde{D} - \left[\widetilde{D}\right]\right) + P_{out}\left(\widetilde{\mathbf{p}}_{[\widetilde{D}]}\right) \end{aligned} \tag{12}$$

where $\widetilde{\mathbf{p}}_L$ is the PDP with L equal-energy multi-paths. A proper $\alpha(0 \le \alpha \le 2,\ \alpha \ne 1)$ can be chosen as the value that minimizes the MSE in Equation (11).

4. Simulation Results

In order to observe the correlation of diversity orders and outage probabilities, PDPs are randomly generated and the corresponding diversity orders and outage probabilities are computed. The ratio of the target data transmission rate and bandwidth (R/W) is assumed to be 2. Figure 1 shows outage probabilities versus diversity orders calculated by Equation (6) when PDPs with 5 multi-paths are generated with random energy values. In the figure, "+" represents the points where the diversity order is clearly defined, in other words, all paths have equal energy values. It can be seen that outage probabilities are widely spread, meaning that diversity orders obtained by Equation (6) is not tightly correlated with the outage probabilities. Increasing the diversity order using Equation (6) may not mean that the outage probability can be reduced. If we use the diversity order definition in Equation (5), all cases in Figure 1 correspond to the diversity order of 5 and the diversity order is not correlated with the outage probabilities.

Figure 2 shows the results generated by Equation (10) with α = 0.57. For each value of a diversity order, the outage probability distribution is far narrower than that in Figure 1 and thus we can say that Equation (10) can provide diversity orders tightly correlated with the corresponding outage probabilities compared to the conventional schemes. The diversity order using Equation (10) with a proper α allows quick performance evaluation of diversity techniques.

A proper value of α according to each target operating SNR can be found by evaluating Equation (11) and selecting the value resulting in the minimum MSE. Figures 3–5 show the MSEs in Equation (11) according to α with different SNR values, and each figure includes three curves representing 3, 5, and 7 multi-paths, respectively. Note that the optimal α is highly related to the SNR, but independent of the number of multi-paths. If the operating SNR or target outage probability is given, a proper value of α can be determined regardless of the number of multi-paths.

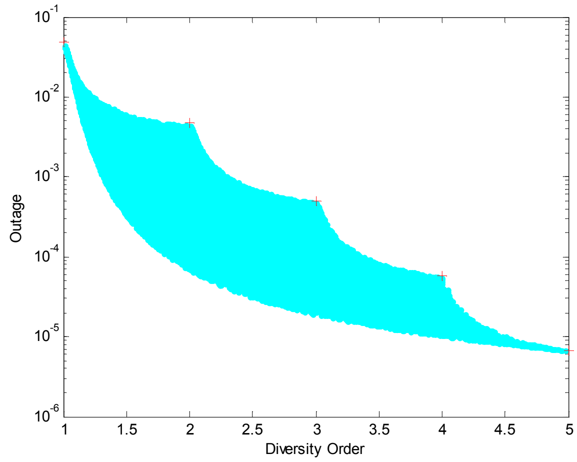

Figure 1. Outage probabilities versus diversity orders calculated using Equation (6) (or Equation (10) with $\alpha = 2$), SNR = 17.8 dB.

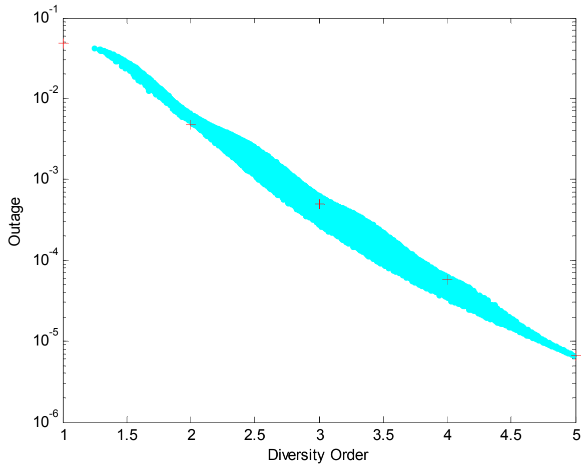

Figure 2. Outage probabilities versus diversity orders calculated using Equation 10 with $\alpha = 0.57$, SNR = 17.8 dB.

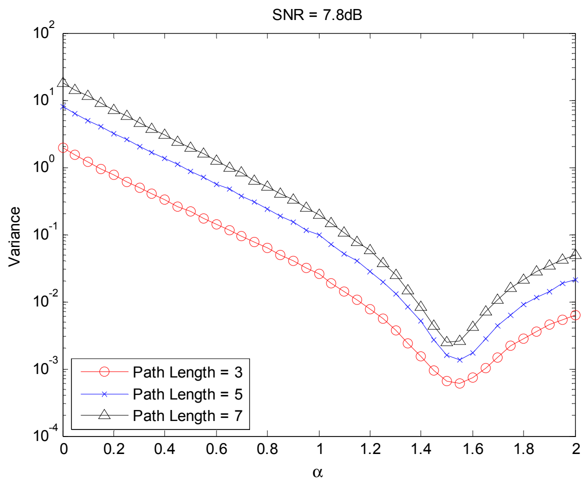

Figure 3. Mean square errors according to α (SNR = 7.8 dB).

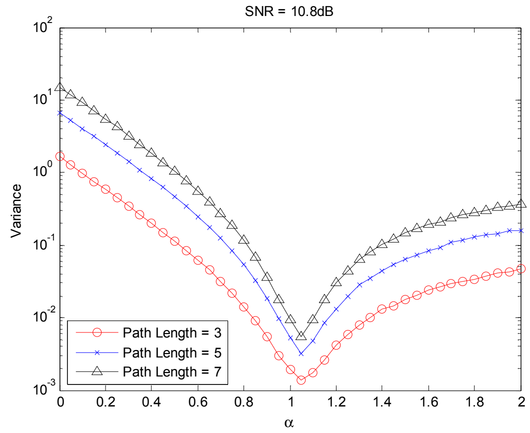

Figure 4. Mean square errors according to α (SNR = 10.8 dB).

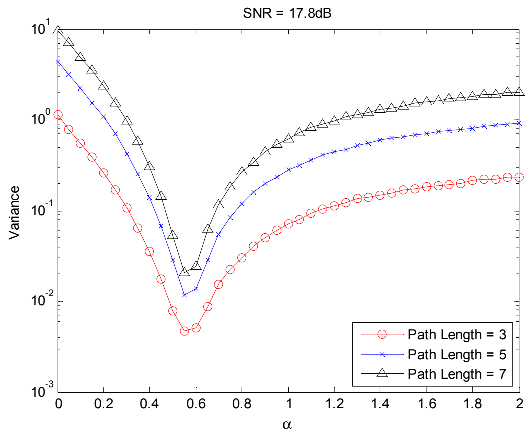

Figure 5. Mean square errors according to α (SNR = 17.8 dB).

Figure 6 shows the optimal α values according to SNR. A large value of α is required for low operating SNRs, where the outage probability is relatively large and paths with low average energy values make little contribution to the outage probability. On the other hand, a small value of α is preferred for high operating SNRs, where the average channel energy is sufficiently strong and even a path with a relatively low average energy value can help the reduction of the outage probability.

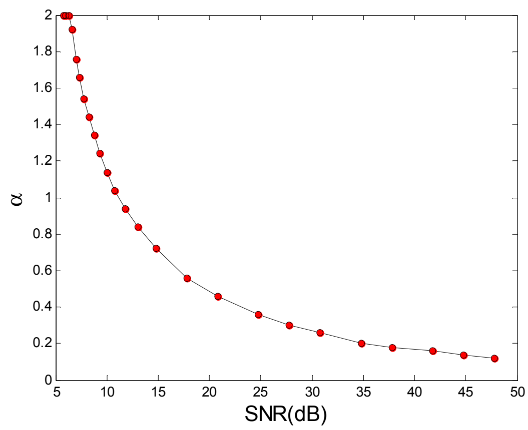

Figure 6. Optimal α according to operating SNR.

5. Application of Diversity Order

The proposed definition of a diversity order can be applied for quick evaluation of system performance with multiple candidates of diversity techniques without the determination of detailed system parameters or complicated calculation of exact outage probabilities. For example, the proposed definition can be used to determine the optimal delay value for a delay diversity technique in a multiple input single output (MISO) system. Consider a MISO system with two transmission antennas and one reception antenna, shown in Figure 7. Suppose that the PDP of each antenna is given as:

$$\mathbf{p}_{\text{single}} = p_{total}\left[p_0, \, p_1, \, \cdots, \, p_{L-1}\right], \tag{13}$$

and the PDPs of the two antennas are independent. When the delay diversity technique with the delay of K symbols is applied, the resulting PDP at the receiver can be represented as follows:

$$\mathbf{p}_{MISO} = \frac{1}{2}p_{total}\,\mathbf{p}_{\sin\text{gle}} + \frac{1}{2}p_{total}\,[\underbrace{0, \, \cdots, \, 0,}_{K} \, \mathbf{p}_{\text{single}}] \tag{14}$$

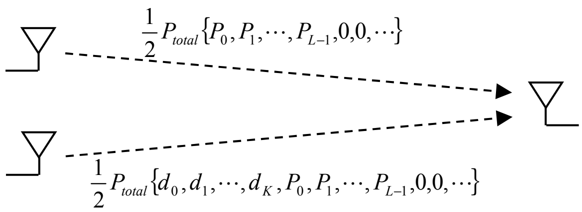

Figure 7. Power delay profile for each antenna with delay diversity.

Without calculating the exact outage probabilities, the optimal delay value K_{opt} $(0 \leq K_{opt} \leq K_{\max})$ can be quickly found by evaluating the diversity order for each K and selecting the maximum:

$$K_{opt} = \underset{0 \leq K \leq K_{\max}}{\operatorname{argmax}} \, D_\alpha(\mathbf{p}_{MISO}(K)) \tag{15}$$

In the simulation, it was assumed that a single-antenna channel has a cluster exponential distribution, as shown in Figure 8 and the maximum delay value $K_{\max}$ is 5. Figures 9 and 10 illustrate the diversity orders and the outage probabilities, respectively, according to the delay value K. Comparing Figures 9 and 10, we can see that maximizing the diversity order can achieve minimizing the outage probability. Therefore, using the new definition of a diversity order, quick evaluation of a large number of diversity techniques can be performed without complicated outage calculation.

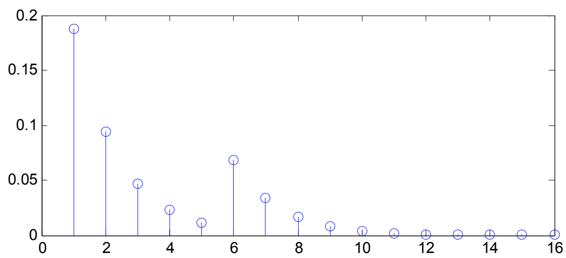

Figure 8. PDP for a single transmission antenna.

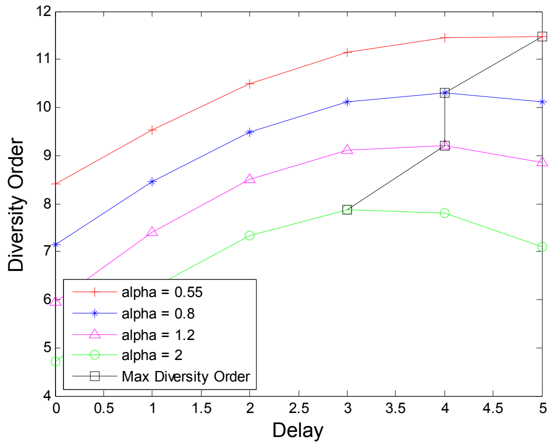

Figure 9. Diversity orders according to delay.

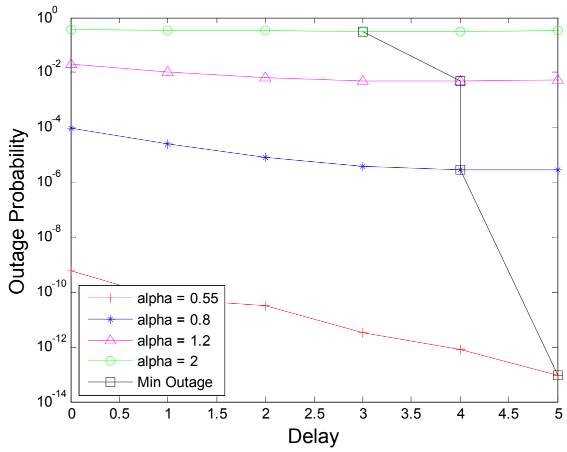

Figure 10. Outage probabilities according to delay.

6. Conclusions

Diversity orders can be used to roughly but quickly compare multiple candidates of diversity techniques in a wide range of operating environments. For a system transmitting over a frequency selective fading channel, however, if the average energy values of multi-paths are different, the diversity order has not been adequately defined. In this paper, we proposed a new definition of diversity order for a frequency selective fading channel. The proposed scheme is based on Renyi entropy and it produces the diversity order tightly correlated with the outage probability. Hence, the proposed definition of a diversity order can be used to quickly evaluate various frequency diversity techniques without calculating the exact outage probabilities. For example, as described in Section 5, the optimal delay value of a delay diversity technique can be quickly found by computing the diversity orders according to delay values. In the future, more theoretical analysis will be performed for the relationships between diversity orders and outage probabilities.

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Conflicts of Interest: The authors declare no conflict of interest.

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