

Supplementary Material for

Soft Sensor design via Switching Observers

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Table S1. Scenarios of nominal conditions

| Scenario | U [day ⁻¹] | X_1 [kg/m ³] | X_2 [kg/m ³] |
|----------|--------------------------|----------------------------|----------------------------|
| 1 | 0.1393 | 0.6502 | 1.2651 |
| 2 | 0.2293 | 1.1696 | 1.1864 |
| 3 | 0.3117 | 1.7370 | 1.1004 |
| 4 | 0.3871 | 2.3571 | 1.0065 |
| 5 | 0.4584 | 3.0598 | 0.9000 |
| 6 | 0.5235 | 3.8299 | 0.7833 |
| 7 | 0.5828 | 4.6741 | 0.6554 |
| 8 | 0.6369 | 5.5995 | 0.5152 |
| 9 | 0.6863 | 6.6139 | 0.3615 |
| 10 | 0.7273 | 7.6154 | 0.2098 |

Table S2. Optimal observer parameter selection for $a = 3.16309$

| Scenario | $\rho_{F,1}$ | $\rho_{F,2}$ | $J_{e,A}$ | $\tilde{J}_{e,O}$ |
|----------|--------------|--------------|-----------|-------------------|
| 1 | 3.16309+ | 8.11101 | 0.5 | 2.26608 |
| 2 | 3.16309+ | 5.33798 | 0.5 | 0.54718 |
| 3 | 3.16309+ | 4.77700 | 0.5 | 0.19104 |
| 4 | 3.16309+ | 4.57362 | 0.5 | 0.07841 |
| 5 | 3.16309+ | 4.48782 | 0.5 | 0.03386 |
| 6 | 3.16309+ | 4.45649 | 0.5 | 0.01506 |
| 7 | 3.16309+ | 4.45175 | 0.5 | 0.00660 |
| 8 | 3.16309+ | 4.46083 | 0.5 | 0.00268 |
| 9 | 3.16309+ | 4.47730 | 0.5 | 0.00090 |
| 10 | 3.16309+ | 4.49556 | 0.5 | 0.00022 |

Table S3. Optimal observer parameter selection for $a = 4.74463$

| Scenario | $\rho_{F,1}$ | $\rho_{F,2}$ | $J_{e,A}$ | $\tilde{J}_{e,O}$ |
|----------|--------------|--------------|-----------|-------------------|
| 1 | 4.74463+ | 5.82279 | 0.5 | 2.30452 |
| 2 | 4.74463+ | 5.38508 | 0.5 | 0.57777 |
| 3 | 4.74463+ | 5.23672 | 0.5 | 0.20871 |
| 4 | 4.74463+ | 5.16769 | 0.5 | 0.08826 |
| 5 | 4.74463+ | 5.13005 | 0.5 | 0.03912 |
| 6 | 4.74463+ | 5.10918 | 0.5 | 0.01775 |
| 7 | 4.74463+ | 5.09744 | 0.5 | 0.00788 |
| 8 | 4.74463+ | 5.09103 | 0.5 | 0.00323 |
| 9 | 4.74463+ | 5.08786 | 0.5 | 0.00109 |
| 10 | 4.74463+ | 5.08679 | 0.5 | 0.00027 |

Table S4. Minimum and maximum metric values for $a = 3.16309$

| Scenario | Minimum $J_{e,A}$ | Maximum $J_{e,A}$ | Minimum $\tilde{J}_{e,O}$ | Maximum $\tilde{J}_{e,O}$ |
|----------|-------------------|-------------------|---------------------------|---------------------------|
| 1 | 0.4452 | 0.5 | 2.26608 | 2.2968 |
| 2 | 0.2899 | 0.5 | 0.54718 | 0.6001 |
| 3 | 0.2317 | 0.5 | 0.19104 | 0.2255 |
| 4 | 0.2032 | 0.5 | 0.07841 | 0.0988 |
| 5 | 0.1870 | 0.5 | 0.03386 | 0.0453 |
| 6 | 0.1775 | 0.5 | 0.01506 | 0.0212 |
| 7 | 0.1716 | 0.5 | 0.00660 | 0.0097 |
| 8 | 0.1680 | 0.5 | 0.00268 | 0.0041 |
| 9 | 0.1657 | 0.5 | 0.00090 | 0.0014 |
| 10 | 0.1644 | 0.5 | 0.00022 | 0.00035 |

Table S5. Minimum and maximum metric values for $a = 4.74463$

| Scenario | Minimum $J_{e,A}$ | Maximum $J_{e,A}$ | Minimum $\tilde{J}_{e,O}$ | Maximum $\tilde{J}_{e,O}$ |
|----------|-------------------|-------------------|---------------------------|---------------------------|
| 1 | 0.1747 | 0.5 | 2.30452 | 2.4033 |
| 2 | 0.1102 | 0.5 | 0.57777 | 0.6290 |
| 3 | 0.0850 | 0.5 | 0.20871 | 0.2367 |
| 4 | 0.0721 | 0.5 | 0.08826 | 0.1041 |
| 5 | 0.0643 | 0.5 | 0.03912 | 0.0479 |
| 6 | 0.0595 | 0.5 | 0.01775 | 0.0225 |
| 7 | 0.0563 | 0.5 | 0.00788 | 0.0103 |
| 8 | 0.0541 | 0.5 | 0.00323 | 0.0043 |
| 9 | 0.0526 | 0.5 | 0.00109 | 0.0015 |
| 10 | 0.0516 | 0.5 | 0.00027 | 0.00037 |

Table S6. Identification process results

| Scenario | h_D | \hat{h}_D | Fluctuation | h_N | \hat{h}_N | Fluctuation |
|----------|--------|-------------|-------------|--------|-------------|-------------|
| 1 | 1.5815 | 1.55785 | 11.23% | 8.3499 | 8.2062 | 10.61% |
| 2 | 1.2422 | 1.21168 | 2.66% | 7.8307 | 7.5833 | 2.74% |
| 3 | 0.9648 | 0.95675 | 1.94% | 7.2631 | 7.20247 | 1.86% |
| 4 | 0.7390 | 0.735441 | 1.57% | 6.6428 | 6.61295 | 1.46% |
| 5 | 0.5500 | 0.548011 | 1.30% | 5.9403 | 5.92086 | 1.42% |
| 6 | 0.3983 | 0.397059 | 0.99% | 5.1695 | 5.15708 | 1.22% |
| 7 | 0.2774 | 0.276654 | 1.00% | 4.3259 | 4.31716 | 1.03% |
| 8 | 0.1816 | 1.180927 | 1.31% | 3.4013 | 3.39306 | 1.01% |
| 9 | 0.1060 | 0.105466 | 2.04% | 2.3868 | 2.3823 | 1.34% |
| 10 | 0.0521 | 0.0516387 | 3.61% | 1.3839 | 1.37502 | 1.84% |

Table S7. Identification error norms and steady state estimation error

| Scenario | $J_{e,O}^* / u_w^2$ ($a = 3.16309$) | $J_{e,O}^* / u_w^2$ ($a = 4.74463$) |
|----------|---------------------------------------|---------------------------------------|
| 1 | 2.11381 | 2.14967 |
| 2 | 0.533495 | 0.563329 |
| 3 | 0.187899 | 0.205286 |
| 4 | 0.0778057 | 0.0875864 |
| 5 | 0.0336782 | 0.0389032 |
| 6 | 0.0150149 | 0.0176936 |
| 7 | 0.00658277 | 0.00786332 |
| 8 | 0.00267343 | 0.00321651 |
| 9 | 0.000898796 | 0.00108247 |
| 10 | 0.000219048 | 0.000262766 |

Table S8. Observer parameter and target operating areas

| Scenario | U_i | $\rho_{F,1}$ | $\rho_{F,2}$ | $U_i - u_{i,\max}$ | $U_i + u_{i,\max}$ |
|----------|--------|--------------|--------------|--------------------|--------------------|
| 1 | 0.1393 | 3.5+ | 6.7289 | 0.1183 | 0.1603 |
| 2 | 0.2293 | 3.5+ | 5.1269 | 0.1869 | 0.2718 |
| 3 | 0.3117 | 3.5+ | 4.7294 | 0.2401 | 0.3833 |
| 4 | 0.3871 | 3.5+ | 4.5732 | 0.2757 | 0.4985 |
| 5 | 0.4584 | 3.5+ | 4.5018 | 0.2894 | 0.6274 |
| 6 | 0.5235 | 3.5+ | 4.4716 | 0.2707 | 0.7742 |
| 7 | 0.5828 | 3.5+ | 4.4621 | 0.2013 | 0.7742 |
| 8 | 0.6369 | 3.5+ | 4.4637 | 0.0384 | 0.7742 |
| 9 | 0.6863 | 3.5+ | 4.4716 | 0 | 0.7742 |
| 10 | 0.7273 | 3.5+ | 4.4817 | 0 | 0.7742 |

Table S9. Observer parameter and target operating areas

| Area | U_i | h_D | h_N | \hat{h}_D | \hat{h}_N | $\rho_{F,1}$ | $\rho_{F,2}$ | $U_i - u_{i,\max}$ | $U_i + u_{i,\max}$ |
|------|-------|--------|--------|-------------|-------------|--------------|--------------|--------------------|--------------------|
| 1 | 0.065 | 1.8904 | 8.7165 | 1.8479 | 8.4729 | 3.5+ | 488.6804 | 0.0566 | 0.0734 |
| 2 | 0.080 | 1.8259 | 8.6464 | 1.7883 | 8.4360 | 3.5+ | 18.4288 | 0.0694 | 0.0906 |
| 3 | 0.105 | 1.7209 | 8.5253 | 1.6952 | 8.3719 | 3.5+ | 9.2990 | 0.09029 | 0.1197 |
| 4 | 0.140 | 1.5788 | 8.3462 | 1.5515 | 8.1911 | 3.5+ | 6.7010 | 0.1189 | 0.1611 |
| 5 | 0.190 | 1.3857 | 8.0688 | 1.3698 | 7.9680 | 3.5+ | 5.5310 | 0.1579 | 0.2221 |
| 6 | 0.280 | 1.0678 | 7.4935 | 1.0614 | 7.4450 | 3.5+ | 4.8404 | 0.2210 | 0.3390 |
| 7 | 0.520 | 0.4059 | 5.2147 | 0.4046 | 55.2005 | 3.5+ | 4.4726 | 0.2728 | 0.7672 |

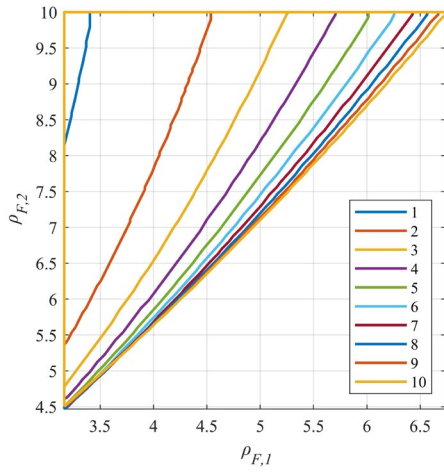


Figure S1. Acceptable observer poles' regions for $a = 3.16309$ and all scenarios of nominal points

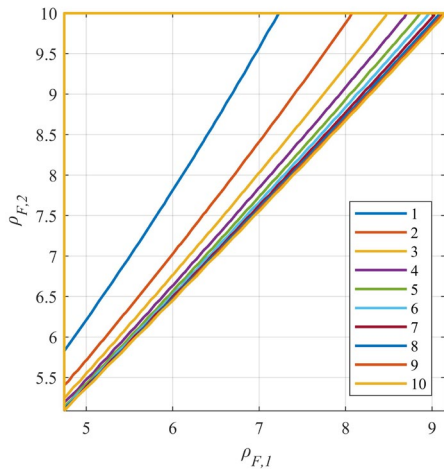


Figure S2. Acceptable observer poles' regions for $a = 4.74463$ and all scenarios of nominal points

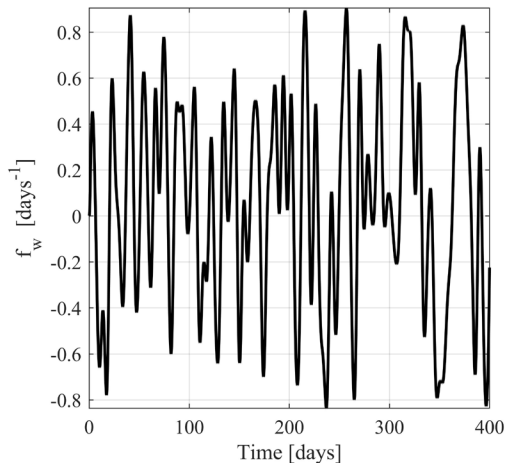


Figure S3. Actuation signal $f_w(t)$

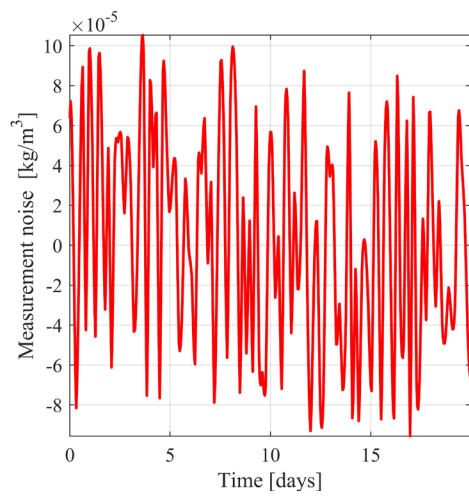


Figure S4. Additive measurement noise

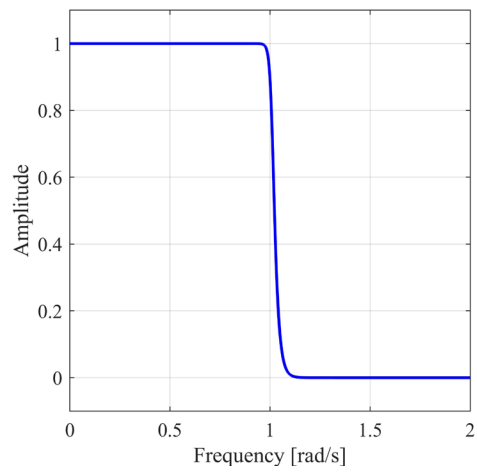


Figure S5. Butterworth filter Bode plot

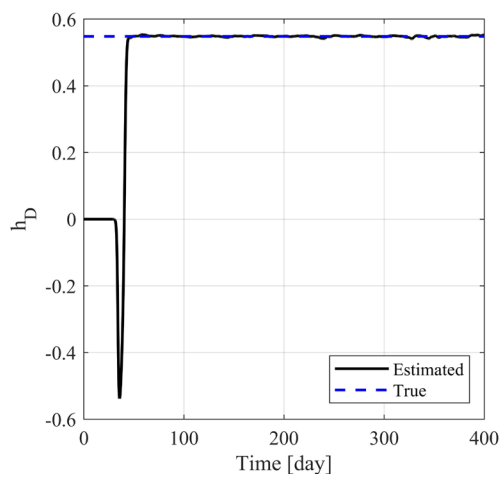


Figure S6. Estimation of h_D for scenario 5.

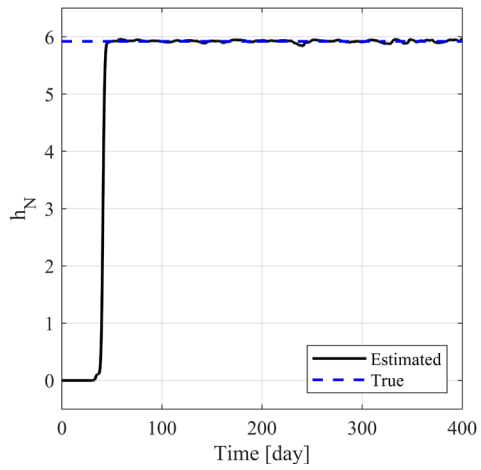


Figure S7. Estimation of h_N for scenario 5.

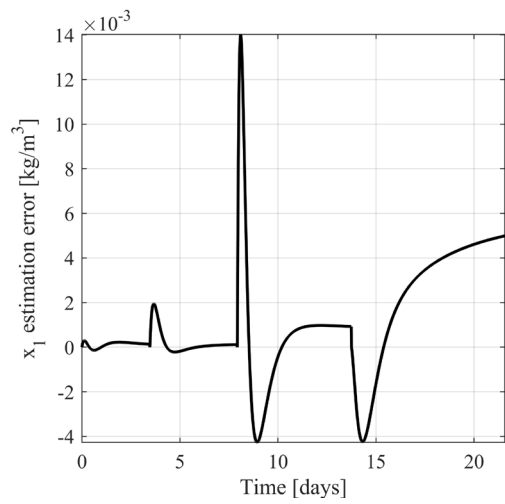


Figure S8. Substrate concentration estimation error.

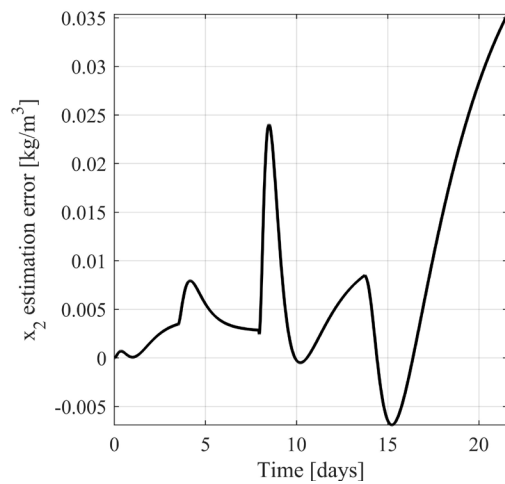


Figure S9. Microorganism concentration estimation error.

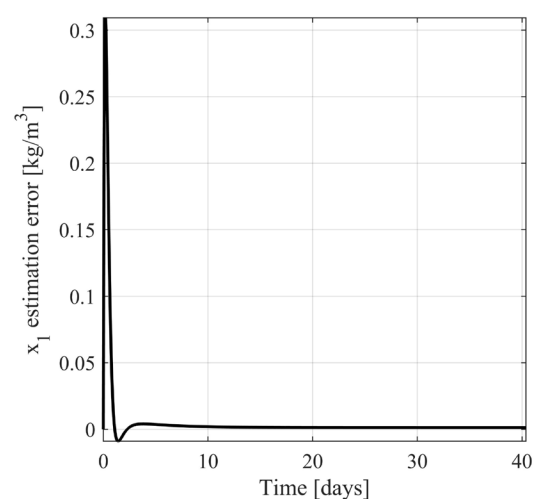


Figure S10. Substrate concentration estimation error for single step transition.

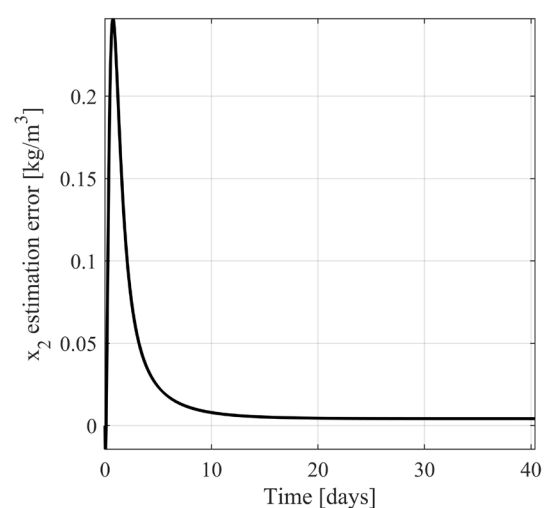


Figure S11. Microorganism concentration estimation error for single step transition.

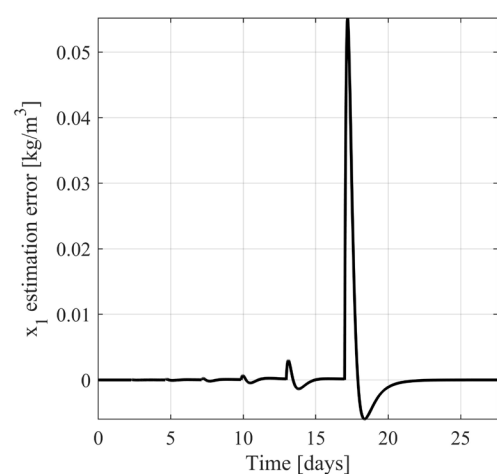


Figure S12. Substrate concentration estimation error.

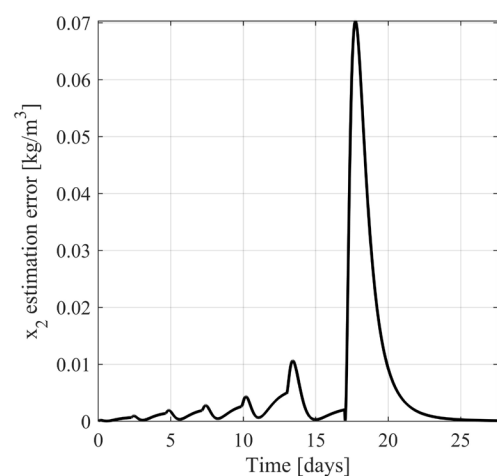


Figure S13. Microorganism concentration estimation error.