

This note provides the simulation details of the hybrid simulation on ibmqx2. We show the ibmqx2 results of the excited state population in the following table, which is compared to the exact numerical results.

	Sub-channel	Exact	ibmqx2		Sub-channel	Exact	ibmqx2
$\delta\tau = 0.5,$ $N = 2$	Initial	0.26894	0.25391	$\delta\tau = 10,$ $N = 2$	Initial	0.26894	0.26624
	$\uparrow$	0.66729	0.62988		$\uparrow$	1.00000	0.95227
	$\downarrow$	0.12240	0.16260		$\downarrow$	0.00000	0.02869
$\delta\tau = 0.5,$ $N = 3$	Initial	0.26894	0.24768	$\delta\tau = 10,$ $N = 3$	Initial	0.26894	0.26001
	$\uparrow$	0.69001	0.65955		$\uparrow$	1.00000	0.95825
	$\downarrow$	0.11404	0.15076		$\downarrow$	0.00000	0.03088
	$\uparrow\uparrow$	0.85104	0.78357		$\uparrow\uparrow$	1.00000	0.95007
	$\uparrow\downarrow$	0.33157	0.36841		$\uparrow\downarrow$	0.00000	0.04944
	$\downarrow\uparrow$	0.57427	0.51660		$\downarrow\uparrow$	1.00000	0.93567
	$\downarrow\downarrow$	0.05480	0.11548		$\downarrow\downarrow$	0.00000	0.03137
$\delta\tau = 0.5,$ $N = 4$	Initial	0.26894	0.26123	$\delta\tau = 10,$ $N = 4$	Initial	0.26894	0.25891
	$\uparrow$	0.70323	0.67090		$\uparrow$	1.00000	0.94946
	$\downarrow$	0.10917	0.16821		$\downarrow$	0.00000	0.03284
	$\uparrow\uparrow$	0.86494	0.80188		$\uparrow\uparrow$	1.00000	0.94153
	$\uparrow\downarrow$	0.32005	0.35596		$\uparrow\downarrow$	0.00000	0.04114
	$\downarrow\uparrow$	0.59458	0.51721		$\downarrow\uparrow$	1.00000	0.95654
	$\downarrow\downarrow$	0.04969	0.09521		$\downarrow\downarrow$	0.00000	0.03052
	$\uparrow\uparrow\uparrow$	0.93362	0.87085		$\uparrow\uparrow\uparrow$	1.00000	0.93958
	$\uparrow\uparrow\downarrow$	0.42511	0.36462		$\uparrow\uparrow\downarrow$	0.00000	0.03894
	$\uparrow\downarrow\uparrow$	0.66581	0.67371		$\uparrow\downarrow\uparrow$	1.00000	0.94092
	$\uparrow\downarrow\downarrow$	0.15730	0.18115		$\uparrow\downarrow\downarrow$	0.00000	0.03552
	$\downarrow\uparrow\uparrow$	0.80074	0.75549		$\downarrow\uparrow\uparrow$	1.00000	0.93787
	$\downarrow\uparrow\downarrow$	0.29223	0.25769		$\downarrow\uparrow\downarrow$	0.00000	0.03308
	$\downarrow\downarrow\uparrow$	0.53293	0.56665		$\downarrow\downarrow\uparrow$	1.00000	0.93896
	$\downarrow\downarrow\downarrow$	0.02442	0.06262		$\downarrow\downarrow\downarrow$	0.00000	0.03613

In the hybrid simulation, we need to implement all the possible choices of the sub-channels. The populations of the system qubit are measured at  $t = 0, \delta\tau, \dots, N\delta\tau$  in a  $N$ -step isothermal process. With two choices of sub-channel in one step, there are  $2^j$  choices of the sub-channels at the  $j$ -th step. Since no work is done in the last step, we only need to obtain the excited state population at  $t = 0, \delta\tau, \dots, (N-1)\delta\tau$ , where  $\delta\tau$  is the time consuming of the isochoric process. We can in principal just implement the  $N-1$  steps of elementary processes and the  $N$ -th adiabatic process. We do not implement the last isochoric process to reduce the number of the circuits.

We show the circuit implemented on ibmqx2 for different step number  $N = 2, 3$ , and  $4$  with the time consuming  $\delta\tau = 0.5$  and  $\delta\tau = 10$  in the following. Every circuit is run once with 8192 shots.

In the result, the index of the qubits is organized as 1, 2, 3, 4 and 5. The qubit 3 represents the system qubit, while the rest are the ancillary qubits. The measurement is performed on the system qubit only, with other qubits undetected. In the histograms, the state 0 on qubit (1, 2, 4, 5) is marked for simplicity.

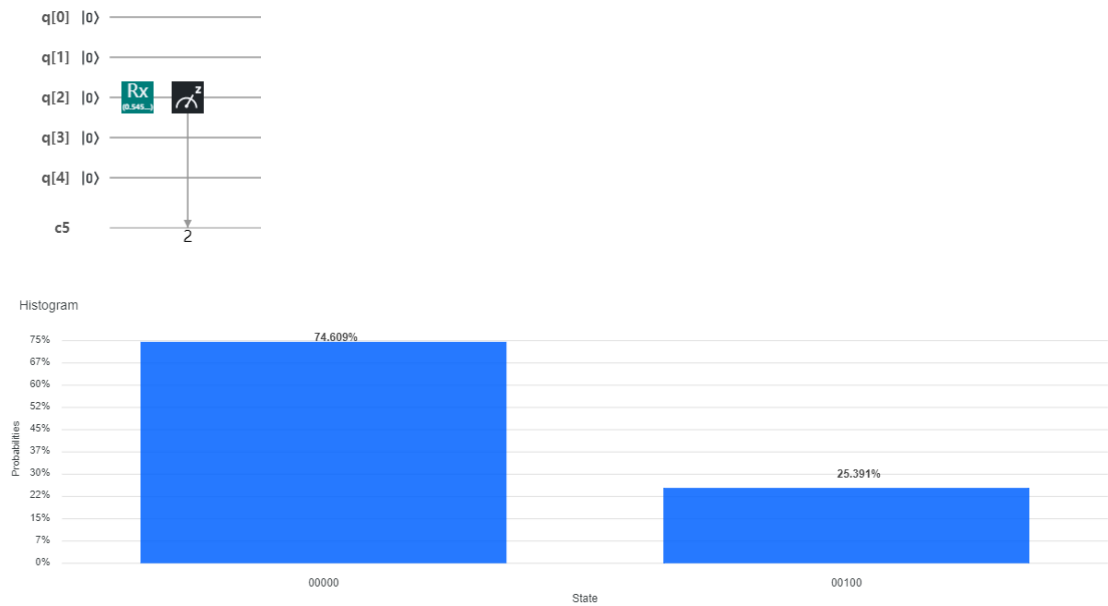
## Short-time dynamics with $\delta\tau = 0.5$

We show the ibmqx2 results for the short-time discrete isothermal process with the step number  $N = 2, 3$ , and 4.

### I. Two-step isothermal process with $\delta\tau = 0.5$

For a two-step isothermal process, we need to obtain the excited state population at  $t = 0, \delta\tau$ .

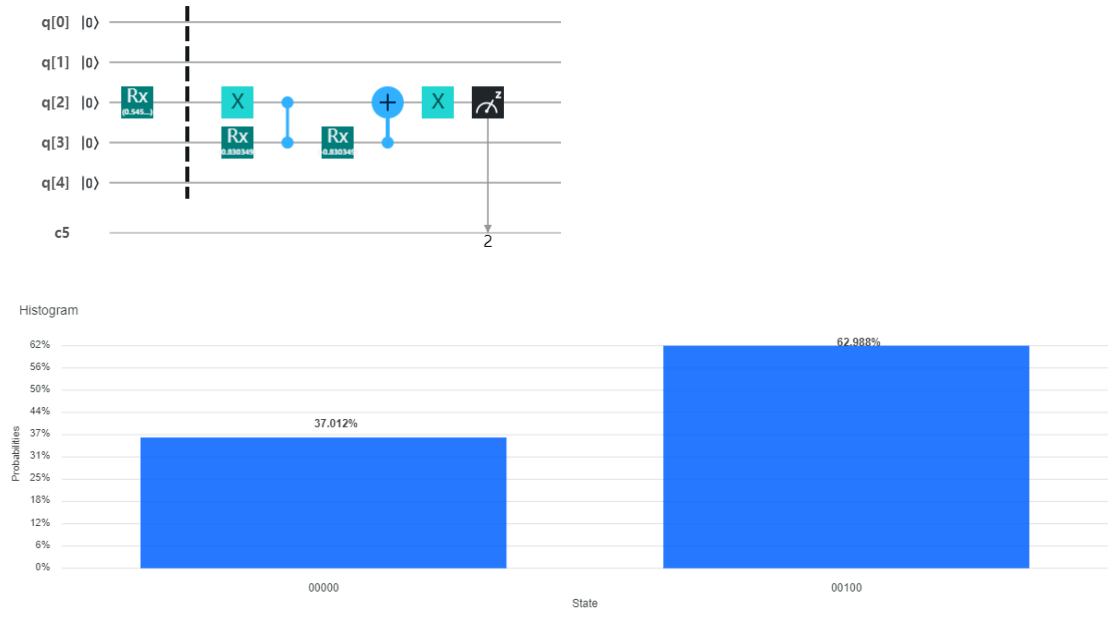
#### 1. Prepare the initial state at $t = 0$



The initial state is prepared with the  $R_x(0.54521 \times 2)$  operation. The excited state population is  $p_e(0) = 0.25391$ . The raw data was obtained on April 16<sup>th</sup>, 2020.

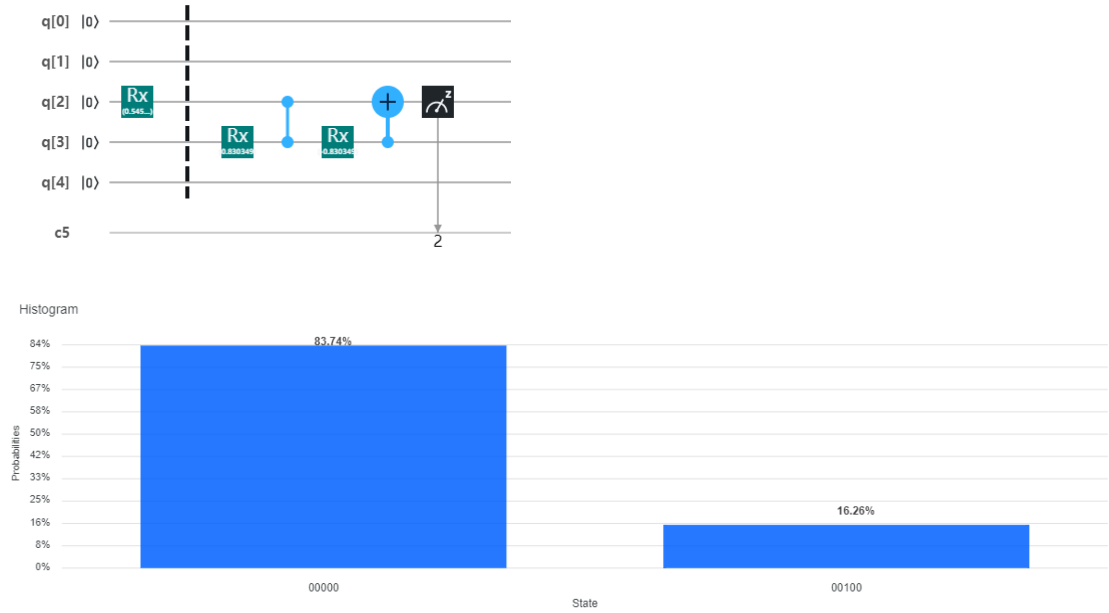
#### 2. The first step $t = \delta\tau$ .

For the sub-channel choice  $\uparrow$ , the circuit and the result are



The excited state population is  $p_e(\delta\tau) = 0.62988$ . The raw data was obtained on April 16<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow$ , the circuit and the result are



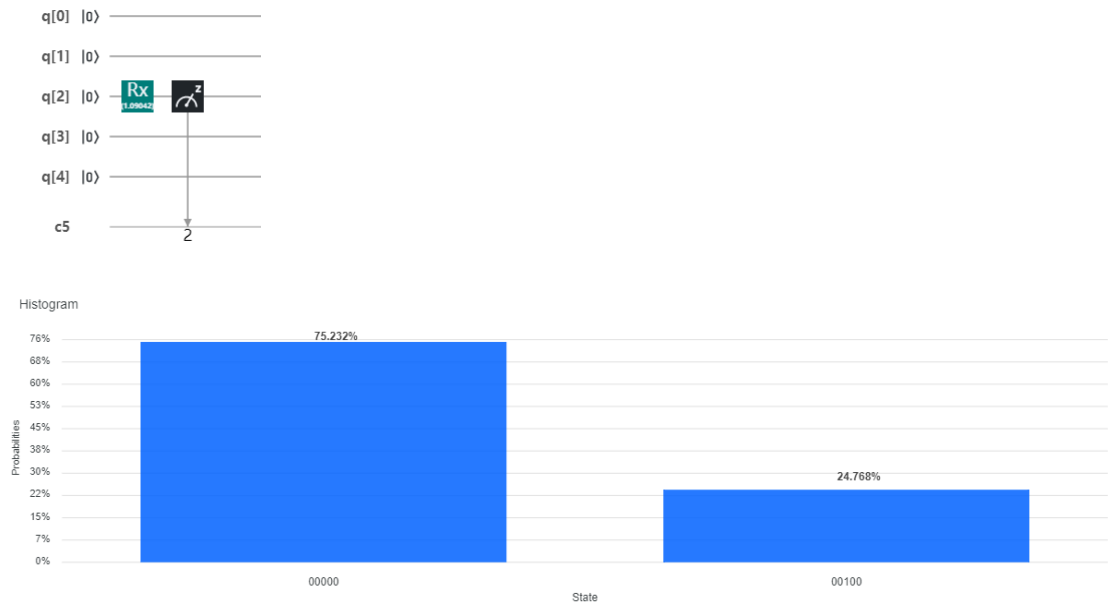
The excited state population is  $p_e(\delta\tau) = 0.16260$ . The raw data was obtained on April 16<sup>th</sup>, 2020.

## II. Three-step isothermal process with $\delta\tau = 0.5$

For the three-step isothermal process, we need to obtain the excited state population at  $t =$

$0, \delta\tau, 2\delta\tau$ .

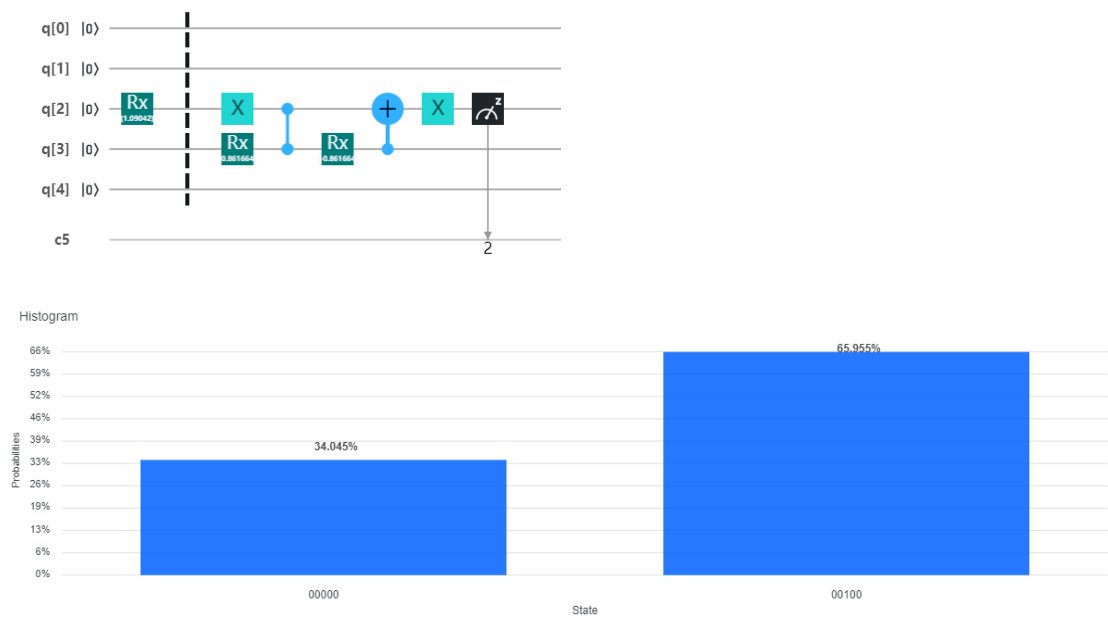
## 1. Prepare the initial state at $t = 0$



The initial state is prepared with the  $R_x(0.54521 \times 2)$  operation. The excited state population is  $p_e(0) = 0.24768$ . The raw data was obtained on April 16<sup>th</sup>, 2020.

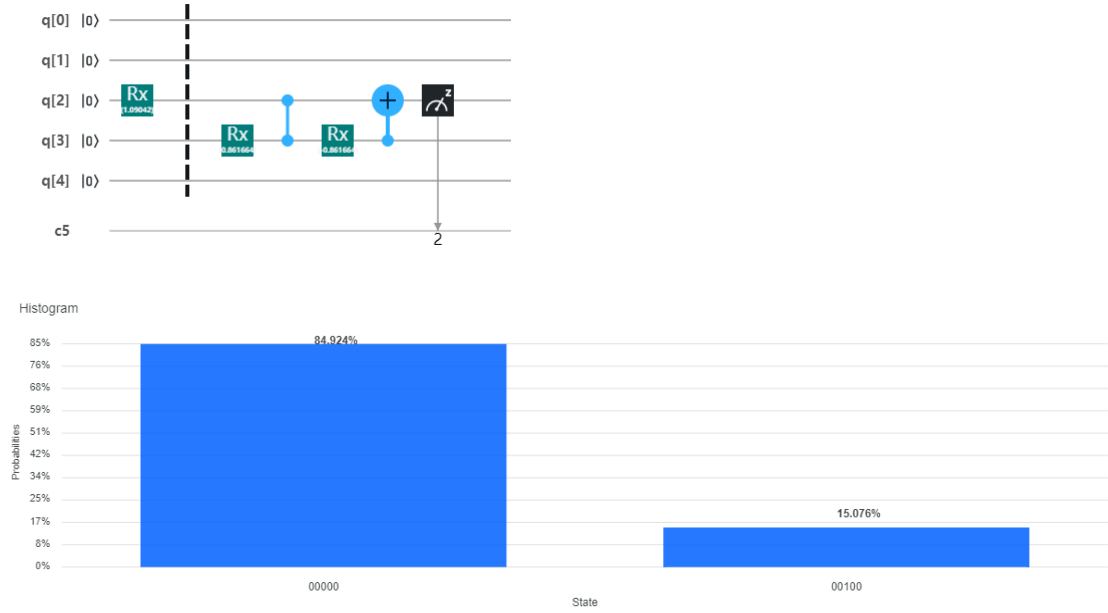
## 2. The first step $t = \delta\tau$ .

For the sub-channel choice  $\uparrow$ , the circuit and the result are



The excited state population is  $p_e(\delta\tau) = 0.65955$ . The raw data was obtained on April 16<sup>th</sup>, 2020.

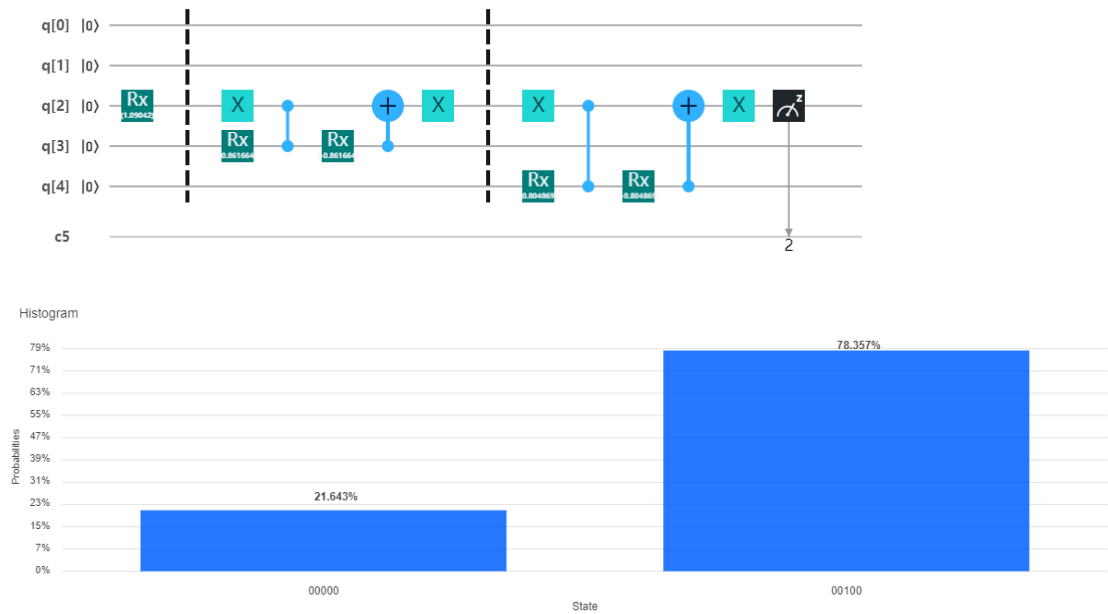
For the sub-channel choice  $\downarrow$ , the circuit and the results are



The excited state population is  $p_e(\delta\tau) = 0.15076$ . The raw data was obtained on April 16<sup>th</sup>, 2020.

### 3. The second step $t = 2\delta\tau$ .

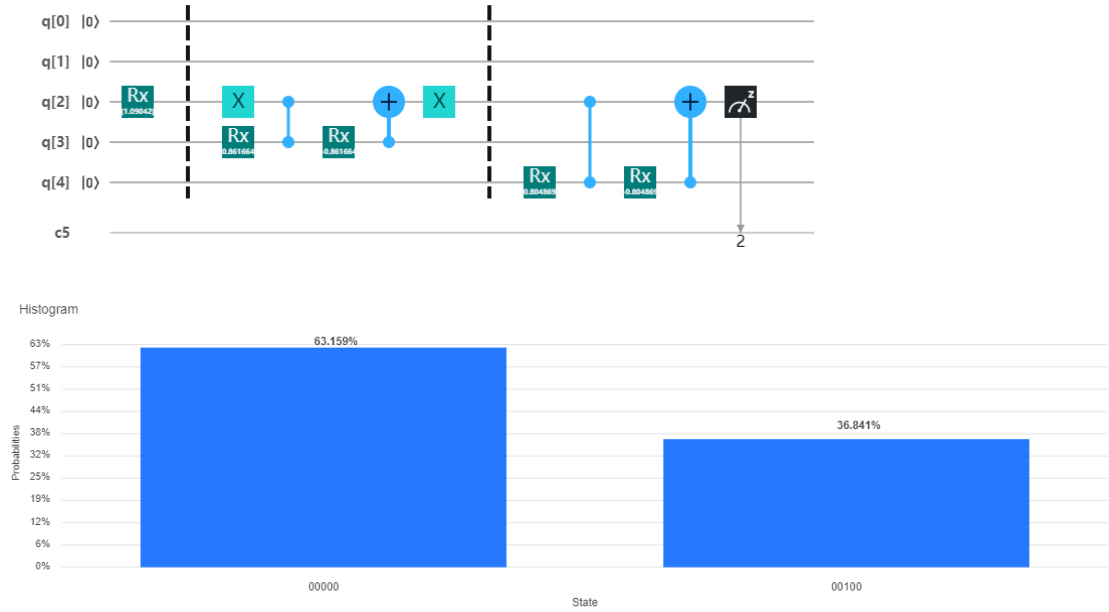
For the sub-channel choice  $\uparrow\uparrow$ , the circuit and the result are



The excited state population  $p_e(2\delta\tau) = 0.78357$ . The raw data was obtained on April 16<sup>th</sup>,

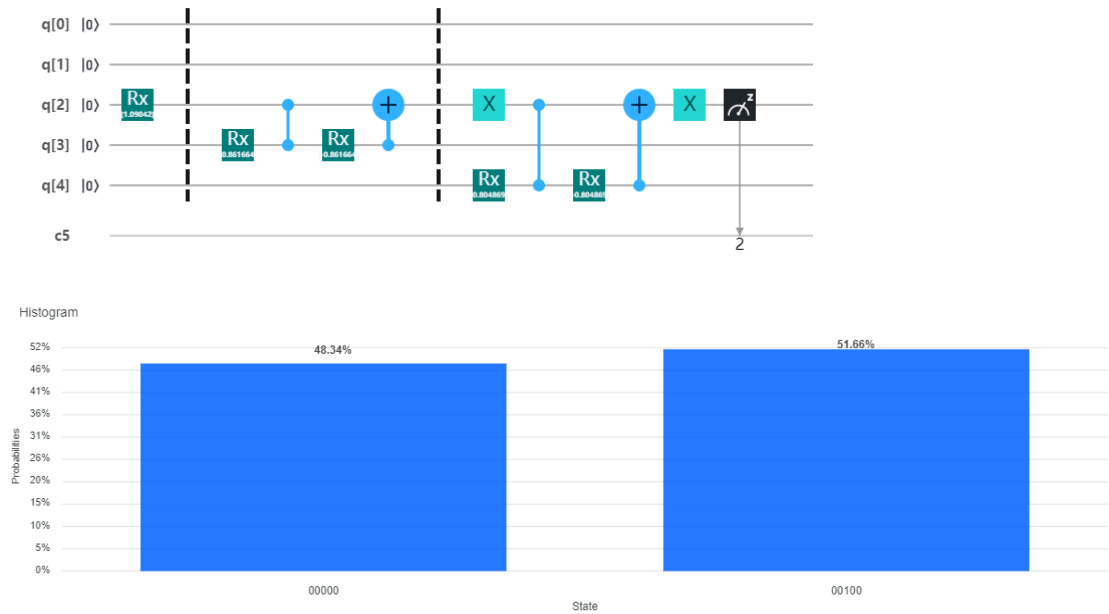
2020.

For the sub-channel choice  $\uparrow\downarrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.36841$ . The raw data was obtained on April 16<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.51660$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

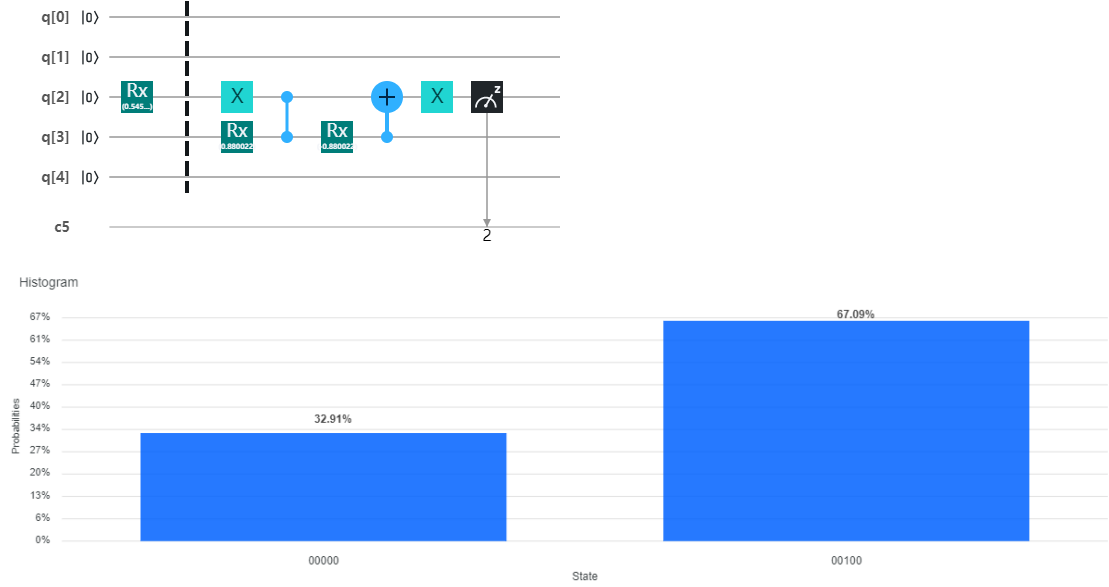
For the sub-channel choice  $\downarrow\downarrow$ , the circuit and the result are



is  $p_e(0) = 0.26123$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

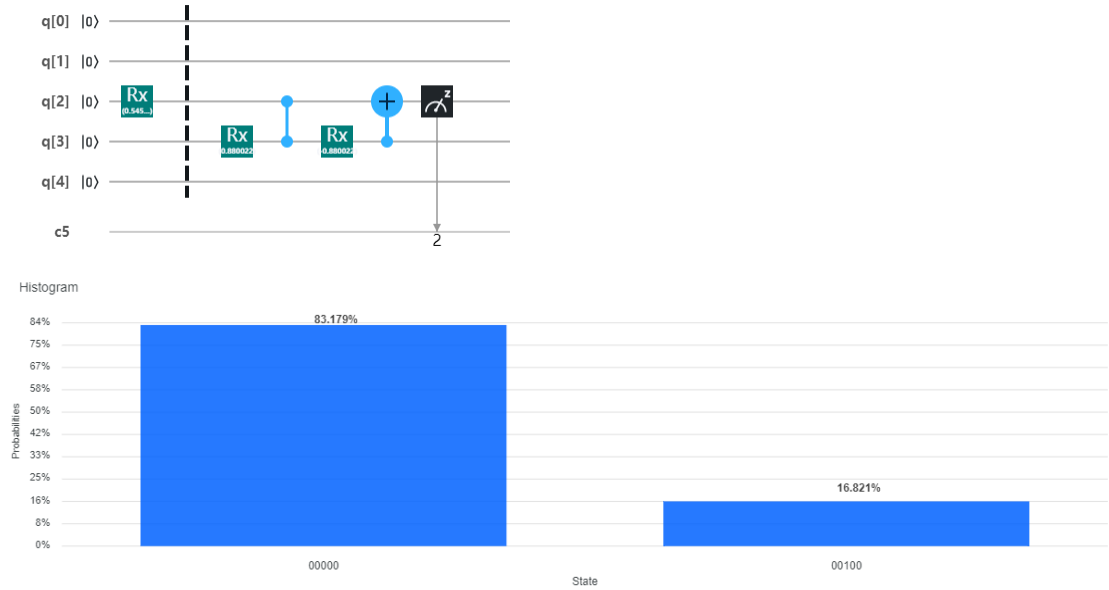
## 2. The first step $t = \delta\tau$ .

For the sub-channel choice  $\uparrow$ , the circuit and the result are



The excited state population is  $p_e(\delta\tau) = 0.67090$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow$ , the circuit and the result are

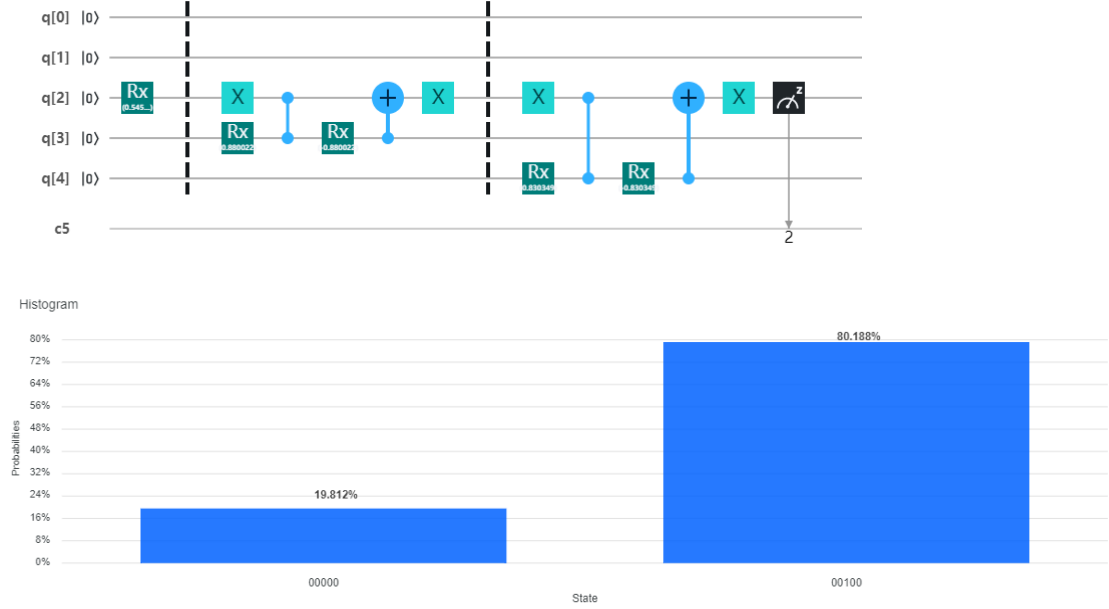


The excited state population  $p_e(\delta\tau) = 0.16821$ . The raw data was obtained on April 17<sup>th</sup>, 2020.



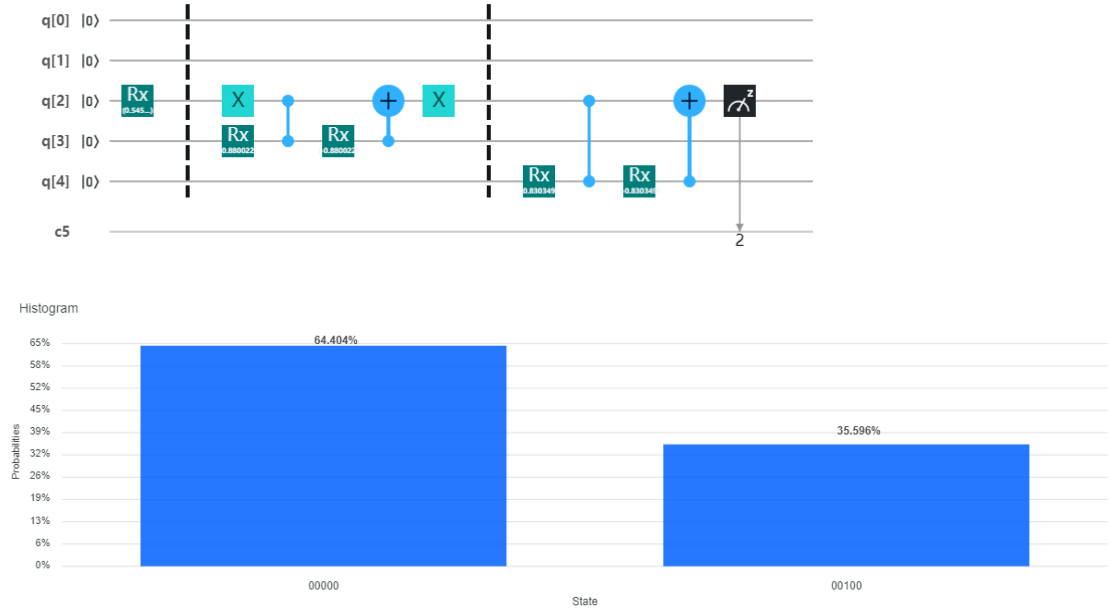
### 3. The second step $t = 2\delta\tau$ .

For the sub-channel choice  $\uparrow\uparrow$ , the circuit and the result are



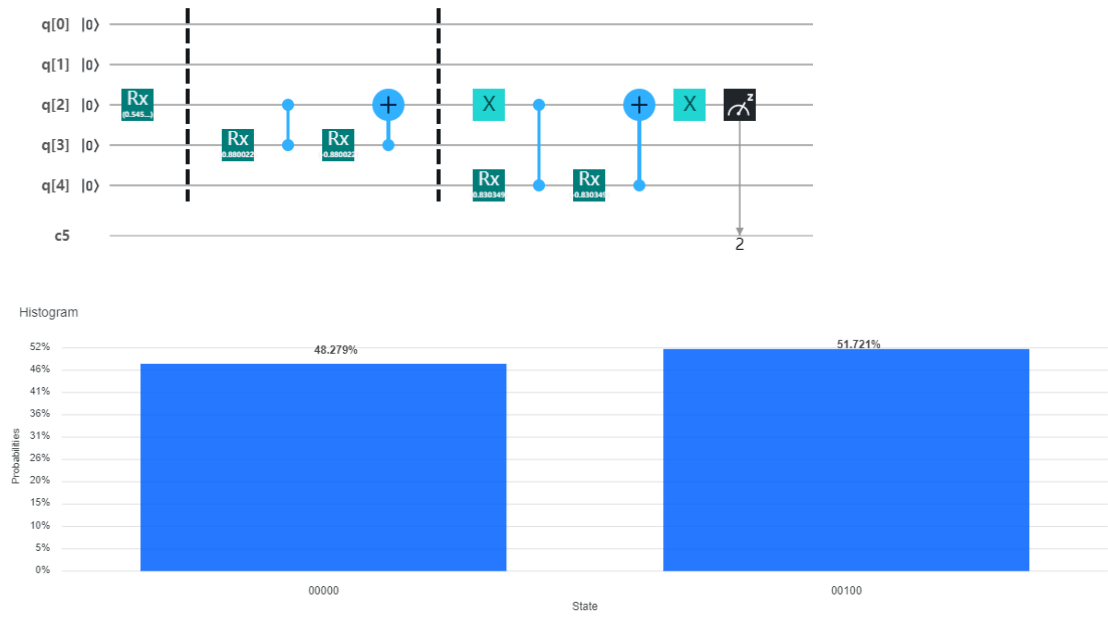
The excited state population is  $p_e(2\delta\tau) = 0.80188$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\uparrow\downarrow$ , the circuit and the result are



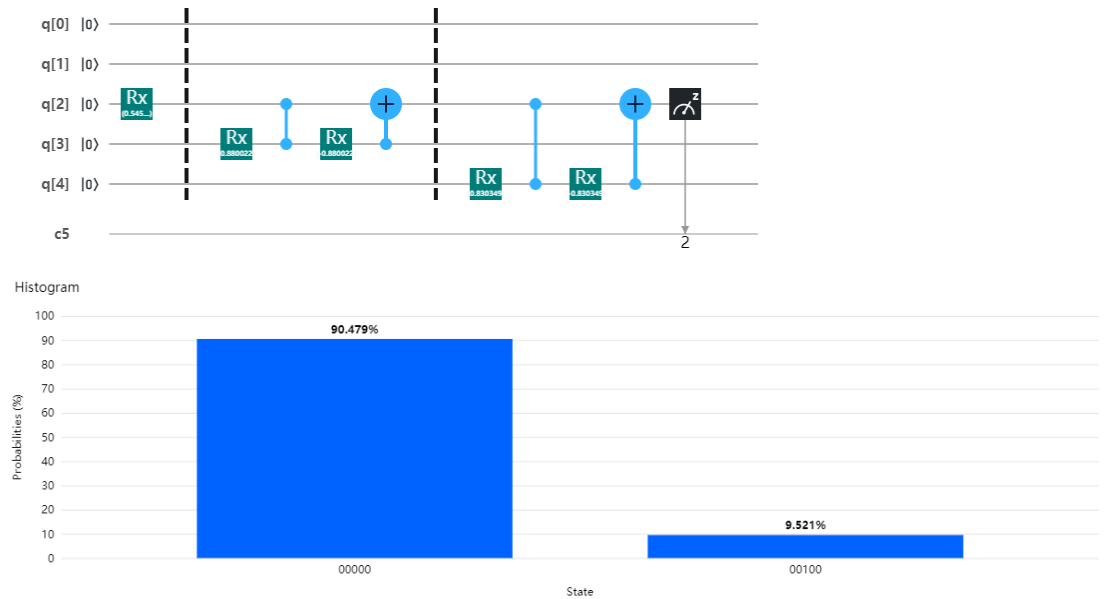
The excited state population is  $p_e(2\delta\tau) = 0.35596$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.51721$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

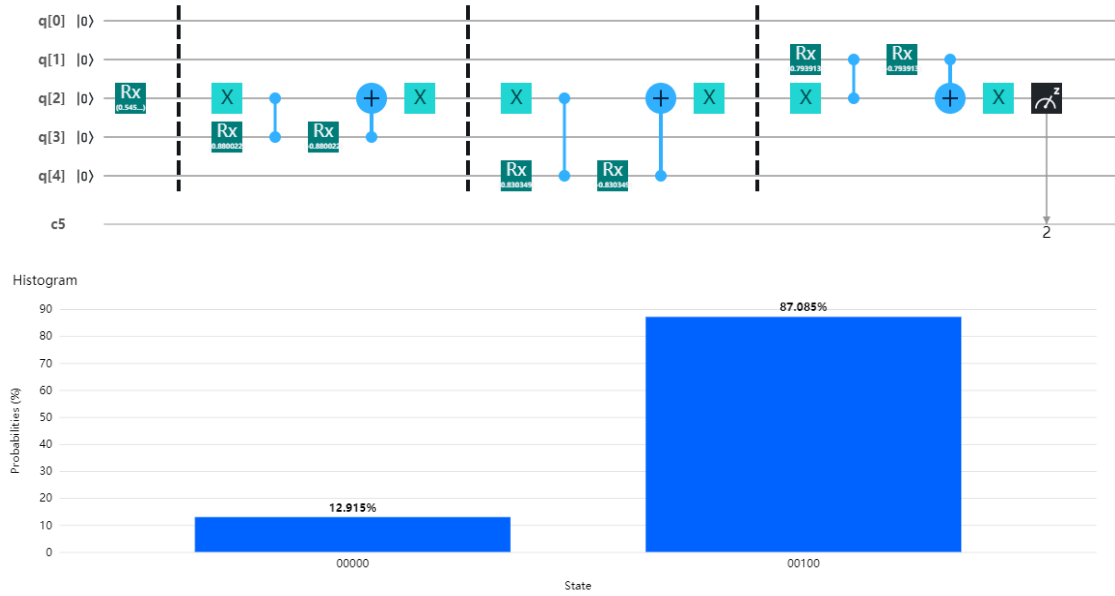
For the sub-channel choice  $\Downarrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.09521$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

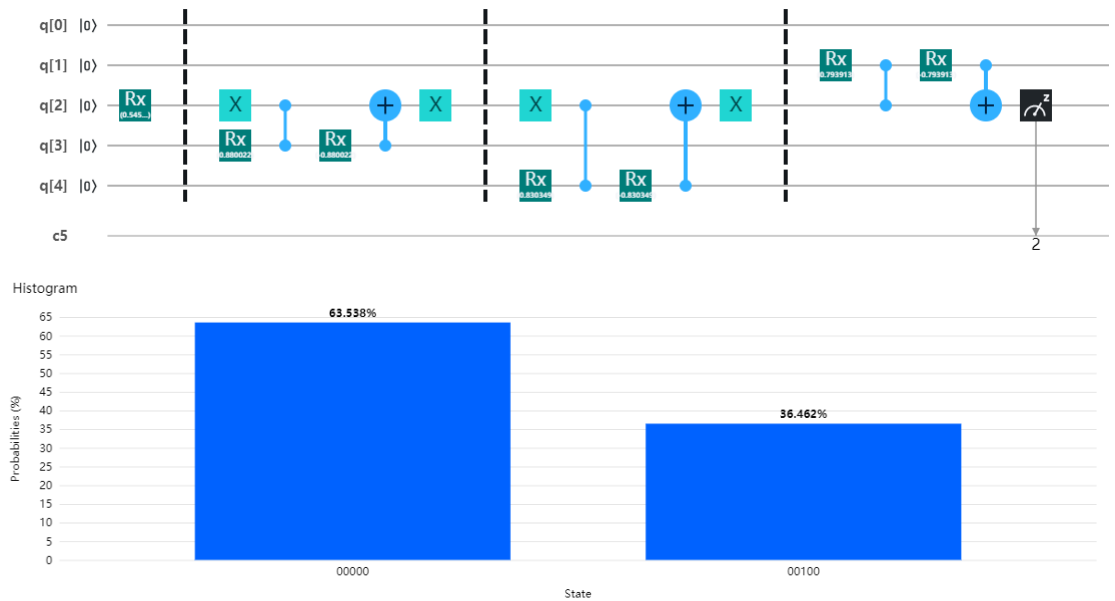
#### 4. The third step $t = 3\delta\tau$ .

For the sub-channel choice  $\Uparrow\Uparrow$ , the circuit and the result are



The excited state population is  $p_e(3\delta\tau) = 0.87085$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\uparrow\downarrow$ , the circuit and the result are



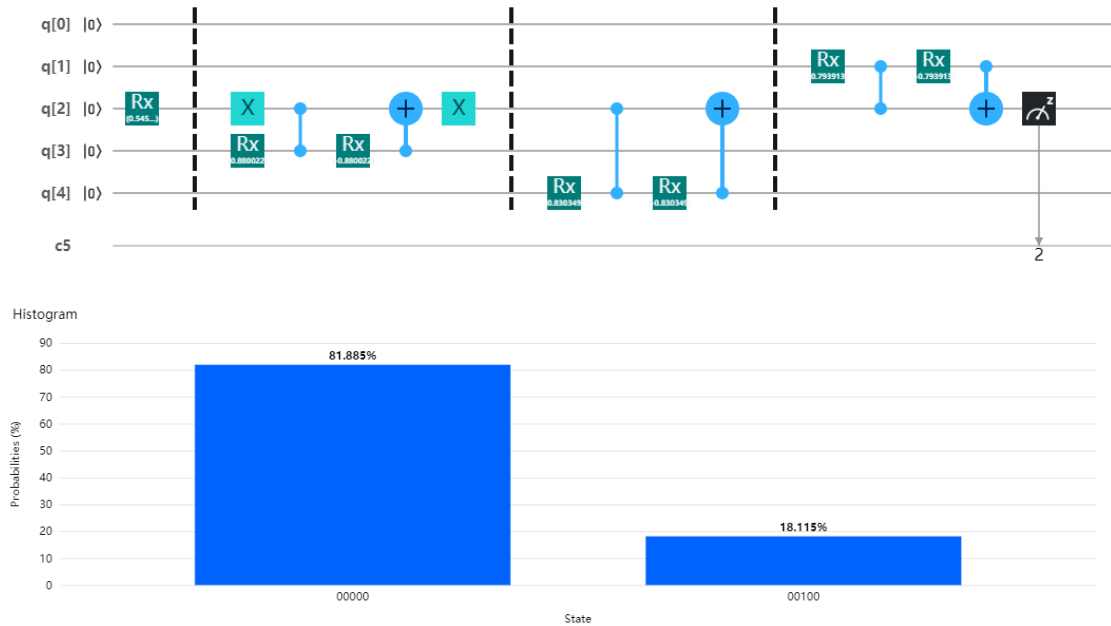
The excited state population is  $p_e(3\delta\tau) = 0.36462$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\uparrow\uparrow$ , the circuit and the result are



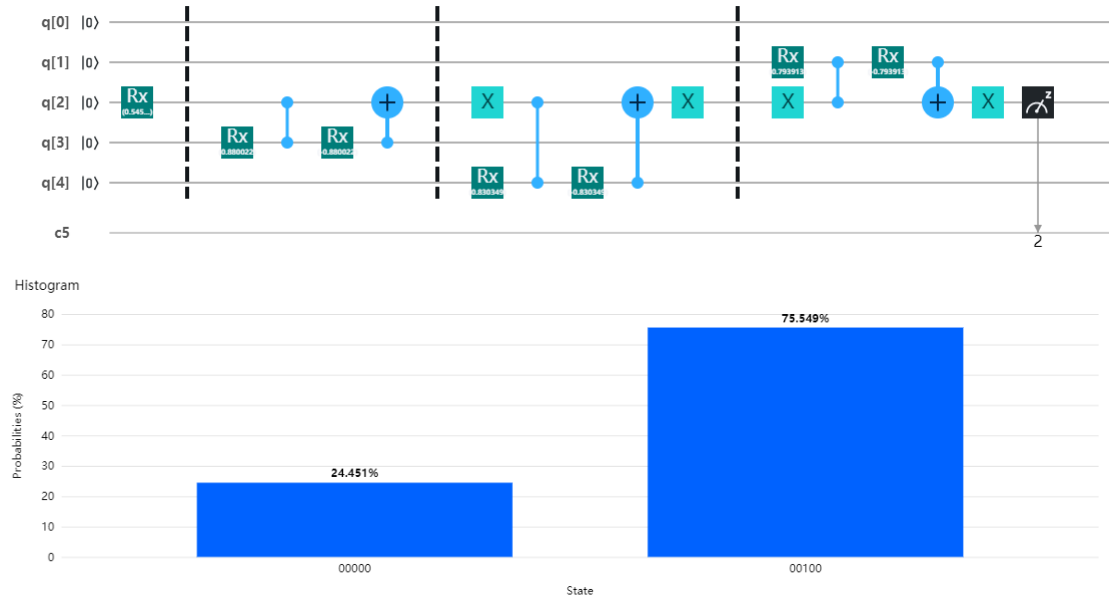
The excited state population is  $p_e(3\delta\tau) = 0.67371$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\uparrow\downarrow$ , the circuit and the result are



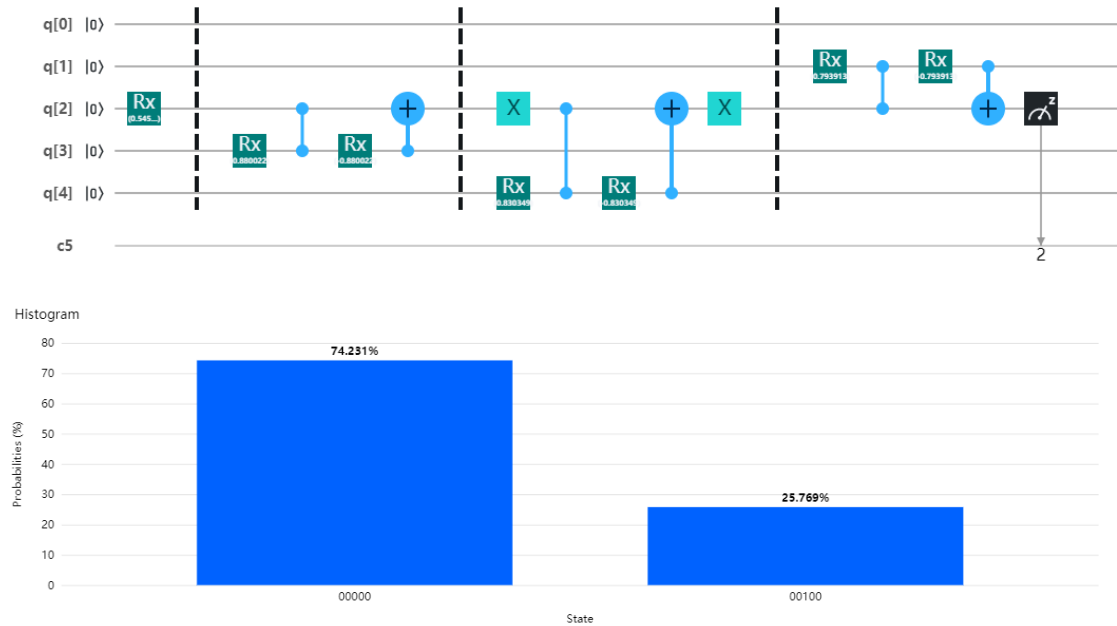
The excited state population is  $p_e(3\delta\tau) = 0.18115$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow$ , the circuit and the result are



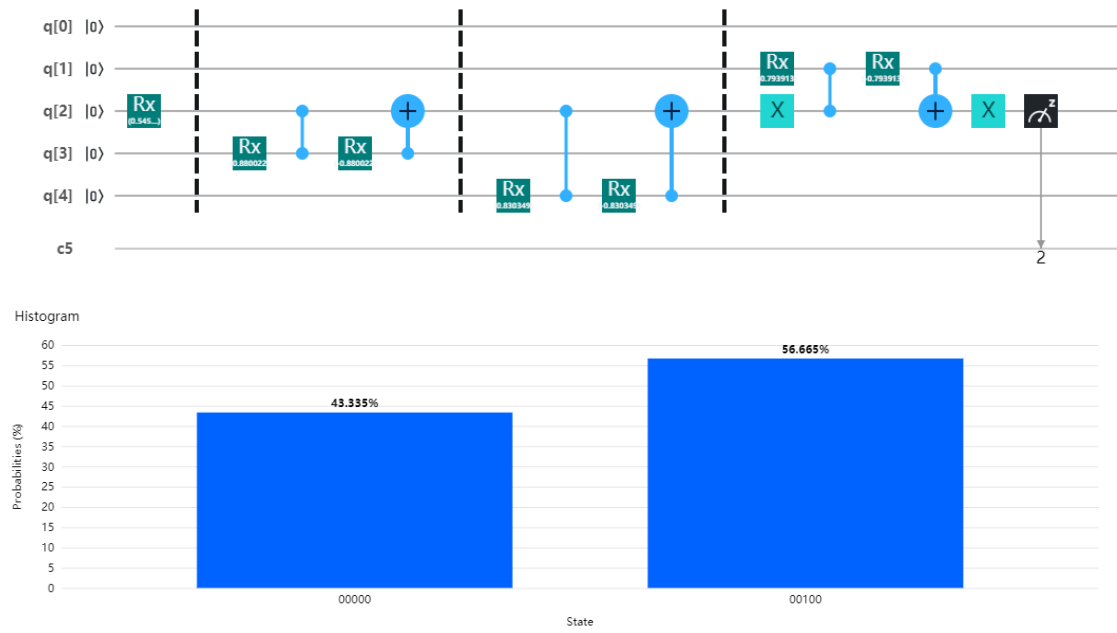
The excited state population is  $p_e(3\delta\tau) = 0.75549$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\downarrow$ , the circuit and the result are



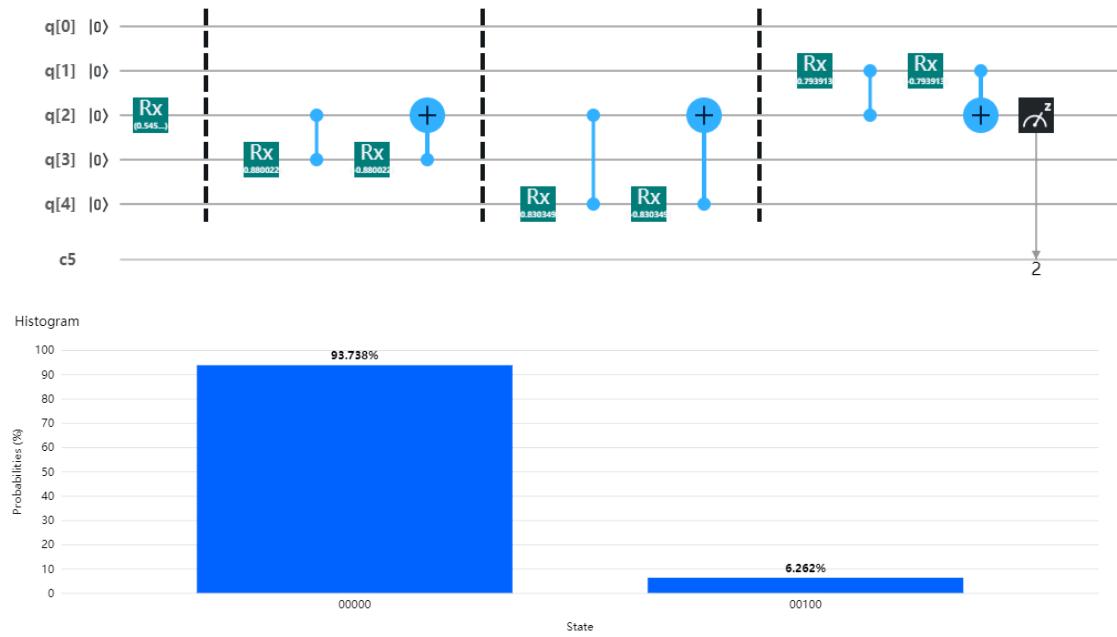
The excited state population is  $p_e(3\delta\tau) = 0.25769$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow$ , the circuit and the result are



The excited state population is  $p_e(3\delta\tau) = 0.56665$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\downarrow\downarrow$ , the circuit and the result are



The excited state population is  $p_e(3\delta\tau) = 0.06262$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

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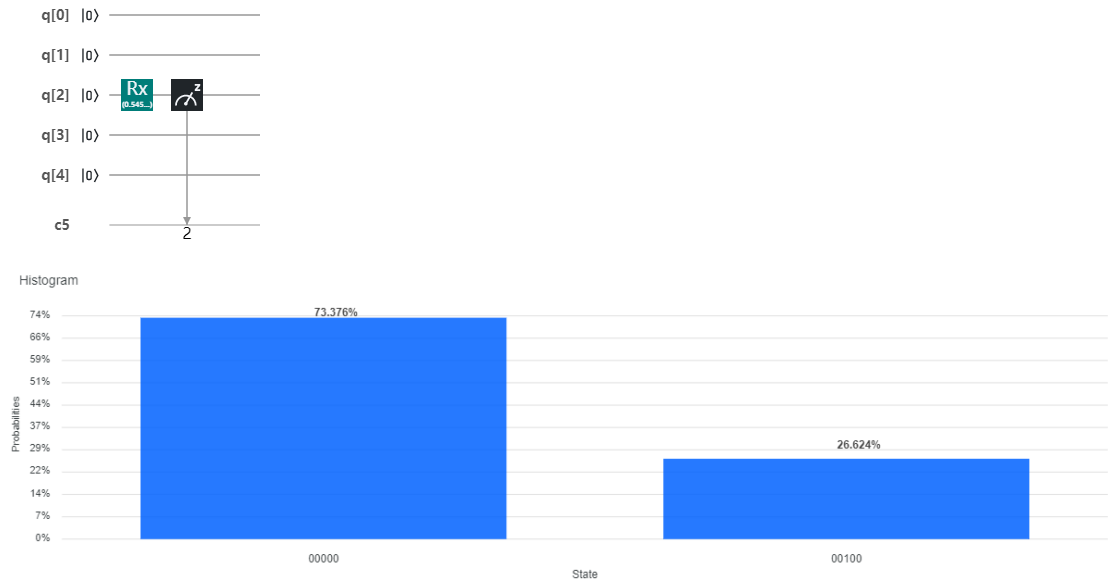
# Long-time dynamics with $\delta\tau = 10$

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We next show the ibmqx2 results for the long-time discrete isothermal process with the step number  $N = 2, 3$ , and 4.

## I. Two-step isothermal process $\delta\tau = 10$

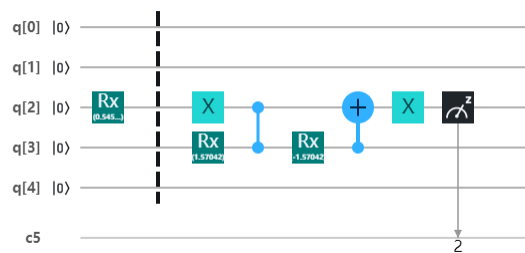
### 1. Prepare the initial state at $t = 0$ .

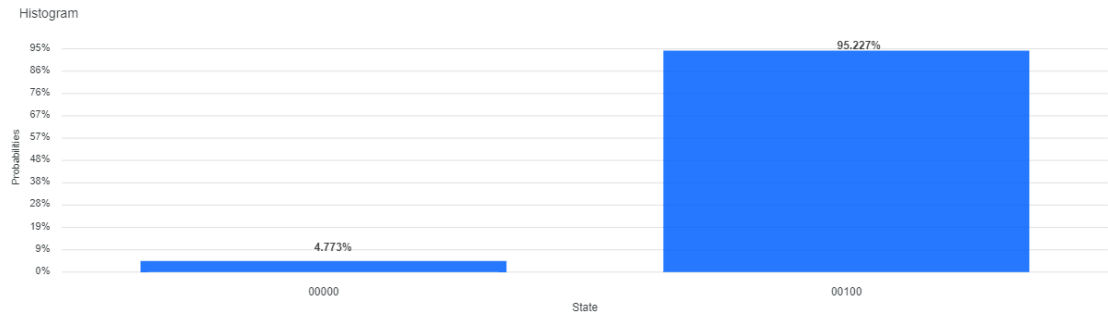


The initial state is prepared with the  $R_x(0.54521 \times 2)$  operation. The excited state population is  $p_e(0) = 0.26624$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

### 2. The first step $t = \delta\tau$ .

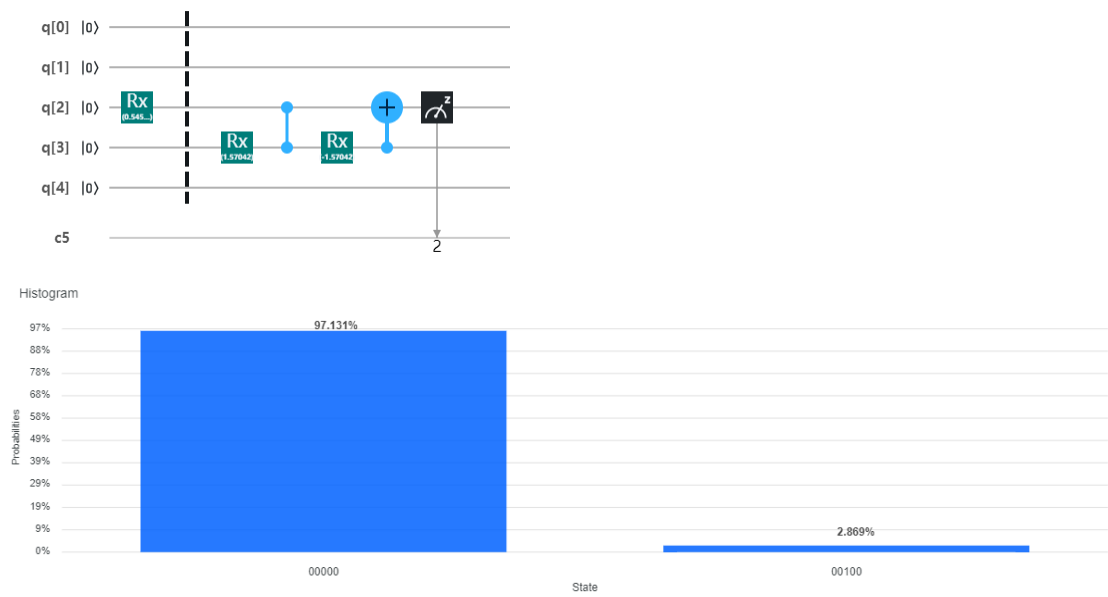
For the sub-channel choice  $\uparrow$ , the circuit and the result are





The excited state population is  $p_e(\delta\tau) = 0.95227$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow$ , the circuit and the result are

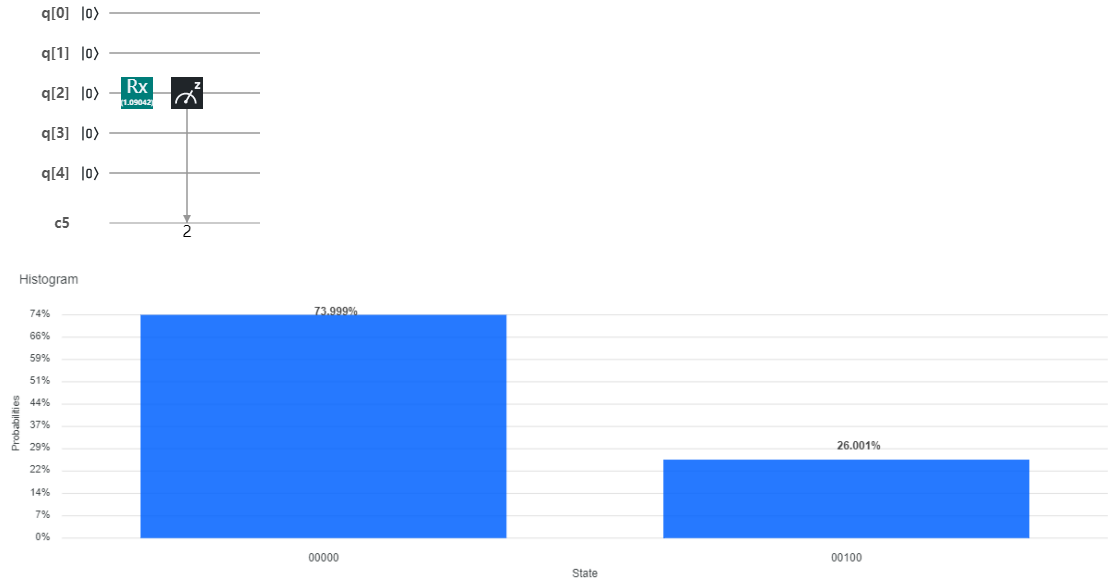


The excited state population is  $p_e(\delta\tau) = 0.02869$ , the raw data was obtained on April 17<sup>th</sup>, 2020.



## II. Three-step isothermal process $\delta\tau = 10$

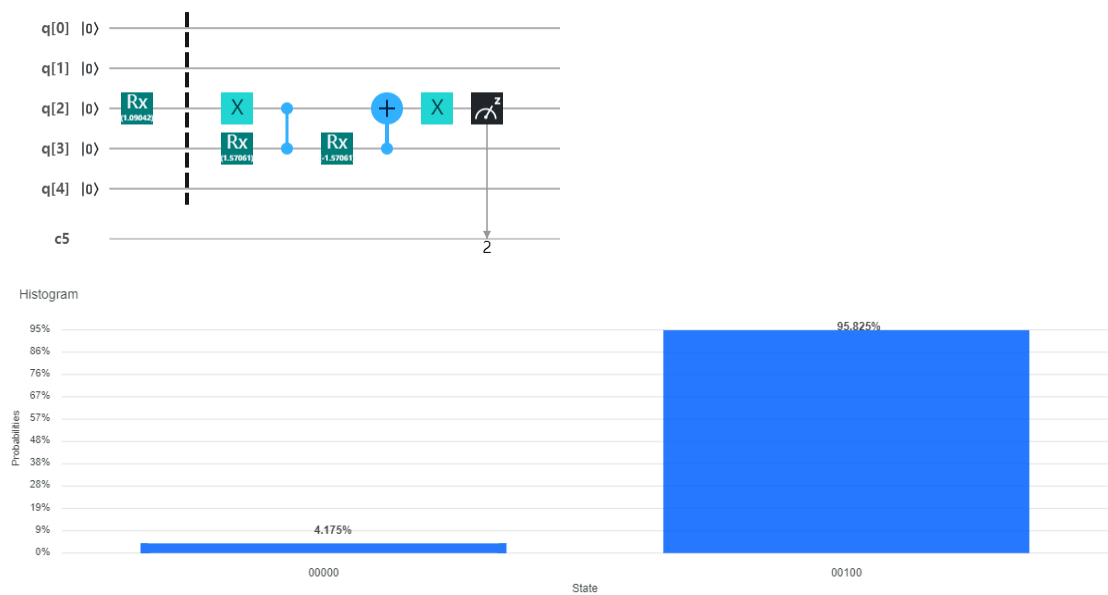
### 1. Prepare the initial state at $t = 0$ .



The initial state is prepared with the  $R_x(1.09042)$  operation. The excited state population is  $p_e(0) = 0.26001$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

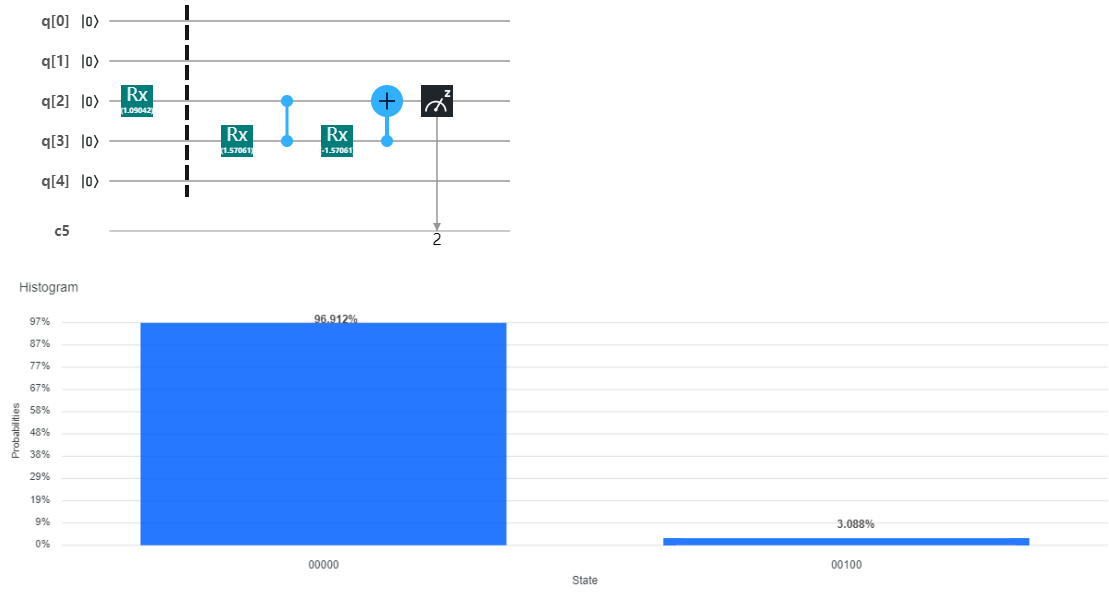
### 2. The first step $t = \delta\tau$ .

For the sub-channel choice  $\uparrow$ , the circuit and the result are



The excited state population is  $p_e(\delta\tau) = 0.95825$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

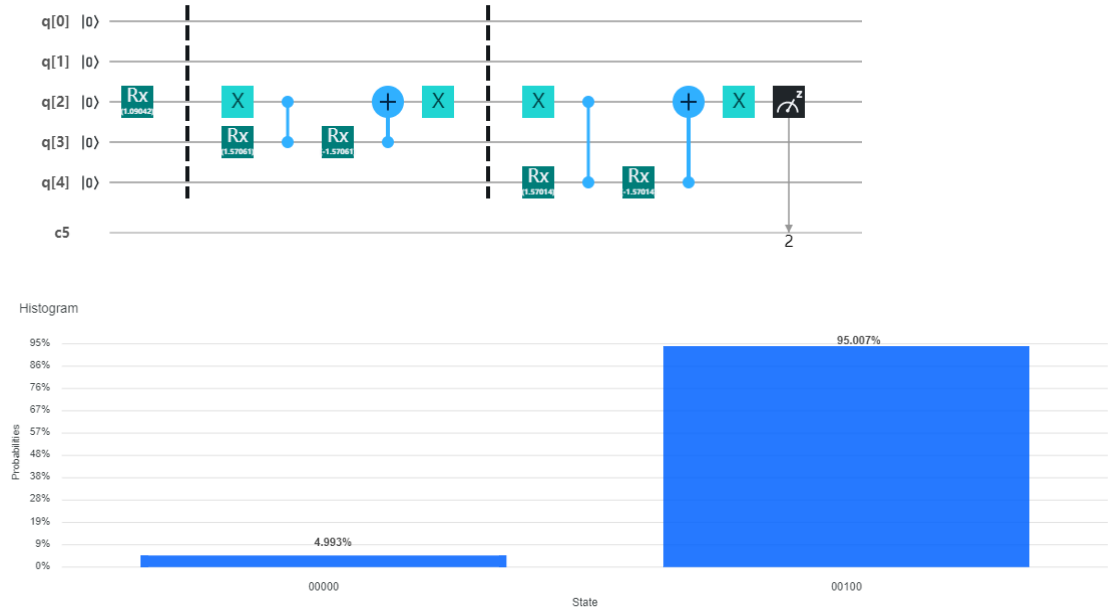
For the sub-channel choice  $\downarrow$ , the circuit and the result are



The excited state population is  $p_e(\delta\tau) = 0.03088$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

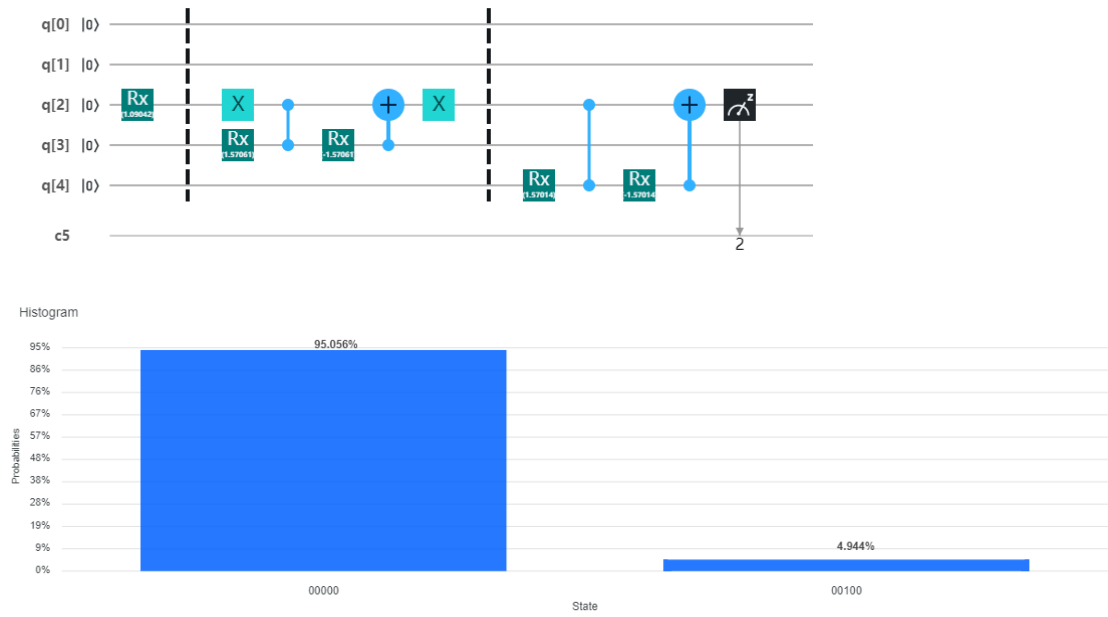
### 3. The second step $t = 2\delta\tau$ .

For the sub-channel choice  $\uparrow\uparrow$ , the circuit and the result are



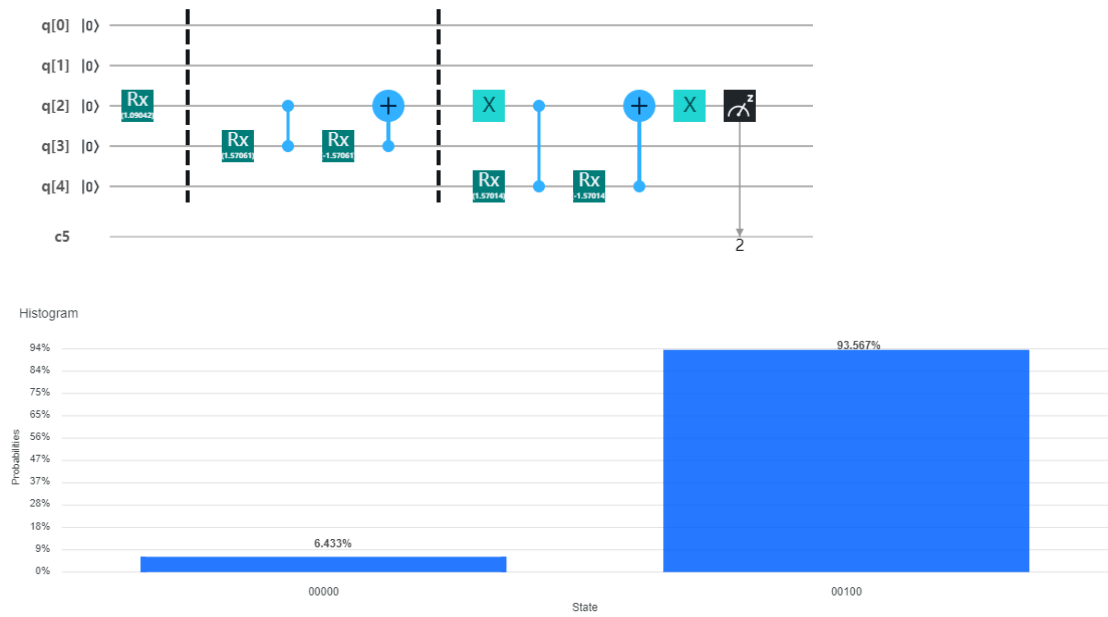
The excited state population is  $p_e(2\delta\tau) = 0.95007$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\uparrow\downarrow$ , the circuit and the result are



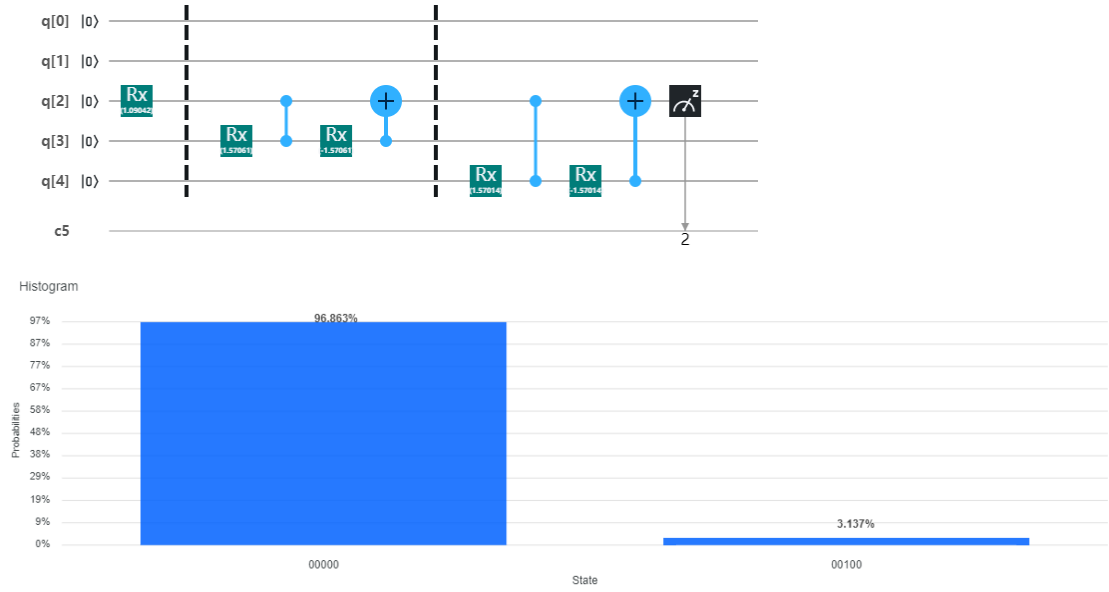
The excited state population is  $p_e(2\delta\tau) = 0.04944$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.93567$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

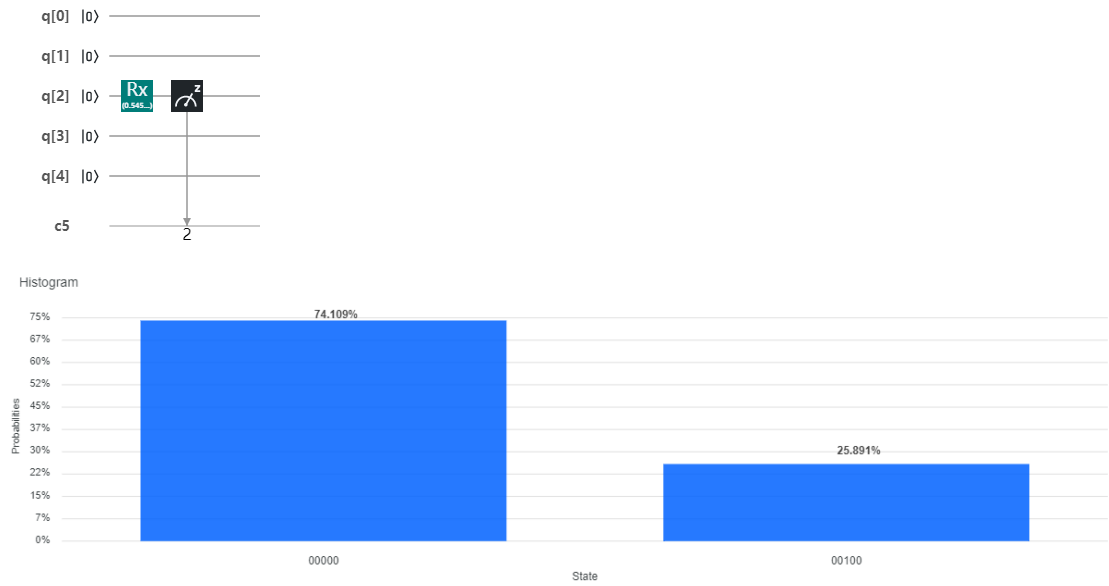
For the sub-channel choice  $\downarrow\downarrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.03137$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

### III. Four-step isothermal process $\delta\tau = 10$

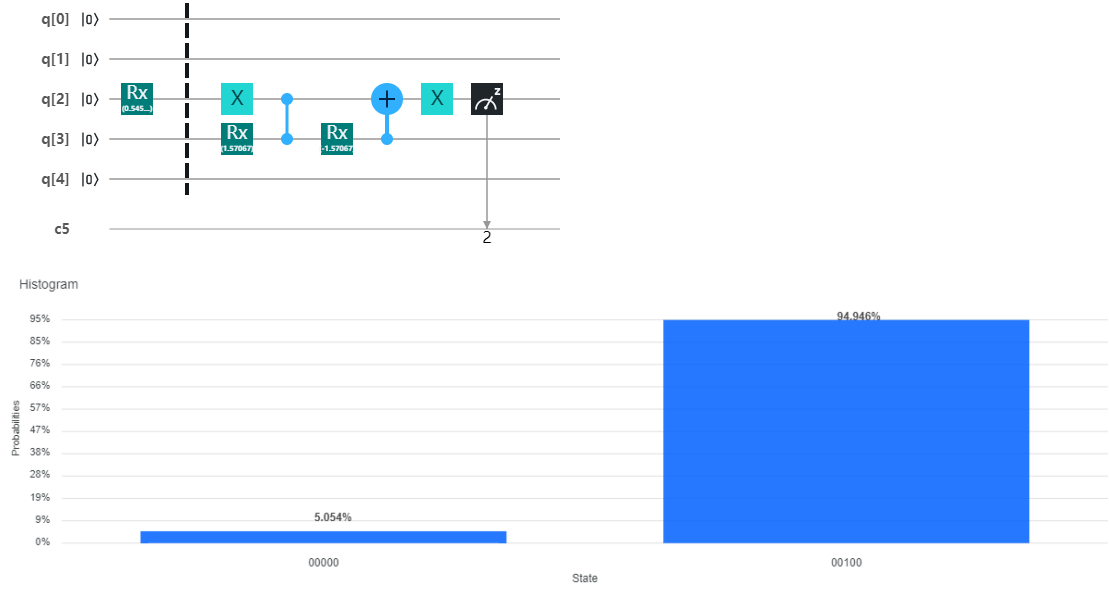
#### 1. Prepare the initial state at $t = 0$ .



The initial state is prepared with the  $R_x(0.54521 \times 2)$  operation. The excited state population is  $p_e(0) = 0.25891$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

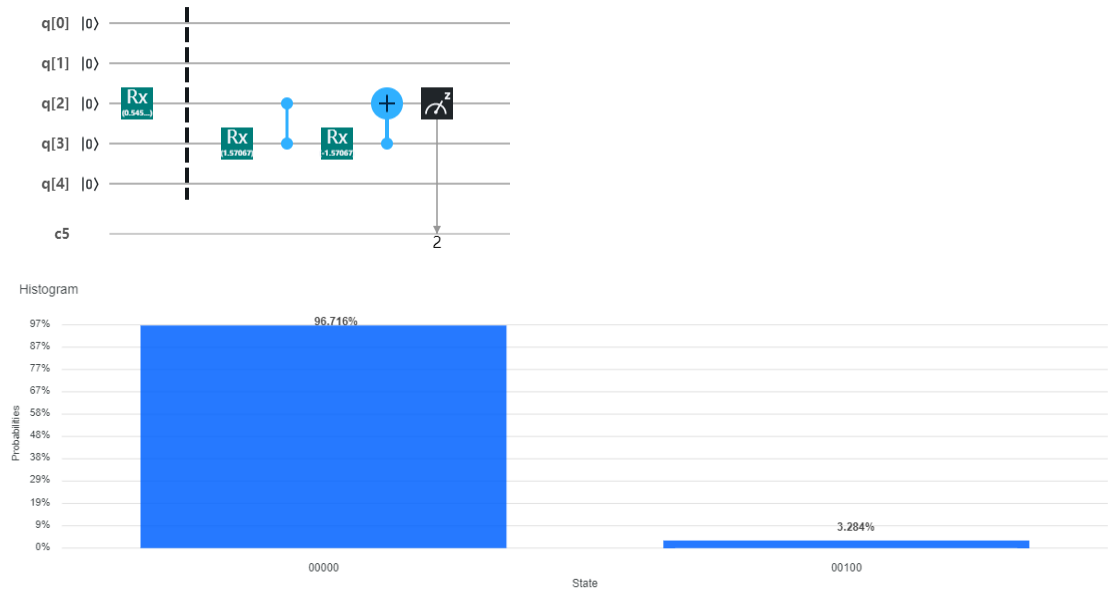
## 2. The first step $t = \delta\tau$ .

For the sub-channel choice  $\uparrow$ , the circuit and the result are



The excited state population is  $p_e(\delta\tau) = 0.94946$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

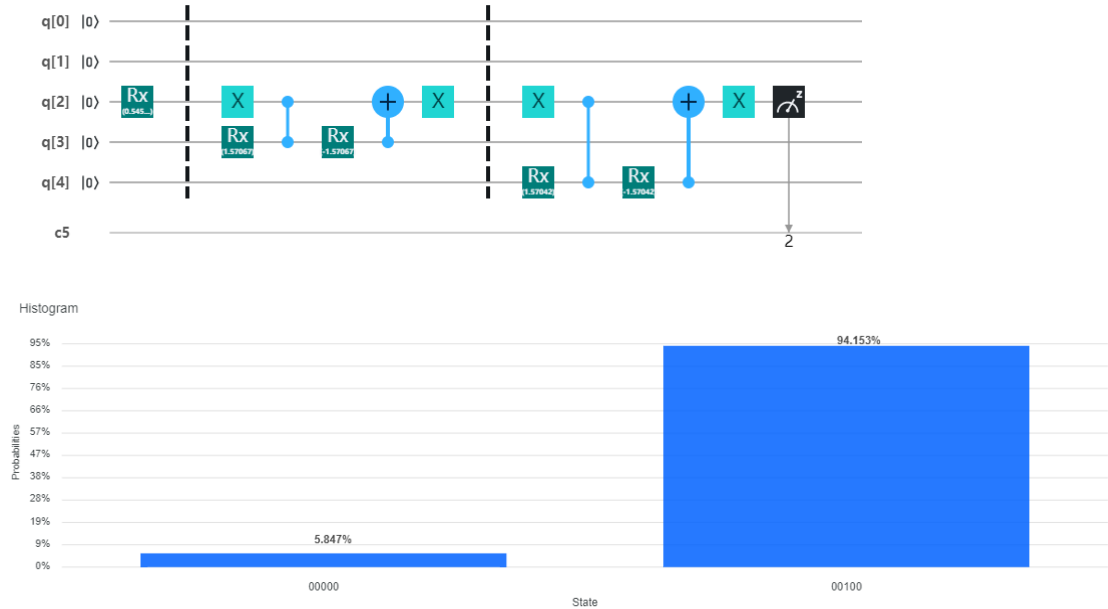
For the sub-channel choice  $\downarrow$ , the circuit and the result are



The excited state population is  $p_e(\delta\tau) = 0.03284$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

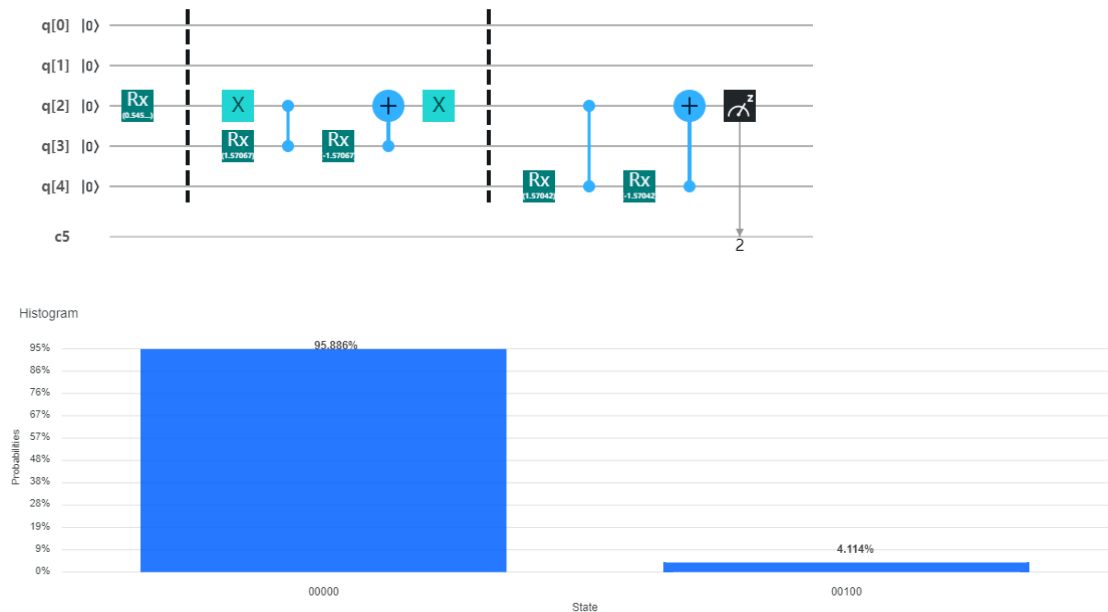
### 3. The second step $t = 2\delta\tau$ .

For the sub-channel choice  $\uparrow\uparrow$ , the circuit and the result are



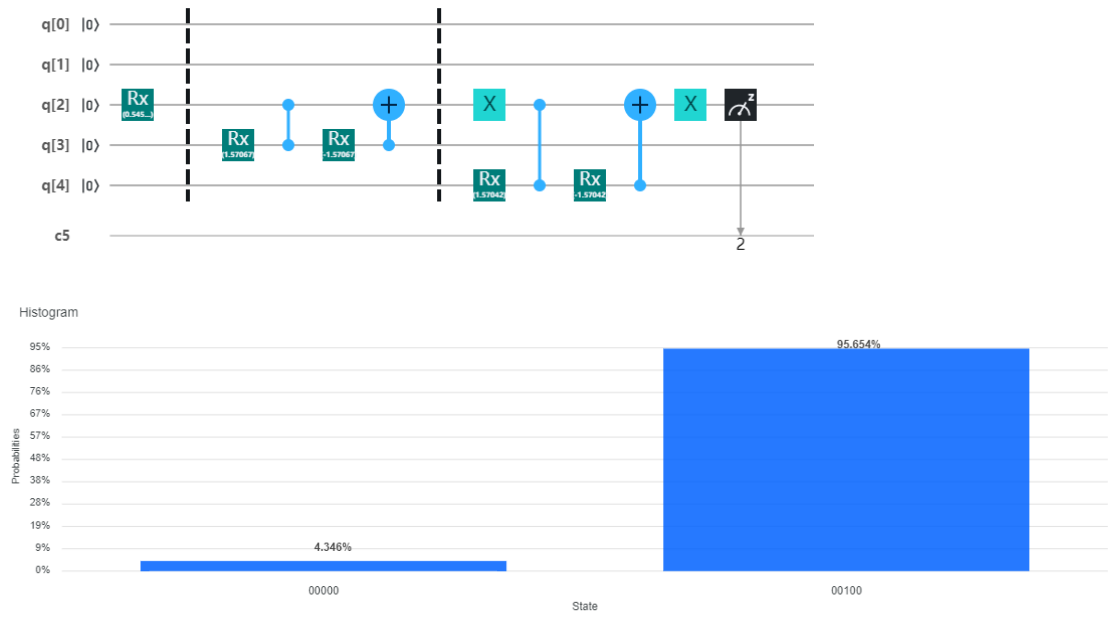
The excited state population is  $p_e(2\delta\tau) = 0.94153$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\uparrow\downarrow$ , the circuit and the result are



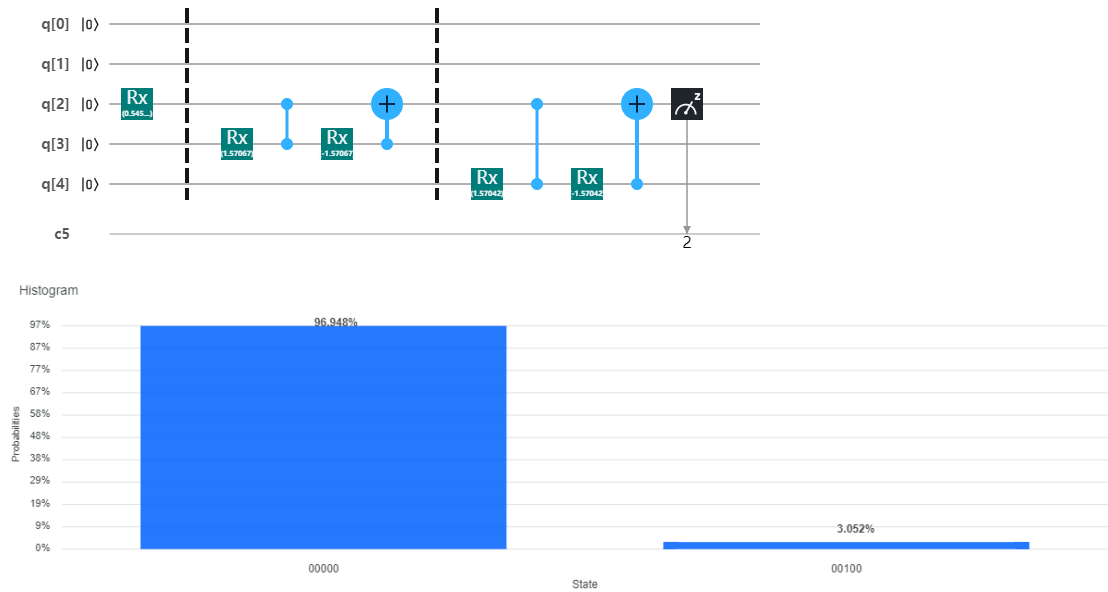
The excited state population is  $p_e(2\delta\tau) = 0.04114$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.95654$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

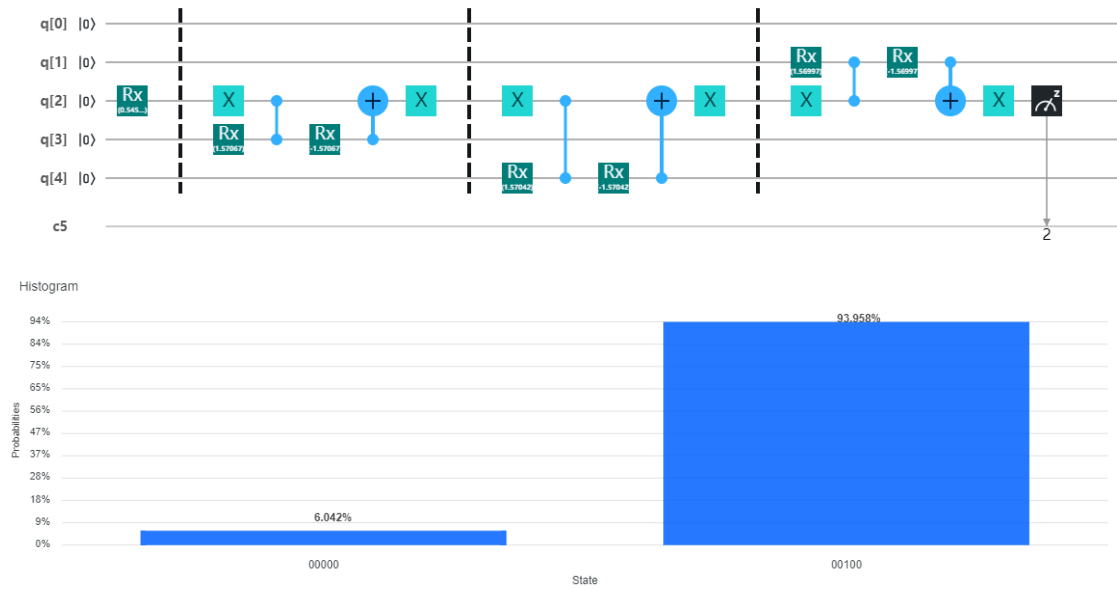
For the sub-channel choice  $\Downarrow$ , the circuit and the result are



The excited state population is  $p_e(2\delta\tau) = 0.03052$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

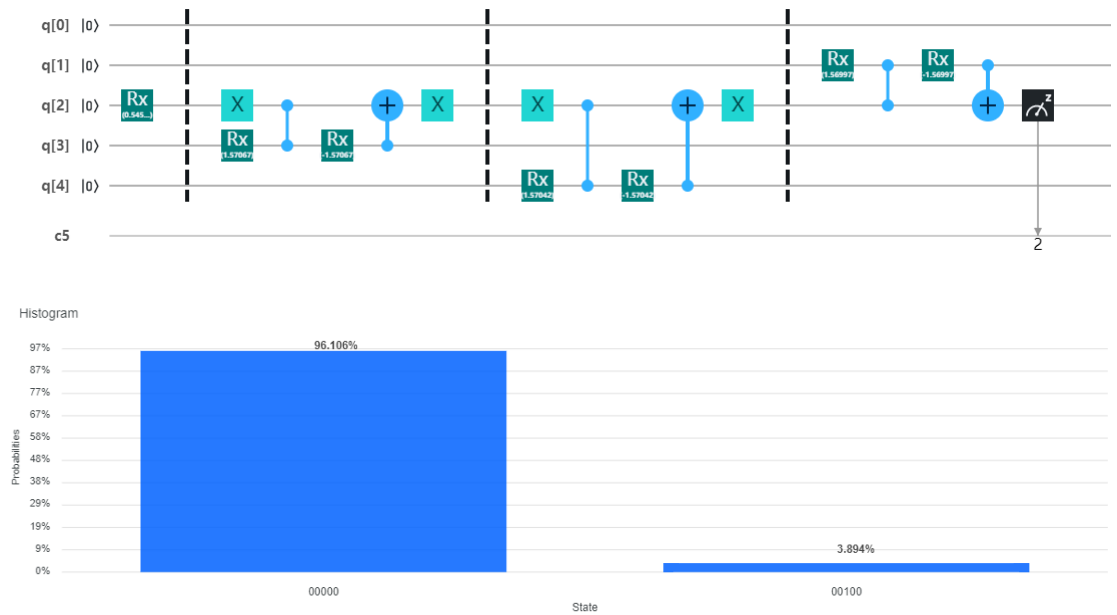
#### 4. The third step $t = 3\delta\tau$ .

For the sub-channel choice  $\Uparrow\Uparrow$ , the circuit and the result are



The excited state population is  $p_e(3\delta\tau) = 0.93958$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

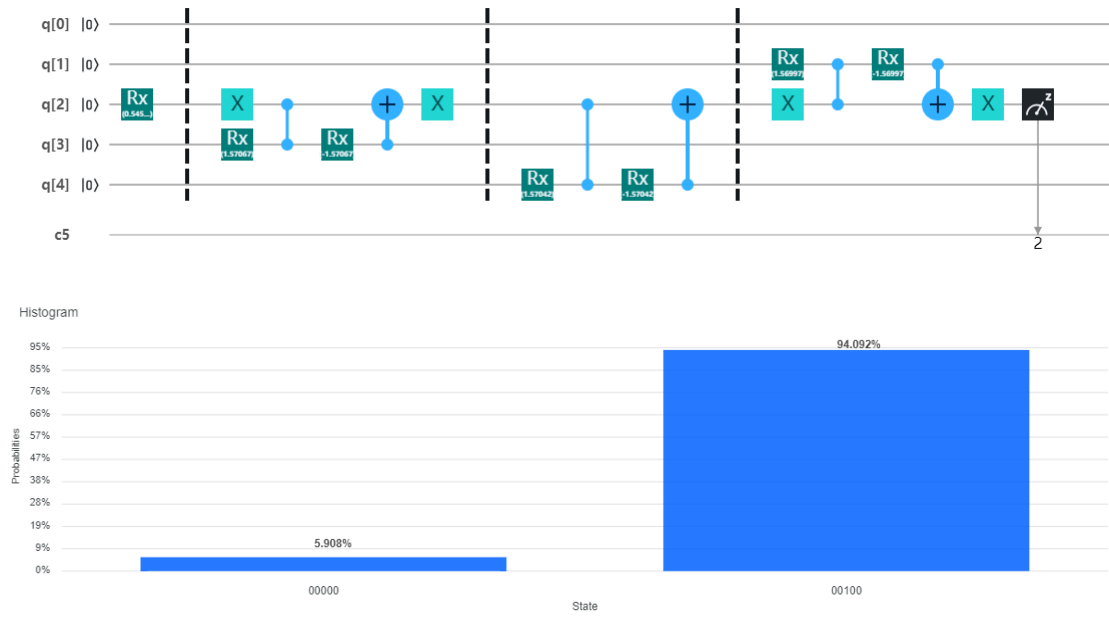
For the sub-channel choice  $\uparrow\downarrow$ , the circuit and the result are



The excited state population is  $p_e(3\delta\tau) = 0.03894$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

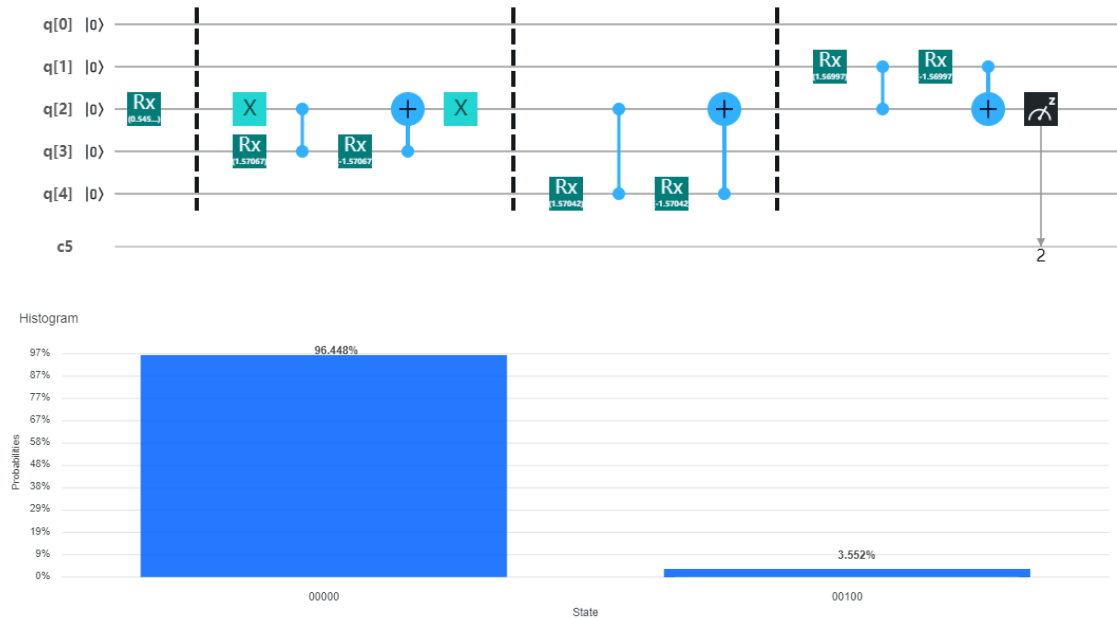
For the sub-channel choice  $\uparrow\uparrow$ , the circuit and the result are





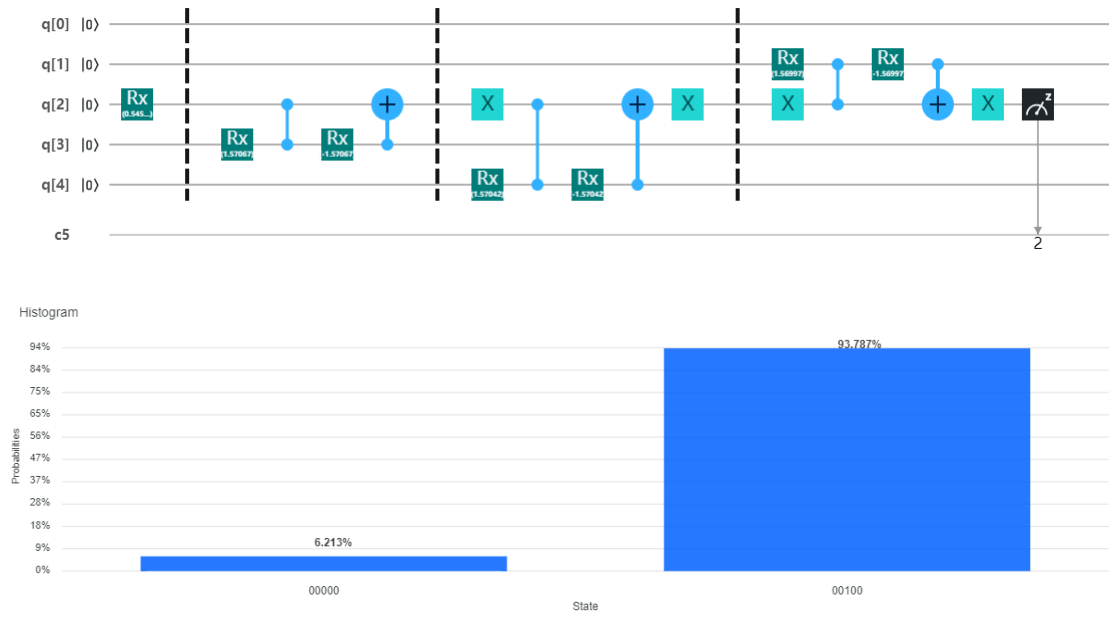
The excited state population  $p_e(3\delta\tau) = 0.94092$ , the raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\uparrow\downarrow\downarrow$ , the circuit and the result are



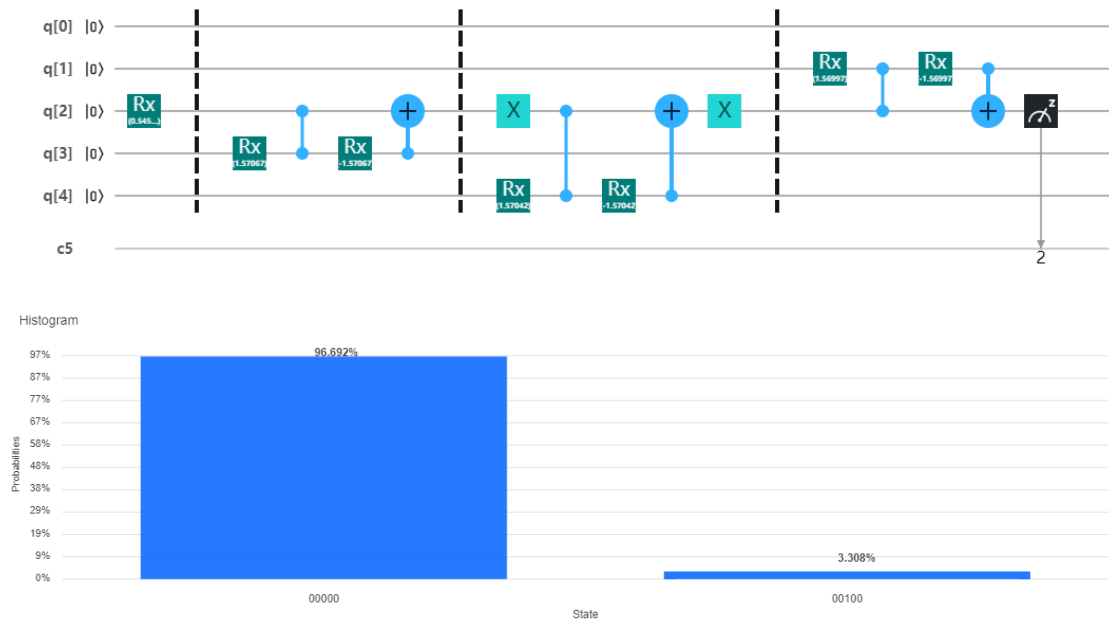
The excited state population is  $p_e(3\delta\tau) = 0.03552$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow\uparrow$ , the circuit and the result are



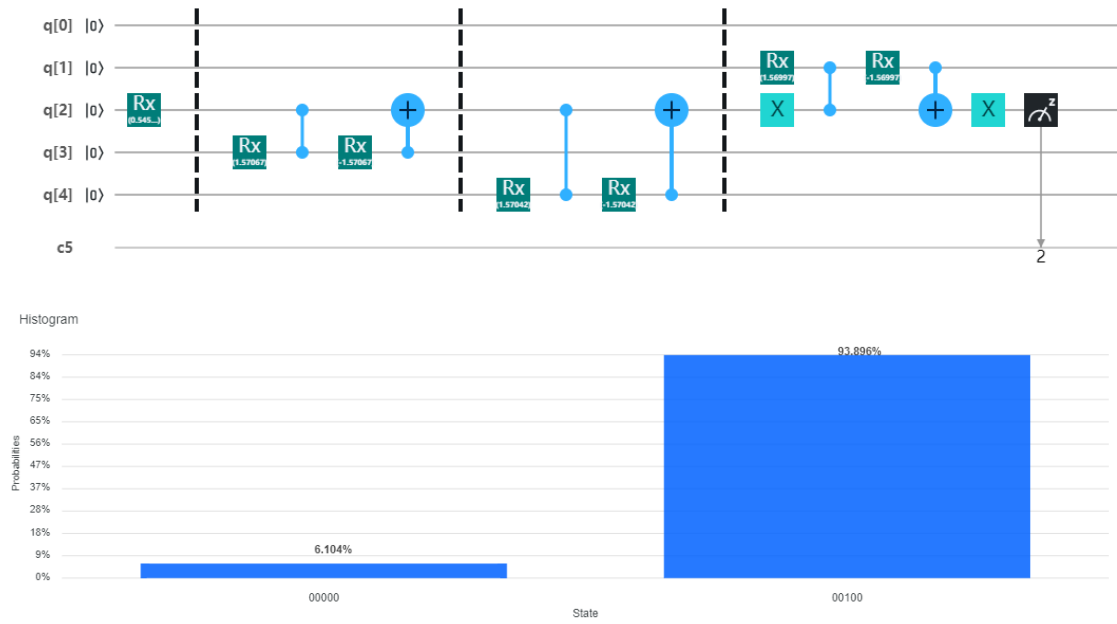
The excited state population is  $p_e(3\delta\tau) = 0.93787$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\uparrow\downarrow$ , the circuit and the result are



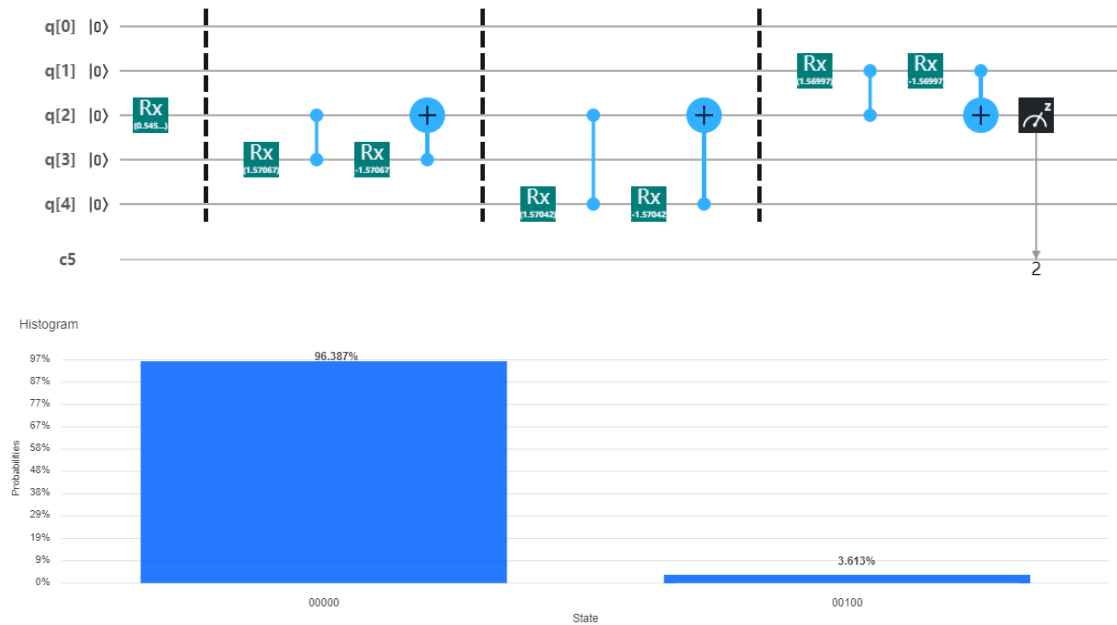
The excited state population is  $p_e(3\delta\tau) = 0.03308$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\downarrow\uparrow$ , the circuit and the result are



The excited state population is  $p_e(3\delta\tau) = 0.93896$ . The raw data was obtained on April 17<sup>th</sup>, 2020.

For the sub-channel choice  $\downarrow\downarrow\downarrow$ , the circuit and the result are



The excited state population is  $p_e(3\delta\tau) = 0.03613$ . The raw data was obtained on April 17<sup>th</sup>, 2020.