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Analysis of a Hybrid Thermoelectric Microcooler: Thomson Heat and Geometric Optimization

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Abstract: In this work, we analyze the thermodynamics and geometric optimization of thermoelectric elements in a hybrid two-stage thermoelectric micro cooler (TEMC). We propose a novel procedure to improve the performance of the micro cooler based on optimum geometric parameters, cross sectional area (A) and length (L), of the semiconductor elements. Our analysis takes into account the Thomson effect to show its role on the performance of the system. We obtain dimensionless temperature spatial distributions, coefficient of performance (COP) and cooling power (Q_c) in terms of the electric current for different values of the geometric ratio $\omega = A/L$. In our analysis we consider two cases: (a) the same materials in both stages (homogeneous system); and (b) different materials in each stage (hybrid system). We introduce the geometric parameter, $W = \omega_1/\omega_2$, to optimize the micro device considering the geometric parameters of both stages, w_1 and w_2. Our results show the optimal configuration of materials that must be used in each stage. The Thomson effect leads to a slight improvement on the performance of the micro cooler. We determine the optimal electrical current to obtain the best performance of the TEMC. Geometric parameters have been optimized and results show that the hybrid system reaches a maximum cooling power 15.9% greater than the one-stage system (with the same electric current $I = 0.49$ A), and 11% greater than a homogeneous system, when $\omega = 0.78$. The optimization of the ratio in the number of thermocouples in each stage shows that (COP) and (Q_c) increase as the number of thermocouples in the second stage increase too, but with $W = 0.94$. We show that when two materials with different performances are placed in each stage, the optimal configuration of materials in the stages of the system must be determined to obtain a better performance of the hybrid two-stage TEMC system. These results are important because we offer a novel procedure to optimize a thermoelectric micro cooler considering the geometry of materials at a micro level.

Keywords: ideal equation; thomson effect; microcooler; thermoelectrics

1. Introduction

Thermoelectric devices are solid state devices. As coolers, they are environmentally friendly because they do not use refrigerant gas. These devices offer good cooling and the absence of moving components results in an increase in reliability, a reduction in maintenance, and in an increase in the system lifetime. Due to these advantages of thermoelectric devices and their large range of applications, the interest in micro and nano technologies has increased in recent years. Therefore, two-stages coolers should be used to improve the cooling power, Q_c [1,2]. A thermoelectric micro cooler (TEMC) uses electricity to pump heat from a cold to a hot reservoir [3]. A thermoelectric element, also called a thermoelectric couple, consists of an n-type and p-type semiconductor. A heat source at the junction causes carriers to flow away from the junction, creating an electrical generator (power generation mode). Similarly, when electrical current is conducted in the appropriate direction through the junction, both types of charge carriers move away from the junction and convey heat away, thus cooling the junction (cooling mode) [4].

Two main research directions in thermoelectrics lie in the semiconductor materials and in thermodynamics. Research on thermoelectric materials focused on the analysis and improvements of established materials or the discovery and prediction of new materials [5,6]. Research on thermodynamics is focused on the analysis and optimizations of the performance of thermoelectric devices based on the properties of established materials; system geometry and energy conversion can then be improved [7,8]. Using thermoelectrics not only requires improvement of energy-conversion efficiency of the materials but also implementation of recent advances in system architecture (geometric optimization).

Micro thermoelectric devices based on non-equilibrium thermodynamics use the temperature difference between the hot and cold junctions of the thermoelectric elements as the working temperature difference of the device [9]. The Peltier effect in a thermoelectric (TE) device is a local effect confined to the junctions of the thermoelectric elements while the joule heating occurs volumetrically over the thermoelectric elements. If a temperature difference exists between any two points of a current-carrying conductor, heat is either absorbed or liberated depending on the direction of the electric current and the material. This is called the Thomson effect (or Thomson heat) [10,11]. At steady state conditions, these two effects, combined with heat conduction from the hot to the cold end, determine the cold side temperature.

In the present work, we analyze a hybrid two-stage thermoelectric microcooler system when the Thomson effect is included and when the ideal equation is used (no Thomson effect). Previous research [12–14] has reported that taking into account the Thomson effect, a better performance in thermoelectric coolers can be achieved, as shown by Chen [15]. Lee [16] also studied a thermoelectric microcooler using: (a) the ideal equation; and (b) including the Thomson effect. Lee concludes that Thomson coefficient lead to an improvement on the performance of the thermoelectric device. Seifert [17] analyzes the temperature profile in a single Peltier element and shows that the temperature dependence effects can be sufficiently approximated by constant values. These last works focus on one-stage analysis for cooling improvement by using a maximum temperature difference, ΔT_{max}, and an optimum electric current, I_{opt}, and do not take into account geometric parameters. Two-stage systems have also been studied. For example, Xuan [18], considered the ratio of the TE couple number between the stages to determine the optimum coefficient of performance. Chen [19] compared the optimal performance of single- and two-stage thermoelectric refrigeration systems. Yang [20] analyzed multistage thermoelectric microcoolers, focusing on the optimization of the maximum temperature difference. Cheng [21] presented a new approach that uses a genetic algorithm (GA) to optimize the arrangement of two-stage thermoelectric coolers (TECs). Yu [22] described a theoretical analysis and simulates the calculation for a basic two-stage thermoelectric module, where results indicated that changing the junction temperature difference in the second stage, and the length of thermocouples and the number of thermocouples in the first stage can improve the cooling performance. Liu [23] took into account Thomson heat in order to discuss its effect on temperature prediction using experimental investigation and numerical simulation. Karimi [24] studied multi-stage thermoelectric coolers, and he found that these devices offer larger temperature differences between the heat source and heat sink than single-stage thermoelectric coolers. Wang [25] presented a three-dimensional multi-physics model to optimize the performance. In more recent research, Kaushik [26] gave the number of thermocouples in the first and second stages of a TEC for the maximum cooling power; energy and exergy efficiency conditions were optimized. Meng [27] showed the effects of thermocouple physical size on the performance of a thermoelectric heat pump (TEH) driven by a thermoelectric generator (TEG). Jaziel [28] addressed theoretically and experimentally the optimization problem of the heat transfer occurring in two coupled thermoelectric devices where the optimization parameters were the applied electric currents and figure of merit.

The studies mentioned above are based on macro systems with constant geometric parameters, and some of them have been studied neglecting the Thomson effect. Many investigations have been conducted to improve the cooling capacity of two-stage TEC and have found that the cooling capacity is closely related to its geometric structure and operating conditions. In fact, there is a lack of data and investigation about hybrid two-stage thermoelectric microcooler systems (systems with different thermoelectric materials in each stage). Investigation is required because some hybrid thermoelectric cooler systems have begun to be produced commercially [29]. Therefore, a theoretical basis is needed to design a hybrid thermoelectric system with maximum performance. The optimal design of a thermoelectric micro cooler (TEMC) requires geometric optimization i.e., different cross-sectional areas for the p-type and n-type legs in both stages. Our analysis on these systems includes geometric optimization (system architecture). We focus on finding a novel procedure to find an optimal configuration for a low-cost production, where two different semiconductor materials with different physical characteristics are used to improve the performance in a thermoelectric micro cooler system. We show the role of the Thomson effect on the performance of a two-stage microcooler system and the optimum design parameters for this system are compared with the one-stage system in given working conditions. Hence, novel methods should be established to further the design and production of micro coolers, in order to improve their performance and cooling power while reducing negative environmental impact.

2. One-Dimensional Model of a Two-Stage Thermoelectric Microcooler (TEMC)

The configuration of a two-stage TE system considered in this work is shown in Figure 1. The two stages are connected electrically and thermally in series. Each stage is made of different thermoelectric semiconductor materials of n and p type. When a voltage is applied to the system, the electric current, I, flows from the positive to the negative terminal. Negative charge carriers, i.e., electrons, in the n-type semiconductor, are attracted by the positive pole of the voltage source, and repelled by the negative pole, absorbing heat from the cold side of the TEMC and transferring or pumping this heat to the hot side of the TEMC. Likewise, positive charge carriers, i.e., the holes in the p-type material, are attracted by the negative potential of the voltage source and repelled by the positive potential and move in the opposite direction to the flow of electrons. These charge carriers transfer heat from one side of the TEMC to the other. The thermoelectric modules are manufactured with several of these pairs of semiconductor elements of type n and p, connected electrically and thermally in series. Arranging pairs of elements in this way allows the heat to be pumped in the same direction.

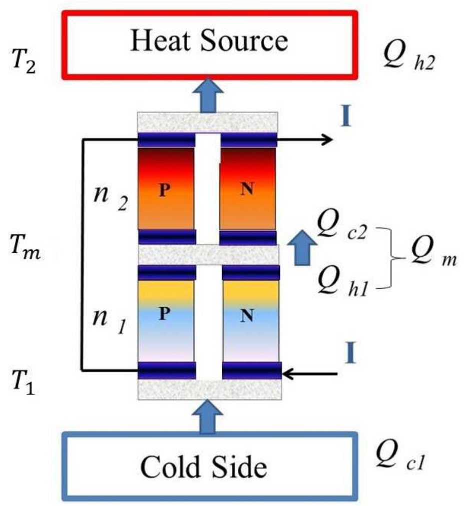

Figure 1. A two-stage thermoelectric microcooler (TEMC).

2.1. Basic Equations of the Two-Stage Thermoelectric Microcooler (TEMC)

According to out-of-equilibrium thermodynamics, when a flow of electrical current density is established, through the semiconductor material with a temperature gradient, we have [17],

$$\vec{\nabla} \cdot \left(\kappa \vec{\nabla} T \right) + j^2 \rho - T \frac{d\alpha}{dT} \vec{J} \cdot \vec{\nabla} T = 0 \tag{1}$$

where, α is the Seebeck coefficient, T is the temperature, ρ is the electric resistivity, κ the thermal conductivity and j is the electric current density flow. In Equation (1), the first term describes the thermal conduction, the second term the joule heating, and the third term the Thomson heat. Thomson heat is given by:

$$\tau = T \frac{d\alpha}{dT} \quad (\mathrm{V/K}) \tag{2}$$

Of course, if the Seebeck coefficient is independent of temperature, the Thomson coefficient τ is zero. Now, if we consider one p-type and n-type thermocouple (see Figure 1) when an electric current flows through the system, from Equation (1), we find that the equation that governs the system for one-dimensional steady state is given by:

$$\frac{d^2 T}{dx^2} - \frac{I \frac{d\alpha}{dT}}{A\kappa} T \frac{dT}{dx} + \frac{I^2 \rho}{A^2 \kappa} = 0 \tag{3}$$

where A is the cross-sectional area of the thermoelement. In order to make Equation (3) dimensionless using the boundary conditions $T(0) = T_1$ and $T(L) = T_2$, in accordance Figure 1 we define the dimensionless temperature, θ, and the ξ-parameter as,

$$\theta = \frac{T - T_1}{T_2 - T_1} \qquad \text{and} \qquad \xi = \frac{x}{L} \tag{4}$$

where L is the element length. Dimensionless differential equation corresponding to Equation (3) is given by:

$$\frac{d^2\theta}{d\xi^2} - \beta \left((\theta - 1)\, \phi + 1 \right) \frac{d\theta}{d\xi} + \gamma = 0 \tag{5}$$

where,

$$\beta = \frac{I T_2 \frac{d\alpha}{dT} \Delta T}{A\kappa \frac{\Delta T}{L}} \tag{6}$$

i.e., β is the ratio of the Thomson heat to the thermal conduction. From Equation (5), if $\beta = 0$ (or $\tau = 0$), we obtain the ideal equation (IE) for the case in which we make the Thomson effect negligible. We also define the dimensionless parameter, γ, as the ratio of the Joule heating to the thermal conduction,

$$\gamma = \frac{I^2 R}{A\kappa \frac{\Delta T}{L}} \tag{7}$$

The value of γ-parameter is determined by the properties of the thermoelectric material and the parameter ϕ, which is the ratio of temperature difference to the high junction temperature, defined as:

$$\phi = \frac{\Delta T}{T_2} \tag{8}$$

Finally, the temperature difference ΔT is as follows:

$$\Delta T = T_2 - T_1 \quad (\mathrm{K}) \tag{9}$$

The cooling power, Q_c, at the cold junction is [17],

$$\dot{Q}_c = \alpha(T_1)T_1 I + \left(-kA\frac{dT}{dx}|_{x=0}\right) \quad (\mathrm{W}) \tag{10}$$

The work per unit is given by:

$$\dot{W} = \bar{\alpha} I \Delta T + I^2 R \quad (\mathrm{W}) \tag{11}$$

and the coefficient of performance, (COP), is defined by:

$$COP = \frac{\dot{Q}_c}{\dot{W}} \tag{12}$$

2.2. Cooling Power: The Ideal Equation (IE) and Thomson Effect (β)

In the case that the Thomson coefficient is negligible (the Seebeck coefficient is independent of temperature), we can easily obtain the exact result for the cooling power at the cold junction from Equation (10),

$$\dot{Q}_c^{IE} = \bar{\alpha} T_1 I - \frac{1}{2} I^2 R - \frac{Ak}{L}(T_2 - T_1) \quad (\mathrm{W}) \tag{13}$$

This last equation is called the ideal equation (IE).

The power cooling, $\dot{Q}_c^{\beta}$, taking into account the Thomson effect, is given by:

$$\dot{Q}_c^{\beta} = \bar{\alpha} T_1 I - \frac{1}{2} I^2 R - \frac{Ak}{L}(T_2 - T_1) + \beta\frac{Ak}{L}(T_2 - T_1) \quad (\mathrm{W}) \tag{14}$$

2.3. Average System Temperature, T_m

A two-stage TEMC consists of n_1 and n_2 thermocouples in the first and second stages, respectively. Each thermocouple is composed of n-type and p-type semiconductor legs. The rates of heat that flows through the hot and the cold sides are Q_{c1} and Q_{h1} in the first stage, and Q_{c2} and Q_{h2} in the second stage, respectively. According to the theory of non-equilibrium thermodynamics, for the TEMC, we have for the first stage [26],

$$Q_{c1} = n_1[\alpha_1 I T_{c1} - K_1(T_m - T_{c1}) - 1/2 R_1 I^2 + \tau_1 I (T_m - T_{c1})] \tag{15}$$

$$Q_{h1} = n_1[\alpha_1 I T_m - K_1(T_m - T_{c1}) + 1/2 R_1 I^2 - \tau_1 I (T_m - T_{c1})] \tag{16}$$

and for the second stage,

$$Q_{c2} = n_2[\alpha_2 I T_m - K_2(T_{h2} - T_m) - 1/2 R_2 I^2 + \tau_2 I (T_{h2} - T_m)] \tag{17}$$

$$Q_{h2} = n_2[\alpha_2 I T_{h2} - K_2(T_{h2} - T_m) + 1/2 R_2 I^2 - \tau_2 I (T_{h2} - T_m)] \tag{18}$$

where,

$$K_1 = k_1 A_1 / L_1, \quad K_2 = k_2 A_2 / L_2 \quad (\mathrm{W/K})$$

and,

$$R_1 = \rho_1 L_1 / A_1, \quad R_2 = \rho_2 L_2 / A_2 \quad (\Omega)$$

For a hybrid system (with different materials in each stage), from Equations (16) and (17), we obtain the temperature between stages, T_m,

$$T_m = \frac{-\frac{1}{2} I^2 (R_1 n_1 + R_2 n_2) + \frac{1}{2} I (\tau_2 n_1 T_{h2} - \tau_1 n_1 T_{c1}) - K_2 T_{h2} n_1 - K_1 T_{c1} n_1}{I n_1 (\alpha_1 - \frac{1}{2}\tau_1) + I n_2 (\frac{1}{2}\tau_2 - \alpha_2) - K_1 n_1 - K_2 n_2} \tag{19}$$

with $\tau_i = \tau_{p_i} - \tau_{n_i}$ and $\alpha_i = \alpha_{p_i} - \alpha_{n_i}$, where the subscript ($i$) can be 1 or 2 as appropriate for material 1, M_1, in the first stage and material 2, M_2, for the second stage.

For a homogeneous two-stage thermoelectric system, i.e., the same thermoelectric material in both stages, we have $\tau_1 = \tau_2$, $K = K_1 = K_2$ and $R = R_1 = R_2$. Thus, the temperature between stage, T_m^H, for the homogeneous case is given by:

$$T_m^H = \frac{-\frac{1}{2}\tau I(T_{h2}n_2 - T_{c1}n_1) - K(n_1 T_{c1} + n_2 T_{h2}) - \frac{1}{2}RI^2(n_1+n_2)}{\alpha I(n_1 - n_2) - K(n_1+n_2) - \frac{1}{2}\tau I(n_1 - n_2)} \tag{20}$$

2.4. Material Properties

In this work we use two different semiconductor materials: Material M_1, which is obtained from commercial module of laird CP10-127-05 and its properties were provided by the manufacturer [17], and as material M_2, $(Bi_{0.5}Sb_{0.5})_2Te_3$ [16], with the properties given in Table 1, where $\bar{\alpha} = \alpha(T_{avg})$ and $T_{avg} = (T_1 + T_2)/2$.

Table 1. Properties of thermoelectric (TE) elements.

Property	Material 1, M_1 = CP-127-05	Material 2, $M_2 = (Bi_{0.5}Sb_{0.5})_2Te_3$	Unit
$\bar{\alpha}_{1,2}$	198.34×10^{-6} (at 288 K)	210.3×10^{-6} (at 288 K)	V/K
$k_{1,2}$	1.6	1.35	W/m K
$\rho_{1,2}$	1.01×10^{-5}	1.5385×10^{-5}	(Ωm)

In order to study the Thomson effect, only Seebeck coefficients are considered to be dependent on temperature while the electrical resistivity and the thermal conductivity are constant. The absolute values of the Seebeck coefficients for p-type and n-type elements are assumed to be the same, but the sign of n-type elements coefficient is negative while the sign of p-type element coefficients is positive. β is obtained with the next equations of the Seebeck coefficient for both materials:

$$\alpha_1 = [0.2068\,T + 138.78] \times 10^{-6} \quad (\mathrm{V/K}) \tag{21}$$

for material 1, and,

$$\alpha_2 = [-62675 + 1610\,T - 2.3\,T^2] \times 10^{-6} \quad (\mathrm{V/K}) \tag{22}$$

for material 2.

2.5. Geometric Parameter between Stages: Area-Length Ratio ($W = \omega_1/\omega_2$)

A common thermocouple is shown in Figure 2 in which L and A are the length and the cross-sectional area of the leg, respectively. It has been shown that an improvement in the performance of thermoelectric devices is possible by optimizing internal physical size of thermocouples [27]. We define a characteristic geometric parameter, ω, which is the area-length ratio to describe the thermocouples physical size in each of the stage of the TEMC,

$$\omega_1 = \frac{A_1}{L_1} \quad (\mathrm{m}) \tag{23}$$

for the first stage and,

$$\omega_2 = \frac{A_2}{L_2} \quad (\mathrm{m}) \tag{24}$$

for the second stage.

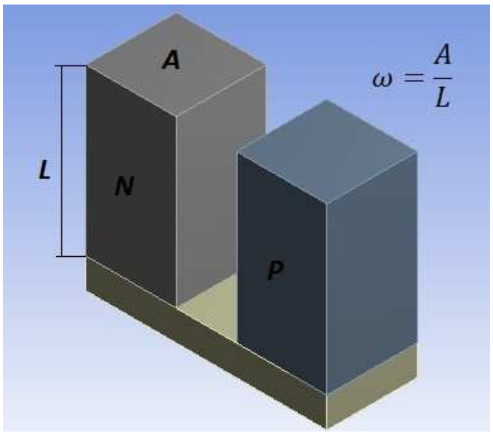

Figure 2. Schematic diagram of a thermocouple.

We define the geometric parameter, W, which allows us to determine the optimal geometric parameters of the stages, which is expressed as,

$$W = \frac{\omega_1}{\omega_2} \tag{25}$$

In terms of the geometric parameters, ω_1 and ω_2, we have:

$$R = R_p + R_n = \frac{L_p}{\sigma_p A_p} + \frac{L_n}{\sigma_n A_n} = \frac{1}{\sigma_p \omega_1} + \frac{1}{\sigma_n \omega_2} \quad (\Omega) \tag{26}$$

and,

$$K = K_p + K_n = \frac{A_p k_p}{A_p} + \frac{A_n k_n}{A_n} = \omega_1 k_p + \omega_2 k_n \quad (\mathrm{W/K}) \tag{27}$$

Substituting Equations (23) and (24) in Equations (13) and (14), we have for the cooling power, Q_c,

$$\dot{Q}_c^{IE} = \alpha(T_{avg})T_1 I - \frac{1}{2}I^2(\frac{1}{\sigma_p\omega_1} + \frac{1}{\sigma_n\omega_2}) - (\omega_1 k_p + \omega_2 k_n)(T_2 - T_1) \tag{28}$$

$$\dot{Q}_c^{\beta} = \alpha(T_{avg})T_1 I - \frac{1}{2}I^2(\frac{1}{\sigma_p\omega_1} + \frac{1}{\sigma_n\omega_2}) - (\omega_1 k_p + \omega_2 k_n)(T_2 - T_1) + \beta(\omega_1 k_p + \omega_2 k_n)(T_2 - T_1) \tag{29}$$

for the cases, ideal equation and Thomson effect, respectively.

Finally, we introduce the ratio, M, of the number of thermocouples in the first stage, n_1, to the number of thermocouples in second stage, n_2,

$$M = \frac{n_1}{n_2} \tag{30}$$

The total number of thermocouples, N, for both stages is given by,

$$N = n_1 + n_2 \tag{31}$$

In this work we focus on the optimization and design of the TEMC. The performance analysis is multiobjective, including geometric parameters, ω_1 and ω_2 and number of thermocouples, n_1 and n_2 for each stage. In the next sections, we analyze the performance of the single-stage system, including different materials, namely, M_1 and M_2 of Table 1, and the geometric parameter, $\omega = A/L$, at different values of electric current I. This analysis shows the performance of each material for the thermoelectric cooling. Consequently, we compare the performance of a hybrid two-stage TEMC system and a homogeneous two-stage TEMC. This analysis takes into account the Thomson effect via the parameter β, which relates the Thomson heat to the thermal conduction. We study a TEMC using: (a) the ideal equation; and (b) including the Thomson effect. This shows the importance of the Thomson heat in the performance of the thermoelectric system. Finally, we establish the ratio of the number of thermocouples that must be between the first and the second stage, to get a better performance. In the next sections, all figures show our results obtained with Thomson effect (solid lines) and with the ideal equation (dashed lines).

3. Special Case: Single-Stage TEMC

It is essential to know the behavior of the semiconductor materials that are going to be used in the analysis of the micro cooler with respect to COP and Q_c in order to establish an optimum geometric configuration. In the next sections, we calculate: (1) dimensionless spatial temperature distribution vs. dimensionless distance; (2) COP and Q_c vs. electric current; and (3) COP and Q_c vs. geometric parameter (ω).

3.1. Dimensionless Spatial Temperature Distribution, $\theta(\xi)$

The analysis of the spatial temperature distribution along the material lets us know the maximum temperature inside a semiconductor material when a temperature difference is established in the hot and cold sides of the TEMC. In the analysis reported in this work, we use the temperature difference $\Delta T = 20$ K and $T_2 = 298$ K. For the analysis of a single-stage system, we use a cross-sectional area of $A = 4.9 \times 10^{-9}$ m^2 and element length of $L = 30 \times 10^{-6}$ m, with a total number of thermocouples of $N = 100$.

According to Equations (5) and (4), Figure 3 shows the dimensionless spatial temperature distribution, $\theta(\xi)$, for materials M_1 and M_2 in black and blue lines, respectively. The values of the parameters β , γ and ϕ are given by Equations (6)–(8), respectively. From Figure 3, notice that in both materials when Thomson effect is taken into account, the temperature distribution values are lower than those reached without the Thomson effect, i.e., when we use the ideal equation (IE).

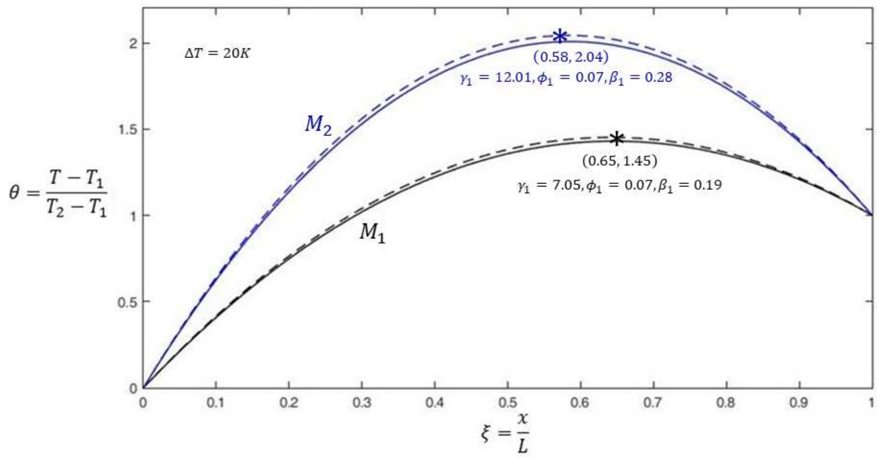

Figure 3. Single stage. Dimensionless temperature distribution, $\theta(\xi)$, for both materials M_1 and M_2. Solid and dashed lines corresponding to Temperature distributions calculated with Thomson effect and ideal equation, respectively. A difference of 40.70% is observed between the maximum values of θ for M_1 and M_2 materials.

An increase of 40.70% is observed for maximum values of the dimensionless temperature distribution, $\theta(\xi)$, between material M_1 and material M_2. In fact, one lower temperature distribution in the thermocouple obtained taking into account the Thomson effect, results in the improved cooling power at the cold side [17]. Therefore, the material M_1 can achieve more cooling power than material M_2, according to Figure 3. Notice that the amount of heat that is absorbed or released in the semiconductors depends directly on the value of β (see Equation (6)).

3.2. Coefficient of Performance (COP) and Cooling Power (Q_c) for Single-Stage TEMC

In this section, we analyze two important parameters, which characterize the performance of the TEMC: (a) the cooling power, Q_c; and (b) the coefficient of performance, COP, when we use both materials.

According to Equations (12)–(14), we obtain Figure 4 that shows the COP_1, $Q_{c,1}$ and COP_2, $Q_{c,2}$ for materials, M_1 and M_2 respectively, as a function of the electric current. From these results, we determine the optimal electric currents, $I^{COP}_{opt,1} = 0.18$ A and $I^{COP}_{opt,2} = 0.16$ A for materials M_1 and M_2 respectively, for which the maximum values of COP are obtained. Notice that the maximum values of COP and Q_c are reached when the Thomson effect is considered.

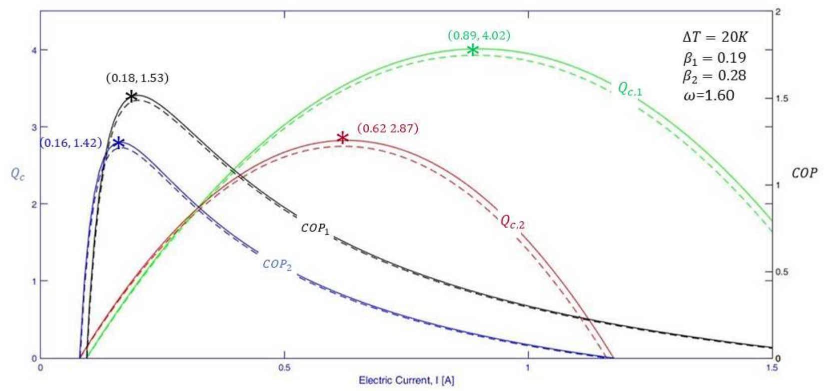

Figure 4. Single stage. Coefficient of performance, $COP(I)$, and cooling power, $Q_c(I)$, for both materials M_1 and material M_2. The COP for material M_1 is 15.1% more than for material M_2 and Q_c for material M_1 is 40.12% more than for material M_2.

An important parameter for the performance of the TECs is the cooling power Q_c. Thus, the maximum values of Q_c for the materials M_1 and M_2 are obtained at the electric currents $I^{Q_c}_{opt,1} = 0.89$ A and $I^{Q_c}_{opt,2} = 0.62$ A, respectively. Notice that the material M_1 offers a better cooling power Q_c and COP than material M_2, although the material M_1 needs a higher electric current to reach its maximum values. We also noticed that the values of cooling power for material M_1 are higher than values of material M_2, for electric current values from $I = 0.37$ A to $I = 1.5$ A. As mentioned before, better cooling power is obtained with lower values of β, and this fact is confirmed with the results shown in this section. In next sections we clarify that an improvement in the micro cooler can be achieved with small values of β.

3.3. Analysis of Geometric Parameter, $\omega = A/L$

Q_c is a very important characteristic in micro coolers and it is necessary to know the optimum geometric parameters in order to improve Q_c. In this section, we analyze the importance of the semiconductor geometric parameters by introducing the geometric parameter $\omega = A/L$, which relates the cross-sectional area, A, with the element length, L.

We use the optimal electric current values determined in the previous section to calculate the coefficient of performance, $COP(\omega)$, and the cooling power, $Q_c(\omega)$, in terms of the geometric parameter, ω shown in Figures 5 and 6. These Figures 5 and 6 were obtained using Equations (12), (28) and (29) for ideal equation and Thomson effect, respectively. Notice that Figure 5 is obtained with the optimum electric currents $I^{COP}_{opt,1}$ and $I^{COP}_{opt,2}$, for materials M_1 and M_2, respectively.

For material M_1 we have the following maximum values: $COP^{\beta}_{1,max} = 1.53$ and $COP^{IE}_{1,max} = 1.48$, with the geometric parameter value of $\omega = 1.6$. The maximum cooling power values are $Q^{\beta}_{c1,max} = 1.11$ and $Q^{IE}_{c1,max} = 1.1$, with the geometric parameter value of $\omega = 0.74$.

For material M_2, we have a $COP^{\beta}_{2,max} = 1.25$ and $COP^{IE}_{2,max} = 1.21$, with the geometric parameter value of $\omega = 1.62$. The maximum cooling power values are $Q^{\beta}_{c2,max} = 0.96$ and $Q^{IE}_{c2,max} = 0.95$ with the geometric parameter value of $\omega = 0.86$.

As shown the previous section, material M_1 has better results, in both cases, than material M_2 for COP and Q_c values.

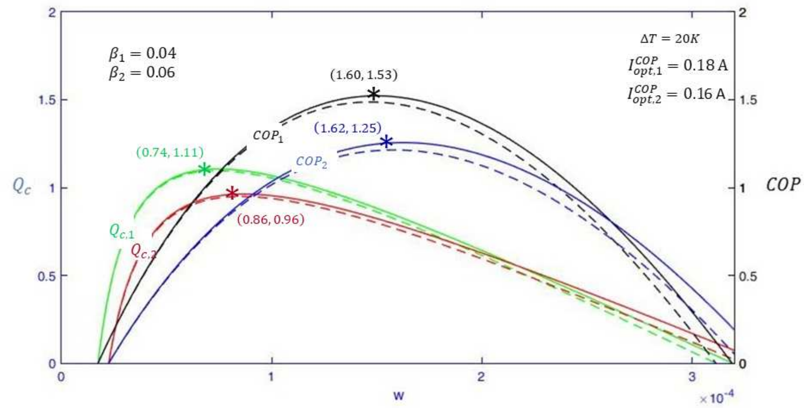

Figure 5. Single stage. $COP(\omega)$ and $Q_c(\omega)$ for both materials, using optimal electric currents I^{COP}_{opt}. The COP of material M_1 is 21.18% higher than for material M_2 and the Q_c value in material M_1 is 14.85% higher than for material M_2.

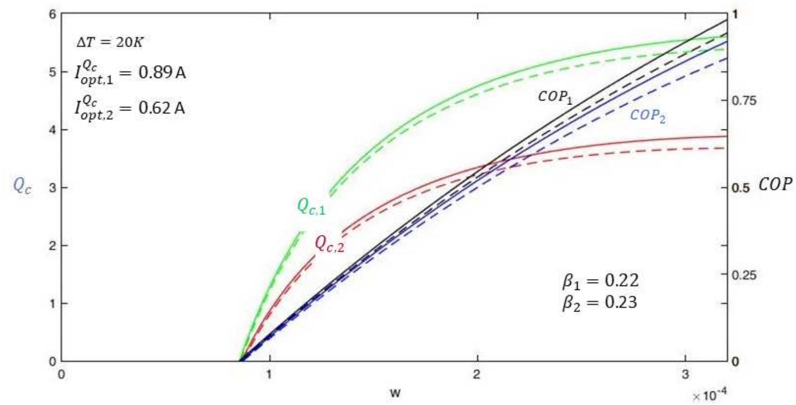

Figure 6. Single stage. $COP(\omega)$ and $Q_c(\omega)$ for both materials, using optimal electric currents, $I^{Q_c}_{opt,1}$ and $I^{Q_c}_{opt,2}$.

The optimal electric currents to obtain the best cooling power Q_c of TEMC are determined from the Figure 4 for each material. Figure 6 is obtained using the values $I^{Q_C}_{opt,1}$ and $I^{Q_C}_{opt,2}$. Thus, for material M_1, $Q^{\beta}_{c1,max} = 5.6$ and $Q^{IE}_{c1,max} = 5.38$, while for material M_2, $Q^{\beta}_{c2,max} = 3.87$ and $Q^{IE}_{c2,max} = 3.67$, with the geometric parameter value of $\omega = 2.3$.

Figure 6 shows that Thomson effect improves the performance of the system up to 4.08%. In this analysis, we consider number of thermocouples of $N = 100$ in both cases. The increase or decrease in the number of thermoelectric elements affects the cooling power but the COP remains constant.

4. Homogeneous and Hybrid Two-Stage TEMC Systems

Now, we analyze a two-stage TEMC for two cases: (a) firstly, we consider a homogeneous TEMC system, i.e, a system with the same material in both stages (two-stage TEMC); and (b) we consider a hybrid TEMC system (hybrid two-stage TEMC), i.e, a system with a different thermoelectric material in each stage. Thus, we determine the optimum thermoelectric material arrangement for the best performance of the TEMC system. We use the following values: a cross-sectional area of $A = 4.9\times 10^{-9}$ m^2 and element length of $L = 15 \times 10^{-6}$ m, with a total number of thermocouples of $n_1 = 100$ and $n_2 = 100$ for the first and second stage, respectively.

4.1. Homogeneous Two-Stage TEMC System (Two-Stage TEMC)

We analyze a homogenous two-stage thermoelectric micro cooler, i.e, single thermoelectric material in both stages. In the next sections, we calculate: (1) dimensionless spatial temperature distribution vs. dimensionless distance; (2) COP and Q_c vs. electric current; and (3) COP and Q_c vs. geometric parameter (W).

4.1.1. Dimensionless Spatial Temperature Distribution for Two-stage TEMC: Thomson Effect ($\beta \neq 0$) and Ideal Equation ($\beta = 0$)

Figure 7 shows the dimensionless spatial temperature distribution for material M_1 and material M_2. In this two-stage TEMC system, an increase in the values of θ compared with the one-stage TEMC system, is observed, with maximum values in the second stage. As expected, the values of θ are lower including the Thomson effect than those obtained using the ideal equation.

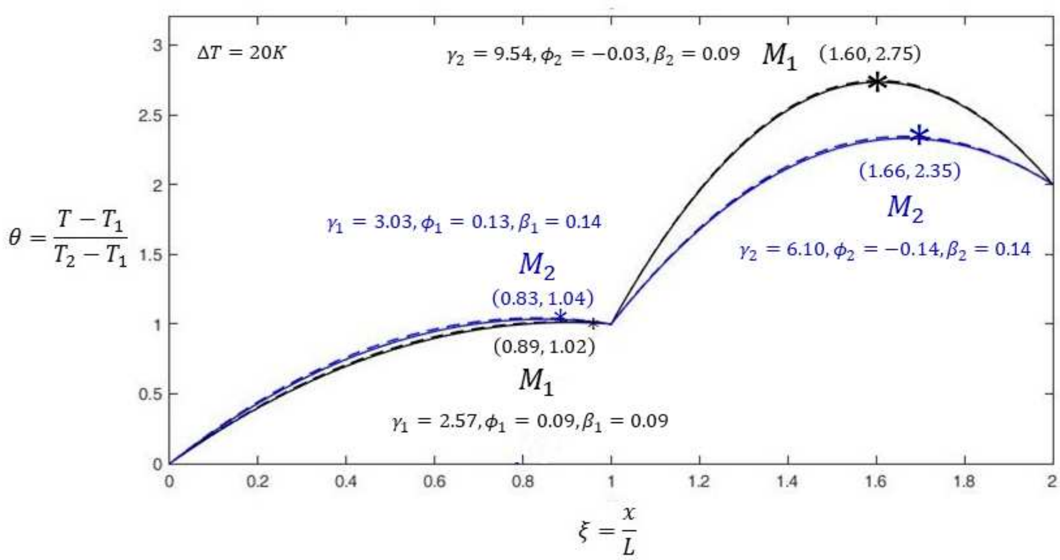

Figure 7. Homogeneous Two-stages TEMC. Dimensionless temperature distribution, $\theta(\xi)$. The same material is used in each stage.

4.1.2. Coefficient of Performance (COP) and Cooling Power (Q_c) for Two-Stage TEMC

Figure 8, shows the maximum values for COP and Q_c for both materials. Notice that material M_1 has higher values of Q_c than those obtained with the material M_2 at the same currents. The optimum electric currents for the COP_{max} are $I^{COP}_{opt,1} = 0.46$ A and $I^{COP}_{opt,2} = 0.55$ A for materials M_1 and $M2$, respectively, while for the maximum cooling power $Q^{\beta}_{c,max}$, we have, $I^{Q_c}_{opt,1} = 1.24$ A. Notice that maximum values of COP decrease in both materials compared with a single-stage system while maximum values of Q_c increase.

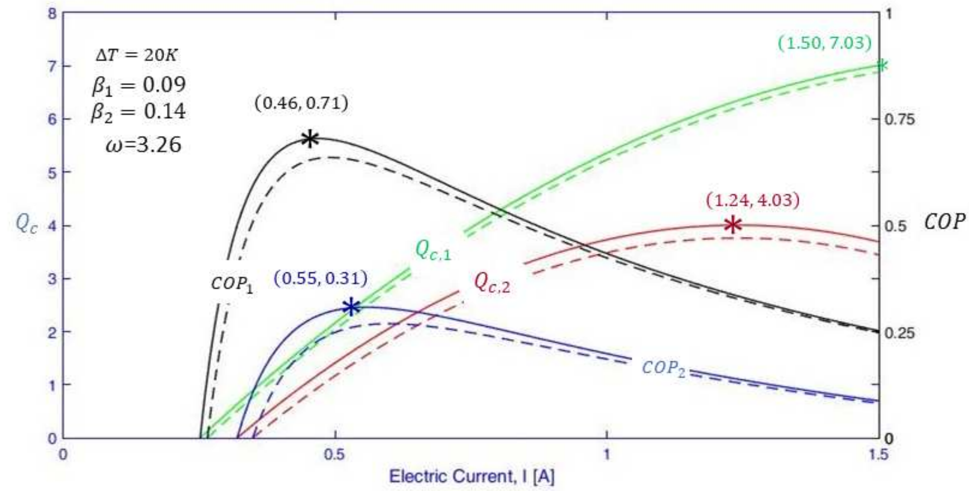

Figure 8. Homogeneous Two-stages TEMC. $COP(I)$ and $Q_c(I)$ with $\omega = \omega_1 = \omega_2$.

4.1.3. Analysis of Geometric Parameter ω in Two-Stage TEMC

In this section, we study the effect of the geometric parameters $\omega = A/L$ on COP and Q_c of the homogeneous two-stage TEMC. Figure 9 (see Equations (11), (12), (28) and (29)), shows the behaviour of COP and Q_c for each material versus $W = \omega_1/\omega_2$, when we take into account: (a) the Thomson effect; and (b) the ideal equation. Our results for the material M_1 are: (a) $COP^{\beta}_{1,max} = 1.29$ (including Thomson effect); and (b) $COP^{IE}_{1,max} = 1.25$ (using ideal equation), with a value of $\omega = 5.19$. Besides, $Q^{\beta}_{c1,max} = 3.03$ and $Q^{IE}_{c1,max} = 3.01$, with a value of $\omega = 2.11$ using $I^{COP}_{opt,1} = 0.46$ A.

For material M_2: (a) $COP^{\beta}_{2,max} = 0.58$ (Thomson effect) and $COP^{IE}_{2,max} = 0.53$ (ideal equation) with a value of $\omega = 4.75$. We obtain $Q^{\beta}_{c2,max} = 2.89$ and $Q^{IE}_{c2,max} = 2.80$, with a value of $\omega = 2.63$ using $I^{COP}_{opt,2} = 0.55$ A.

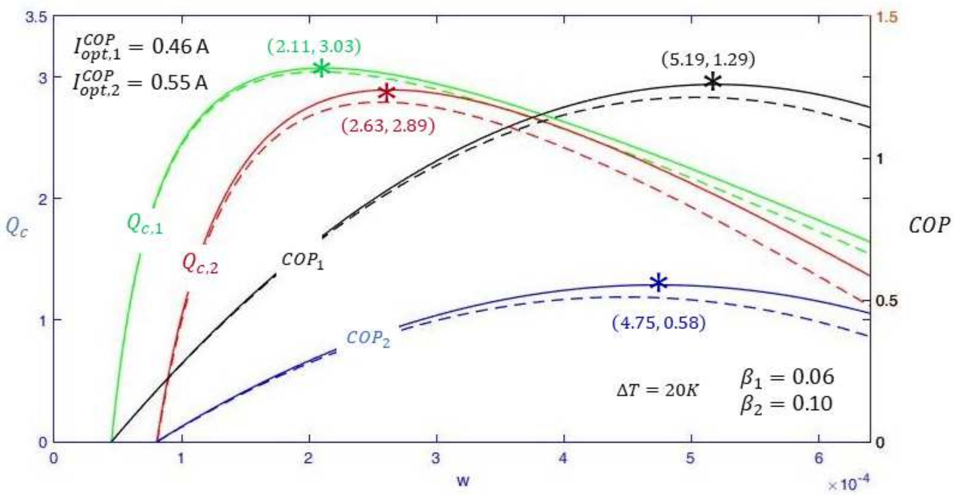

Figure 9. Homogeneous Two-stages TEMC. $COP(\omega)$ and $Q_c(\omega)$ for both materials, using optimal electric currents, $I^{COP}_{opt,1}$ and $I^{COP}_{opt,2}$.

4.2. Hybrid Two-Stage TEMC System

The main objective in this work is to propose a novel procedure to optimize the performance of a hybrid two-stage thermoelectric micro cooler, based on the equations of out-of-equilibrium thermodynamics and mainly on a new analysis of the geometric parameters of the elements type n and p. From results of the previous section, it is clear that some materials offer better performance in cooling power, while using different values in the electric current. In the next sections, we determine: (a) the optimal configuration of the thermoelectric materials to be used in each stage of the system; and (b) the optimal geometric parameters to improve the performance in the cooling power and coefficient of performance.

We consider two configurations of materials for a hybrid two-stage TEMC system, namely: (a) the material M_1 placed in the first stage and material M_2 in the second stage; and (b) reverse order of materials, i.e., the material M_2 placed for the first stage and material M_1 in the second stage.

4.2.1. Dimensionless Temperature distribution for Hybrid Two-stage TEMC: Ideal Equation $\beta = 0$ and Thomson Effect $\beta \neq 0$

For the hybrid two-stage TEMC system, the best configuration of semiconductor thermoelectric materials and its optimal geometric parameters is about to be found. Two different cases are analyzed according to the two different materials we work with.

Figure 10, according to Equation (5), shows the dimensionless spatial temperature distributions, for cases (a) and (b). In thermoelectric effects, it is known that the moving charged electrons transport thermal energy, absorbing or liberating heat depending on the β value [17]. In Figure 10, notice that before the maximum value for θ, namely, θ_{max}, we have $d\theta/d\xi > 0$; the Thomson heat acts by absorbing heat, while after θ_{max}, it acts by liberating heat when $d\theta/d\xi < 0$ is negative. We note that in stage one, both materials can absorb heat at lower values of θ, which is desirable for a better cooling power. In the second stage of the system, it is required that the semiconductor be capable of liberating heat for optimizing the system. For case (a) the material M_2 can liberate more heat since $d\theta^{(a)}_{M_2}/d\xi > \theta^{(b)}_{M_1}/d\xi$ from $\xi = 1.57$ (value close to the midpoint of the second stage) compared to material M_1 in case (b), in which it liberates less heat. According to this last statement, case (a) results the best configuration of materials to improve the TEMC. This will be verified in the next section.

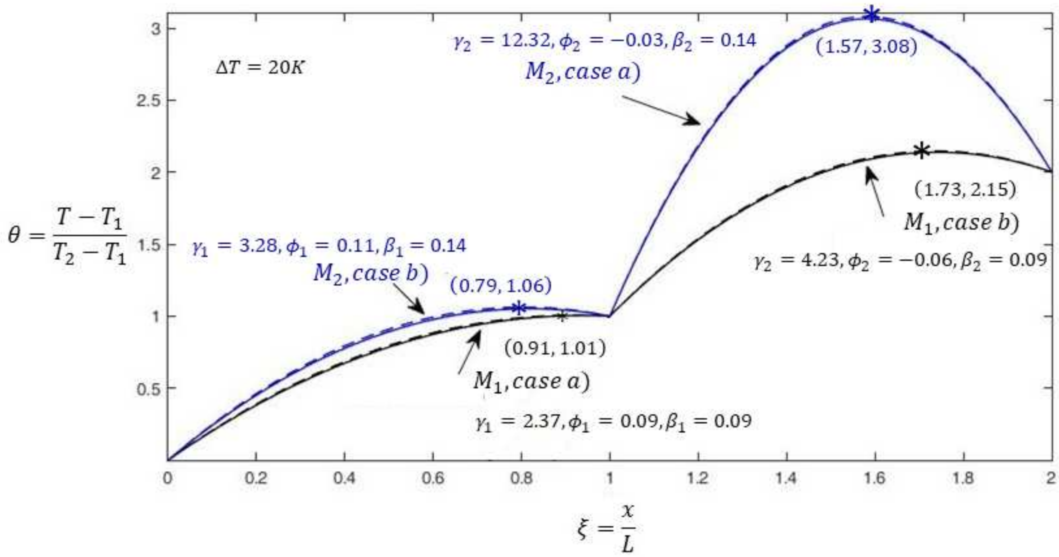

Figure 10. Hybrid Two-stages TEMC. Dimensionless temperature distribution, $\theta(\xi)$. Case (a): material M_1 is placed in stage one (black line) and material M_2 in stage two (blue line). Case (b): material M_2 is placed in stage one (blue line) and material M_1 in stage two (black line).

4.2.2. Coefficient of Performance (COP) and Cooling Power (Q_c) for Hybrid Two-Stage TEMC

Figure 11 shows the COP and Q_c for the hybrid two-stage TEMC system for cases (a) and (b) described previously. It is found that case (a) is the best option for the cooling power and the coefficient of performance. For this case, we have an optimal electric current $I_{opt}^{COP}(a) = 0.49$ A, while for the case (b), we have an optimal electric current $I_{opt}^{COP}(b) = 0.51$ A. Notice that the $COP_{max}^{\beta}(a)$ in case (a), when M_1 is placed in the stage one, is 19.05% better than $COP_{max}^{\beta}(b)$ in case (b), in which M_2 is placed in stage one. The values of the cooling power, Q_c, for the case (a) are always over those of the case (b) at the same current values.

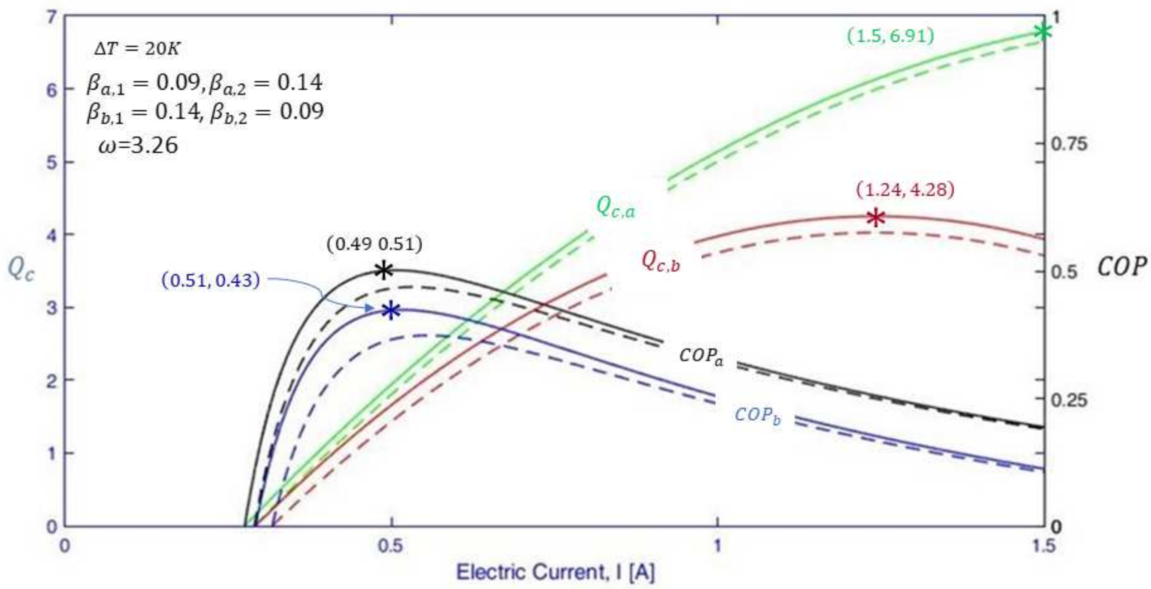

Figure 11. Hybrid Two-stages TEMC. $COP(I)$ and $Q_c(I)$ for cases (a) and (b) with $\omega = \omega_1 = \omega_2$.

4.2.3. Analysis of the Geometric Parameter ω for Hybrid Two-Stage TEMC

In the results from previous sections, it has been shown that an optimal cooling power in a thermoelectric micro cooler system is possible when we consider geometric parameters. In fact, cooling power in a two-stage TEMC system can be improved with different considerations such as temperature difference, material properties and geometry in the semiconductor elements of the TEMC. In this work, we study the performance of a two-stage TEMC system with an alternative approach: We optimize the two-stage TEMC system, COP, and Q_c with a new geometric parameter, $W = \omega_1/\omega_2$, i.e., we study the performance of the two-stage TEMC system when the thermocouples' physical sizes in the two stages are related each other. The physical size refers to the length and the cross-sectional area of the thermocouples.

The effect of the parameter W on COP and Q_c is analyzed when (1) $\omega_1 = \omega_2 = \omega$; and (2) $\omega_1 \neq \omega_2$. Figures 12 and 13 show the behaviour of the $COP(\omega)$ and $Q_c(\omega)$ at optimal electric current, $I = 0.49$ A and $I = 0.51$ A for cases (1) and (2), respectively (see Figure 11).

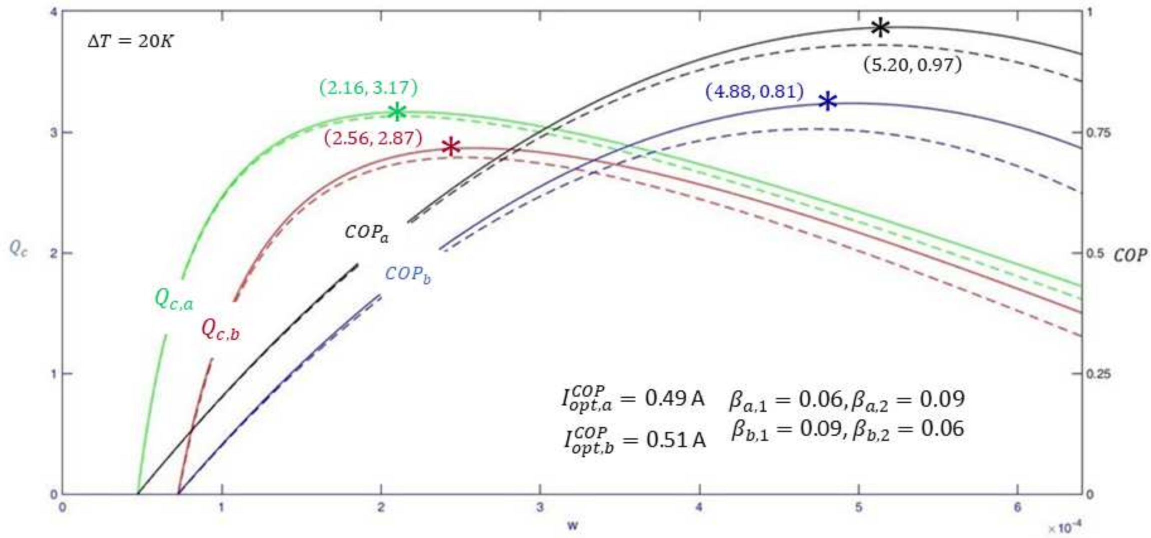

Figure 12. Hybrid Two-stages TEMC. $COP(\omega)$ and $Q_c(\omega)$ with optimal electric current $I^{COP}_{opt,a}$ and $I^{COP}_{opt,b}$, for case (a) and case (b), respectively. $\omega_1 = \omega_2 = \omega$.

From Figure 12, we can see that in the configuration of thermoelectric materials in case (a), the material M_1 is placed in the first stage and material M_2 in the second stage, and COP and Q_c reach values higher than those for case (b). This means that large area–length ratio values are not necessary to improve Q_c since the cooling power is improved for small values of ω. COP increases by 19% and Q_c increases 10.5% in case (a) compared with case (b).

Finally, we analyze the most important case when $\omega_1 \neq \omega_2$. In this case, we set $\omega_2 = 3.26 \times 10^{-4}$ m to be a constant value and the results are shown in terms of the geometric parameter $W = \omega_1/\omega_2$. Figure 13 shows $COP(W)$ and $Q_c(W)$ for the hybrid two-stage TEMC system. In case (a), we use a value of $I^{COP}_{opt}(a) = 0.49$ A and for the case (b) $I^{COP}_{opt}(b) = 0.51$ A. COP increases by 8.9% and Q_c increases 6.27% in case (a) compared with case (b).

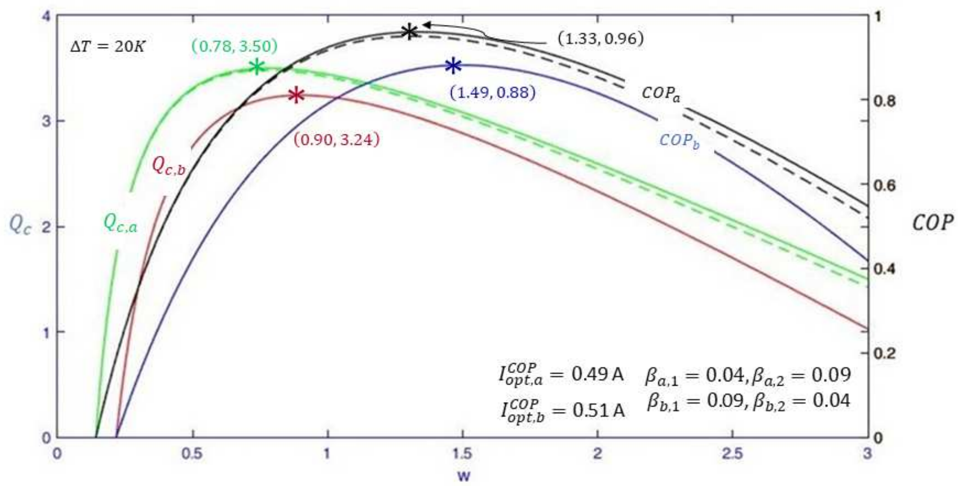

Figure 13. Hybrid Two-stages TEMC. $COP(W)$ and $Q_c(W)$ using $I^{COP}_{opt,a}$ and $I^{COP}_{opt,b}$ for case (a) and case (b), respectively, with $\omega_1 \neq \omega_2$.

4.2.4. Ratio of the Total Number of Thermocouples for Hybrid Two-Stage TEMC

Finally, we briefly analyze the influence of the number of thermocouples in the first and second stage on the COP and Q_c of the TEMC system. In this analysis, the number of thermocouples in the first stage is fixed at $n_1 = 100$ and for the second stage, we use values of $n_2 = 75, 100, 125$. The total heat transfer area is the sum of the area in the cold and hot side of the total number of thermocouples in the TEMC. It is clear from Figure 13 that when the number of thermocouples in the first stage is less than the number of thermocouples in the second stage: (1) the amount of heat rejected at the hot side of the second stage is higher than the heat absorbed at the cold side; and (2) the heat transfer area in the hot side should be higher than the cold side.

In Figure 14, we use the best configuration, according to our previous results, and we find that increasing the number of thermocouples in the second stage leads to an improvement in the COP and Q_c.

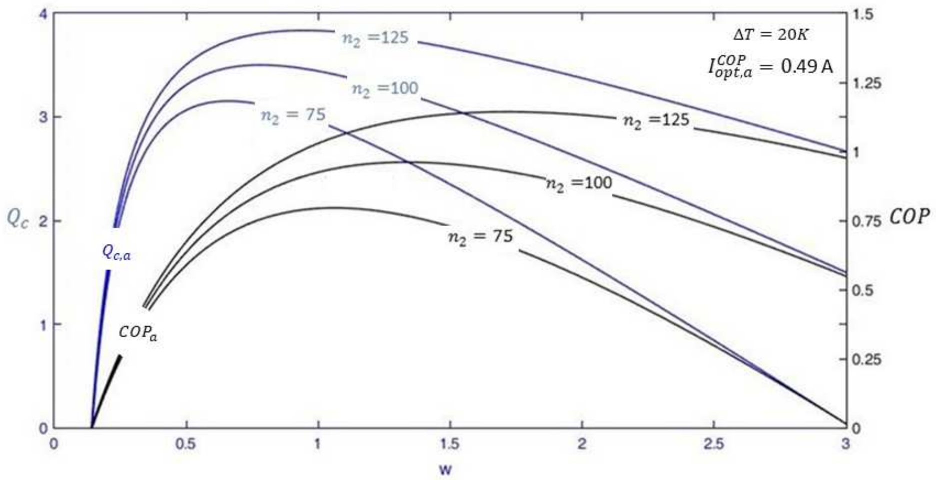

Figure 14. Hybrid Two-stages TEMC. $COP(W)$ and $Q_c(W)$ with $n_1 \neq n_2$ and $\omega_2 = 3.26 \times 10^{-4}$ m.

5. Conclusions

We determine the performance of a hybrid two-stage thermoelectric microcooler (TEMC), in which a different semiconductor material is placed in each stage. The stages are electrically connected in series. Results are presented for the cases when $\beta = 0$ (ideal equation) and when $\beta \neq 0$ (Thomson effect) to show the contribution of the Thomson effect in the performance of the TEMC. Our results show that the geometric optimization for the hybrid TEMC system with the parameter $W = \omega_1/\omega_2$ improves the cooling power of the system compared with that of the homogeneous TEMC system.

From the dimensionless spatial temperature distribution, we determined the best thermoelectric material to absorb or release heat. It was observed that material 1 was the best choice in order to improve the TEMC, because it has lower temperature distribution with lower values of β, as seen in Figures 3 and 7. For the hybrid system, the best configuration of materials to be used in the first and the second stage (see Figure 10 case (a)) was found according to the requirements of maximum absorbing heat in the first stage, and maximum release heat in the second stage. Additionally, we show that in case (a), which has higher values of performance than case (b), the COP increases 19% and Q_c increases 10.5% compared with case (b). It should be noted that the configuration of materials, with different performance in the stages of the system is very important. In our study, we note that material M_1 has higher values of performance than M_2. If we place M_1 in the second stage it helps to improve the performance in the system (see Figure 12 case (b)).

Our results show that the Thomson effect increases the maximum cooling power and coefficient of performance with a maximum of 4.08% and 6.5% in single-stage and two-stage systems, respectively. Also, applying the geometric optimization to the TEMC systems for a single-stage and homogeneous two-stage TEMC, material M_1 can achieve 5% more in cooling power with values of ω smaller than those of material M_2 (see Figure 9). Besides, the geometric optimization for COP and Q_c was achieved for the hybrid two-stage TEMC in case (a) when $\omega_1 \neq \omega_2$, at optimal electric current, $I_{opt}^{COP}(a)$ and the value of the parameter $W = 0.78$. An improvement of 15.9% in Q_c is achieved with a variation in COP of 7% compared to single-stage system when working at the same electric current. Finally, the optimum number of thermocouples for the maximum cooling power and coefficient of performance has been evaluated. We show that increasing the number of thermocouples in the second stage leads to an increase in COP and Q_c.

In this work, the alternative approach to improve cooling power and performance in a hybrid two-stage thermoelectric micro cooler determines the optimal geometry in semiconductor elements of the TEMC and the optimal configuration of the materials that must be used in each stage in the system. This study will be useful in the optimal design of the architecture of the two-stage thermoelectric micro coolers.

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