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A Fuzzy Comprehensive Evaluation Method Based on AHP and Entropy for a Landslide Susceptibility Map

Hongliang Zhao [1], Leihua Yao [1,*], Gang Mei [1], Tianyu Liu [1] and Yuansong Ning [2]

1 School of Engineering and Technology, China University of Geosciences (Beijing), Beijing 100083, China; zhl@cugb.edu.cn (H.Z.); gang.mei@cugb.edu.cn (G.M.); Liuty@cugb.edu.cn (T.L.)
2 Beijing General Research Institute of Mining & Metallurgy, Beijing 100160, China; ningyuansong@bgrimm.com
* Correspondence: yaolh@cugb.edu.cn; Tel.: +86-010-8232-1912

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Abstract: Landslides are a common type of natural disaster in mountainous areas. As a result of the comprehensive influences of geology, geomorphology and climatic conditions, the susceptibility to landslide hazards in mountainous areas shows obvious regionalism. The evaluation of regional landslide susceptibility can help reduce the risk to the lives of mountain residents. In this paper, the Shannon entropy theory, a fuzzy comprehensive method and an analytic hierarchy process (AHP) have been used to demonstrate a variable type of weighting for landslide susceptibility evaluation modeling, combining subjective and objective weights. Further, based on a single factor sensitivity analysis, we established a strict criterion for landslide susceptibility assessments. Eight influencing factors have been selected for the study of Zhen'an County, Shan'xi Province: the lithology, relief amplitude, slope, aspect, slope morphology, altitude, annual mean rainfall and distance to the river. In order to verify the advantages of the proposed method, the landslide index, prediction accuracy P, the R-index and the area under the curve were used in this paper. The results show that the proposed model of landslide hazard susceptibility can help to produce more objective and accurate landslide susceptibility maps, which not only take advantage of the information from the original data, but also reflect an expert's knowledge and the opinions of decision-makers.

Keywords: Shannon entropy; fuzzy comprehensive evaluation; AHP; landslide

1. Introduction

Landslides are one of the most important types of natural disasters. They are characterized by their wide distribution, high frequency, fast movement and serious disaster-related losses. Landslide hazards endanger the safety of human lives and property, destroying the environment and natural resources. Generally, the damage can be much more serious in densely-populated areas. For example, the Abe Barek landslide, which occurred around 11 a.m. on 2 May 2014 in the Ago District of Badakhshan Province, Afghanistan, buried 86 houses and took the lives of almost 2700 people [1]. Moreover, on 29 October 2014, a major landslide occurred in Koslanda, in the district of Badulla, burying 150 houses, killing 16 people and leaving 192 people missing. The Qin-ba mountain area is the largest east-west mountain range in Central China. It has extremely complex terrain conditions and various lithologies and rock structures, as well as four distinct seasons and hydrological conditions. This region has one of the highest incidences of geological disasters in China. For example, at 0:30 a.m. on 12 August 2015 in the town of Yanjiagou, a village of Shanyang County in China's Shan'xi Province, a sudden onset of landslides, with a volume of 1,500,000 m^3, buried 15 dormitories and there houses and left 64 people missing.

The production of a landslide susceptibility map (LSM) in the early period is of great significance to the prevention and control of geological hazards. Landslide susceptibility describes the likelihood of a landslide occurring in an area and is controlled by local terrain conditions [2,3]. Diverse research methods for LSM have made great progress from early qualitative descriptions to semi-quantitative studies to recent sophisticated quantitative assessment modeling [4]. Over the past century (since the 1990s), with the development of multidisciplinary improvements and various theoretical methods, the LSM have placed more emphasis on regional quantitative evaluations, especially with the application of geospatial technology, which can quickly obtain regional, large-scale landslide survey results and collect environmental factors, as well as other information that is closely related to the occurrence of landslide hazards, effectively promoting the research and application of non-deterministic methods for landslide susceptibility assessments. At present, universally-applied geological hazard assessment models include subjective inference analysis models, statistical analysis models, deterministic models, pattern recognition models and the like. Then, different scholars use a variety of theories to study landslide susceptibility assessments, including such techniques as artificial neural networks [5–7], logistic regression [8–11], analytic hierarchy processes (AHP) [2,12,13], the information value method [14–16], the certainty factor [17–19], fuzzy logic [20–22] and an index of entropy [23–25]. These methods are mainly based on the analysis of the distributions of landslide hazards and the relationship between the influencing factors.

Despite the effectiveness of the previously-used methods that consider complex classification problems, these methods mostly assign unvarying weight values to the whole study area. In other words, the currently-reported research work did not consider the difference of the main influencing factors at different sites among the study area.

In this paper, to improve the previous methods for landslide susceptibility assessment and derive more reasonable evaluation results, a method for LSM is proposed that combines subjective and objective weights. Therefore, the weights obtained from the entropy method and AHP are combined with the fuzzy comprehensive method before being applied to landslide susceptibility zoning evaluation, especially with the introduction of the Shannon entropy algorithm, which can highly enhance the objectivity of statistical data from the field investigation. This method has been used in development strategy research [26], website usability evaluations [27], comprehensive project decisions [28], as well as quantitative evaluation on the characteristics of activated sludge granules and flocs [29]. However, differing very much from previous studies, in our research, the fuzzy evaluation matrices R decide the objective weights with the entropy method so that the final comprehensive weights change along with the evaluation units, such that the proposed methodology is a variable comprehensive weight model for LSM. Specifically, this study will assist in performing a more accurate and reasonable LSM and reducing the loss of landslide disasters.

In summary, the following contributions have been made in this paper:

1. The subjective and objective weights are combined such that the information from the original statistical data is used, and meanwhile, the knowledge of experts and the opinions of decision-makers (DMs) are also reflected.
2. The comprehensive weight used in this paper is variable with respect to the changes of the evaluated units. However, in previously-published research, each an evaluation factor was given a single weight for the whole region.

This paper is organized as follows. Section 2 introduces the principle and procedure of the proposed entropy-FAHP method which combined the entropy algorithm and AHP-fuzzy (FAHP) method. Section 3 selects Zhen'an County of Shan'xi Province as the study area for the application of the proposed method for LSM. Section 4 shows the results of landslide susceptibility assessment. Section 5 discusses the evaluation, comparison and validation of the LSM methods. Section 6 draws some conclusions.

2. Proposed Method

Any evaluation method will be affected by subjective factors, so the objectivity of an evaluation cannot be fully established. There are many ways to determine weights, such as the analytic hierarchy process (AHP), principal component analysis [30,31], entropy weighting [32,33], TOPSIS [34] and coefficients of variation [35]. Among all of these methods, the most widely used one that determines the subjective weight is AHP. This kind of weighting is reasonable, but it cannot overcome the subjective arbitrariness. Objective weights are usually determined by the Shannon entropy method. Entropy weights may have an objective result when fully exploiting the information contained in the original data, but can also cause large false positives (i.e., showing very few pixels with a very high susceptible class) [33], and it cannot reflect the knowledge and practical experience of experts and the opinions of DMs. In addition, a combination of subjective and objective weights can make the evaluation results more comprehensive and reasonable (Figure 1).

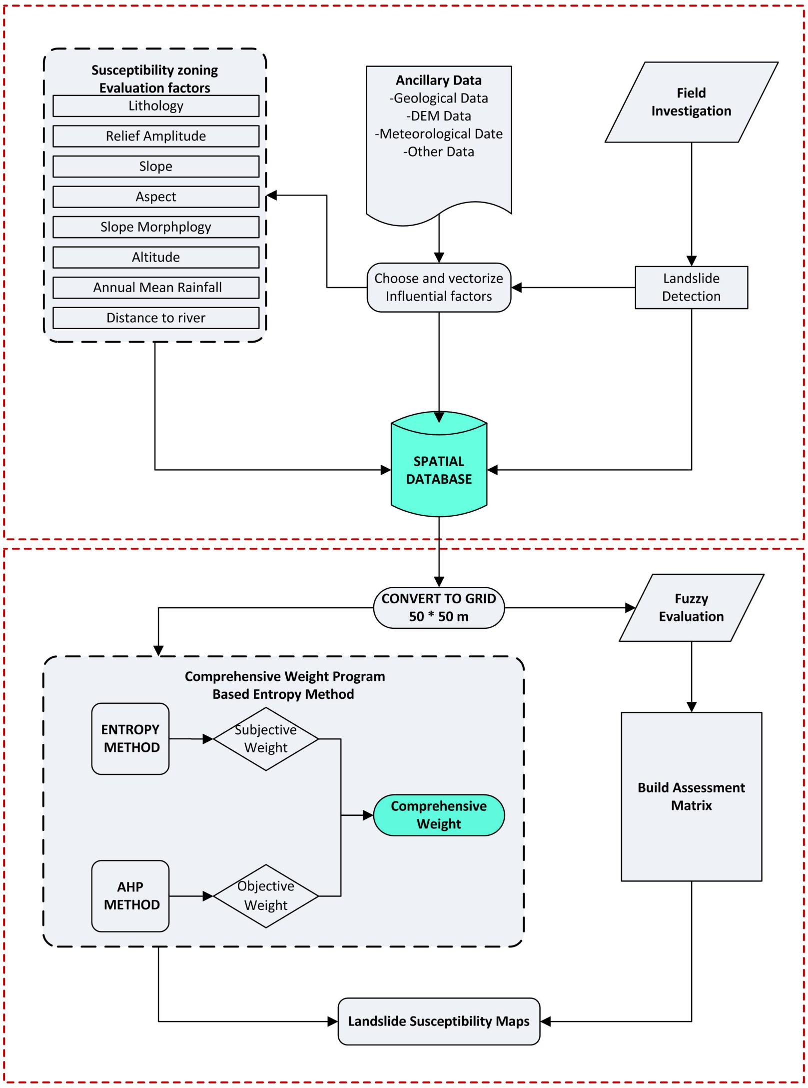

Figure 1. The flowchart of the proposed method.

To depict the proposed method, it is best to consider a four-step procedure: in Step 1, select an appropriate side length, and divide the region into grid units. After establishing a strict fuzzy comprehensive evaluation index system, the attributes of each unit are extracted in an ARCGIS (version 10.3, Esri Co. Ltd.) environment. After that, we import the data collected above into MATLAB (version 2014a, MathWorks Co. Ltd.) and construct a fuzzy judgment matrix. Accordingly, in Step 2, AHP is used to quantify the influences of n factors on the target u (subjective weight). In Step 3, the entropy method is used to calculate the entropy weights (objective weights) with a fuzzy judgment matrix as a research object in the MATLAB (version 2014a, MathWorks Co. Ltd.) environment. Finally, in Step 4, the comprehensive weights are calculated from the subjective and objective weights. Then, according to the principle of maximum membership degree, the results of the evaluated units are determined.

2.1. Building a Fuzzy Matrix

The fuzzy comprehensive evaluation method is a quantitative evaluation method proposed by Zadeh [36]. That is, the method makes a general evaluation of processes or objects subjected to a variety of factors [33,37]. First of all, supposing there are n evaluation ratings and m evaluation factors, the evaluation rating domain U can be expressed as $U = (u_1, u_2, \cdots, u_n)$, the evaluation factor domain V can be expressed as $V = (V_1, V_2, \cdots, V_m)$. Furthermore, the critical point to its success lies in correctly prescribing the domain of the fuzzy evaluation and constructing a reasonable fuzzy evaluation matrix. Then, according to the fuzzy relation between the comment set (i.e., evaluation rating) and the evaluation factor, the fuzzy evaluation matrix R is established (Equation (1)).

$$R = \begin{array}{c} V_1 \\ V_2 \\ \vdots \\ V_m \end{array} \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} = \left(r_{ij}\right)_{m \times n}, i = 1, 2, \cdots, m; j = 1, 2, \cdots, n \tag{1}$$

2.2. Determining the Subjective Weight Using AHP

AHP was originally proposed by Saaty [38] , which is a simple, flexible and practical multi-criteria decision-making method for qualitative analysis. First of all, AHP method is used to decompose a problem into a ladder-shaped and ordered structural model. The importance weights ω_i of every evaluation factor V_i on the target u are different. As the following matrix (Equation (2)) shows, according to the DM's judgment of the objects' real characteristics, we compare the influences of m factors on the target u according to the degree of their impact. Then, the relative importance of each factor is quantitatively described. Finally, the weights of the relative importance of all of the factors are calculated [37].

$$A = \begin{bmatrix} \omega_1/\omega_1 & \omega_1/\omega_2 & \cdots & \omega_1/\omega_m \\ \omega_2/\omega_1 & \omega_2/\omega_2 & \cdots & \omega_2/\omega_m \\ \vdots & \vdots & \ddots & \vdots \\ \omega_m/\omega_1 & \omega_m/\omega_2 & \cdots & \omega_m/\omega_m \end{bmatrix} \tag{2}$$

A is the judgment matrix. If A satisfies the consistency judgment condition, we can find the weight values $\omega = \omega_1, \omega_2, \cdots, \omega_m$, calculated with the equation $A\omega = \lambda\omega$. In addition, we can then normalize ω. The result is the weight of the evaluation factors $V_1, V_2, \cdots, V_m$ of the target u.

2.3. Determining the Objective Weights Using Entropy

The information entropy, introduced by Shannon [39], describes the uncertainty, the degree of disorder and the measurement of the disorder of a system. In the case of certain evaluation factors, the entropy weight represents the relative intensity coefficient in the competitive sense. The smaller the entropy of an evaluation factor, the greater the amount of information provided by that factor and the greater the role it plays in a comprehensive evaluation; thus, it has a higher weight [29,33,37].

The fuzzy evaluation matrix R (Equation (1)) is used as the research object. Meanwhile, $H(I)$ is the entropy of the i-th evaluation factors in the domain of the evaluating factors, as shown in Equation (3):

$$H(I) = -\frac{1}{ln(n)} \sum_{j=1}^{n} r_{ij} ln(r_{ij}), i = 1, 2, \cdots, m; j = 1, 2, \cdots, n \tag{3}$$

where n is the number of evaluation ratings; r_{ij} is satisfied such that $\sum_{j=1}^{n} r_{ij} = 1$. We stipulate that $H(I) = 0$ when $r_{ij} = 0$. The entropy weight of the i-th evaluation factor is stated as Equation (4).

$$B_i = \frac{1 - H(I)}{\sum_{i=1}^{m}(1 - H(I))}, i = 1, 2, \cdots, m \tag{4}$$

where m is the number of evaluation factors. Similarly, the entropy weight of each evaluation factor can be obtained as $B = b_1, b_2, \cdots, b_m$.

2.4. Calculating the Comprehensive Weight

Suppose that there is an evaluation factor domain $V = V_1, V_2, \cdots, V_m$. The weights calculated by the AHP method and entropy weight method are $\omega = \omega_1, \omega_2, \cdots, \omega_m$ and $B = b_1, b_2, \cdots, b_m$, respectively. Then, the comprehensive weight of the m evaluation factor can be expressed as Equation (5).

$$W_i = \frac{\omega_i B_i}{\sum_{i=1}^{m} \omega_i B_i}; i = 1, 2, \cdots, m \tag{5}$$

The final membership matrix A is synthesized by combining the weight W with the fuzzy matrix R, which can be expressed as Equation (6). In addition, according to the principle of the maximum membership degree, the results of the evaluated units are determined.

$$\begin{aligned} A &= W \times R \\ &= \begin{bmatrix} W_1 & W_2 & \cdots & W_m \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \\ &= \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \end{aligned} \tag{6}$$

3. Brief Introduction to the Study Region

Zhen'an County is located in the southern part of the Qinling Mountains, 98 kilometers away from Xi'an city in Shan'xi Province and 178 kilometers away from Shangluo city, which is bounded by 108°34′35″ E–109°36′51″ E and 33°08′44″ N–33°48′57″ N, with an area of 3453 square kilometers (Figure 2). In the study area, flat land resources are scarce and the terrain has three hills, two valleys and a river. The whole terrain tilts from northwest to southeast. Zhen'an County has a maximum elevation of 2601.6 meters above sea level (m.a.s.l) and a minimum altitude of 344 m.a.s.l. The elevation difference is approximately 2257.6 m, leading to abrupt changes in precipitation, gradually decreasing from west to east (Figure 3g). During the year, the distribution of precipitation is also very uneven. It is the greatest from June to August, followed by that from September to November; the precipitation from June to October accounted for 68.7% of the annual precipitation.

According to the statistical data from the field investigation, Zhen'an County has had 286 landslide disasters, with characteristically wide distributions, high frequencies, besides intensive development of the local areas. Furthermore, the landslide disasters in Zhen'an County have also destroyed roads and buildings, troubling economic activity, such that the total economic losses due to landslides was approximated at US $30 million between 2001 and 2013.

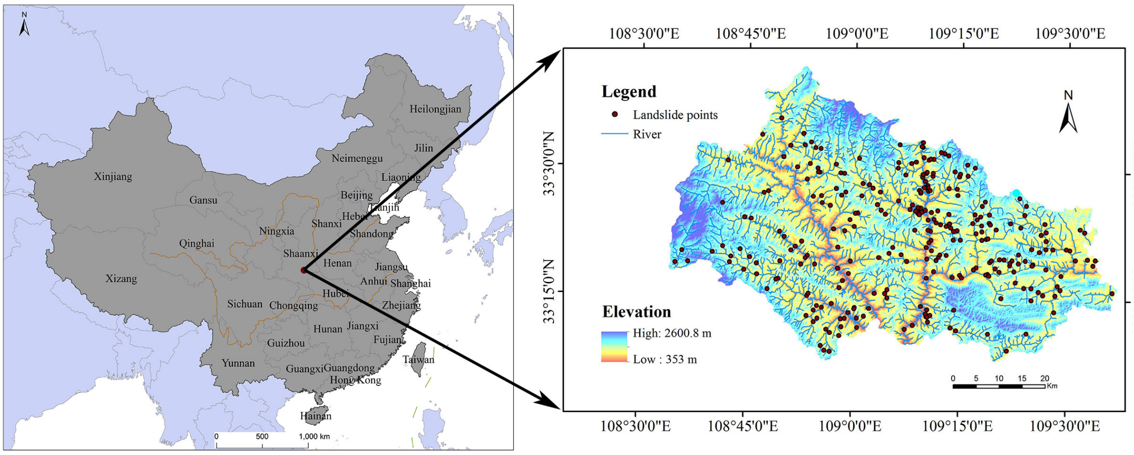

Figure 2. Location of study area in Shan′xi Province of China.

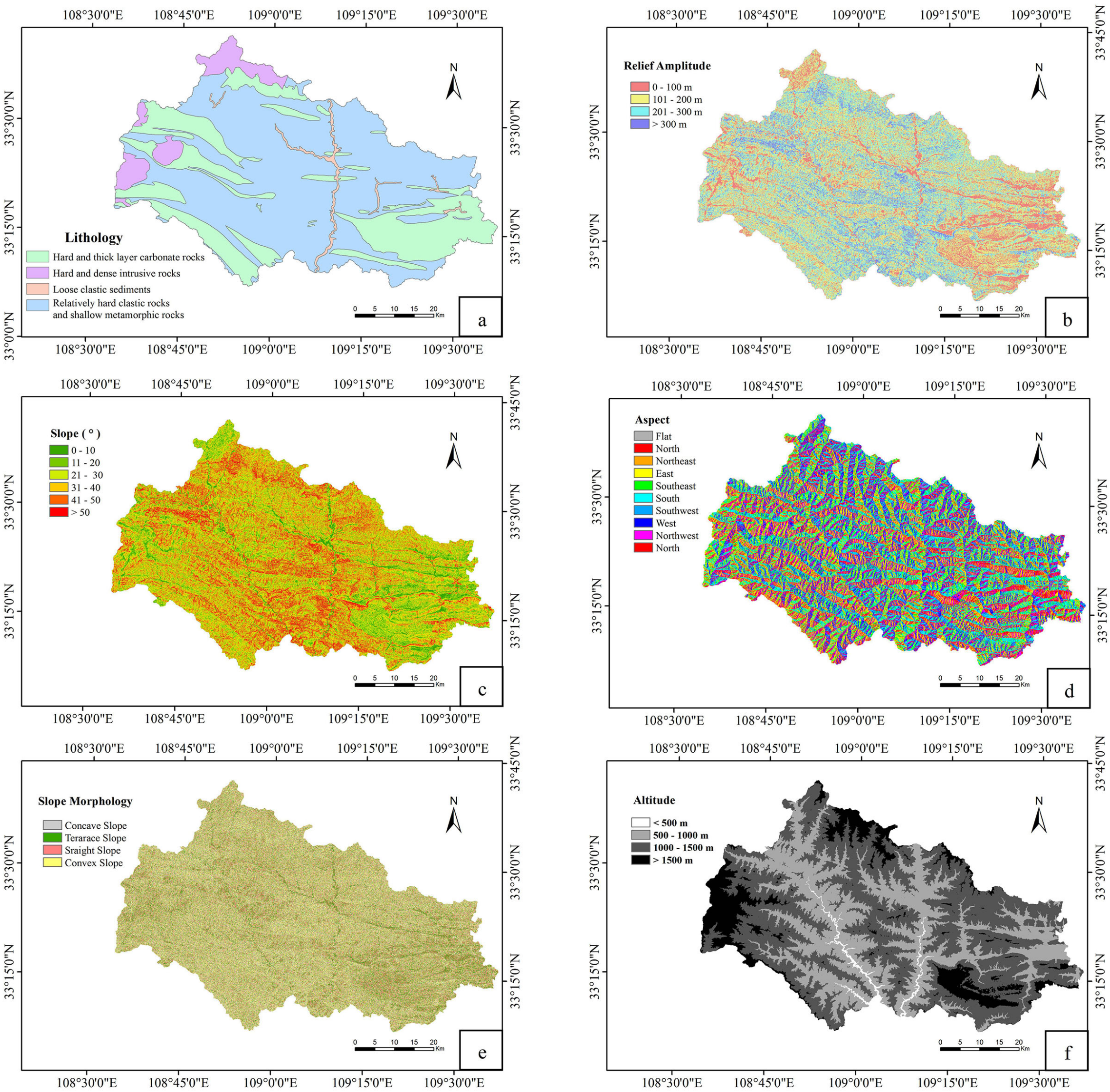

Figure 3. *Cont.*

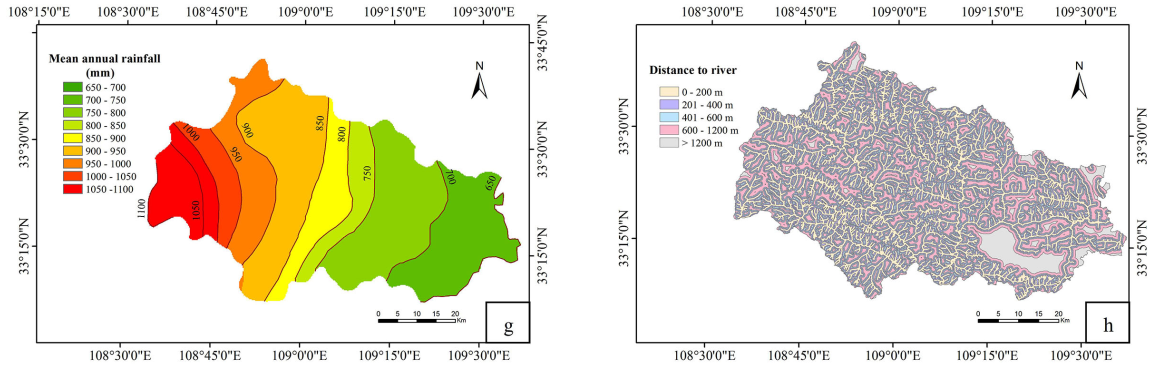

Figure 3. Eight applied evaluation factors used in the landslide susceptibility map (LSM) of Zhen'an County involving: (**a**) lithology; (**b**) relief amplitude; (**c**) slope; (**d**) aspect; (**e**) slope morphology; (**f**) altitude; (**g**) annual mean rainfall; (**h**) distance to River.

4. Results

4.1. Landslide Influencing Data Layers

In this study, eight evaluation factors were selected according to the specific characteristics of the regional landslide disasters, the single factor sensitivity analysis (Table 1) and the results of previous studies (Table 2 and Figure 2).

The study region uses the influencing factors as described in Table 2 to get the LSMs. The lithology factors (Figure 3a) are obtained from the Geologic Map of Zhen'an County (1:200,000). The distance to the river factor (Figure 3h) is calculated by a multiple ring buffer using the data extracted from the Geologic Map of Zhen'an County (1:200,000). In addition, the relief amplitude (Figure 3b), slope (Figure 3c), aspect (Figure 3d), slope morphology (Figure 3e) and altitude (Figure 3f) data are analyzed and extracted from the ASTER-GDEM data (Advanced Spaceborne Thermal Emission and Reflection Radiometer Global Digital Elevation Model). The annual mean rainfall is calculated by kriging interpolation in an ARCGIS (version 10.3, Esri Co. Ltd.) environment based on the most recent 20 years' precipitation data from the China Meteorological Administration (Figure 3g).

Table 1. The sensitivity calculation of evaluation factors.

Evaluation Factors	Categories	Area (km^2)	Landslide Count	Comprehensive Influencing Factor	Assignment	Intermediate Value
Lithology	Hard thick layer carbonate rocks	1006.15	57	0.248	3	S1 0.098
	Hard and dense intrusive rocks	212.5	2	0.011	1	S2 0.272
	Loose clastic sediments	55.6	19	0.036	2	S3 0.445
	Relatively hard clastic rocks and shallow metamorphic rocks	2178.75	208	0.705	4	S4 0.619
Relief Amplitude	0–100 m	1593.5	164	0.548	4	S1 0.079
	101–200 m	1776.25	102	0.374	3	S2 0.213
	201–300 m	77	15	0.065	2	S3 0.374
	>301 m	6.25	5	0.012	1	S4 0.481
Slope (°)	0–10	116.5	21	0.004	1	S1 0.059
	11–20	421.5	54	0.075	3	S2 0.168
	21–30	998	88	0.339	5	S3 0.277
	31–40	1253.75	85	0.441	6	S4 0.386
	41–50	583.75	37	0.129	4	-
	>50	79.5	1	0.011	2	-
Aspect	(Flat)	1.25	-	-	-	S1 0.079
	North	422.25	27	0.097	3	S2 0.126
	Northeast	434.5	24	0.1	4	S3 0.193
	East	447.75	36	0.127	5	S4 0.250
	Southeast	461.5	44	0.278	8	-
	South	422	43	0.147	7	-
	Southwest	413.25	39	0.142	6	-
	West	404.75	38	0.059	3	-
	Northwest	445.75	35	0.051	1	-

Table 1. *Cont.*

Evaluation Factors	Categories	Area (km^2)	Landslide Count	Comprehensive Influencing Factor	Assignment	Intermediate Value
Slope Morphology	Concave Slope	1200.25	95	0.141	1	S1 0.162
	Terrace Slope	465.5	53	0.269	2	S2 0.204
	Straight Slope	510.75	53	0.281	3	S3 0.246
	Convex Slope	1276.5	85	0.309	4	S4 0.288
Altitude	<500 m	38.25	1	0.002	1	S1 0.075
	500–1000 m	1223	188	0.585	4	S2 0.221
	1000–1500 m	1770.5	93	0.404	3	S3 0.366
	1500–2000 m	421.25	4	0.009	2	S4 0.512
Annual Mean rainfall	650–750 mm	1291	120	0.485	5	S1 0.083
	750–850 mm	562	84	0.171	3	S2 0.198
	850–950 mm	1064.75	63	0.271	4	S3 0.313
	950–1050 mm	308.5	11	0.047	2	S4 0.428
	>1050 mm	226.75	8	0.026	1	-
Distance to river	0–200 m	1094	127	0.391	5	S1 0.069
	201–400 m	898.75	85	0.229	4	S2 0.166
	401–600 m	633.5	43	0.16	2	S3 0.263
	600–1200 m	650.75	30	0.218	3	S4 0.360
	>1200 m	176	1	0.002	1	-

Table 2. Evaluation factors, data sources and classifications.

Evaluation Factors	Source	Resolution/Scale	Description	Classification
Lithology	Geologic Map of Zhen'an County	1:200,000	-	Four categories: Hard thick layer carbonate rocks, Hard and dense intrusive rocks, Loose clastic sediments, Relatively hard clastic rocks and Shallow metamorphic rocks
Relief Amplitude	ASTER-GDEM v2	30 m	Calculated from ASTER-GDEM	Four classes: 0–100 m, 101–200 m, 201–300 m, >300 m
Slope	ASTER-GDEM v2	30 m	Calculated from ASTER-GDEM	Six classes: 0–10, 11–20, 21–30, 31–40, 41–50, >50
Aspect	ASTER-GDEM v2	30 m	Calculated from ASTER-GDEM	Eight categories: north, northeast, east, southeast, south, southwest, west, northwest
Slope Morphology	ASTER-GDEM v2	30 m	Calculated from ASTER-GDEM	Four categories: concave slope, terrace slope, straight slope, convex slope
Altitude	ASTER-GDEM v2	30 m	Calculated from ASTER-GDEM	Four classes: <500 m, 500–1000 m, 1000–1500 m, >1500 m
Annual Mean Rainfall	China Meteorological Administration	1:200,000	Kriging interpolation	Five classes: 650–750 mm, 759–850 mm, 850–950 mm, 950–1050 mm, >1050 mm
Distance to River	Geologic Map of Zhen'an County	1:200,000	Multiple Ring Buffer	Five classes: 0–200 m 201–400 m, 401–600 m, 600–1200 m, >1200 m

The research region is divided into 1,381,200 units in a 50 m $\times$ 50 m grid. The evaluation factor attributes of each cell are extracted via ARCGIS (version 10.3, Esri Co. Ltd.). Then, the above data are imported into MATLAB (version 2014a, MathWorks Co. Ltd.) for fuzzification. After all of this, fuzzy evaluations of the *R* matrices of each unit are calculated in a MATLAB (version 2014a, MathWorks Co. Ltd.) environment. In addition, the entropy weight method is then used to obtain the objective weights of each of the unit evaluation factors.

4.2. Single Factor Sensitivity Analysis

The susceptibility at each classification of every evaluation factor can be calculated by taking the average of the landslide-area ratio (i.e., the area of landslides in one classification divided by that in the whole region), the landslide-volume (i.e., the volume of landslides in one classification divided by that in the whole region) ratio and the landslide-number ratio (i.e., the number of landslides in one classification divided by that in the whole region). After that, Table 1 shows the regions with frequent landslide disasters in Zhen'an County, including those areas with loose clastic sediments assemblages, relatively hard clastic rocks and shallow metamorphic rock assemblages; slope gradients in the range of 20–40°; southeast and southern slope aspects; straight and convex slope morphologies; altitudes in the range of 500–1000 m, which are called the middle mountains; annual mean rainfalls of 650–750 mm; or distance within 200 m from the river.

4.3. Membership Degrees of Evaluation Factors

The evaluation ratings domain refers to the collection of evaluation results that may be given by the landslide evaluation factors. In this study, the comment set is divided into four grades: $U = U_1, U_2, U_3, U_4$, where U_1 is low susceptibility, U_2 is moderate susceptibility, U_3 is high susceptibility and U_4 is very high susceptibility. Then, the membership function is used to quantitatively describe the membership degree of the evaluation factors for the LSM, which is a key step in the fuzzy comprehensive evaluation. The membership functions are written as Equations (7) and (8), according to the existing related structural membership function combined with the geological environmental conditions of the study area, using a "small and semi-trapezoidal" distribution.

$$U_1(x) = \begin{cases} 1, & x \le S_1 \\ \frac{S_2 - x}{S_2 - S_1}, & S_1 < x \le S_2 \\ 0, & x > S_2 \end{cases} \qquad U_2(x) = \begin{cases} 0, & x \le S_1, x > S_3 \\ \frac{x - S_1}{S_2 - S_1}, & S_1 < x \le S_2 \\ \frac{S_3 - x}{S_3 - S_2}, & S_2 < x \le S_3 \end{cases} \tag{7}$$

$$U_3(x) = \begin{cases} 0, & x \le S_2, x > S_4 \\ \frac{x - S_2}{S_3 - S_2}, & S_2 < x \le S_3 \\ \frac{S_4 - x}{S_4 - S_3}, & S_3 < x \le S_4 \end{cases} \qquad U_4(x) = \begin{cases} 0, & x < S_3 \\ \frac{x - S_3}{S_4 - S_3}, & S_3 \le x < S_4 \\ 1, & x \ge S_4 \end{cases} \tag{8}$$

S_1, S_2, S_3 and S_4 are the values of the representative grades corresponding to the low susceptibility areas, moderate susceptibility areas, high susceptibility areas and very high susceptibility areas, respectively (Table 1). In addition, $U_1(x)$, $U_2(x)$, $U_3(x)$ and $U_4(x)$ are the membership values of evaluation unit x in to the low susceptibility areas, moderate susceptibility areas, high susceptibility areas and very high susceptibility areas, respectively. The affiliation of the evaluation factors to the grade of landslide susceptibility is calculated by the membership function, which constitutes the fuzzy evaluation matrix R. Assuming that each evaluation factor Vi has a fuzzy evaluation matrix $R(r_{ij}) = r_{i1}, r_{i2}, r_{i3}, r_{i4}$, then eight evaluation factors will have eight evaluation matrices $R_1, R_2, \cdots, R_8$, which can be combined as shown in Equation (9).

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ \cdots & \cdots & \cdots & \cdots \\ r_{81} & r_{82} & r_{83} & r_{84} \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ \cdots \\ R_8 \end{bmatrix} \tag{9}$$

4.4. Comprehensive Weights of Evaluation Factors

Based on the different contributions of each of the evaluation factor to landslide occurrences, the weighted value of each of the evaluation factors is different. First, AHP is adopted to construct the matrix of this relationship by a pairwise comparison between the evaluation factors (Equation (2)). Second, the evaluation factor relationship matrix is solved using MATLAB (version 2014a, MathWorks Co. Ltd.). The maximum eigenvalue λ_{max} of the matrix is 8.622. The consistency index (CI) of the matrix is 0.089 and $CR = CI/RI = 0.06 < 0.1$, which indicates that the consistency ratio of this matrix is acceptable. The subjective weighting values for each factor are shown in Table 3.

Then, the objective weights $(B_i = b_{i1}, b_{i2}, \cdots, b_{i8},\ i = 1,381,200)$ are calculated from the fuzzy evaluation matrix R by using the evaluation unit as the object (Equations (3) and (4)).

Finally, in the MATLAB (version 2014a, MathWorks Co. Ltd.) environment, the comprehensive weight W of each cell's eight evaluation factors is calculated (Equation (5)) across the whole study area.

Table 3. Pair-wise comparison matrix, factor weights and consistency ratio of the data layers.

Evaluation Factors	Lithology	Relief Amplitude	Slope	Aspect	Slope Morphology	Altitude	Annual Mean Rainfall	Distance to River
Lithology	1							
Relief Amplitude	1/3	1						
Slope	1/2	5	1					
Aspect	1/2	3	1	1				
Slope Morphology	1	3	1/3	3	1			
Altitude	1/3	1	1/3	1/3	1/3	1		
Annual Mean Rainfall	2	3	2	3	2	5	1	
Distance to River	2	3	3	3	3	5	3	1
Subjective Weight	0.1327	0.0429	0.1307	0.0827	0.1116	0.0378	0.1847	0.2771
Consistency Ratio: 0.06 < 0.1								

4.5. Landslide Susceptibility Assessment Results

The final membership matrix A is synthesized by combining the weight W with the fuzzy matrix R, which can be expressed as Equation (6). Then, according to the principle of maximum membership degree, the results of the unit evaluations are determined. However, it is worth mentioning that the difference between AHP-fuzzy and entropy-FAHP is whether the Shannon entropy algorithm is applied in weight computing. In an ARCGIS (version 10.3, Esri Co. Ltd.) environment, the 1,381,200 results from the above of unit evaluations are converted to landslide susceptibility raster layers (Figure 4).

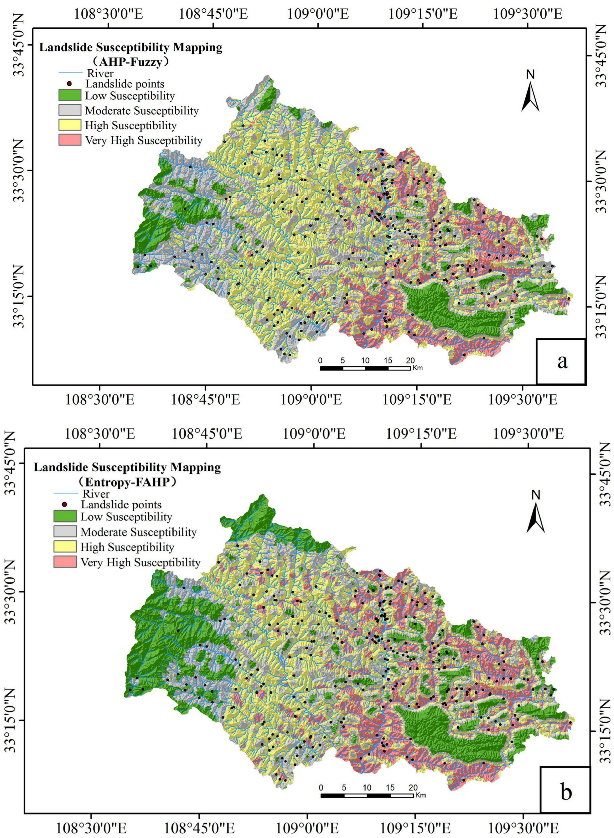

Figure 4. Landslide susceptibility maps: (**a**) LSM produced by AHP-fuzzy; (**b**) LSM produced by entropy-FAHP .

5. Discussion

The accuracy of assessment models is considered a major concern in the majority of environmental modeling applications including LSM [33]. Nevertheless, the accuracy of previously-reported LSM methods can be easily affected by the DMs' point of view. Moreover, with unvarying weight values, the influence of DMs' subjective thoughts cannot be avoided or weakened during both data quantization and criteria weighting, even though that may not be completely appropriate for the whole study region. Dividing the study area into mini units with a 50 m × 50 m grid, by applying a more objective entropy algorithm, we attempted to fully extract the original information of spatial data. Furthermore, the proposed method shows a set of independent weights of evaluation factors in every unit, which can reduce the influence of the subjective decision of DMs. In other words, each unit has a unique sequencing of influencing factors to get a more reliable assessment result. According to the obtained results, the accuracy of the proposed method is improved significantly compared with the accuracy of previous research.

5.1. Evaluation, Comparison and Precision of the LSM Methods

To evaluate the accuracy of the LSM models, this article introduces the concept of a landslide index (Li), which is shown as Equation (10).

$$Li = ((Si/Ai)/(\sum_{1}^{n} Si/Ai)) \times 100 \tag{10}$$

where Li is the index for the danger rating in each susceptibility zone (percent). The higher the value is, the higher the risk is [40]. Si is the landslide area in each susceptibility zone, as well as Ai is the area of each zone. In addition, the landslide index is the percentage of sliding area in each zone relative to the total area of that zone. Further, to compare the obtained LSMs, the parameter (P) is considered for the precision of the predicted results, as shown in Equation (11).

$$p = Ks/S \tag{11}$$

where Ks is the area of the sliding zone with an upper moderate susceptibility level and S is the area of the whole landslide region.

As shown in Table 4, the entropy-FAHP model for LSM is more accurate and reasonable, and the density of the landslides gradually increases with the level of the susceptibility zones. Meanwhile, Li is also gradually increased. The precision of the entropy-FAHP for the LSM is 81%. Although the AHP-fuzzy model for LSM has a similarly high precision (79%), the density of the landslides and landslide index in the very high susceptibility zone is even less than that in the high susceptibility zone, which is unreasonable. In addition, it is also inconsistent with the observational data. This further demonstrates the capability of the proposed entropy-FAHP model for the prediction of landslide susceptibility values.

Table 4. Comparison of the information obtained from crossing each of the susceptibility maps.

Susceptibility Maps	Susceptibility Classes	Si (km^2)	Ai (km^2)	Density of Landslide in Any Class	Landslide Index(Li) in Any Class Percent	Ks (km^2)	S (km^2)	p
AHP-Fuzzy	Low	0.22	666.25	0.01	0.04	5.896	7.439	0.79
	Moderate	1.33	981.75	0.05	0.17			
	High	3.98	965	0.15	0.51			
	Very high	1.91	840	0.1	0.28			
Entropy-FAHP	Low	0.7	1043	0.02	0.08	6.01	7.439	0.81
	Moderate	0.73	725.75	0.05	0.11			
	High	2.91	828.25	0.12	0.39			
	Very high	3.1	856	0.15	0.42			

5.2. Validation of Landslide Susceptibility Maps Using the Area under the Curve (AUC) and R-Index Methods

To evaluate the quality of the LSM, in this research, the distributions of landslides are compared with the LSMs. Then, the cumulative percentage of the predicted susceptibility areas is taken as the abscissa, and the cumulative percentage of the actual landslide number is taken as the ordinate (Figure 5). Further, the area under the curve (AUC) can be used to quantitatively indicate the success rate of the susceptibility prediction and to evaluate the fitting degree of the prediction model and the actual landslides [4]. In addition, this paper introduces the R index (relative landslide density) to validate the landslide susceptibility evaluation results [40,41].

$$R = ((n_i/N_i)/\sum(n_i/N_i)) \times 100 \tag{12}$$

where n_i is the number of landslides that occurred at the sensitivity level i, as well as N_i is the number of pixels at the same sensitivity level i.

As shown in Figure 5, the test curve is convex in shape, which can indicate favorable landslide susceptibility assessment results. Moreover, the closer to one the AUC is, the better the susceptibility assessment prediction result [42]. Then, the areas under the test curves are calculated to be 0.6885 and 0.634, respectively. That is, the success rate of the landslide prediction model is 68.85% when using the entropy-FAHP model and 63.4% for the AHP-fuzzy model. In other words, these results indicate that the proposed method (entropy-FAHP) for LSM can achieve superior prediction accuracy compared with the unvarying weight model (AHP-fuzzy).

The R index of the moderate susceptibility zone and that of the high susceptibility zone in the entropy-FAHP model are smaller than those in the AHP-fuzzy model. However, in the entropy-FAHP model, the R index of the high susceptibility zone is always greater than those of the lower zones. This is distinctly different in AHP-fuzzy model, in which the R index of its very high susceptibility zone is lower than that in the high susceptibility zone (Figure 6). In other words, it is not consistent with the facts. In spite of the similarly high precision, the R index shows the difference between the two LSM methods. Further, the conclusions could be summarized, with the combination of the subjective weight and the objective weight, as the LSM from entropy-FAHP method is more reasonable and objective than that from the AHP-fuzzy method.

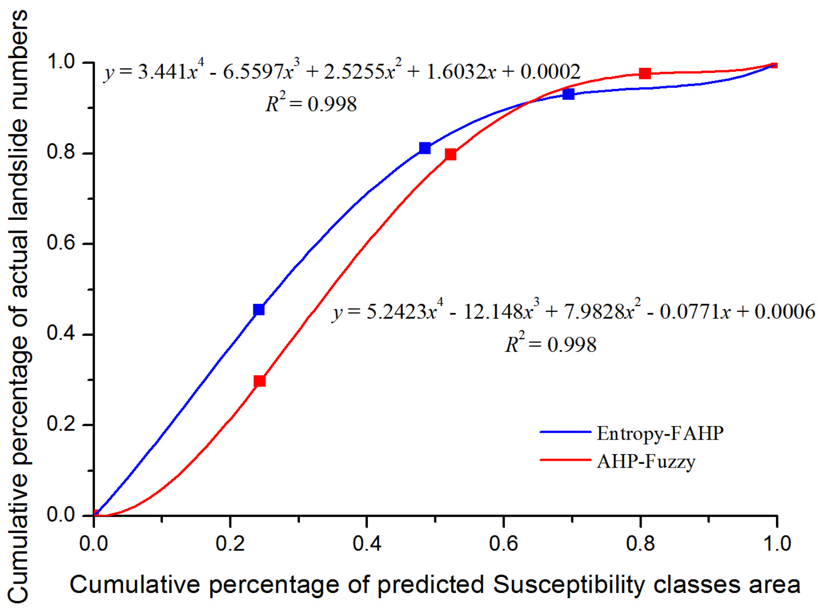

Figure 5. Test curve of the constructed landslide susceptibility models.

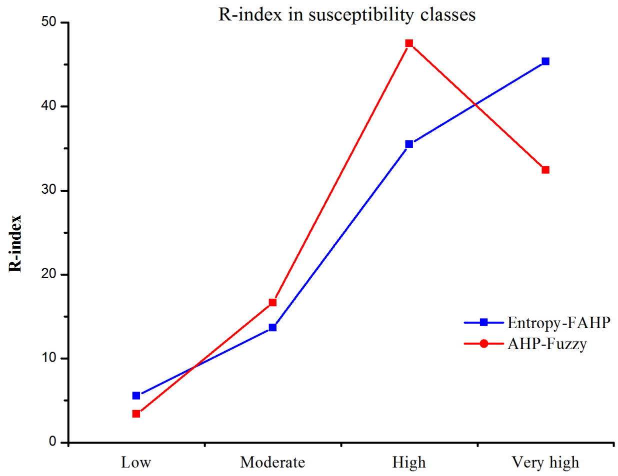

Figure 6. *R*-index for validating both LSM methods: AHP-fuzzy and entropy-FAHP.

5.3. Comparison to the Previous Studies

Considering the high frequency of landslides occurring in local areas of Qinba mountain, a more accurate landslide susceptibility map is strongly demanded. The accuracy of LSM based not only on the presence of concise and perceptible data, but also on the selection of the appropriate methodology of data processing and modeling [33]. The method proposed in this paper has been used in the development of strategic research [26], the usability evaluation of websites [27], and so on. This paper introduces it as a method of combining subjective and objective weights for landslide susceptibility evaluations. It is also used to divide the research area into evaluation units, based on the new regionally-variable attribute, which is very different from previous research methods. The previous methods mostly assign unvarying weight values to the whole study area, even though that may not be completely appropriate for the whole study region. In other words, the previous research did not consider the diversity at different sites among the study area. The model validation results seen in the above show that it is more reasonable to use the entropy-FAHP method to obtain the LSM in the study area of Zhen'an County.

5.4. Outlook and Future Work

While Shannon's information entropy theory-based methods such as the one proposed in our research have shown considerable potential in predictive landslide susceptibility, they do have their own limitations. In other words, even though the application of the proposed methodology as a comprehensive weighting scheme is not completely dependent on DMs' expertise and judgment, it is conditional on the approach of mathematical computations. We find two limitations in the use of the entropy-FAHP model. First, the choice of the mathematical method used to combine the subjective weight and objective weight influences the evaluation results of the regional landslide susceptibility. Second, the choice of the membership function affects the construction of the fuzzy matrix for each evaluation unit, which will affect the calculation of the objective weights. These problems are not only limited to our study; therefore, we believe that further research on more effective calculation methods is vital for developing more robust LSM methods.

In this paper, our study area is large, and this size of study area, coupled with the grid unit scale accuracy limit, causes a large size effect, which affects the evaluation of the quantitative factors. In future work, in order to better use the proposed methodology from this paper in different cases, we should choose a typical landslide disaster area, from the method of combining the subjective and objective weights, as well as the fitting of the membership function, to develop a new hybrid GIS-based landslide susceptibility evaluation.

6. Conclusions

In the present research, a landslide susceptibility assessment with a new Shannon' information entropy theory-based method was performed in Zhen'an County of the Qinba mountain area, China. The research shows that the entropy-FAHP model, as applied to the LSM, has three merits. First, in the process of standardizing the evaluation factors of landslides in the research area, the landslide numbers, areas and volumes are used to set up a sample. The actual development of landslides due to different evaluation factors in the study area is objectively statistically analyzed, and the inherent subjectivity of a DM's preference is weakened. Second, the LSM method proposed in this paper is a combination of subjective and objective methods. Compared with subjective (such as the AHP-fuzzy method) and objective weightings (such as the entropy method), the LSM method is used to extract the information from the original data. Expert knowledge and advice from a DM can also be reflected in this method. Finally, unlike the unvarying values for the whole study area seen in currently-reported research work, this paper divides the research area into 1,381,200 evaluation units, and the synthetic weights of the evaluation factors are changed with respect to the evaluation units. The results of our research show that the combination of fuzzy matrix and entropy weighting, objective and subjective weights can help to produce a more reliable LSM.

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