Review

# The Weak Reality That Makes Quantum Phenomena More Natural: Novel Insights and Experiments 

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#### Abstract

While quantum reality can be probed through measurements, the Two-State Vector Formalism (TSVF) reveals a subtler reality prevailing between measurements. Under special preand post-selections, odd physical values emerge. This unusual picture calls for a deeper study. Instead of the common, wave-based picture of quantum mechanics, we suggest a new, particle-based perspective: Each particle possesses a definite location throughout its evolution, while some of its physical variables (characterized by deterministic operators, some of which obey nonlocal equations of motion) are carried by "mirage particles" accounting for its unique behavior. Within the time interval between pre- and post-selection, the particle gives rise to a horde of such mirage particles, of which some can be negative. What appears to be "no-particle", known to give rise to interaction-free measurement, is in fact a self-canceling pair of positive and negative mirage particles, which can be momentarily split and cancel out again. Feasible experiments can give empirical evidence for these fleeting phenomena. In this respect, the Heisenberg ontology is shown to be conceptually advantageous compared to the Schrödinger picture. We review several recent advances, discuss their foundational significance and point out possible directions for future research.


Keywords: foundations of quantum mechanics; time (a)symmetry; Two-State Vector Formalism; weak measurements; delayed measurements; interaction-free measurements

## 1. Introduction

For many years, the Two-State Vector Formalism (TSVF) [1-3] has been unearthing more and more hidden aspects of quantum reality never conceived before. The basic premise is simple: quantum theory, like classical physics, is time-symmetric, save for the "wavefunction collapse" introduced by measurement. This gives the notion of quantum measurement a profound twist. The measurement's effect goes not only forward in time but backwards as well. Consequently the particle's physical properties between two measurements are affected by both past (pre-selection) and future (post-selection) effects. The resulting picture is fully consistent with standard quantum theory, and yet reveals hitherto unnoticed aspects of the process, namely "weak values" [4-7]. The latter constitute a "weak reality" which offers a deeper understanding of quantum reality and how it is related to the classical one [8-10].

The underlying mathematics is simple and intuitive. To determine some physical property $A$ of the system at time $t$, we evolve the initial state $\left|\psi\left(t_{i}\right)\right\rangle$, prepared at time $t_{i}<t$, from past to future, and
then evolve the final state of the system $\left|\phi\left(t_{f}\right)\right\rangle$, determined at a later time $t_{f}>t$, from future to past. We then combine, at each moment $t$, the two evolutions using the "two-state" $\langle\phi(t)| \quad|\psi(t)\rangle$ to infer the weak value of any operator $A$ defined as:

$$
\begin{equation*}
\langle A\rangle_{w}(t)=\frac{\langle\phi(t)| A|\psi(t)\rangle}{\langle\phi(t) \mid \psi(t)\rangle} \tag{1}
\end{equation*}
$$

Such values can be extraordinary, very large, very small or even complex, lying outside the spectrum of the measured operator $A$ [9-14]. These weak values manifest themselves as effective interaction terms between the pre- and post-selected systems (and any other system coupled to it weakly enough) [15]. However, they prevail between rather than upon quantum measurements, thereby being inaccessible to direct inference via the standard measurement techniques. Several methods, described below, were invented to bypass this difficulty, and eventually vindicated a plethora of surprising predictions (see e.g., [4-20]). TSVF is therefore much more than just an interpretation of quantum mechanics. On the one hand, it is fully consistent with the conventional, one-vector formalism; hence all its predictions are obliged by the latter as well. Nonetheless, this affirmation by the conventional formalism always comes with hindsight. In other words, none of the TSVF's intriguing predictions have ever been proposed by the standard approach. This computational efficiency lends support to TSVF's ontological soundness as well.

Gradually, a broad and self-consistent landscape began emerging from the formalism and its offshoots. Weak values, it turns out to be, underlie the ordinary quantum values [10,21,22], offering a novel, yet very natural explanation to quantum oddities, considered so far axiomatic or even banned from realistic inquiry by Copenhagen-like interpretations.

A deeper understanding of the dynamics of these weak values is offered by "mirage particles", momentary particles springing from the initial particle during the above "between-measurements" interval. This concept was already alluded in earlier works of ours [14,19-22] and colleagues [23,24]. Among these mirage particles, there are some of which the presence has a minus sign, implying that, upon a weak enough interaction, their properties, including mass and charge, reverse their sign [15,25,26]. We also refer to mirage particles having negative weak values as "nega-particles" [14]. The formalism shows how positive and negative mirage particles can cancel one another into an apparent "nothing" [21,22], somewhat similarly to particle-antiparticle annihilation but with no energy output, and with the possibility of parting again out of the vacuum. Feasible experiments, awaiting laboratory realization, have already been proposed for demonstrating these predictions, some of which are described below.

The consequences for quantum theory are far-reaching. The particle's hypothesized multiplication, disappearance and reappearance prior to measurement [19,20] offer an intuitive account of the wavefunction's oddities like "collapse," nonlocality and temporal anomalies. The present article sketches this evolving formalism of quantum mechanics and points out new directions for future research.

The paper's outline is as follows. Section 2 uses the simplest quantum mechanical type of measurement for illustrating TSVF's approach. In Section 3, we describe an interesting prediction of TSVF related to an extensively studied nested Mach-Zehnder interferometer (MZI) setup, where mirage and nega-mirage particles are involved, and the challenges it poses to the theory and experiment. Section 4 presents two customary methods of validation, namely weak measurements as well as projective (strong) ones. Section 5 describes an equivalent experiment that illuminates additional aspects of mirage particles' dynamics. In Section 6, we further discuss the physical meaning of negative weak values. In Section 7, we present a very preliminary outline for generalizing this formalism, and point out further avenues for research. In Section 8, we show how these predictions are best understood using the Heisenberg particle-based rather than the Schrödinger wave approach.

## 2. How "Void" Are the Wavefunction's Non-Observed Parts?

For an intuitive introduction to TSVF, the simplest quantum-mechanical measurement setup is considered. One photon hits a beam splitter, its wavefunction splits into transmitted and reflected halves, and is finally detected as a single photon by one of two equidistant detectors (Figure 1).


Figure 1. A particle split by a beam splitter is predicted to go through one out of two possible paths and eventually be detected in one, with the other becoming "void" (a). Similarly to the time-reversed retrodiction (b): the wavefunction splits again towards the past, with one half leading to an obviously void "origin".

There are two "void" branches in this process. One is familiar, namely (i) the path leading to the detector that eventually did not click (Figure 1a) and (ii) a subtler one branches from the backwards path, returning from the clicking detector to the beam-splitter: one half returns to the source but the other goes to the opposite direction from which the photon could have never come (Figure 1b).

These two void parts of the particle's evolution epitomize quantum mechanics' two major contrasts with classical physics, namely (i) indeterminism and (ii) the time-asymmetry inflicted by measurement. They seem to present mere mathematical curiosities with no physical content. TSVF, however, can extract from them a surprising physics: a combination of such future and past void branches within one evolution gives rise to a temporary particle in a location where it seems to have never gone. Other unusual phenomena then follow, described in the following sections.

## 3. Can a Particle Be Where It Never Went?

Consider [16,17] an MZI within which a smaller one is nested (Figure 2). The first beam splitter $\mathrm{BS}_{1}$ splits the beam into $1 / 3$ and $2 / 3$, and the last, $\mathrm{BS}_{4}$, splits the $1 / 3$ of the original beam into $1 / 9-2 / 9$. On the right, $2 / 3 \operatorname{arm} E$, goes past a smaller, standard MZI with two $50 / 50 \mathrm{BS}$ dividing the beam into two equal parts.

Let a photon go through the setting. This preparation gives rise to the initial state:

$$
\begin{equation*}
|\psi\rangle=\frac{1}{\sqrt{3}}(|A\rangle+|B\rangle+|C\rangle), \tag{2}
\end{equation*}
$$

Superposed over arms $A, B$ and $C$. If the photon takes the right path $E$ and enters the smaller MZI, then, by constructive interference, it must exit towards detector $D_{1}$ and never goes through $F$ to the final beam splitter $\mathrm{BS}_{4}$ and the last two detectors $\mathrm{D}_{2}$ and $\mathrm{D}_{3}$.

Then, the cases where $D_{1}$ did not click are selected. This entire part of the wavefunction now becomes "void" based on the introduction of the previous section: the photon seems to have never gone through this $E$ arm, but takes the $C$ arm.


Figure 2. Vaidman's nested MZI [16,17]. From $\mathrm{BS}_{1}$, the path goes to a smaller MZI between $\mathrm{BS}_{2}$ and $\mathrm{BS}_{3}$. The path emerging from the nested MZI in case of constructive interference goes to detector $\mathrm{D}_{1}$, of which non-clicking cancels the entire right-hand path, implying that the photon never passes through $B S_{1}$ but is rather reflected to the left towards $B S_{4}$ and detectors $D_{2}$ and $D_{3}$.

Next, the remaining $1 / 9$ of cases where $D_{3}$ has clicked are selected. This amounts to post-selection of

$$
\begin{equation*}
|\phi\rangle=\frac{1}{\sqrt{3}}(|A\rangle-|B\rangle+|C\rangle) \tag{3}
\end{equation*}
$$

Then, again by interference, this backward state vector "leaves" the nested MZI through another "exit", say towards a wall, which, of course, could have never been the photon's source.

However, on this segment of its way back to the past (see Figure 3), this void branch is going over the earlier void part of the forward-moving wavefunction which comes from the source through arm $F$ into the nested MZI:

$$
\begin{equation*}
|F\rangle \rightarrow \frac{1}{\sqrt{2}}(|A\rangle-|B\rangle) \tag{4}
\end{equation*}
$$



Figure 3. The backward evolution from the actual detection. Again, a void branch goes to the nested MZI and exits towards an obviously void source.

Combining (2) and (3), a surprising result is given in the form of the two-state vector:

$$
\begin{equation*}
\langle\phi| \quad|\psi\rangle=\frac{1}{3}(\langle A|-\langle B|+\langle C|)(|A\rangle+|B\rangle+|C\rangle) \tag{5}
\end{equation*}
$$

indicating by the minus sign assigned to the particle in the $B$ arm (while the particle must be found in either $A$ or $C$ with certainty).

The corresponding weak values, as defined in Equation (1), are:

$$
\begin{gather*}
\left\langle\Pi_{B}\right\rangle_{w}=-\left\langle\Pi_{A}\right\rangle_{w}=-\left\langle\Pi_{C}\right\rangle_{w}=-1  \tag{6}\\
\left\langle\Pi_{E}\right\rangle_{w}=\left\langle\Pi_{F}\right\rangle_{w}=\left\langle\Pi_{A}\right\rangle_{w}+\left\langle\Pi_{B}\right\rangle_{w}=1-1=0 \tag{7}
\end{gather*}
$$

where for all $i, \Pi_{i}$ is defined as a projection operator onto arm $i$ of the interferometer (amounting to the question: if we look for the photon in arm $i$, will we find it there?).

In other words, in the middle segment of the right-hand path, where the two void histories overlap, an additional (detectable) particle appears (Figure 4). The appearance is short-lived, only along the right path $A$ of the nested MZI, with neither entry nor an exit into or from it.


Figure 4. The purple trajectory is the overlap of the blue and red lines of the earlier forward and backward state vectors. A momentary additional particle appears on the right path's middle segment.

## 4. In Search of Validation: Weak and Strong Measurements

This is certainly a striking derivation. Once $D_{1}$ has failed to click, one would regard this part of the wavefunction completely ruled out, "collapsed" into nothingness. Lo and behold, when $\mathrm{D}_{3}$ later clicks, then, within the middle segment of this never-traversed trajectory, the particle is revived.

This deviation is striking indeed, yet apparently banned from validation by the fact that this is a retrodiction, holding for the past, prior to the final click. In other words, the derivation holds only if we have refrained from measuring the particle's whereabouts within the MZI. Is this derivation, then, doomed to remain inaccessible to empirical proof?

One bypass is offered by weak measurement [4]. Let the coupling between the particle and detector be very weak, thereby highly plagued by quantum uncertainty. After a sufficient number of trials, the averaged result gives the weak value with arbitrarily high precision, yet with no visible disturbance. Several experiments have already been carried out, and all TSVF predictions have indeed been verified by this method.

What would weak measurements reveal for the present case? Let us (gedankenly, ignoring technical issues) make the two solid mirrors of the nested MZI small enough and movable, such that they can react with the slightest recoil to make the photon take the right or left path. Post-selection for all cases where $D_{3}$ has clicked, apparently implies that the nested MZI has never been traversed. Because the mirrors' momenta are subject to quantum uncertainty, it is necessary to repeat the experiment many times to overcome the noise. The predicted result offers the first affirmation to the TSVF prediction; moreover, the right-hand mirror indicates a recoil upon the photon's overall hits, while the left hand undergoes a negative recoil, namely a "pull" rather than a "push".

Skeptics, however, have objected to weak measurements as a means of revealing true quantum properties of the system [27-29] (see the reply in [30]), often explaining away their outcomes as noise inflicted by the measuring device's uncertainty. Even stronger objections have been raised against Vaidman et al.'s version of weak measurements in the present case [17], because it has employed classically vibrating mirrors and classical beams (rather than single photons), see e.g., the discussions in [31-34].
"Extraordinary claims require extraordinary evidence [35]." While we find the objections to weak measurements ill-reasoned, we want to face the challenge head-on. A particle's alleged fleeting appearance, in the middle of a path it seems never to have entered into or exited from, is extraordinary enough to merit a more unequivocal validation. Such a validation will, in turn, add credibility to weak measurements as well.

Fortunately, such a method has been introduced for a TSVF prediction analogous to the nested MZI. This is a standard, projective measurement, hence immune to all objections against weak measurement. First, here is a brief account of the method's development.

Okamoto and Takeuchi [36], following an earlier suggestion of Aharonov and Vaidman [37], have realized a photon that acts like a "shutter" that reflects a probe photon "hitting" it using a novel photonic quantum router. They took the TSVF analysis of a photon superposed over three locations, where, upon the appropriate post-selection, it is predicted to act as a shutter with certainty in two of them at the same time. A probe photon, also superposed, directed towards the superposed shutter, has become entangled with it, as if being reflected from both locations. This is a TSVF prediction verified with a standard quantum measurement. Elitzur et al. [20] took this technique one step further for testing, with a finer temporal resolution, another intriguing TSVF retrodiction [19]. The experiment involves a similar three-box setting, within which the particle is retrodicted to disappear and reappear at different instances across distant boxes. Then, a probe photon, superposed in both space and time, interacts with the three boxes at the times the shutter photon is supposed to be present and absent. The two photons become correlated only if the shutter reflects the probe photon when the former is expected to be present, and let the probe photon pass through its box when absent. As these instances of the shutter's presence and absence occur one after another in the same boxes, it seems to disappear and reappear time and again.

This method of validation can be applied to the present setting, namely the nested MZI. A feasible optical setup was given in [20], and the following discussion is on the pure gedanken level (Figure 5). Let the photon going through the entire nested MZI device be a shutter photon. Let a probe photon be split in both space and time such that it interrogates the whereabouts of the shutter within the device over time. For this purpose, the four split branches of the probe photon go one by one:
(i) at $t_{1}$ : to a mirror placed just behind the trajectory $E$ leading to the nested MZI;
(ii) at $t_{2}$ : to the nested MZI's right-hand path $A$ where the mirage photon is expected to be;
(iii) at $t_{2}$ : to the large MZI's left path $C$ where the photon is simultaneously expected to be;
(iv) at $t_{3}$ : to a mirror placed behind the exit trajectory $F$ from the nested MZI towards $\mathrm{BS}_{4}$.

We expect the probe photon to hit the first mirror without being disturbed by the any shutter photon on its way to the nested MZI; then to be reflected by both the mirage photon within the nested MZI and the shutter photon on the large MZI's left path; and then again to be reflected by the second mirror, indicating that no shutter photon has left the nested MZI along $F$. The resulting shutter-probe entanglement is Bell-like: one can either check correlations between their paths, or between their interference patterns [20]. Upon a successful post-selection, the latter option would indicate a constructive interference at $\mathrm{D}_{5}$ of all the probe's wavepackets returning from $E, A, C$ and $F$ with their original amplitudes $\alpha_{i}$, that is

$$
\begin{equation*}
|\psi\rangle_{p}=\left(\alpha_{1}\left|E_{T}\left(t_{1}\right)\right\rangle+\alpha_{2}\left|A_{R}\left(t_{2}\right)\right\rangle+\alpha_{3}\left|C_{R}\left(t_{2}\right)\right\rangle+\alpha_{4}\left|F_{T}\left(t_{3}\right)\right\rangle\right) \tag{8}
\end{equation*}
$$

where $T$ denotes transmittance through a void part and $R$ is the reflection from a particle.


Figure 5. A probe photon (drawn in blue), superposed in both space and time, interacts via quantum routers with the photon traversing the nested MZI in three moments at four places, where the shutter photon is expected to be either present (the probe being reflected by the shutter) or absent (the probe being reflected by a mirror). Correlation between probe and shutter detectors emerges only if the probe photon is reflected by the mirage shutter photons where they are expected to pass $\left(D_{2}\right.$, and $\left.D_{5}\right)$, and by the mirrors where no shutter photon is expected $\left(D_{3}\right.$, and $\left.D_{4}\right)$.

Let us conclude with the foundational significance of this experiment. Whereas the photon seems to have never taken the MZI's right path, TSVF reveals a much deeper account. This path has been taken by a pair of mirage and nega-mirage photons, which through mutual cancellation gives the appearance of no particle. Vaidman's setting in [16,17], as well as those in [19,20], thus enable a momentary resolution of this apparent "nothing" into its two subtle components, followed again by self-cancellation.

Of special interest is the right path's $F$ segment, leading from the nested MZI to final $\mathrm{BS}_{4}$ and detectors $D_{2}$ and $D_{3}$. The photon is not expected to pass there by the basic laws of optics, and indeed the probe photon's 4th part is expected not to find it there. However, this segment should remain open, that is, any obstruction along it would make the experiment fail [17], for the future effect of the post-selection at $\mathrm{D}_{3}$. This account, while demanding a great conceptual sacrifice, is the most intuitive for us.

## 5. The "Spooky Particle" Experiment

The nested MZI is a variant of an earlier version, formulated in the form of a particle in three boxes [8]. The experiment (see Section 4 above) has been described in great detail in [20] (see also the popular version [38], which gave it its name), so suffice it here to mention only its most salient features.

A particle is initially superposed over three boxes, of which $A$ and $B$ are close enough to allow it to move between them, while $C$ is arbitrarily far away. Here, the dynamics is cyclic. Under appropriate pre- and post-selections, the particle is expected to reside:
(i) in $A$ and $C$ at $t_{1}$;
(ii) only in $C$ at $t_{2}$;
(iii) in $B$ and $C$ at $t_{3}$;
(iv) and then again in $A$ and $C$ at $t_{4}$.

In other words, the particle totally vanishes from $A$ and $B$, to reside only in $C$ (where it could never tunnel), then returns to $B$, then tunnels back to $A$ and then all over again. Here, the disappearance-reappearance cycle is due to the nega-mirage particle being first in $B$, then joining the positive mirage particle in $A$ to make it disappear, and then parting from it to make it reappear.

There is an additional intriguing feature of this setting. Consider the time $t_{1}$ when the two mirage particles coexist in $A$ and $C$ (as already implied by the Okamoto-Takeuchi experiment [36]). The nega-photon is now in $B$. At this instant, the following retrodiction holds: had we joined $B$ with $C$ rather than with $A$, the particle would have vanished from $C$ and "collapsed" into $A$.

This is a unique situation. So far, the two possible measurement outcomes "click" and "no click" have been random. Here, however, it is possible in retrospect to point out their cause. Can this insight be generalized to all measurements? To address this question, we have at present only a few hints, proposed in the following sections.

Significantly, this derivation of the click/no-click "cause" is only retrospective. This holds for all TSVF predictions, such as the above disappearance-reappearance cycle. The reason is clear: having access to phenomena associated with weak values in real time would entail the bluntest causality violations.

## 6. On Negative Weak Values: Can a Mirror be "Pushed" Inwards?

In the weak measurement version of the above nested MZI experiment, we encountered a curious negative recoil of the MZI mirror that the nega-photon is expected to hit. A similar derivation has been presented in earlier works [25,26,39]. This effect of weak reality adds another piece to our emerging picture, which we have earlier referred to in a paper titled "1-1 = Counterfactual" [22]. Quantum non-events, known for their curious causal efficacy, can be better understood as a sum of positive and negative weak values.

Indeed, as discussed earlier in [13], negative weak values are quite abundant: whenever the weak value of some projection operator exceeds unity, there exists at least one other projection operator with a negative weak value. Only for a measure zero of post-selected states, no negative weak values are expected.

Negative recoil is not the only inverse effect of the nega-particle. For example, when photons are absorbed by excited nega-atoms in the process of stimulated emission, the nega-atoms become ground without a subsequent emission. Alternatively, spontaneously emitted radiation from excited atoms, some of which having negative weak values, can have an extraordinary spatial distribution, indicating interference between photons emitted from nega-atoms and positive ones. Such phenomena are analyzed in detail in [14].

The effects of nega-particles go even further: Weak values determine the effective potential whenever a weak coupling to a pre- and post-selected system is created [15]. Consequently, a negative value of some operator $A$ implies that when we couple weakly to $A$, the sign of the interaction term is flipped. In the current paper, this physical understanding is attached to all nega-particles, but in [13], an interesting alternative was proposed: under two plausible consistency conditions (which create the connection to the standard formalism of quantum mechanics), all strange weak values (not only the negative ones) can be interpreted as complex conditional probabilities corresponding to counterfactual scenarios.

In our perspective, then, all weak values are real in the sense that they represent physical, measurable quantities (which were in fact measured in numerous experiments with various systems and multiple methods). However, some of the mirage and nega-mirage particles have a fleeting, transient existence, like in Vaidman's nested MZI experiment or in the authors' "case of the disappearing particle". Our next step is to explore some generalizations of the proposed dynamics.

## 7. Generalizing: Interaction-Free and Positive Measurements as Sums of Weak Values

The plethora of weak values revealed by TSVF, only a few of which we presented here, can now be viewed in a broader context. These values, we submit, constitute a coherent weak reality that underlies quantum reality, thereby offering novel insights into the latter's riddles. For a given pre- and post-selected ensemble, the weak value of every projector in the system's Hilbert space is defined, and this produces a noncontextual value assignment to all observables of the system, which necessarily includes complex and negative values for some projectors.

Quantum reality, on the other hand, as known from measurements, is mostly discrete: elementary particles are indivisible. However, the formalism describes a continuum underlying reality: A wave propagates in a deterministic fashion, only to give rise to a discrete and indeterministic outcome, namely a particle in an unpredictable location, upon measurement. Weak reality now makes the dual picture much richer. Prior to measurement, mirage copies of the particle, some of which may be too large, too small or even complex, can momentarily appear between pre- and post-selections, and their varying interactions with one another determine the particle's familiar quantum value upon measurement.

How does this transition occur? Let us begin with the case where measurement indicates that the particle is not at that location. It is one of the wonders of quantum mechanics that, unlike classical physics, this apparent non-event is not devoid of causal efficacy: The interaction that has not occurred exerts nonlocal effects on the entire wavefunction just as if it has. This is interaction-free measurement (IFM) [40], where even a detector's non-click destroys the interference. Within the TSVF, however, this is very natural. Consider again the nested MZI (Figure 4): What appears to be no particle hitting detector $D_{1}$ and no detection turns out to be a self-cancelling pair of mirage particles, of which one is a nega-particle. The nested MZI, then, enables a momentary resolution of this "nothing". Suppose, for example, that detector $D_{1}$ is a movable mirror, which, by not recoiling, indicates that the particle does not go that way. According to the TSVF, the mirror's non-recoil is simply the sum of positive and negative recoils. Similarly, if detector $D_{1}$ is a photographic plate, the "no dot" would be a mirage photon accompanied by a nega-photon, as implicated in the above "negative absorption" experiment [14]. Indeed, Quantum Oblivion [41], which underlies several quantum phenomena from the quantum Zeno to the Aharonov-Bohm effects, has shown, also with pre- and post-selections and "strong" measurements, how each of these apparent non-events can be decomposed into its occurrence followed by "un-occurrence" [21,22]. Nega-particles may thus become a common currency in quantum transactions. A profound time-symmetry of quantum reality seems to underlie the (in)famous asymmetry of measurement and classical reality.

## 8. Discussion: Time-Symmetric Causality and the Particle-Based Heisenberg Representation

Finally, it is instructive to point out that this picture of weak reality with the phenomena derived from it accords well with the Heisenberg approach advocated in [42-44]. We have pointed out that the Schrödinger wavefunction is often conceptually confusing and the Heisenberg-operator-based formalism is more natural. According to this Heisenbergian view, a set of deterministic operators carries the same amount of information stored in the wavefunction, but in contrast to the latter, can be viewed as a proper description of the single particle. Interestingly, these operators often obey nonlocal equations of motion [42-44], naturally accounting for quantum phenomena such as the Aharonov-Bohm effect, IFM and many others. In the above examples, it is the particle, with its host
of mirage particles, rather than wave-like properties, which explain curious effects such as these manifested in the above experiments.

Several other derivations based on the TSVF, some of which have been already experimentally validated, have demonstrated an interesting "Cheshire cat" effect [18,45]: An apparently intrinsic property of the particle, such as its spin, can traverse an MZI path other than that traversed by the particle itself. These phenomena can be more naturally understood in a particle-based framework: a massive particle traverses one arm of the MZI, while a pair of mirage-nega-mirgae, having zero mass but non-zero spin, traverses the other.

Deriving pairs of mirage particles and their accompanying nega-mirage particles, we can maintain an intuitive picture of continuous trajectories within the weak reality of quantum mechanics. Since the positive and negative mirage particles can hide one another, this picture allows us to think of the pre-selection event as the source for all the extra particles emerging from the original. Each then follows some definite trajectory through space-time until they all meet again at the post-selection where they are re-absorbed. The reabsorption is essential because this is where the back action from the pointer system on each of the different mirage particles collectively affects the original particle.

This picture has naturally emerged for the two-vector account of quantum processes, which is equivalent to the mainstream, one-vector account. It is worth comparing the two views.
(i) Quantum measurement outcomes are not fully determined by the past. The future also takes part in shaping them. When the initial and final boundary conditions are not likely to be an pair, Nature, so to speak, "goes out of its way" to reconcile between the forward and backward components of the resulting evolution, by giving rise to weak values like mirage and nega-mirage particles. Delicate measurements can later validate such phenomena that have occurred between pre- and post-selections.
(ii) Among all (forward-in-time) possible quantum evolutions, there are some that involve anomalous weak values. These, however, are mixed with all the other weak values, stemming from all possible post-selections, and cannot be distinguished in real time (therefore giving rise to the customary expectation values). It is only the actual post-selection in a given experiment which informs us, in retrospect, which are the cases where these values were certainly involved.

Which account is more natural is a matter of personal choice. We only point out that it was the two-vector account which has revealed these phenomena based on weak values, the conventional alternative following only with hindsight.

To conclude, we have discussed a few experiments leading to a new perspective on the TSVF in particular, and on quantum mechanics in general. This particle-based approach is time-symmetric and realistic. Admittedly, the outlined picture, based on mirage and nega-mirage particles, is still far from being complete. Further derivations, experiments and generalizations are currently under investigation.

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